Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.3.1-a+b-cos-^m-c+d-cos-^n-A+B-cos-

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [644]. This is test number [92].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (644)	% 0. (0)
Mathematica	% 98.45 (634)	% 1.55 (10)
Maple	% 98.45 (634)	% 1.55 (10)
Maxima	% 29.35 (189)	% 70.65 (455)
Fricas	% 48.6 (313)	% 51.4 (331)
Sympy	% 9.47 (61)	% 90.53 (583)
Giac	% 30.9 (199)	% 69.1 (445)

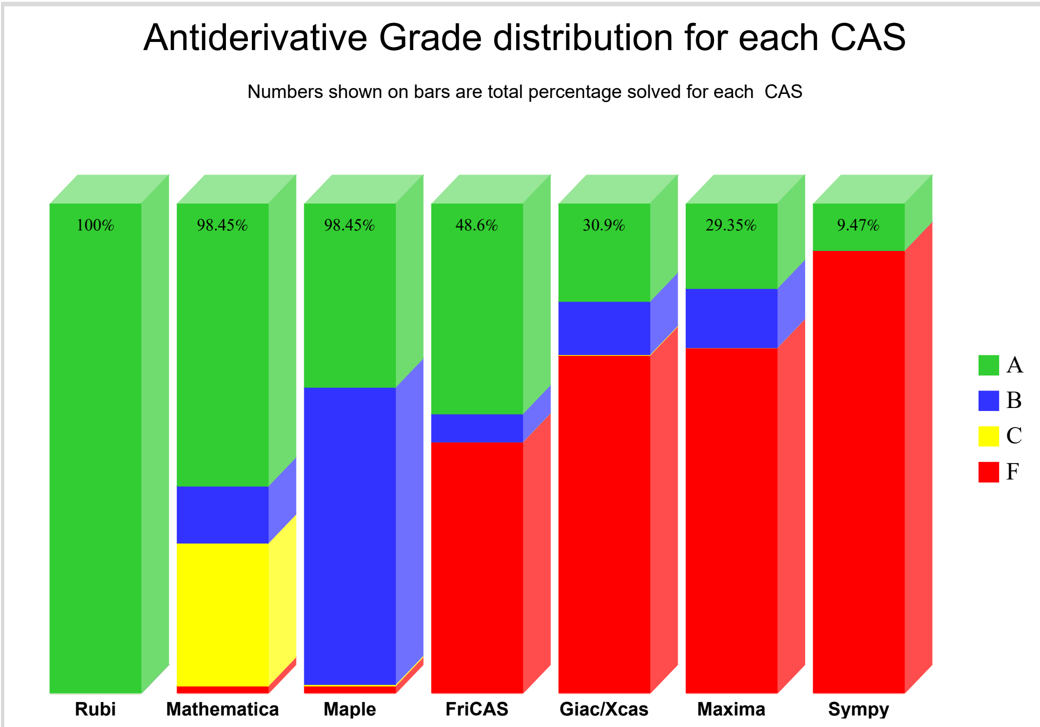
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

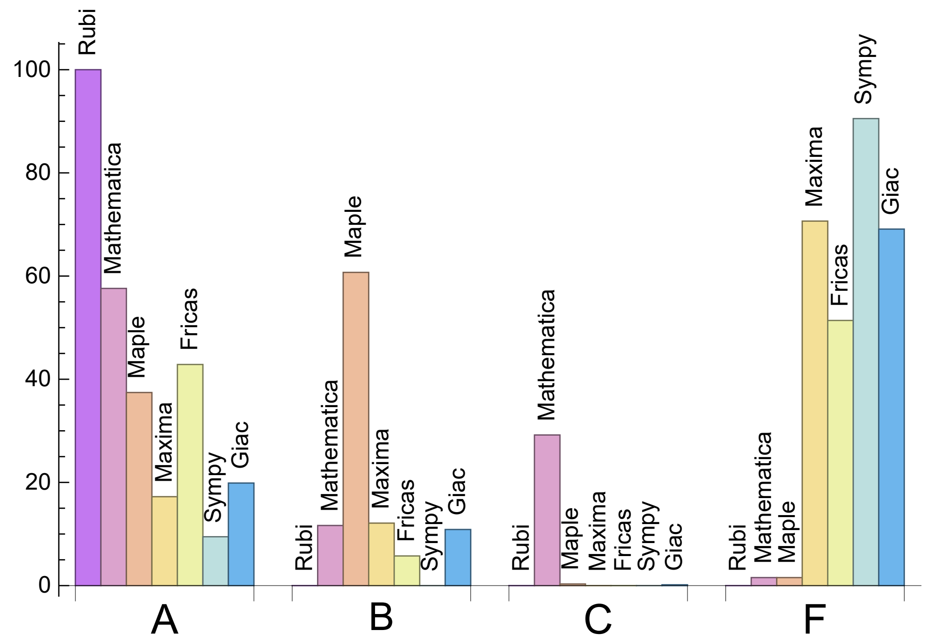
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	57.61	11.65	29.19	1.55
Maple	37.42	60.71	0.31	1.55
Maxima	17.24	12.11	0.	70.65
Fricas	42.86	5.75	0.	51.4
Sympy	9.47	0.	0.	90.53
Giac	19.88	10.87	0.16	69.1

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.61	222.34	0.98	187.5	1.
Mathematica	4.24	530.69	2.05	223.	1.09
Maple	2.77	887.73	3.2	423.5	2.51
Maxima	1.96	1110.15	6.66	311.	2.14
Fricas	6.18	565.84	3.49	425.	2.78
Sympy	12.07	414.85	2.97	264.	2.5
Giac	1.61	360.78	2.29	254.	2.03

1.4 list of integrals that has no closed form antiderivative

{449, 455, 456, 457, 458, 635, 641, 642, 643, 644}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {55, 64, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 151, 152, 157, 158, 159, 165, 194, 195, 196, 200, 201, 318, 332, 338, 339, 340, 395, 397, 399, 401, 402, 403, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 421, 424, 426, 430, 431, 432, 434, 435, 436, 454, 492, 522, 524, 529, 531, 532, 546, 547, 549, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 590, 591, 592, 593, 595, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 612, 614, 615, 616, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 633, 634, 638, 639, 640}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

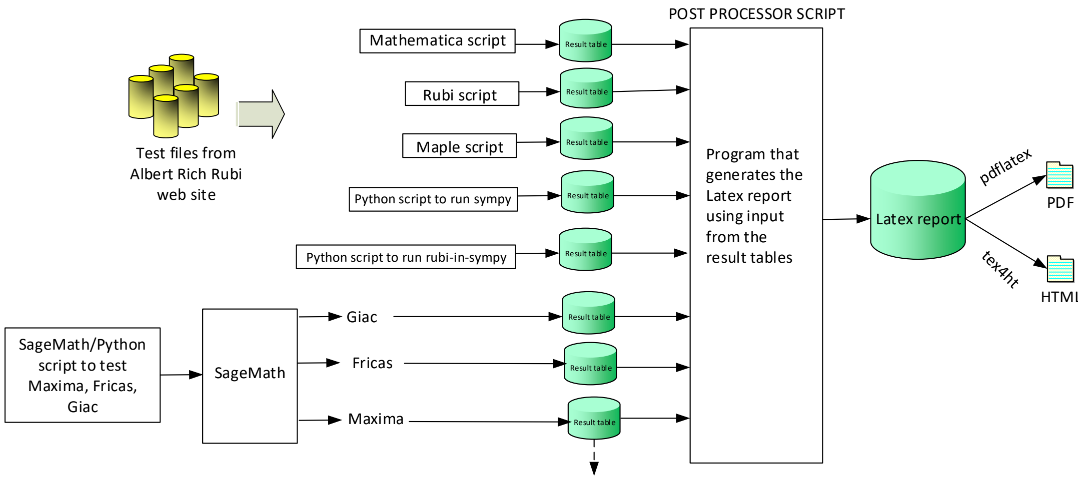
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 49, 51, 60, 61, 62, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 304, 305, 306, 311, 312, 313, 319, 320, 321, 322, 323, 326, 327, 328, 329, 333, 334, 335, 336, 337, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 398, 399, 422, 423, 424, 438, 439, 440, 449, 450, 451, 452, 453, 455, 456, 457, 458, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 528, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594, 599, 601, 613, 615, 616, 617, 621, 622, 628, 631, 632, 633, 635, 636, 637, 638, 639, 641, 642, 643, 644 }

B grade: { 16, 17, 23, 25, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 236, 246, 256, 257, 273, 274, 283, 369, 393, 397, 454, 573, 574, 575, 576, 590, 595, 596, 597, 598, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 614, 619, 620, 624, 625, 626, 627, 630, 640 }

C grade: { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 317, 318, 324, 325, 330, 331, 332, 338, 339, 340, 344, 395, 396, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 520, 521, 522, 524, 526, 527, 529, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 618, 623, 629, 634 }

F grade: { 441, 442, 443, 444, 445, 446, 447, 448, 523, 530}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 82, 83, 84, 85, 91, 92, 93, 100, 101, 102, 107, 130, 131, 137, 138, 139, 145, 146, 147, 148, 149, 151, 158, 159, 170, 171, 172, 173, 178, 179, 180, 181, 188, 189, 190, 192, 193, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 253, 254, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 300, 322, 323, 329, 330, 342, 343, 344, 367, 368, 387, 390, 391, 422, 437, 438, 439, 449, 455, 456, 457, 458, 461, 464, 467, 468, 469, 474, 475, 477, 478, 479, 480, 481, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 504, 507, 508, 509, 510, 511, 512, 513, 517, 518, 519, 525, 526, 527, 528, 534, 535, 552, 555, 567, 568, 569, 570, 585, 617, 632, 633, 634, 635, 641, 642, 643, 644 }

B grade: { 38, 39, 40, 44, 45, 46, 47, 54, 55, 78, 79, 80, 81, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 140, 141, 142, 143, 144, 150, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 289, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 440, 441, 442, 443, 444, 445, 446, 447, 448, 459, 460, 462, 463, 465, 466, 470, 471, 472, 473, 476, 482, 487, 497, 505, 506, 514, 515, 516, 520, 521, 522, 523, 524, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631 }

C grade: { 341, 386 }

F grade: { 450, 451, 452, 453, 454, 636, 637, 638, 639, 640 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 50, 51, 52, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 91, 92, 93, 94, 104, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 449, 455, 456, 457, 458, 635, 641, 642, 643, 644

B grade: { 6, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 63, 64, 79, 80, 81, 87, 88, 95, 97, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 219, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518 }

C grade: { }

F grade: { 89, 90, 96, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 182, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 516, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 109, 110, 114, 115, 116, 117, 122, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 261, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 449, 455, 456, 457, 458, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 635, 641, 642, 643, 644 }

B grade: { 6, 53, 72, 78, 79, 103, 104, 105, 106, 111, 112, 113, 118, 119, 120, 121, 219, 255, 256, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 273, 274, 275, 276, 277, 285, 294 }

C grade: { }

F grade: { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 271, 272, 278, 279, 280, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 528, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 10, 11, 12, 13, 19, 20, 21, 28, 29, 30, 38, 39, 40, 41, 42, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 215, 216, 217, 223, 224, 225, 231, 232, 233, 240, 241, 242, 253, 281, 282, 283, 284, 285, 286, 288, 456, 457, 458, 642, 643 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 52, 53, 54, 55, 62, 63, 64, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 644 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 94, 100, 101, 102, 103, 107, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 215, 216, 217, 223, 224, 225, 231, 232, 233, 235, 240, 241, 242, 244, 251, 252, 253, 254, 255, 258, 260, 261, 262, 264, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 294, 295, 457, 458, 635, 641, 642, 643, 644 }

B grade: { 5, 6, 7, 15, 78, 79, 80, 81, 86, 87, 88, 89, 90, 95, 96, 97, 98, 99, 104, 105, 106, 112, 113, 114, 121, 122, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 234, 236, 237, 238, 239, 243, 245, 246, 247, 248, 249, 250, 256, 257, 259, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 285, 293 }

C grade: { 284 }

F grade: { 74, 75, 76, 77, 82, 83, 84, 85, 91, 92, 93, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	77	128	167	239	333	151
normalized size	1	1.	0.62	1.02	1.34	1.91	2.66	1.21
time (sec)	N/A	0.167	0.244	0.05	1.012	1.76	3.587	1.101

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	107	136	193	252	120
normalized size	1	1.	0.77	1.1	1.4	1.99	2.6	1.24
time (sec)	N/A	0.149	0.223	0.052	1.007	1.433	1.805	1.147

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	65	85	107	146	168	92
normalized size	1	1.	0.84	1.1	1.39	1.9	2.18	1.19
time (sec)	N/A	0.078	0.166	0.053	0.984	1.375	0.928	1.105

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	74	99	94	61
normalized size	1	1.	0.94	1.21	1.57	2.11	2.	1.3
time (sec)	N/A	0.021	0.098	0.046	0.987	1.333	0.412	1.168

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	46	56	63	139	0	107
normalized size	1	1.	1.44	1.75	1.97	4.34	0.	3.34
time (sec)	N/A	0.091	0.024	0.076	0.99	1.451	0.	1.233

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	43	65	99	220	0	113
normalized size	1	1.	1.34	2.03	3.09	6.88	0.	3.53
time (sec)	N/A	0.103	0.019	0.092	1.038	1.473	0.	1.213

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	128	239	0	167
normalized size	1	1.	1.34	1.54	2.29	4.27	0.	2.98
time (sec)	N/A	0.137	0.027	0.093	0.99	1.409	0.	1.241

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	56	128	171	288	0	208
normalized size	1	1.	0.65	1.49	1.99	3.35	0.	2.42
time (sec)	N/A	0.154	0.303	0.101	1.029	1.423	0.	1.25

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	171	220	339	0	254
normalized size	1	1.	0.73	1.61	2.08	3.2	0.	2.4
time (sec)	N/A	0.165	0.364	0.151	0.982	1.41	0.	1.271

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	134	217	292	327	600	224
normalized size	1	1.	0.7	1.14	1.53	1.71	3.14	1.17
time (sec)	N/A	0.31	0.59	0.057	1.052	1.429	6.142	1.197

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	108	186	240	271	459	185
normalized size	1	1.	0.68	1.16	1.5	1.69	2.87	1.16
time (sec)	N/A	0.278	0.398	0.076	0.975	1.4	3.417	1.222

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	86	154	194	213	338	149
normalized size	1	1.	0.67	1.19	1.5	1.65	2.62	1.16
time (sec)	N/A	0.176	0.331	0.051	0.991	1.439	2.195	1.187

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	61	116	149	165	199	115
normalized size	1	1.	0.65	1.23	1.59	1.76	2.12	1.22
time (sec)	N/A	0.059	0.18	0.046	1.	1.406	0.792	1.198

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	96	108	127	194	0	196
normalized size	1	1.	1.17	1.32	1.55	2.37	0.	2.39
time (sec)	N/A	0.193	0.171	0.081	0.971	1.444	0.	1.225

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	143	107	142	278	0	209
normalized size	1	1.	1.93	1.45	1.92	3.76	0.	2.82
time (sec)	N/A	0.21	0.311	0.092	1.001	1.424	0.	1.243

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	277	113	192	297	0	208
normalized size	1	1.	3.15	1.28	2.18	3.38	0.	2.36
time (sec)	N/A	0.219	1.183	0.106	0.98	1.392	0.	1.323

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	451	141	235	315	0	240
normalized size	1	1.	3.99	1.25	2.08	2.79	0.	2.12
time (sec)	N/A	0.27	5.642	0.099	1.019	1.379	0.	1.233

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	262	187	311	362	0	286
normalized size	1	1.	1.82	1.3	2.16	2.51	0.	1.99
time (sec)	N/A	0.304	1.134	0.115	1.021	1.481	0.	1.242

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	134	266	354	332	695	224
normalized size	1	1.	0.67	1.32	1.76	1.65	3.46	1.11
time (sec)	N/A	0.432	0.546	0.073	0.998	1.459	8.225	1.252

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	108	223	288	278	530	184
normalized size	1	1.	0.7	1.45	1.87	1.81	3.44	1.19
time (sec)	N/A	0.229	0.409	0.052	0.976	1.453	4.328	1.21

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	86	176	225	216	371	151
normalized size	1	1.	0.74	1.52	1.94	1.86	3.2	1.3
time (sec)	N/A	0.098	0.303	0.047	1.026	1.334	1.61	1.184

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	153	190	254	0	243
normalized size	1	1.	1.02	1.38	1.71	2.29	0.	2.19
time (sec)	N/A	0.304	0.249	0.111	1.	1.497	0.	1.289

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	272	145	189	323	0	259
normalized size	1	1.	2.47	1.32	1.72	2.94	0.	2.35
time (sec)	N/A	0.308	1.69	0.1	1.026	1.416	0.	1.323

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	208	144	223	342	0	259
normalized size	1	1.	1.82	1.26	1.96	3.	0.	2.27
time (sec)	N/A	0.338	1.839	0.111	1.01	1.471	0.	1.262

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	786	158	286	356	0	255
normalized size	1	1.	6.29	1.26	2.29	2.85	0.	2.04
time (sec)	N/A	0.337	6.347	0.152	1.004	1.421	0.	1.329

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	273	188	363	366	0	286
normalized size	1	1.	1.77	1.22	2.36	2.38	0.	1.86
time (sec)	N/A	0.418	1.236	0.121	1.007	1.404	0.	1.339

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	294	234	455	431	0	332
normalized size	1	1.	1.59	1.26	2.46	2.33	0.	1.79
time (sec)	N/A	0.447	1.415	0.114	1.028	1.478	0.	1.271

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	156	358	481	396	960	261
normalized size	1	1.	0.65	1.49	2.	1.64	3.98	1.08
time (sec)	N/A	0.593	0.793	0.067	1.018	1.432	11.627	1.317

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	134	306	401	329	765	224
normalized size	1	1.	0.72	1.65	2.17	1.78	4.14	1.21
time (sec)	N/A	0.304	0.49	0.059	1.019	1.466	6.802	1.233

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	108	248	319	279	544	188
normalized size	1	1.	0.72	1.65	2.13	1.86	3.63	1.25
time (sec)	N/A	0.139	0.336	0.051	1.03	1.313	4.469	1.238

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	138	199	267	306	0	289
normalized size	1	1.	0.91	1.32	1.77	2.03	0.	1.91
time (sec)	N/A	0.413	0.372	0.105	1.012	1.814	0.	1.299

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	312	190	252	383	0	305
normalized size	1	1.	2.08	1.27	1.68	2.55	0.	2.03
time (sec)	N/A	0.453	1.548	0.148	1.011	1.676	0.	1.344

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	343	182	269	390	0	311
normalized size	1	1.	2.12	1.12	1.66	2.41	0.	1.92
time (sec)	N/A	0.476	4.283	0.127	1.028	1.452	0.	1.335

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	380	189	317	405	0	306
normalized size	1	1.	2.3	1.15	1.92	2.45	0.	1.85
time (sec)	N/A	0.514	6.203	0.124	1.019	1.545	0.	1.303

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	326	204	414	408	0	301
normalized size	1	1.	1.88	1.18	2.39	2.36	0.	1.74
time (sec)	N/A	0.523	1.719	0.122	1.256	1.436	0.	1.274

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	306	234	508	431	0	332
normalized size	1	1.	1.55	1.18	2.57	2.18	0.	1.68
time (sec)	N/A	0.587	1.571	0.152	1.174	1.395	0.	1.307

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	358	280	626	481	0	378
normalized size	1	1.	1.56	1.22	2.73	2.1	0.	1.65
time (sec)	N/A	0.65	2.079	0.125	1.066	1.416	0.	1.316

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	311	351	532	301	1794	244
normalized size	1	1.	2.03	2.29	3.48	1.97	11.73	1.59
time (sec)	N/A	0.207	0.607	0.069	1.537	1.376	13.353	1.184

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	249	281	419	242	1161	204
normalized size	1	1.	2.04	2.3	3.43	1.98	9.52	1.67
time (sec)	N/A	0.172	0.534	0.066	1.521	1.466	7.635	1.241

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	99	197	211	304	204	665	167
normalized size	1	1.1	2.19	2.34	3.38	2.27	7.39	1.86
time (sec)	N/A	0.123	0.438	0.069	1.506	1.426	4.29	1.179

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	126	108	193	147	264	105
normalized size	1	1.	2.33	2.	3.57	2.72	4.89	1.94
time (sec)	N/A	0.139	0.231	0.061	1.614	1.337	2.186	1.213

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	72	56	99	105	49	58
normalized size	1	1.	2.12	1.65	2.91	3.09	1.44	1.71
time (sec)	N/A	0.05	0.117	0.053	1.501	1.312	1.108	1.167

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	109	78	134	197	0	96
normalized size	1	1.	2.48	1.77	3.05	4.48	0.	2.18
time (sec)	N/A	0.078	0.238	0.082	1.015	1.39	0.	1.227

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	201	163	265	320	0	149
normalized size	1	1.	2.91	2.36	3.84	4.64	0.	2.16
time (sec)	N/A	0.156	1.1	0.106	1.018	1.397	0.	1.266

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	289	252	381	385	0	212
normalized size	1	1.	2.7	2.36	3.56	3.6	0.	1.98
time (sec)	N/A	0.169	3.081	0.115	1.025	1.373	0.	1.254

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	490	340	497	419	0	246
normalized size	1	1.	3.74	2.6	3.79	3.2	0.	1.88
time (sec)	N/A	0.18	4.114	0.104	1.325	1.454	0.	1.189

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	369	322	502	387	1425	259
normalized size	1	1.	2.17	1.89	2.95	2.28	8.38	1.52
time (sec)	N/A	0.322	0.617	0.066	2.01	1.389	21.057	1.195

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	315	252	382	343	843	221
normalized size	1	1.	2.14	1.71	2.6	2.33	5.73	1.5
time (sec)	N/A	0.341	0.757	0.061	1.953	1.388	12.303	1.245

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	137	149	258	294	411	161
normalized size	1	1.	1.38	1.51	2.61	2.97	4.15	1.63
time (sec)	N/A	0.276	0.677	0.079	1.895	1.366	7.01	1.267

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	153	97	162	228	105	116
normalized size	1	1.	2.19	1.39	2.31	3.26	1.5	1.66
time (sec)	N/A	0.155	0.333	0.052	1.847	1.359	3.511	1.242

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	126	144	94	81
normalized size	1	1.	1.17	0.92	1.94	2.22	1.45	1.25
time (sec)	N/A	0.054	0.169	0.045	1.234	1.363	2.852	1.214

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	170	119	196	338	0	153
normalized size	1	1.	2.15	1.51	2.48	4.28	0.	1.94
time (sec)	N/A	0.18	0.487	0.082	1.27	1.38	0.	1.235

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	264	205	329	502	0	209
normalized size	1	1.	2.47	1.92	3.07	4.69	0.	1.95
time (sec)	N/A	0.295	1.512	0.095	1.049	1.463	0.	1.214

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	496	294	454	564	0	267
normalized size	1	1.	3.26	1.93	2.99	3.71	0.	1.76
time (sec)	N/A	0.314	3.106	0.109	1.044	1.447	0.	1.246

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	609	382	574	617	0	305
normalized size	1	1.	3.4	2.13	3.21	3.45	0.	1.7
time (sec)	N/A	0.365	4.847	0.125	1.048	1.479	0.	1.203

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	491	362	556	539	1584	308
normalized size	1	1.	2.25	1.66	2.55	2.47	7.27	1.41
time (sec)	N/A	0.515	0.908	0.065	1.511	1.404	45.615	1.215

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	435	292	435	497	966	270
normalized size	1	1.	2.25	1.51	2.25	2.58	5.01	1.4
time (sec)	N/A	0.468	0.791	0.066	1.508	1.473	27.862	1.178

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	361	189	312	429	496	209
normalized size	1	1.	2.46	1.29	2.12	2.92	3.37	1.42
time (sec)	N/A	0.457	0.833	0.065	1.568	1.364	16.565	1.212

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	241	137	216	351	148	162
normalized size	1	1.	2.08	1.18	1.86	3.03	1.28	1.4
time (sec)	N/A	0.321	0.537	0.056	1.49	1.378	9.228	1.238

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	155	227	117	101
normalized size	1	1.	1.32	0.63	1.52	2.23	1.15	0.99
time (sec)	N/A	0.188	0.32	0.05	1.017	1.312	5.7	1.217

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	96	64	155	227	114	101
normalized size	1	1.	0.94	0.63	1.52	2.23	1.12	0.99
time (sec)	N/A	0.079	0.259	0.044	1.022	1.338	3.662	1.206

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	197	159	252	481	0	200
normalized size	1	1.	1.68	1.36	2.15	4.11	0.	1.71
time (sec)	N/A	0.311	0.912	0.094	1.007	1.364	0.	1.269

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	482	245	386	670	0	257
normalized size	1	1.	3.32	1.69	2.66	4.62	0.	1.77
time (sec)	N/A	0.469	2.924	0.108	1.029	1.468	0.	1.263

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	610	334	509	756	0	315
normalized size	1	1.	3.11	1.7	2.6	3.86	0.	1.61
time (sec)	N/A	0.541	4.673	0.112	1.002	1.444	0.	1.229

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	555	332	491	647	1085	315
normalized size	1	1.	2.42	1.45	2.14	2.83	4.74	1.38
time (sec)	N/A	0.672	1.255	0.064	1.49	1.425	91.689	1.287

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	481	229	366	572	578	254
normalized size	1	1.	2.6	1.24	1.98	3.09	3.12	1.37
time (sec)	N/A	0.679	0.855	0.058	1.478	1.316	42.492	1.284

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	329	177	271	475	192	209
normalized size	1	1.	2.14	1.15	1.76	3.08	1.25	1.36
time (sec)	N/A	0.498	0.741	0.059	1.479	1.386	24.311	1.215

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	193	90	236	308	182	158
normalized size	1	1.	1.42	0.66	1.74	2.26	1.34	1.16
time (sec)	N/A	0.348	0.435	0.053	1.009	1.367	16.207	1.185

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	163	88	235	309	178	158
normalized size	1	1.	1.18	0.64	1.7	2.24	1.29	1.14
time (sec)	N/A	0.215	0.359	0.053	1.01	1.297	11.417	1.177

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	109	88	236	311	177	158
normalized size	1	1.	0.79	0.64	1.71	2.25	1.28	1.14
time (sec)	N/A	0.138	0.324	0.049	1.004	1.402	8.089	1.19

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	239	199	308	624	0	246
normalized size	1	1.	1.63	1.35	2.1	4.24	0.	1.67
time (sec)	N/A	0.466	1.419	0.095	1.039	1.482	0.	1.259

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	595	285	440	838	0	302
normalized size	1	1.	3.4	1.63	2.51	4.79	0.	1.73
time (sec)	N/A	0.672	4.879	0.113	1.039	1.413	0.	1.29

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	798	374	566	938	0	360
normalized size	1	1.	3.44	1.61	2.44	4.04	0.	1.55
time (sec)	N/A	0.688	6.446	0.115	1.038	1.513	0.	1.282

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	103	121	196	263	0	0
normalized size	1	1.	0.55	0.65	1.05	1.41	0.	0.
time (sec)	N/A	0.304	0.629	1.147	1.901	1.333	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	80	102	159	219	0	0
normalized size	1	1.	0.56	0.71	1.1	1.52	0.	0.
time (sec)	N/A	0.265	0.335	0.998	1.935	1.224	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	64	83	119	170	0	0
normalized size	1	1.	0.63	0.82	1.18	1.68	0.	0.
time (sec)	N/A	0.202	0.186	1.095	1.824	1.343	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	62	77	126	0	0
normalized size	1	1.	0.74	1.	1.24	2.03	0.	0.
time (sec)	N/A	0.059	0.075	1.125	1.792	1.333	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	210	28	344	0	201
normalized size	1	1.	1.	3.18	0.42	5.21	0.	3.05
time (sec)	N/A	0.138	0.085	3.296	1.635	1.521	0.	3.985

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	85	642	959	406	0	351
normalized size	1	1.	1.25	9.44	14.1	5.97	0.	5.16
time (sec)	N/A	0.163	0.2	3.951	1.87	1.564	0.	2.851

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	101	991	4525	460	0	637
normalized size	1	1.	0.86	8.47	38.68	3.93	0.	5.44
time (sec)	N/A	0.22	0.781	3.84	20.601	1.514	0.	2.765

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	129	1311	6778	508	0	861
normalized size	1	1.	0.81	8.19	42.36	3.18	0.	5.38
time (sec)	N/A	0.293	1.787	3.984	21.461	1.614	0.	2.793

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	125	142	250	351	0	0
normalized size	1	1.	0.53	0.61	1.07	1.5	0.	0.
time (sec)	N/A	0.529	0.956	1.046	2.	1.331	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	103	123	208	286	0	0
normalized size	1	1.	0.54	0.65	1.1	1.51	0.	0.
time (sec)	N/A	0.446	0.528	1.463	1.969	1.404	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	81	104	166	236	0	0
normalized size	1	1.	0.59	0.75	1.2	1.71	0.	0.
time (sec)	N/A	0.25	0.357	1.29	1.86	1.616	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	65	85	126	182	0	0
normalized size	1	1.	0.64	0.84	1.25	1.8	0.	0.
time (sec)	N/A	0.087	0.18	1.117	1.811	1.612	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	85	272	53	397	0	265
normalized size	1	1.	0.81	2.59	0.5	3.78	0.	2.52
time (sec)	N/A	0.265	0.192	3.507	1.659	2.158	0.	5.903

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	98	696	1775	451	0	401
normalized size	1	1.	0.95	6.76	17.23	4.38	0.	3.89
time (sec)	N/A	0.284	0.301	3.605	1.907	2.387	0.	3.129

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	109	991	4508	474	0	639
normalized size	1	1.	0.92	8.33	37.88	3.98	0.	5.37
time (sec)	N/A	0.325	0.511	3.779	2.492	1.978	0.	2.972

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	132	1310	0	531	0	861
normalized size	1	1.	0.8	7.99	0.	3.24	0.	5.25
time (sec)	N/A	0.4	0.919	4.168	0.	1.993	0.	2.923

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	151	1631	0	581	0	1083
normalized size	1	1.	0.72	7.8	0.	2.78	0.	5.18
time (sec)	N/A	0.485	1.471	4.114	0.	2.237	0.	3.214

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	127	142	279	366	0	0
normalized size	1	1.	0.54	0.6	1.18	1.54	0.	0.
time (sec)	N/A	0.648	1.085	1.161	3.02	1.659	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	105	123	232	302	0	0
normalized size	1	1.	0.6	0.7	1.33	1.73	0.	0.
time (sec)	N/A	0.28	0.671	1.134	2.791	1.629	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	83	104	188	248	0	0
normalized size	1	1.	0.6	0.75	1.36	1.8	0.	0.
time (sec)	N/A	0.109	0.319	1.	2.65	1.618	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	104	311	82	462	0	306
normalized size	1	1.	0.73	2.19	0.58	3.25	0.	2.15
time (sec)	N/A	0.414	0.38	4.168	2.646	1.764	0.	3.833

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	120	756	10954	516	0	460
normalized size	1	1.	0.83	5.25	76.07	3.58	0.	3.19
time (sec)	N/A	0.447	0.505	3.766	4.808	1.945	0.	2.828

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	126	1016	0	522	0	686
normalized size	1	1.	0.81	6.51	0.	3.35	0.	4.4
time (sec)	N/A	0.474	0.62	4.404	0.	2.027	0.	3.129

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	131	1310	10792	544	0	861
normalized size	1	1.	0.8	7.99	65.8	3.32	0.	5.25
time (sec)	N/A	0.526	1.05	4.138	22.504	2.047	0.	3.278

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	152	1630	0	608	0	1083
normalized size	1	1.	0.73	7.8	0.	2.91	0.	5.18
time (sec)	N/A	0.609	1.656	4.174	0.	2.182	0.	3.409

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	176	1951	0	670	0	1304
normalized size	1	1.	0.69	7.68	0.	2.64	0.	5.13
time (sec)	N/A	0.713	2.024	4.312	0.	2.32	0.	3.568

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	111	281	0	494	0	244
normalized size	1	1.	0.55	1.39	0.	2.45	0.	1.21
time (sec)	N/A	0.578	0.642	2.797	0.	1.665	0.	1.772

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	94	240	0	446	0	213
normalized size	1	1.	0.59	1.51	0.	2.81	0.	1.34
time (sec)	N/A	0.384	0.32	2.135	0.	1.723	0.	1.742

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	78	194	0	400	0	153
normalized size	1	1.	0.66	1.64	0.	3.39	0.	1.3
time (sec)	N/A	0.21	0.156	2.51	0.	1.664	0.	1.726

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	60	160	0	363	0	119
normalized size	1	1.	0.77	2.05	0.	4.65	0.	1.53
time (sec)	N/A	0.071	0.063	1.862	0.	1.644	0.	1.672

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	72	268	123	464	0	227
normalized size	1	1.	0.79	2.95	1.35	5.1	0.	2.49
time (sec)	N/A	0.166	0.077	4.029	2.026	1.802	0.	2.741

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	95	812	0	699	0	433
normalized size	1	1.	0.8	6.82	0.	5.87	0.	3.64
time (sec)	N/A	0.308	0.335	4.62	0.	2.156	0.	2.938

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	114	1240	0	753	0	722
normalized size	1	1.	0.69	7.52	0.	4.56	0.	4.38
time (sec)	N/A	0.48	0.771	5.027	0.	2.136	0.	3.091

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	167	448	0	653	0	343
normalized size	1	1.	0.64	1.72	0.	2.5	0.	1.31
time (sec)	N/A	0.786	1.066	2.718	0.	1.791	0.	1.992

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	142	407	0	601	0	273
normalized size	1	1.	0.66	1.88	0.	2.78	0.	1.26
time (sec)	N/A	0.595	0.871	2.473	0.	1.799	0.	1.929

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	97	327	0	544	0	227
normalized size	1	1.	0.57	1.91	0.	3.18	0.	1.33
time (sec)	N/A	0.42	0.704	2.164	0.	1.682	0.	1.916

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	104	256	0	493	0	177
normalized size	1	1.	0.88	2.17	0.	4.18	0.	1.5
time (sec)	N/A	0.223	0.393	2.435	0.	1.767	0.	1.893

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	63	220	0	455	0	136
normalized size	1	1.	0.72	2.53	0.	5.23	0.	1.56
time (sec)	N/A	0.077	0.184	2.506	0.	1.729	0.	1.742

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	131	374	0	737	0	289
normalized size	1	1.	1.03	2.94	0.	5.8	0.	2.28
time (sec)	N/A	0.315	0.646	4.757	0.	1.934	0.	3.089

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	141	1051	0	880	0	504
normalized size	1	1.	0.83	6.18	0.	5.18	0.	2.96
time (sec)	N/A	0.521	1.038	4.868	0.	2.301	0.	3.227

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	205	1540	0	944	0	787
normalized size	1	1.	0.93	6.97	0.	4.27	0.	3.56
time (sec)	N/A	0.705	1.462	5.48	0.	2.353	0.	3.275

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	139	467	0	741	0	347
normalized size	1	1.	0.53	1.79	0.	2.84	0.	1.33
time (sec)	N/A	0.799	1.461	2.654	0.	1.723	0.	2.281

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	117	397	0	687	0	275
normalized size	1	1.	0.54	1.84	0.	3.18	0.	1.27
time (sec)	N/A	0.611	1.009	2.477	0.	1.776	0.	2.205

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	100	327	0	628	0	244
normalized size	1	1.	0.59	1.93	0.	3.72	0.	1.44
time (sec)	N/A	0.42	0.685	2.579	0.	1.694	0.	2.185

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	87	292	0	586	0	181
normalized size	1	1.	0.69	2.32	0.	4.65	0.	1.44
time (sec)	N/A	0.23	0.462	2.665	0.	1.676	0.	2.121

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	80	292	0	579	0	181
normalized size	1	1.	0.63	2.32	0.	4.6	0.	1.44
time (sec)	N/A	0.103	0.455	2.23	0.	1.726	0.	1.871

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	126	445	0	902	0	338
normalized size	1	1.	0.77	2.71	0.	5.5	0.	2.06
time (sec)	N/A	0.466	1.379	4.747	0.	1.911	0.	3.289

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	142	1122	0	1067	0	552
normalized size	1	1.	0.69	5.42	0.	5.15	0.	2.67
time (sec)	N/A	0.715	3.052	5.265	0.	2.462	0.	3.768

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	178	1610	0	1131	0	837
normalized size	1	1.	0.67	6.1	0.	4.28	0.	3.17
time (sec)	N/A	0.923	5.656	5.76	0.	2.54	0.	3.768

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	914	411	0	0	0	0
normalized size	1	1.	5.75	2.58	0.	0.	0.	0.
time (sec)	N/A	0.199	6.314	3.214	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	872	383	0	0	0	0
normalized size	1	1.	6.61	2.9	0.	0.	0.	0.
time (sec)	N/A	0.175	6.247	3.418	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	830	355	0	0	0	0
normalized size	1	1.	8.22	3.51	0.	0.	0.	0.
time (sec)	N/A	0.158	6.198	3.14	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	309	321	0	0	0	0
normalized size	1	1.	4.41	4.59	0.	0.	0.	0.
time (sec)	N/A	0.144	5.734	3.188	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	256	240	0	0	0	0
normalized size	1	1.	3.88	3.64	0.	0.	0.	0.
time (sec)	N/A	0.148	5.82	3.251	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	813	426	0	0	0	0
normalized size	1	1.	8.56	4.48	0.	0.	0.	0.
time (sec)	N/A	0.169	6.33	7.918	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	865	661	0	0	0	0
normalized size	1	1.	6.55	5.01	0.	0.	0.	0.
time (sec)	N/A	0.177	6.374	10.085	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	944	413	0	0	0	0
normalized size	1	1.	4.87	2.13	0.	0.	0.	0.
time (sec)	N/A	0.319	6.263	3.137	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	898	385	0	0	0	0
normalized size	1	1.	5.58	2.39	0.	0.	0.	0.
time (sec)	N/A	0.289	6.227	3.511	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	852	357	0	0	0	0
normalized size	1	1.	6.76	2.83	0.	0.	0.	0.
time (sec)	N/A	0.274	6.271	3.53	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	623	388	0	0	0	0
normalized size	1	1.	5.28	3.29	0.	0.	0.	0.
time (sec)	N/A	0.269	6.334	3.598	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	624	513	0	0	0	0
normalized size	1	1.	5.2	4.28	0.	0.	0.	0.
time (sec)	N/A	0.28	6.394	3.663	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	883	741	0	0	0	0
normalized size	1	1.	5.55	4.66	0.	0.	0.	0.
time (sec)	N/A	0.306	6.463	10.34	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	925	851	0	0	0	0
normalized size	1	1.	4.77	4.39	0.	0.	0.	0.
time (sec)	N/A	0.337	6.533	12.111	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	990	441	0	0	0	0
normalized size	1	1.	4.18	1.86	0.	0.	0.	0.
time (sec)	N/A	0.479	6.298	4.067	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	944	413	0	0	0	0
normalized size	1	1.	4.63	2.02	0.	0.	0.	0.
time (sec)	N/A	0.447	6.26	3.167	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	898	385	0	0	0	0
normalized size	1	1.	5.25	2.25	0.	0.	0.	0.
time (sec)	N/A	0.427	6.321	3.609	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	888	519	0	0	0	0
normalized size	1	1.	5.25	3.07	0.	0.	0.	0.
time (sec)	N/A	0.434	6.412	4.093	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	879	654	0	0	0	0
normalized size	1	1.	5.46	4.06	0.	0.	0.	0.
time (sec)	N/A	0.43	6.482	4.144	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	890	916	0	0	0	0
normalized size	1	1.	5.2	5.36	0.	0.	0.	0.
time (sec)	N/A	0.462	6.528	10.02	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	925	929	0	0	0	0
normalized size	1	1.	4.53	4.55	0.	0.	0.	0.
time (sec)	N/A	0.491	6.563	13.096	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	967	1178	0	0	0	0
normalized size	1	1.	4.08	4.97	0.	0.	0.	0.
time (sec)	N/A	0.521	6.631	14.807	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	1182	281	0	0	0	0
normalized size	1	1.	7.58	1.8	0.	0.	0.	0.
time (sec)	N/A	0.199	6.557	3.599	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	1129	262	0	0	0	0
normalized size	1	1.	9.18	2.13	0.	0.	0.	0.
time (sec)	N/A	0.18	6.488	2.849	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	1098	244	0	0	0	0
normalized size	1	1.	12.92	2.87	0.	0.	0.	0.
time (sec)	N/A	0.15	6.408	3.473	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	1094	243	0	0	0	0
normalized size	1	1.	13.18	2.93	0.	0.	0.	0.
time (sec)	N/A	0.15	6.423	3.329	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	1130	319	0	0	0	0
normalized size	1	1.	9.5	2.68	0.	0.	0.	0.
time (sec)	N/A	0.174	6.606	6.605	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	1167	493	0	0	0	0
normalized size	1	1.	7.63	3.22	0.	0.	0.	0.
time (sec)	N/A	0.195	6.912	10.125	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	1262	465	0	0	0	0
normalized size	1	1.	6.22	2.29	0.	0.	0.	0.
time (sec)	N/A	0.407	6.77	3.945	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	1218	435	0	0	0	0
normalized size	1	1.	7.34	2.62	0.	0.	0.	0.
time (sec)	N/A	0.389	6.682	3.277	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	1184	421	0	0	0	0
normalized size	1	1.	8.71	3.1	0.	0.	0.	0.
time (sec)	N/A	0.314	6.577	3.768	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	815	350	0	0	0	0
normalized size	1	1.	6.74	2.89	0.	0.	0.	0.
time (sec)	N/A	0.277	6.472	3.144	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	815	350	0	0	0	0
normalized size	1	1.	6.74	2.89	0.	0.	0.	0.
time (sec)	N/A	0.342	6.486	3.663	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	1217	494	0	0	0	0
normalized size	1	1.	7.24	2.94	0.	0.	0.	0.
time (sec)	N/A	0.359	6.69	4.243	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	1258	750	0	0	0	0
normalized size	1	1.	6.26	3.73	0.	0.	0.	0.
time (sec)	N/A	0.364	7.194	11.656	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	1346	493	0	0	0	0
normalized size	1	1.	5.3	1.94	0.	0.	0.	0.
time (sec)	N/A	0.548	7.028	3.471	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	1306	465	0	0	0	0
normalized size	1	1.	5.96	2.12	0.	0.	0.	0.
time (sec)	N/A	0.518	6.857	3.536	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	1273	451	0	0	0	0
normalized size	1	1.	6.77	2.4	0.	0.	0.	0.
time (sec)	N/A	0.478	6.793	3.592	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	1265	451	0	0	0	0
normalized size	1	1.	7.03	2.51	0.	0.	0.	0.
time (sec)	N/A	0.47	6.699	3.637	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	1264	451	0	0	0	0
normalized size	1	1.	7.1	2.53	0.	0.	0.	0.
time (sec)	N/A	0.464	6.587	3.708	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	1265	451	0	0	0	0
normalized size	1	1.	6.95	2.48	0.	0.	0.	0.
time (sec)	N/A	0.48	6.659	3.121	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	1305	685	0	0	0	0
normalized size	1	1.	5.9	3.1	0.	0.	0.	0.
time (sec)	N/A	0.519	6.927	4.433	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	1346	876	0	0	0	0
normalized size	1	1.	5.3	3.45	0.	0.	0.	0.
time (sec)	N/A	0.603	7.474	4.655	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	135	428	11097	423	0	0
normalized size	1	1.	0.61	1.94	50.21	1.91	0.	0.
time (sec)	N/A	0.35	0.985	0.678	4.527	2.23	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	118	356	4024	373	0	0
normalized size	1	1.	0.67	2.02	22.86	2.12	0.	0.
time (sec)	N/A	0.298	0.531	0.721	3.625	1.991	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	100	284	2499	324	0	0
normalized size	1	1.	0.76	2.17	19.08	2.47	0.	0.
time (sec)	N/A	0.227	0.288	0.681	2.727	1.959	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	83	164	1268	274	0	0
normalized size	1	1.	1.06	2.1	16.26	3.51	0.	0.
time (sec)	N/A	0.169	0.153	0.692	2.665	1.888	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	86	109	331	298	0	0
normalized size	1	1.	1.13	1.43	4.36	3.92	0.	0.
time (sec)	N/A	0.165	0.162	0.657	2.309	1.623	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	62	390	177	0	0
normalized size	1	1.	0.67	0.73	4.59	2.08	0.	0.
time (sec)	N/A	0.163	0.143	0.647	1.835	1.461	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	78	86	578	223	0	0
normalized size	1	1.	0.6	0.66	4.45	1.72	0.	0.
time (sec)	N/A	0.218	0.239	0.715	2.014	1.428	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	102	108	705	270	0	0
normalized size	1	1.	0.58	0.62	4.03	1.54	0.	0.
time (sec)	N/A	0.286	0.368	0.605	2.043	1.42	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	136	429	12020	451	0	0
normalized size	1	1.	0.6	1.89	52.95	1.99	0.	0.
time (sec)	N/A	0.504	1.099	0.517	5.038	2.321	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	119	357	4081	401	0	0
normalized size	1	1.	0.66	1.98	22.67	2.23	0.	0.
time (sec)	N/A	0.412	0.654	0.689	3.478	1.904	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	101	283	2543	343	0	0
normalized size	1	1.	0.76	2.13	19.12	2.58	0.	0.
time (sec)	N/A	0.33	0.355	0.677	2.556	1.874	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	107	300	2431	362	0	0
normalized size	1	1.	0.85	2.38	19.29	2.87	0.	0.
time (sec)	N/A	0.331	0.311	0.596	2.451	2.059	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	106	211	1517	359	0	0
normalized size	1	1.	0.85	1.69	12.14	2.87	0.	0.
time (sec)	N/A	0.315	0.369	0.596	2.11	1.935	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	80	87	464	231	0	0
normalized size	1	1.	0.6	0.65	3.46	1.72	0.	0.
time (sec)	N/A	0.34	0.296	0.567	1.673	1.659	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	102	109	649	285	0	0
normalized size	1	1.	0.56	0.6	3.59	1.57	0.	0.
time (sec)	N/A	0.431	0.504	0.576	1.678	1.78	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	124	131	774	335	0	0
normalized size	1	1.	0.54	0.57	3.39	1.47	0.	0.
time (sec)	N/A	0.505	0.653	0.639	1.751	1.86	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	159	503	0	540	0	0
normalized size	1	1.	0.58	1.84	0.	1.97	0.	0.
time (sec)	N/A	0.709	1.872	0.561	0.	2.485	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	137	431	12710	478	0	0
normalized size	1	1.	0.6	1.9	55.99	2.11	0.	0.
time (sec)	N/A	0.713	1.201	0.642	5.007	2.001	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	121	357	4146	414	0	0
normalized size	1	1.	0.67	1.98	23.03	2.3	0.	0.
time (sec)	N/A	0.547	0.732	0.708	4.671	1.672	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	126	336	2808	432	0	0
normalized size	1	1.	0.71	1.89	15.78	2.43	0.	0.
time (sec)	N/A	0.552	0.663	0.689	3.37	1.662	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	130	484	3200	436	0	0
normalized size	1	1.	0.75	2.8	18.5	2.52	0.	0.
time (sec)	N/A	0.533	0.709	0.514	2.882	1.584	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	130	306	2090	424	0	0
normalized size	1	1.	0.76	1.78	12.15	2.47	0.	0.
time (sec)	N/A	0.506	0.725	0.674	3.198	1.347	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	104	111	535	297	0	0
normalized size	1	1.	0.57	0.61	2.96	1.64	0.	0.
time (sec)	N/A	0.552	0.6	0.601	2.005	1.269	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	126	133	720	351	0	0
normalized size	1	1.	0.55	0.58	3.16	1.54	0.	0.
time (sec)	N/A	0.703	0.829	0.621	2.452	1.192	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	147	155	845	414	0	0
normalized size	1	1.	0.53	0.56	3.07	1.51	0.	0.
time (sec)	N/A	0.714	0.959	0.635	2.359	1.126	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	348	346	0	516	0	0
normalized size	1	1.	1.83	1.82	0.	2.72	0.	0.
time (sec)	N/A	0.595	1.826	0.73	0.	20.854	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	222	216	0	466	0	0
normalized size	1	1.	1.57	1.53	0.	3.3	0.	0.
time (sec)	N/A	0.397	1.217	0.62	0.	10.981	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	82	149	0	278	0	0
normalized size	1	1.	0.82	1.49	0.	2.78	0.	0.
time (sec)	N/A	0.242	0.138	0.749	0.	8.538	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	203	230	0	396	0	0
normalized size	1	1.	2.05	2.32	0.	4.	0.	0.
time (sec)	N/A	0.192	1.521	0.593	0.	1.287	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	627	383	0	447	0	0
normalized size	1	1.	4.42	2.7	0.	3.15	0.	0.
time (sec)	N/A	0.336	6.755	0.609	0.	1.372	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	1728	519	0	491	0	0
normalized size	1	1.	9.24	2.78	0.	2.63	0.	0.
time (sec)	N/A	0.609	7.916	0.68	0.	1.304	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	362	379	0	644	0	0
normalized size	1	1.	1.84	1.92	0.	3.27	0.	0.
time (sec)	N/A	0.636	2.101	0.602	0.	40.727	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	226	298	0	571	0	0
normalized size	1	1.	1.56	2.06	0.	3.94	0.	0.
time (sec)	N/A	0.403	1.825	0.579	0.	34.571	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	212	247	0	447	0	0
normalized size	1	1.	1.98	2.31	0.	4.18	0.	0.
time (sec)	N/A	0.216	1.115	0.55	0.	1.64	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	423	299	0	531	0	0
normalized size	1	1.	2.71	1.92	0.	3.4	0.	0.
time (sec)	N/A	0.384	3.684	0.577	0.	1.746	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	1054	443	0	586	0	0
normalized size	1	1.	5.19	2.18	0.	2.89	0.	0.
time (sec)	N/A	0.555	6.8	0.623	0.	1.726	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	376	647	0	830	0	0
normalized size	1	1.	1.53	2.63	0.	3.37	0.	0.
time (sec)	N/A	0.837	3.306	0.632	0.	98.471	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	246	515	0	736	0	0
normalized size	1	1.	1.27	2.65	0.	3.79	0.	0.
time (sec)	N/A	0.582	2.03	0.593	0.	69.35	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	198	413	0	571	0	0
normalized size	1	1.	1.29	2.68	0.	3.71	0.	0.
time (sec)	N/A	0.377	1.406	0.563	0.	1.375	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	200	413	0	578	0	0
normalized size	1	1.	1.28	2.65	0.	3.71	0.	0.
time (sec)	N/A	0.439	1.369	0.561	0.	1.387	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	217	443	0	666	0	0
normalized size	1	1.	1.07	2.18	0.	3.28	0.	0.
time (sec)	N/A	0.567	2.574	0.632	0.	1.404	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	239	571	0	728	0	0
normalized size	1	1.	0.96	2.28	0.	2.91	0.	0.
time (sec)	N/A	0.752	3.333	0.527	0.	1.443	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	396	887	0	1018	0	0
normalized size	1	1.	1.35	3.03	0.	3.47	0.	0.
time (sec)	N/A	1.044	5.517	0.665	0.	141.259	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	266	703	0	905	0	0
normalized size	1	1.	1.1	2.92	0.	3.76	0.	0.
time (sec)	N/A	0.765	3.04	0.635	0.	98.454	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	217	549	0	702	0	0
normalized size	1	1.	1.08	2.73	0.	3.49	0.	0.
time (sec)	N/A	0.587	2.249	0.649	0.	1.72	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	215	549	0	705	0	0
normalized size	1	1.	1.07	2.73	0.	3.51	0.	0.
time (sec)	N/A	0.579	1.992	0.62	0.	1.66	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	216	549	0	716	0	0
normalized size	1	1.	1.06	2.7	0.	3.53	0.	0.
time (sec)	N/A	0.591	2.012	0.606	0.	1.686	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	240	581	0	810	0	0
normalized size	1	1.	0.96	2.32	0.	3.24	0.	0.
time (sec)	N/A	0.804	2.73	0.625	0.	1.733	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	262	715	0	878	0	0
normalized size	1	1.	0.88	2.41	0.	2.96	0.	0.
time (sec)	N/A	1.032	5.105	0.484	0.	1.888	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	136	205	252	120
normalized size	1	1.	0.87	1.02	1.3	1.95	2.4	1.14
time (sec)	N/A	0.17	0.22	0.042	0.998	1.394	1.305	1.435

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	75	85	107	149	168	92
normalized size	1	1.	0.89	1.01	1.27	1.77	2.	1.1
time (sec)	N/A	0.09	0.152	0.037	1.117	1.365	0.669	1.327

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	74	104	94	61
normalized size	1	1.	0.98	1.1	1.42	2.	1.81	1.17
time (sec)	N/A	0.023	0.084	0.038	1.01	1.356	0.316	1.366

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	56	63	142	0	107
normalized size	1	1.	1.31	1.6	1.8	4.06	0.	3.06
time (sec)	N/A	0.105	0.027	0.06	1.099	1.434	0.	1.592

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	65	99	225	0	113
normalized size	1	1.	1.23	1.86	2.83	6.43	0.	3.23
time (sec)	N/A	0.114	0.014	0.07	1.1	1.509	0.	1.283

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	128	247	0	204
normalized size	1	1.	1.23	1.41	2.1	4.05	0.	3.34
time (sec)	N/A	0.148	0.022	0.1	0.98	1.506	0.	1.443

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	67	128	171	298	0	284
normalized size	1	1.	0.72	1.38	1.84	3.2	0.	3.05
time (sec)	N/A	0.163	0.264	0.073	1.034	1.41	0.	1.357

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	85	171	220	352	0	410
normalized size	1	1.	0.75	1.5	1.93	3.09	0.	3.6
time (sec)	N/A	0.179	0.584	0.076	1.09	1.417	0.	1.362

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	146	184	238	350	459	211
normalized size	1	1.	0.77	0.97	1.26	1.85	2.43	1.12
time (sec)	N/A	0.311	0.457	0.043	1.098	1.403	3.445	1.435

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	118	152	192	274	338	167
normalized size	1	1.	0.69	0.89	1.13	1.61	1.99	0.98
time (sec)	N/A	0.234	0.431	0.047	1.093	1.428	1.751	1.561

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	146	201	199	126
normalized size	1	1.	0.84	1.07	1.36	1.88	1.86	1.18
time (sec)	N/A	0.094	0.213	0.039	1.041	1.338	0.84	1.385

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	120	120	124	213	0	240
normalized size	1	1.	1.4	1.4	1.44	2.48	0.	2.79
time (sec)	N/A	0.176	0.222	0.07	1.044	1.542	0.	1.297

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	109	104	139	294	0	205
normalized size	1	1.	1.82	1.73	2.32	4.9	0.	3.42
time (sec)	N/A	0.169	0.473	0.071	0.969	1.405	0.	1.52

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	67	133	189	335	0	257
normalized size	1	1.	0.84	1.66	2.36	4.19	0.	3.21
time (sec)	N/A	0.2	0.266	0.088	1.037	1.413	0.	1.599

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	92	174	232	371	0	397
normalized size	1	1.	0.79	1.5	2.	3.2	0.	3.42
time (sec)	N/A	0.27	0.451	0.083	1.058	1.459	0.	1.497

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	120	241	308	443	0	645
normalized size	1	1.	0.77	1.54	1.97	2.84	0.	4.13
time (sec)	N/A	0.293	0.694	0.084	1.107	1.47	0.	1.498

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	289	270	359	517	721	311
normalized size	1	1.	1.07	1.	1.33	1.92	2.68	1.16
time (sec)	N/A	0.507	0.653	0.052	1.	1.589	7.03	1.305

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	176	227	293	423	551	254
normalized size	1	1.	0.72	0.93	1.21	1.74	2.27	1.05
time (sec)	N/A	0.333	0.671	0.125	1.118	1.527	3.852	1.341

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	140	180	231	321	386	200
normalized size	1	1.	0.82	1.05	1.35	1.88	2.26	1.17
time (sec)	N/A	0.197	0.382	0.045	1.163	1.347	1.761	1.366

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	159	207	196	317	0	424
normalized size	1	1.	1.16	1.51	1.43	2.31	0.	3.09
time (sec)	N/A	0.324	0.37	0.077	1.134	1.469	0.	1.602

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	217	168	194	369	0	316
normalized size	1	1.	1.66	1.28	1.48	2.82	0.	2.41
time (sec)	N/A	0.332	0.65	0.075	1.167	1.564	0.	1.546

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	277	172	228	401	0	323
normalized size	1	1.	2.23	1.39	1.84	3.23	0.	2.6
time (sec)	N/A	0.339	2.035	0.085	1.144	1.464	0.	1.706

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	108	223	292	458	0	454
normalized size	1	1.	0.74	1.54	2.01	3.16	0.	3.13
time (sec)	N/A	0.346	0.566	0.086	1.154	1.504	0.	1.486

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	140	290	369	510	0	791
normalized size	1	1.	0.74	1.54	1.96	2.71	0.	4.21
time (sec)	N/A	0.458	0.803	0.098	1.149	1.577	0.	1.367

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	181	382	460	612	0	975
normalized size	1	1.	0.77	1.62	1.95	2.59	0.	4.13
time (sec)	N/A	0.489	3.213	0.092	1.116	1.529	0.	1.524

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	408	368	494	717	1017	423
normalized size	1	1.	1.11	1.01	1.35	1.96	2.78	1.16
time (sec)	N/A	0.838	0.864	0.047	1.171	1.699	11.88	1.467

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	333	316	414	587	811	355
normalized size	1	1.	1.02	0.97	1.27	1.81	2.5	1.09
time (sec)	N/A	0.509	1.055	0.084	1.046	1.573	6.869	1.719

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	263	258	332	478	580	286
normalized size	1	1.	1.09	1.07	1.38	1.98	2.41	1.19
time (sec)	N/A	0.338	0.62	0.04	1.153	1.521	3.673	1.427

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	210	319	281	447	0	814
normalized size	1	1.	1.05	1.6	1.4	2.23	0.	4.07
time (sec)	N/A	0.547	0.58	0.076	1.125	1.548	0.	1.54

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	257	255	266	471	0	501
normalized size	1	1.	1.32	1.31	1.36	2.42	0.	2.57
time (sec)	N/A	0.57	1.019	0.084	1.129	1.602	0.	1.528

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	310	236	282	479	0	710
normalized size	1	1.	1.48	1.13	1.35	2.29	0.	3.4
time (sec)	N/A	0.615	2.384	0.092	1.09	1.529	0.	1.557

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	415	262	331	524	0	522
normalized size	1	1.	2.1	1.32	1.67	2.65	0.	2.64
time (sec)	N/A	0.58	5.963	0.102	1.162	1.545	0.	1.618

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	160	338	428	603	0	857
normalized size	1	1.	0.74	1.56	1.98	2.79	0.	3.97
time (sec)	N/A	0.597	1.014	0.098	1.133	1.542	0.	1.862

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	198	431	521	687	0	1148
normalized size	1	1.	0.74	1.61	1.95	2.57	0.	4.3
time (sec)	N/A	0.723	4.151	0.116	1.053	1.576	0.	1.702

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	244	550	640	797	0	1601
normalized size	1	1.	0.75	1.7	1.98	2.46	0.	4.94
time (sec)	N/A	0.802	2.642	0.098	1.026	1.671	0.	1.7

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	152	641	0	1164	0	486
normalized size	1	1.	0.85	3.6	0.	6.54	0.	2.73
time (sec)	N/A	0.493	0.435	0.115	0.	1.823	0.	1.545

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	367	0	918	0	306
normalized size	1	1.	0.9	2.74	0.	6.85	0.	2.28
time (sec)	N/A	0.287	0.289	0.109	0.	1.894	0.	1.337

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	172	0	689	0	192
normalized size	1	1.	0.96	1.93	0.	7.74	0.	2.16
time (sec)	N/A	0.174	0.195	0.116	0.	1.862	0.	1.295

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	524	524	136
normalized size	1	1.	1.01	1.69	0.	7.82	7.82	2.03
time (sec)	N/A	0.078	0.112	0.098	0.	1.824	127.075	1.58

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	689	0	171
normalized size	1	1.	1.47	1.78	0.	9.07	0.	2.25
time (sec)	N/A	0.115	0.155	0.151	0.	4.658	0.	1.485

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	129	228	0	1049	0	236
normalized size	1	1.	1.3	2.3	0.	10.6	0.	2.38
time (sec)	N/A	0.183	0.518	0.181	0.	1.877	0.	1.364

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	300	410	0	1334	0	363
normalized size	1	1.	2.1	2.87	0.	9.33	0.	2.54
time (sec)	N/A	0.49	1.594	0.151	0.	22.09	0.	1.648

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	422	688	0	1634	0	556
normalized size	1	1.	2.26	3.68	0.	8.74	0.	2.97
time (sec)	N/A	0.77	2.072	0.175	0.	5.02	0.	1.732

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	184	643	0	2120	0	456
normalized size	1	1.	0.7	2.44	0.	8.06	0.	1.73
time (sec)	N/A	0.659	0.983	0.145	0.	1.594	0.	1.629

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	147	445	0	1701	0	502
normalized size	1	1.	0.95	2.87	0.	10.97	0.	3.24
time (sec)	N/A	0.44	0.782	0.125	0.	1.433	0.	1.307

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	119	320	0	1210	0	269
normalized size	1	1.	0.98	2.62	0.	9.92	0.	2.2
time (sec)	N/A	0.241	0.528	0.131	0.	1.291	0.	1.6

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	234	0	846	0	215
normalized size	1	1.	0.97	2.34	0.	8.46	0.	2.15
time (sec)	N/A	0.089	0.317	0.103	0.	1.288	0.	1.293

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	191	342	0	1534	0	301
normalized size	1	1.	1.44	2.57	0.	11.53	0.	2.26
time (sec)	N/A	0.282	0.59	0.158	0.	24.139	0.	1.268

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	240	502	0	2419	0	545
normalized size	1	1.	1.27	2.66	0.	12.8	0.	2.88
time (sec)	N/A	0.673	1.719	0.174	0.	69.443	0.	1.405

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	438	690	0	2952	0	510
normalized size	1	1.	1.62	2.56	0.	10.93	0.	1.89
time (sec)	N/A	0.975	6.246	0.198	0.	109.609	0.	1.516

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	734	1504	0	4001	0	1821
normalized size	1	1.	1.84	3.78	0.	10.05	0.	4.58
time (sec)	N/A	1.724	3.293	0.132	0.	2.166	0.	1.657

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	232	1301	0	3380	0	733
normalized size	1	1.	0.83	4.65	0.	12.07	0.	2.62
time (sec)	N/A	1.222	2.059	0.169	0.	1.964	0.	1.703

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	204	1023	0	2462	0	614
normalized size	1	1.	0.97	4.85	0.	11.67	0.	2.91
time (sec)	N/A	0.564	1.308	0.125	0.	1.733	0.	1.647

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	172	886	0	1616	0	528
normalized size	1	1.	0.96	4.92	0.	8.98	0.	2.93
time (sec)	N/A	0.29	0.784	0.108	0.	1.398	0.	1.585

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	157	886	0	1616	0	527
normalized size	1	1.	0.96	5.4	0.	9.85	0.	3.21
time (sec)	N/A	0.188	0.607	0.116	0.	1.443	0.	1.61

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	269	1045	0	3032	0	649
normalized size	1	1.	1.26	4.88	0.	14.17	0.	3.03
time (sec)	N/A	0.706	1.24	0.173	0.	128.467	0.	1.502

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	352	1358	0	0	0	775
normalized size	1	1.	1.18	4.54	0.	0.	0.	2.59
time (sec)	N/A	1.764	5.527	0.187	0.	0.	0.	1.599

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	507	1551	0	0	0	1883
normalized size	1	1.	1.26	3.86	0.	0.	0.	4.68
time (sec)	N/A	2.236	2.709	0.206	0.	0.	0.	1.602

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	1278	2787	0	5723	0	1304
normalized size	1	1.	3.12	6.81	0.	13.99	0.	3.19
time (sec)	N/A	5.175	6.612	0.141	0.	4.17	0.	1.491

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	717	2158	0	4095	0	1098
normalized size	1	1.	2.38	7.17	0.	13.6	0.	3.65
time (sec)	N/A	1.207	3.131	0.13	0.	2.567	0.	1.721

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	251	1726	0	2692	0	930
normalized size	1	1.	0.92	6.3	0.	9.82	0.	3.39
time (sec)	N/A	0.636	1.195	0.142	0.	2.215	0.	1.612

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	252	1883	0	2700	0	975
normalized size	1	1.	0.96	7.16	0.	10.27	0.	3.71
time (sec)	N/A	0.53	1.052	0.138	0.	2.154	0.	1.643

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	227	1727	0	2692	0	933
normalized size	1	1.	0.96	7.29	0.	11.36	0.	3.94
time (sec)	N/A	0.478	2.158	0.118	0.	2.19	0.	1.562

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	368	2180	0	0	0	1130
normalized size	1	1.	1.22	7.24	0.	0.	0.	3.75
time (sec)	N/A	1.509	1.604	0.187	0.	0.	0.	1.893

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	549	2844	0	0	0	1345
normalized size	1	1.	1.31	6.77	0.	0.	0.	3.2
time (sec)	N/A	6.221	3.022	0.194	0.	0.	0.	1.715

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	547	781	3042	0	0	0	1472
normalized size	1	1.	1.43	5.56	0.	0.	0.	2.69
time (sec)	N/A	7.304	4.937	0.219	0.	0.	0.	1.781

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	0	61	56	34
normalized size	1	1.	1.	0.82	0.	2.18	2.	1.21
time (sec)	N/A	0.016	0.008	0.051	0.	1.798	1.641	1.473

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	0	61	68	45
normalized size	1	1.	0.89	1.04	0.	2.26	2.52	1.67
time (sec)	N/A	0.015	0.02	0.053	0.	1.503	1.141	1.573

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	0	24	31	15
normalized size	1	1.	2.09	1.09	0.	2.18	2.82	1.36
time (sec)	N/A	0.009	0.007	0.046	0.	1.404	0.747	1.398

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	7	2	14
normalized size	1	1.	1.	1.33	0.	2.33	0.67	4.67
time (sec)	N/A	0.001	0.	0.025	0.	1.167	0.159	1.321

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	0	81	39	63
normalized size	1	1.	1.	1.67	0.	6.75	3.25	5.25
time (sec)	N/A	0.007	0.003	0.056	0.	1.419	4.503	1.478

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	45	32	15
normalized size	1	1.	1.	1.09	0.	4.09	2.91	1.36
time (sec)	N/A	0.012	0.004	0.056	0.	1.314	8.563	1.535

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	40	0	170	0	70
normalized size	1	1.	1.	1.11	0.	4.72	0.	1.94
time (sec)	N/A	0.02	0.008	0.066	0.	1.545	0.	1.604

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	24	25	0	84	42	34
normalized size	1	1.	0.86	0.89	0.	3.	1.5	1.21
time (sec)	N/A	0.016	0.037	0.063	0.	1.361	114.743	1.564

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	98	229	0	759	0	250
normalized size	1	1.	0.86	2.01	0.	6.66	0.	2.19
time (sec)	N/A	0.217	0.229	0.122	0.	1.606	0.	1.467

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	73	105	0	612	0	173
normalized size	1	1.	0.92	1.33	0.	7.75	0.	2.19
time (sec)	N/A	0.132	0.134	0.135	0.	1.576	0.	1.487

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	69	0	505	0	130
normalized size	1	1.	0.97	1.13	0.	8.28	0.	2.13
time (sec)	N/A	0.067	0.073	0.119	0.	1.566	0.	1.282

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	45	0	409	0	105
normalized size	1	1.	0.98	0.9	0.	8.18	0.	2.1
time (sec)	N/A	0.035	0.036	0.102	0.	1.511	0.	1.397

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	103	91	0	670	0	165
normalized size	1	1.	1.47	1.3	0.	9.57	0.	2.36
time (sec)	N/A	0.081	0.077	0.119	0.	1.948	0.	1.418

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	116	139	0	925	0	209
normalized size	1	1.	1.32	1.58	0.	10.51	0.	2.38
time (sec)	N/A	0.145	0.351	0.138	0.	1.995	0.	1.54

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	239	273	0	1121	0	298
normalized size	1	1.	1.94	2.22	0.	9.11	0.	2.42
time (sec)	N/A	0.346	0.963	0.154	0.	2.776	0.	1.566

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	292	1635	0	0	0	0
normalized size	1	1.	0.76	4.24	0.	0.	0.	0.
time (sec)	N/A	0.785	1.48	4.869	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	232	1305	0	0	0	0
normalized size	1	1.	0.77	4.31	0.	0.	0.	0.
time (sec)	N/A	0.542	0.946	4.021	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	179	993	0	0	0	0
normalized size	1	1.	0.77	4.3	0.	0.	0.	0.
time (sec)	N/A	0.408	0.824	4.45	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	146	600	0	0	0	0
normalized size	1	1.	0.85	3.51	0.	0.	0.	0.
time (sec)	N/A	0.22	0.575	4.319	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	107	247	0	0	0	0
normalized size	1	1.	0.6	1.39	0.	0.	0.	0.
time (sec)	N/A	0.36	2.376	4.004	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	372	746	0	0	0	0
normalized size	1	1.	1.75	3.5	0.	0.	0.	0.
time (sec)	N/A	0.607	10.245	6.549	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	420	1290	0	0	0	0
normalized size	1	1.	1.44	4.42	0.	0.	0.	0.
time (sec)	N/A	0.955	4.126	7.812	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	635	2213	0	0	0	0
normalized size	1	1.	1.68	5.85	0.	0.	0.	0.
time (sec)	N/A	1.339	6.509	11.474	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	291	1635	0	0	0	0
normalized size	1	1.	0.77	4.33	0.	0.	0.	0.
time (sec)	N/A	0.733	1.445	4.509	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	233	1305	0	0	0	0
normalized size	1	1.	0.78	4.39	0.	0.	0.	0.
time (sec)	N/A	0.528	1.009	4.073	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	203	993	0	0	0	0
normalized size	1	1.	0.9	4.41	0.	0.	0.	0.
time (sec)	N/A	0.352	0.734	4.003	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	406	738	0	0	0	0
normalized size	1	1.	1.72	3.13	0.	0.	0.	0.
time (sec)	N/A	0.708	2.445	4.046	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	398	1167	0	0	0	0
normalized size	1	1.	1.72	5.03	0.	0.	0.	0.
time (sec)	N/A	0.686	2.445	4.417	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	422	1403	0	0	0	0
normalized size	1	1.	1.43	4.76	0.	0.	0.	0.
time (sec)	N/A	1.058	4.795	9.067	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	634	2327	0	0	0	0
normalized size	1	1.	1.69	6.21	0.	0.	0.	0.
time (sec)	N/A	1.45	6.591	12.028	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	357	1983	0	0	0	0
normalized size	1	1.	0.77	4.29	0.	0.	0.	0.
time (sec)	N/A	0.931	1.986	4.483	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	291	1635	0	0	0	0
normalized size	1	1.	0.78	4.4	0.	0.	0.	0.
time (sec)	N/A	0.796	1.482	4.468	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	254	1305	0	0	0	0
normalized size	1	1.	0.88	4.53	0.	0.	0.	0.
time (sec)	N/A	0.516	1.017	4.409	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	453	1067	0	0	0	0
normalized size	1	1.	1.55	3.65	0.	0.	0.	0.
time (sec)	N/A	1.014	2.801	4.432	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	442	1563	0	0	0	0
normalized size	1	1.	1.49	5.28	0.	0.	0.	0.
time (sec)	N/A	1.109	3.796	4.425	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	451	1742	0	0	0	0
normalized size	1	1.	1.43	5.53	0.	0.	0.	0.
time (sec)	N/A	1.055	5.636	9.44	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	486	2438	0	0	0	0
normalized size	1	1.	1.29	6.48	0.	0.	0.	0.
time (sec)	N/A	1.433	5.808	12.175	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	465	465	729	3548	0	0	0	0
normalized size	1	1.	1.57	7.63	0.	0.	0.	0.
time (sec)	N/A	1.845	6.721	16.885	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	230	1305	0	0	0	0
normalized size	1	1.	0.72	4.08	0.	0.	0.	0.
time (sec)	N/A	0.619	1.015	4.258	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	180	993	0	0	0	0
normalized size	1	1.	0.73	4.04	0.	0.	0.	0.
time (sec)	N/A	0.43	0.891	3.907	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	154	671	0	0	0	0
normalized size	1	1.	0.84	3.67	0.	0.	0.	0.
time (sec)	N/A	0.292	0.656	4.369	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	93	249	0	0	0	0
normalized size	1	1.	0.72	1.92	0.	0.	0.	0.
time (sec)	N/A	0.126	3.121	3.184	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0
normalized size	1	1.	0.69	1.64	0.	0.	0.	0.
time (sec)	N/A	0.313	0.183	3.668	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	320	639	0	0	0	0
normalized size	1	1.	1.48	2.96	0.	0.	0.	0.
time (sec)	N/A	0.655	6.366	5.221	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	420	1182	0	0	0	0
normalized size	1	1.	1.4	3.95	0.	0.	0.	0.
time (sec)	N/A	0.955	5.753	7.652	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	304	1308	0	0	0	0
normalized size	1	1.	0.79	3.38	0.	0.	0.	0.
time (sec)	N/A	0.727	1.716	12.509	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	189	954	0	0	0	0
normalized size	1	1.	0.72	3.64	0.	0.	0.	0.
time (sec)	N/A	0.486	1.386	10.478	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	170	515	0	0	0	0
normalized size	1	1.	0.83	2.52	0.	0.	0.	0.
time (sec)	N/A	0.332	0.788	7.861	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	151	428	0	0	0	0
normalized size	1	1.	0.82	2.31	0.	0.	0.	0.
time (sec)	N/A	0.231	0.541	7.684	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	460	429	0	0	0	0
normalized size	1	1.	2.42	2.26	0.	0.	0.	0.
time (sec)	N/A	0.508	3.787	7.651	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	482	908	0	0	0	0
normalized size	1	1.	1.59	3.	0.	0.	0.	0.
time (sec)	N/A	0.991	5.469	10.223	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	398	398	678	1564	0	0	0	0
normalized size	1	1.	1.7	3.93	0.	0.	0.	0.
time (sec)	N/A	1.431	6.841	12.836	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	372	1746	0	0	0	0
normalized size	1	1.	0.68	3.17	0.	0.	0.	0.
time (sec)	N/A	1.187	3.899	21.393	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	334	1389	0	0	0	0
normalized size	1	1.	0.81	3.36	0.	0.	0.	0.
time (sec)	N/A	0.805	2.739	19.001	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	274	950	0	0	0	0
normalized size	1	1.	0.83	2.87	0.	0.	0.	0.
time (sec)	N/A	0.553	2.161	15.549	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	224	860	0	0	0	0
normalized size	1	1.	0.73	2.8	0.	0.	0.	0.
time (sec)	N/A	0.472	1.859	14.194	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	193	750	0	0	0	0
normalized size	1	1.	0.7	2.73	0.	0.	0.	0.
time (sec)	N/A	0.373	1.523	13.244	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	743	854	0	0	0	0
normalized size	1	1.	2.13	2.45	0.	0.	0.	0.
time (sec)	N/A	1.099	6.728	14.143	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	437	437	750	1341	0	0	0	0
normalized size	1	1.	1.72	3.07	0.	0.	0.	0.
time (sec)	N/A	1.481	7.093	18.753	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	532	532	820	2000	0	0	0	0
normalized size	1	1.	1.54	3.76	0.	0.	0.	0.
time (sec)	N/A	1.934	7.743	23.7	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	76	0	0	0	0
normalized size	1	1.	1.	1.31	0.	0.	0.	0.
time (sec)	N/A	0.045	0.048	0.404	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	167	0	0	0	0
normalized size	1	1.	1.	2.83	0.	0.	0.	0.
time (sec)	N/A	0.146	0.071	3.125	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	84	218	0	0	0	0
normalized size	1	1.	0.78	2.02	0.	0.	0.	0.
time (sec)	N/A	0.082	0.208	3.993	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	403	377	0	0	0	0
normalized size	1	1.	2.25	2.11	0.	0.	0.	0.
time (sec)	N/A	0.424	4.804	4.299	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	125	451	0	0	0	0
normalized size	1	1.	0.74	2.65	0.	0.	0.	0.
time (sec)	N/A	0.207	1.267	3.349	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	103	413	0	0	0	0
normalized size	1	1.	0.74	2.95	0.	0.	0.	0.
time (sec)	N/A	0.184	0.823	3.103	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	86	371	0	0	0	0
normalized size	1	1.	0.8	3.44	0.	0.	0.	0.
time (sec)	N/A	0.168	0.384	3.474	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	326	0	0	0	0
normalized size	1	1.	0.89	4.35	0.	0.	0.	0.
time (sec)	N/A	0.146	0.213	3.522	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	244	0	0	0	0
normalized size	1	1.	0.9	3.44	0.	0.	0.	0.
time (sec)	N/A	0.154	0.329	3.262	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	107	428	0	0	0	0
normalized size	1	1.	1.04	4.16	0.	0.	0.	0.
time (sec)	N/A	0.17	0.446	7.556	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	134	663	0	0	0	0
normalized size	1	1.	0.96	4.74	0.	0.	0.	0.
time (sec)	N/A	0.188	0.772	9.989	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	196	666	0	0	0	0
normalized size	1	1.	0.74	2.52	0.	0.	0.	0.
time (sec)	N/A	0.381	1.711	3.161	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	167	610	0	0	0	0
normalized size	1	1.	0.75	2.74	0.	0.	0.	0.
time (sec)	N/A	0.33	1.371	3.724	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	139	548	0	0	0	0
normalized size	1	1.	0.76	3.01	0.	0.	0.	0.
time (sec)	N/A	0.316	1.068	3.138	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	106	487	0	0	0	0
normalized size	1	1.	0.76	3.48	0.	0.	0.	0.
time (sec)	N/A	0.266	0.575	3.194	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	404	0	0	0	0
normalized size	1	1.	0.84	3.34	0.	0.	0.	0.
time (sec)	N/A	0.246	0.605	3.398	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	105	677	0	0	0	0
normalized size	1	1.	0.83	5.37	0.	0.	0.	0.
time (sec)	N/A	0.305	1.119	7.309	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	175	750	0	0	0	0
normalized size	1	1.	1.02	4.36	0.	0.	0.	0.
time (sec)	N/A	0.351	1.056	10.05	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	235	825	0	0	0	0
normalized size	1	1.	0.77	2.7	0.	0.	0.	0.
time (sec)	N/A	0.545	1.905	3.362	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	197	745	0	0	0	0
normalized size	1	1.	0.77	2.92	0.	0.	0.	0.
time (sec)	N/A	0.496	1.152	3.348	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	158	664	0	0	0	0
normalized size	1	1.	0.77	3.24	0.	0.	0.	0.
time (sec)	N/A	0.478	1.268	3.418	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	150	867	0	0	0	0
normalized size	1	1.	0.74	4.29	0.	0.	0.	0.
time (sec)	N/A	0.464	1.085	3.497	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	165	1212	0	0	0	0
normalized size	1	1.	0.86	6.31	0.	0.	0.	0.
time (sec)	N/A	0.465	1.033	9.1	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	176	997	0	0	0	0
normalized size	1	1.	0.86	4.89	0.	0.	0.	0.
time (sec)	N/A	0.482	2.082	10.698	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	264	1074	0	0	0	0
normalized size	1	1.	1.45	5.9	0.	0.	0.	0.
time (sec)	N/A	0.82	2.372	4.116	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	209	786	0	0	0	0
normalized size	1	1.	1.53	5.74	0.	0.	0.	0.
time (sec)	N/A	0.513	1.414	3.556	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	131	295	0	0	0	0
normalized size	1	1.	1.47	3.31	0.	0.	0.	0.
time (sec)	N/A	0.21	0.863	3.138	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0
normalized size	1	1.	0.95	3.56	0.	0.	0.	0.
time (sec)	N/A	0.144	0.211	3.542	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	210	327	0	0	0	0
normalized size	1	1.	2.44	3.8	0.	0.	0.	0.
time (sec)	N/A	0.309	2.406	6.233	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	262	468	0	0	0	0
normalized size	1	1.	1.75	3.12	0.	0.	0.	0.
time (sec)	N/A	0.77	2.21	9.544	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	322	1066	0	0	0	0
normalized size	1	1.	1.06	3.52	0.	0.	0.	0.
time (sec)	N/A	0.933	3.068	11.879	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	284	849	0	0	0	0
normalized size	1	1.	1.27	3.79	0.	0.	0.	0.
time (sec)	N/A	0.627	2.59	9.332	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	262	808	0	0	0	0
normalized size	1	1.	1.32	4.08	0.	0.	0.	0.
time (sec)	N/A	0.54	2.296	8.609	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	276	721	0	0	0	0
normalized size	1	1.	1.38	3.6	0.	0.	0.	0.
time (sec)	N/A	0.618	2.556	7.84	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	320	883	0	0	0	0
normalized size	1	1.	1.25	3.45	0.	0.	0.	0.
time (sec)	N/A	0.923	3.962	10.801	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	367	1031	0	0	0	0
normalized size	1	1.	1.06	2.99	0.	0.	0.	0.
time (sec)	N/A	1.292	6.316	16.085	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	394	1977	0	0	0	0
normalized size	1	1.	1.07	5.39	0.	0.	0.	0.
time (sec)	N/A	1.013	4.672	14.966	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	364	1937	0	0	0	0
normalized size	1	1.	1.06	5.63	0.	0.	0.	0.
time (sec)	N/A	0.99	3.386	14.346	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	369	1850	0	0	0	0
normalized size	1	1.	1.09	5.49	0.	0.	0.	0.
time (sec)	N/A	0.919	4.197	14.49	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	387	1744	0	0	0	0
normalized size	1	1.	1.12	5.06	0.	0.	0.	0.
time (sec)	N/A	1.06	4.675	13.454	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	462	2002	0	0	0	0
normalized size	1	1.	1.1	4.77	0.	0.	0.	0.
time (sec)	N/A	1.474	4.983	17.036	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	523	574	2158	0	0	0	0
normalized size	1	1.	1.1	4.13	0.	0.	0.	0.
time (sec)	N/A	1.953	7.167	27.283	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	203	0	0	0	0
normalized size	1	1.	0.93	4.61	0.	0.	0.	0.
time (sec)	N/A	0.025	0.047	3.126	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	37	180	0	0	0	0
normalized size	1	1.	0.84	4.09	0.	0.	0.	0.
time (sec)	N/A	0.024	0.043	2.984	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	134	0	0	0	0
normalized size	1	1.	1.	7.88	0.	0.	0.	0.
time (sec)	N/A	0.011	0.023	2.138	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	0	0	0	0
normalized size	1	1.	1.	1.12	0.	0.	0.	0.
time (sec)	N/A	0.011	0.027	0.037	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	102	0	0	0	0
normalized size	1	1.	1.	2.55	0.	0.	0.	0.
time (sec)	N/A	0.022	0.059	3.253	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	37	214	0	0	0	0
normalized size	1	1.	0.84	4.86	0.	0.	0.	0.
time (sec)	N/A	0.023	0.06	2.872	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	161	517	0	0	0	0
normalized size	1	1.	1.39	4.46	0.	0.	0.	0.
time (sec)	N/A	0.403	1.57	3.52	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	85	228	0	0	0	0
normalized size	1	1.	1.09	2.92	0.	0.	0.	0.
time (sec)	N/A	0.167	0.102	3.178	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	189	0	0	0	0
normalized size	1	1.	0.89	3.44	0.	0.	0.	0.
time (sec)	N/A	0.103	0.051	3.593	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	151	0	0	0	0
normalized size	1	1.	1.	5.03	0.	0.	0.	0.
time (sec)	N/A	0.049	0.065	2.624	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	200	355	0	0	0	0
normalized size	1	1.	2.5	4.44	0.	0.	0.	0.
time (sec)	N/A	0.247	2.605	4.01	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	215	452	0	0	0	0
normalized size	1	1.	1.62	3.4	0.	0.	0.	0.
time (sec)	N/A	0.56	4.069	8.419	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	1224	2949	0	0	0	0
normalized size	1	1.	2.19	5.27	0.	0.	0.	0.
time (sec)	N/A	1.507	6.305	0.501	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	1175	2055	0	0	0	0
normalized size	1	1.	2.48	4.34	0.	0.	0.	0.
time (sec)	N/A	1.042	21.019	0.417	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	3054	1693	0	0	0	0
normalized size	1	1.	7.93	4.4	0.	0.	0.	0.
time (sec)	N/A	0.713	17.886	0.573	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	275	1687	0	0	0	0
normalized size	1	1.	0.78	4.81	0.	0.	0.	0.
time (sec)	N/A	0.504	12.925	0.446	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	407	1729	0	0	0	0
normalized size	1	1.	1.43	6.09	0.	0.	0.	0.
time (sec)	N/A	0.505	14.007	0.406	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	1315	2479	0	0	0	0
normalized size	1	1.	3.76	7.08	0.	0.	0.	0.
time (sec)	N/A	0.825	6.342	0.474	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	1408	3427	0	0	0	0
normalized size	1	1.	3.25	7.91	0.	0.	0.	0.
time (sec)	N/A	1.176	6.43	0.611	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	670	670	1284	4048	0	0	0	0
normalized size	1	1.	1.92	6.04	0.	0.	0.	0.
time (sec)	N/A	2.096	6.392	0.616	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	566	566	1227	3139	0	0	0	0
normalized size	1	1.	2.17	5.55	0.	0.	0.	0.
time (sec)	N/A	1.663	6.304	0.505	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	1198	2430	0	0	0	0
normalized size	1	1.	2.54	5.15	0.	0.	0.	0.
time (sec)	N/A	1.147	6.331	0.585	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	1196	2185	0	0	0	0
normalized size	1	1.	2.66	4.87	0.	0.	0.	0.
time (sec)	N/A	1.176	6.327	0.419	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	1236	2318	0	0	0	0
normalized size	1	1.	2.95	5.53	0.	0.	0.	0.
time (sec)	N/A	0.858	6.344	0.41	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	1314	2666	0	0	0	0
normalized size	1	1.	3.72	7.55	0.	0.	0.	0.
time (sec)	N/A	0.924	6.418	0.441	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	1407	3413	0	0	0	0
normalized size	1	1.	3.25	7.88	0.	0.	0.	0.
time (sec)	N/A	1.346	6.502	0.565	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	522	522	1515	4391	0	0	0	0
normalized size	1	1.	2.9	8.41	0.	0.	0.	0.
time (sec)	N/A	1.885	6.586	0.741	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	779	779	1353	5164	0	0	0	0
normalized size	1	1.	1.74	6.63	0.	0.	0.	0.
time (sec)	N/A	3.083	6.507	0.894	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	664	664	1287	4238	0	0	0	0
normalized size	1	1.	1.94	6.38	0.	0.	0.	0.
time (sec)	N/A	2.206	6.387	0.661	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	564	564	1251	3512	0	0	0	0
normalized size	1	1.	2.22	6.23	0.	0.	0.	0.
time (sec)	N/A	1.698	6.478	0.68	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	547	547	1241	3270	0	0	0	0
normalized size	1	1.	2.27	5.98	0.	0.	0.	0.
time (sec)	N/A	1.665	6.473	0.454	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	1269	3204	0	0	0	0
normalized size	1	1.	2.37	5.98	0.	0.	0.	0.
time (sec)	N/A	1.669	6.471	0.475	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	493	493	1319	3274	0	0	0	0
normalized size	1	1.	2.68	6.64	0.	0.	0.	0.
time (sec)	N/A	1.247	6.518	0.491	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	434	434	1409	3628	0	0	0	0
normalized size	1	1.	3.25	8.36	0.	0.	0.	0.
time (sec)	N/A	1.372	6.58	0.563	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	522	522	1517	4392	0	0	0	0
normalized size	1	1.	2.91	8.41	0.	0.	0.	0.
time (sec)	N/A	1.939	6.685	0.735	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	622	622	1640	5373	0	0	0	0
normalized size	1	1.	2.64	8.64	0.	0.	0.	0.
time (sec)	N/A	2.625	6.802	0.94	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	1236	2346	0	0	0	0
normalized size	1	1.	2.96	5.61	0.	0.	0.	0.
time (sec)	N/A	0.95	19.409	0.549	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	1175	1871	0	0	0	0
normalized size	1	1.	2.45	3.91	0.	0.	0.	0.
time (sec)	N/A	1.076	12.376	0.479	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	4017	1005	0	0	0	0
normalized size	1	1.	9.41	2.35	0.	0.	0.	0.
time (sec)	N/A	1.088	17.73	0.53	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	146	197	0	0	0	0
normalized size	1	1.	0.64	0.86	0.	0.	0.	0.
time (sec)	N/A	0.274	1.48	0.464	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	299	935	0	0	0	0
normalized size	1	1.	1.3	4.07	0.	0.	0.	0.
time (sec)	N/A	0.316	12.864	0.388	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	416	1536	0	0	0	0
normalized size	1	1.	1.43	5.3	0.	0.	0.	0.
time (sec)	N/A	0.524	16.293	0.397	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	1319	2480	0	0	0	0
normalized size	1	1.	3.63	6.83	0.	0.	0.	0.
time (sec)	N/A	0.862	6.389	0.467	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	500	500	1234	2881	0	0	0	0
normalized size	1	1.	2.47	5.76	0.	0.	0.	0.
time (sec)	N/A	1.287	6.397	0.427	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	1012	2013	0	0	0	0
normalized size	1	1.	2.43	4.84	0.	0.	0.	0.
time (sec)	N/A	0.612	17.986	0.411	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	1223	1633	0	0	0	0
normalized size	1	1.	4.31	5.75	0.	0.	0.	0.
time (sec)	N/A	0.512	6.36	0.475	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	1281	2280	0	0	0	0
normalized size	1	1.	4.2	7.48	0.	0.	0.	0.
time (sec)	N/A	0.615	6.472	0.472	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	1357	3334	0	0	0	0
normalized size	1	1.	3.45	8.48	0.	0.	0.	0.
time (sec)	N/A	0.98	6.644	0.498	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	674	674	1396	8611	0	0	0	0
normalized size	1	1.	2.07	12.78	0.	0.	0.	0.
time (sec)	N/A	2.189	6.655	0.592	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	545	545	1342	5751	0	0	0	0
normalized size	1	1.	2.46	10.55	0.	0.	0.	0.
time (sec)	N/A	1.402	6.491	0.521	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	1335	4237	0	0	0	0
normalized size	1	1.	3.41	10.84	0.	0.	0.	0.
time (sec)	N/A	0.869	6.422	0.473	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	1384	5203	0	0	0	0
normalized size	1	1.	3.23	12.13	0.	0.	0.	0.
time (sec)	N/A	0.985	6.525	0.856	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	456	456	1431	6498	0	0	0	0
normalized size	1	1.	3.14	14.25	0.	0.	0.	0.
time (sec)	N/A	1.161	6.684	1.06	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	567	567	1499	8093	0	0	0	0
normalized size	1	1.	2.64	14.27	0.	0.	0.	0.
time (sec)	N/A	1.877	6.921	0.74	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	480	623	0	0	0	0
normalized size	1	1.	1.15	1.49	0.	0.	0.	0.
time (sec)	N/A	0.789	1.44	0.611	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	133	160	0	0	0	0
normalized size	1	1.	1.14	1.37	0.	0.	0.	0.
time (sec)	N/A	0.085	0.138	0.479	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	171	124	0	0	0	0
normalized size	1	1.	1.55	1.13	0.	0.	0.	0.
time (sec)	N/A	0.087	0.882	0.565	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	212	613	0	0	0	0
normalized size	1	1.	0.94	2.71	0.	0.	0.	0.
time (sec)	N/A	0.268	2.11	0.46	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	0	658	0	0	0	0
normalized size	1	1.	0.	9.14	0.	0.	0.	0.
time (sec)	N/A	0.088	34.837	0.444	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	601	0	0	0	0
normalized size	1	1.	0.	8.59	0.	0.	0.	0.
time (sec)	N/A	0.104	38.35	0.461	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	611	0	0	0	0
normalized size	1	1.	0.	6.57	0.	0.	0.	0.
time (sec)	N/A	0.206	32.611	0.469	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	0	705	0	0	0	0
normalized size	1	1.	0.	7.42	0.	0.	0.	0.
time (sec)	N/A	0.191	28.693	0.364	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	0	665	0	0	0	0
normalized size	1	1.	0.	9.24	0.	0.	0.	0.
time (sec)	N/A	0.083	37.742	0.448	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	0	663	0	0	0	0
normalized size	1	1.	0.	8.96	0.	0.	0.	0.
time (sec)	N/A	0.099	38.014	0.458	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	0	714	0	0	0	0
normalized size	1	1.	0.	7.29	0.	0.	0.	0.
time (sec)	N/A	0.202	40.906	0.352	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	0	740	0	0	0	0
normalized size	1	1.	0.	7.71	0.	0.	0.	0.
time (sec)	N/A	0.189	30.115	0.366	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	7.554	2.795	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	595	595	487	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	1.984	6.198	2.33	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	269	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	1.053	2.906	1.981	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	217	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.541	1.746	1.823	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	151	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	0.325	1.793	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	286	286	10482	0	0	0	0	0
normalized size	1	1.	36.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	27.058	1.145	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	180	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.527	66.256	0.497	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	9.681	0.457	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	7.962	0.464	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	190	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.5	10.557	0.423	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	292	661	0	0	0	0
normalized size	1	1.	1.7	3.84	0.	0.	0.	0.
time (sec)	N/A	0.222	1.897	10.024	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	225	426	0	0	0	0
normalized size	1	1.	1.67	3.16	0.	0.	0.	0.
time (sec)	N/A	0.192	1.194	7.894	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	157	240	0	0	0	0
normalized size	1	1.	1.48	2.26	0.	0.	0.	0.
time (sec)	N/A	0.179	1.056	3.536	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	148	321	0	0	0	0
normalized size	1	1.	1.35	2.92	0.	0.	0.	0.
time (sec)	N/A	0.184	1.239	2.699	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	148	355	0	0	0	0
normalized size	1	1.	1.05	2.52	0.	0.	0.	0.
time (sec)	N/A	0.203	1.593	2.924	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	182	383	0	0	0	0
normalized size	1	1.	1.06	2.23	0.	0.	0.	0.
time (sec)	N/A	0.224	2.196	3.135	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	299	741	0	0	0	0
normalized size	1	1.	1.5	3.72	0.	0.	0.	0.
time (sec)	N/A	0.345	2.979	10.25	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	279	513	0	0	0	0
normalized size	1	1.	1.74	3.21	0.	0.	0.	0.
time (sec)	N/A	0.318	2.243	3.736	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	302	388	0	0	0	0
normalized size	1	1.	1.89	2.42	0.	0.	0.	0.
time (sec)	N/A	0.323	1.839	3.353	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	153	357	0	0	0	0
normalized size	1	1.	0.92	2.15	0.	0.	0.	0.
time (sec)	N/A	0.34	1.62	3.26	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	193	385	0	0	0	0
normalized size	1	1.	0.96	1.92	0.	0.	0.	0.
time (sec)	N/A	0.369	2.293	2.94	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	435	929	0	0	0	0
normalized size	1	1.	1.78	3.81	0.	0.	0.	0.
time (sec)	N/A	0.505	4.096	11.729	0.	0.	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	268	916	0	0	0	0
normalized size	1	1.	1.27	4.34	0.	0.	0.	0.
time (sec)	N/A	0.495	3.161	10.273	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	202	654	0	0	0	0
normalized size	1	1.	1.02	3.29	0.	0.	0.	0.
time (sec)	N/A	0.491	1.892	3.417	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	207	519	0	0	0	0
normalized size	1	1.	0.98	2.46	0.	0.	0.	0.
time (sec)	N/A	0.495	1.597	3.493	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	194	385	0	0	0	0
normalized size	1	1.	0.92	1.82	0.	0.	0.	0.
time (sec)	N/A	0.508	2.387	3.111	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	196	413	0	0	0	0
normalized size	1	1.	0.8	1.69	0.	0.	0.	0.
time (sec)	N/A	0.535	2.768	3.016	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	650	493	0	0	0	0
normalized size	1	1.	3.37	2.55	0.	0.	0.	0.
time (sec)	N/A	0.304	7.182	9.714	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	400	319	0	0	0	0
normalized size	1	1.	2.52	2.01	0.	0.	0.	0.
time (sec)	N/A	0.276	4.369	5.949	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	200	244	0	0	0	0
normalized size	1	1.	1.63	1.98	0.	0.	0.	0.
time (sec)	N/A	0.245	1.093	3.193	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	422	244	0	0	0	0
normalized size	1	1.	3.38	1.95	0.	0.	0.	0.
time (sec)	N/A	0.253	2.576	3.398	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	444	262	0	0	0	0
normalized size	1	1.	2.72	1.61	0.	0.	0.	0.
time (sec)	N/A	0.281	4.933	3.086	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	518	281	0	0	0	0
normalized size	1	1.	2.64	1.43	0.	0.	0.	0.
time (sec)	N/A	0.3	3.216	3.461	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	303	494	0	0	0	0
normalized size	1	1.	1.46	2.38	0.	0.	0.	0.
time (sec)	N/A	0.431	3.145	4.286	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	256	350	0	0	0	0
normalized size	1	1.	1.59	2.17	0.	0.	0.	0.
time (sec)	N/A	0.388	2.004	3.796	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	256	350	0	0	0	0
normalized size	1	1.	1.52	2.08	0.	0.	0.	0.
time (sec)	N/A	0.391	2.443	3.863	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	475	421	0	0	0	0
normalized size	1	1.	2.7	2.39	0.	0.	0.	0.
time (sec)	N/A	0.406	6.37	3.718	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	777	435	0	0	0	0
normalized size	1	1.	3.77	2.11	0.	0.	0.	0.
time (sec)	N/A	0.434	6.84	3.851	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	358	685	0	0	0	0
normalized size	1	1.	1.37	2.62	0.	0.	0.	0.
time (sec)	N/A	0.612	5.397	4.529	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	793	451	0	0	0	0
normalized size	1	1.	3.57	2.03	0.	0.	0.	0.
time (sec)	N/A	0.581	6.887	3.971	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	792	451	0	0	0	0
normalized size	1	1.	3.67	2.09	0.	0.	0.	0.
time (sec)	N/A	0.566	6.865	4.217	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	793	451	0	0	0	0
normalized size	1	1.	3.57	2.03	0.	0.	0.	0.
time (sec)	N/A	0.58	6.956	3.7	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	817	451	0	0	0	0
normalized size	1	1.	3.58	1.98	0.	0.	0.	0.
time (sec)	N/A	0.577	7.11	3.719	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	259	589	465	0	0	0	0
normalized size	1	1.	2.27	1.8	0.	0.	0.	0.
time (sec)	N/A	0.624	4.559	3.979	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	124	138	890	316	0	0
normalized size	1	1.	0.56	0.63	4.05	1.44	0.	0.
time (sec)	N/A	0.486	0.538	0.802	1.984	1.723	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	102	116	767	273	0	0
normalized size	1	1.	0.58	0.66	4.38	1.56	0.	0.
time (sec)	N/A	0.406	0.428	0.801	2.179	1.643	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	78	94	641	225	0	0
normalized size	1	1.	0.6	0.72	4.93	1.73	0.	0.
time (sec)	N/A	0.334	0.272	0.759	1.842	1.897	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	70	513	177	0	0
normalized size	1	1.	0.67	0.82	6.04	2.08	0.	0.
time (sec)	N/A	0.266	0.182	0.806	1.754	1.984	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	86	171	1223	259	0	0
normalized size	1	1.	0.9	1.78	12.74	2.7	0.	0.
time (sec)	N/A	0.273	0.214	0.777	2.309	1.947	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	103	168	1268	274	0	0
normalized size	1	1.	1.05	1.71	12.94	2.8	0.	0.
time (sec)	N/A	0.271	0.201	0.766	2.813	1.923	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	120	238	2499	348	0	0
normalized size	1	1.	0.79	1.58	16.55	2.3	0.	0.
time (sec)	N/A	0.338	0.378	0.858	3.385	1.949	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	138	308	4024	397	0	0
normalized size	1	1.	0.7	1.57	20.53	2.03	0.	0.
time (sec)	N/A	0.414	0.69	0.833	3.337	1.97	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	146	161	961	402	0	0
normalized size	1	1.	0.53	0.59	3.49	1.46	0.	0.
time (sec)	N/A	0.723	0.775	0.777	2.304	1.481	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	124	139	836	338	0	0
normalized size	1	1.	0.54	0.61	3.67	1.48	0.	0.
time (sec)	N/A	0.65	0.692	0.684	2.152	1.433	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	102	117	711	288	0	0
normalized size	1	1.	0.56	0.65	3.93	1.59	0.	0.
time (sec)	N/A	0.561	0.545	0.652	2.508	1.463	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	80	95	589	234	0	0
normalized size	1	1.	0.6	0.71	4.4	1.75	0.	0.
time (sec)	N/A	0.469	0.33	0.569	1.993	1.46	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	106	287	1974	355	0	0
normalized size	1	1.	0.73	1.98	13.61	2.45	0.	0.
time (sec)	N/A	0.451	0.395	0.666	2.87	1.615	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	107	308	2431	323	0	0
normalized size	1	1.	0.73	2.11	16.65	2.21	0.	0.
time (sec)	N/A	0.466	0.322	0.655	2.98	1.859	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	121	233	2543	365	0	0
normalized size	1	1.	0.79	1.52	16.62	2.39	0.	0.
time (sec)	N/A	0.455	0.441	0.696	2.771	1.967	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	141	309	4081	423	0	0
normalized size	1	1.	0.7	1.54	20.4	2.12	0.	0.
time (sec)	N/A	0.542	0.504	0.658	4.384	1.87	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	158	381	12016	473	0	0
normalized size	1	1.	0.64	1.54	48.65	1.91	0.	0.
time (sec)	N/A	0.644	0.773	0.549	6.195	2.338	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	171	185	1030	487	0	0
normalized size	1	1.	0.53	0.57	3.2	1.51	0.	0.
time (sec)	N/A	0.939	0.891	0.651	2.196	1.512	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	147	163	907	417	0	0
normalized size	1	1.	0.53	0.59	3.3	1.52	0.	0.
time (sec)	N/A	0.848	1.279	0.674	2.424	1.481	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	126	141	782	354	0	0
normalized size	1	1.	0.55	0.62	3.43	1.55	0.	0.
time (sec)	N/A	0.768	0.938	0.654	2.398	1.393	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	104	119	659	300	0	0
normalized size	1	1.	0.57	0.66	3.64	1.66	0.	0.
time (sec)	N/A	0.676	0.7	0.655	2.33	1.443	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	130	389	2313	425	0	0
normalized size	1	1.	0.68	2.03	12.05	2.21	0.	0.
time (sec)	N/A	0.623	0.815	0.724	3.29	1.617	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	130	492	3753	432	0	0
normalized size	1	1.	0.67	2.55	19.45	2.24	0.	0.
time (sec)	N/A	0.654	0.734	0.688	3.743	1.939	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	126	344	0	393	0	0
normalized size	1	1.	0.64	1.74	0.	1.98	0.	0.
time (sec)	N/A	0.664	0.781	0.71	0.	1.916	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	141	305	4146	436	0	0
normalized size	1	1.	0.7	1.52	20.73	2.18	0.	0.
time (sec)	N/A	0.65	0.969	0.799	3.188	1.91	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	159	383	12677	500	0	0
normalized size	1	1.	0.64	1.55	51.32	2.02	0.	0.
time (sec)	N/A	0.752	0.969	0.658	5.214	2.371	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	181	455	0	562	0	0
normalized size	1	1.	0.62	1.55	0.	1.91	0.	0.
time (sec)	N/A	0.866	1.427	0.589	0.	2.439	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	272	793	0	539	0	0
normalized size	1	1.	0.92	2.69	0.	1.83	0.	0.
time (sec)	N/A	1.057	9.506	0.641	0.	1.7	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	250	657	0	493	0	0
normalized size	1	1.	1.	2.63	0.	1.97	0.	0.
time (sec)	N/A	0.838	6.797	0.769	0.	1.724	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	1718	521	0	443	0	0
normalized size	1	1.	8.3	2.52	0.	2.14	0.	0.
time (sec)	N/A	0.647	7.97	0.774	0.	1.688	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	0	384	0	393	0	0
normalized size	1	1.	0.	2.37	0.	2.43	0.	0.
time (sec)	N/A	0.453	0.	0.774	0.	1.627	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	203	231	0	306	0	0
normalized size	1	1.	1.71	1.94	0.	2.57	0.	0.
time (sec)	N/A	0.306	1.51	0.717	0.	1.612	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	102	153	0	278	0	0
normalized size	1	1.	0.73	1.09	0.	1.99	0.	0.
time (sec)	N/A	0.35	0.205	0.753	0.	10.438	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	467	232	0	466	0	0
normalized size	1	1.	2.58	1.28	0.	2.57	0.	0.
time (sec)	N/A	0.513	1.36	0.727	0.	13.466	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	412	300	0	539	0	0
normalized size	1	1.	1.79	1.3	0.	2.34	0.	0.
time (sec)	N/A	0.699	1.413	0.786	0.	25.88	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	143	317	0	0	0	0
normalized size	1	1.	0.74	1.65	0.	0.	0.	0.
time (sec)	N/A	0.665	0.45	0.799	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	2966	731	0	645	0	0
normalized size	1	1.	9.36	2.31	0.	2.03	0.	0.
time (sec)	N/A	1.106	10.334	0.72	0.	2.907	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	270	0	595	0	590	0	0
normalized size	1	1.	0.	2.2	0.	2.19	0.	0.
time (sec)	N/A	0.887	0.	0.703	0.	2.779	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	981	457	0	531	0	0
normalized size	1	1.	4.4	2.05	0.	2.38	0.	0.
time (sec)	N/A	0.702	6.845	0.727	0.	2.172	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	443	312	0	435	0	0
normalized size	1	1.	2.52	1.77	0.	2.47	0.	0.
time (sec)	N/A	0.519	4.451	0.675	0.	2.002	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	196	236	0	397	0	0
normalized size	1	1.	1.54	1.86	0.	3.13	0.	0.
time (sec)	N/A	0.337	1.622	0.656	0.	1.965	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	243	288	0	571	0	0
normalized size	1	1.	1.31	1.56	0.	3.09	0.	0.
time (sec)	N/A	0.544	1.645	0.657	0.	43.247	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	836	370	0	667	0	0
normalized size	1	1.	3.53	1.56	0.	2.81	0.	0.
time (sec)	N/A	0.736	6.696	1.02	0.	47.803	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	261	729	0	732	0	0
normalized size	1	1.	0.82	2.3	0.	2.31	0.	0.
time (sec)	N/A	1.125	8.177	0.747	0.	1.845	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	243	585	0	672	0	0
normalized size	1	1.	0.9	2.17	0.	2.49	0.	0.
time (sec)	N/A	0.929	3.589	0.638	0.	1.835	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	219	457	0	570	0	0
normalized size	1	1.	0.98	2.05	0.	2.56	0.	0.
time (sec)	N/A	0.729	2.193	0.578	0.	1.897	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	216	376	0	551	0	0
normalized size	1	1.	1.23	2.14	0.	3.13	0.	0.
time (sec)	N/A	0.526	1.737	0.682	0.	1.844	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	213	375	0	544	0	0
normalized size	1	1.	1.22	2.16	0.	3.13	0.	0.
time (sec)	N/A	0.511	1.77	0.582	0.	1.714	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	264	476	0	759	0	0
normalized size	1	1.	1.13	2.03	0.	3.24	0.	0.
time (sec)	N/A	0.738	2.509	0.605	0.	77.079	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	929	609	0	853	0	0
normalized size	1	1.	3.25	2.13	0.	2.98	0.	0.
time (sec)	N/A	0.981	7.157	0.615	0.	99.568	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	267	729	0	822	0	0
normalized size	1	1.	0.84	2.3	0.	2.59	0.	0.
time (sec)	N/A	1.147	5.5	0.649	0.	1.847	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	242	595	0	714	0	0
normalized size	1	1.	0.9	2.2	0.	2.64	0.	0.
time (sec)	N/A	0.95	3.181	0.647	0.	1.77	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	228	512	0	686	0	0
normalized size	1	1.	1.02	2.3	0.	3.08	0.	0.
time (sec)	N/A	0.728	3.021	0.707	0.	1.736	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	233	512	0	679	0	0
normalized size	1	1.	1.05	2.32	0.	3.07	0.	0.
time (sec)	N/A	0.725	2.872	0.661	0.	1.747	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	488	512	0	675	0	0
normalized size	1	1.	2.21	2.32	0.	3.05	0.	0.
time (sec)	N/A	0.723	7.165	0.598	0.	1.849	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	281	667	0	927	0	0
normalized size	1	1.	1.	2.37	0.	3.3	0.	0.
time (sec)	N/A	0.919	3.912	0.675	0.	98.877	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	1017	855	0	1041	0	0
normalized size	1	1.	3.05	2.57	0.	3.13	0.	0.
time (sec)	N/A	1.205	7.577	0.71	0.	172.239	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	132	663	0	0	0	0
normalized size	1	1.	0.73	3.68	0.	0.	0.	0.
time (sec)	N/A	0.223	1.825	10.268	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	104	428	0	0	0	0
normalized size	1	1.	0.73	2.99	0.	0.	0.	0.
time (sec)	N/A	0.201	0.797	8.347	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	85	244	0	0	0	0
normalized size	1	1.	0.77	2.2	0.	0.	0.	0.
time (sec)	N/A	0.18	0.259	3.681	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	90	326	0	0	0	0
normalized size	1	1.	0.78	2.83	0.	0.	0.	0.
time (sec)	N/A	0.188	0.227	3.579	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	108	371	0	0	0	0
normalized size	1	1.	0.73	2.51	0.	0.	0.	0.
time (sec)	N/A	0.208	0.527	3.483	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	125	413	0	0	0	0
normalized size	1	1.	0.69	2.29	0.	0.	0.	0.
time (sec)	N/A	0.234	0.967	3.358	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	171	750	0	0	0	0
normalized size	1	1.	0.77	3.39	0.	0.	0.	0.
time (sec)	N/A	0.38	2.368	10.918	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	125	677	0	0	0	0
normalized size	1	1.	0.71	3.82	0.	0.	0.	0.
time (sec)	N/A	0.352	1.119	7.943	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	404	0	0	0	0
normalized size	1	1.	0.77	2.51	0.	0.	0.	0.
time (sec)	N/A	0.32	0.728	3.925	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	128	487	0	0	0	0
normalized size	1	1.	0.75	2.85	0.	0.	0.	0.
time (sec)	N/A	0.336	0.882	3.695	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	161	548	0	0	0	0
normalized size	1	1.	0.76	2.57	0.	0.	0.	0.
time (sec)	N/A	0.372	1.332	3.781	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	225	944	0	0	0	0
normalized size	1	1.	0.76	3.2	0.	0.	0.	0.
time (sec)	N/A	0.597	3.508	13.595	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	192	997	0	0	0	0
normalized size	1	1.	0.79	4.09	0.	0.	0.	0.
time (sec)	N/A	0.577	1.555	10.882	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	166	1212	0	0	0	0
normalized size	1	1.	0.69	5.07	0.	0.	0.	0.
time (sec)	N/A	0.566	1.869	10.385	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	172	867	0	0	0	0
normalized size	1	1.	0.73	3.66	0.	0.	0.	0.
time (sec)	N/A	0.533	1.376	4.28	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	180	664	0	0	0	0
normalized size	1	1.	0.73	2.71	0.	0.	0.	0.
time (sec)	N/A	0.544	1.256	3.599	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	219	745	0	0	0	0
normalized size	1	1.	0.74	2.53	0.	0.	0.	0.
time (sec)	N/A	0.579	1.782	3.452	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	229	468	0	0	0	0
normalized size	1	1.	1.09	2.23	0.	0.	0.	0.
time (sec)	N/A	0.808	3.408	9.955	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	127	327	0	0	0	0
normalized size	1	1.	1.01	2.6	0.	0.	0.	0.
time (sec)	N/A	0.459	1.265	7.079	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	217	0	0	0	0
normalized size	1	1.	0.77	2.15	0.	0.	0.	0.
time (sec)	N/A	0.283	0.522	3.706	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	224	295	0	0	0	0
normalized size	1	1.	1.5	1.98	0.	0.	0.	0.
time (sec)	N/A	0.346	6.142	3.76	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	282	786	0	0	0	0
normalized size	1	1.	1.43	3.99	0.	0.	0.	0.
time (sec)	N/A	0.56	6.516	3.815	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	741	1031	0	0	0	0
normalized size	1	1.	1.83	2.55	0.	0.	0.	0.
time (sec)	N/A	1.289	7.056	16.78	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	316	316	687	883	0	0	0	0
normalized size	1	1.	2.17	2.79	0.	0.	0.	0.
time (sec)	N/A	0.942	6.936	10.849	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	645	721	0	0	0	0
normalized size	1	1.	2.48	2.77	0.	0.	0.	0.
time (sec)	N/A	0.613	6.819	8.523	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	632	808	0	0	0	0
normalized size	1	1.	2.45	3.13	0.	0.	0.	0.
time (sec)	N/A	0.605	6.776	9.402	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	661	849	0	0	0	0
normalized size	1	1.	2.33	2.99	0.	0.	0.	0.
time (sec)	N/A	0.658	6.886	10.895	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	363	707	1066	0	0	0	0
normalized size	1	1.	1.95	2.94	0.	0.	0.	0.
time (sec)	N/A	0.962	6.993	13.194	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	480	850	2002	0	0	0	0
normalized size	1	1.	1.77	4.17	0.	0.	0.	0.
time (sec)	N/A	1.45	7.165	19.237	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	803	1744	0	0	0	0
normalized size	1	1.	1.98	4.31	0.	0.	0.	0.
time (sec)	N/A	1.036	6.923	15.027	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	790	1850	0	0	0	0
normalized size	1	1.	1.97	4.6	0.	0.	0.	0.
time (sec)	N/A	1.09	6.907	15.087	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	400	400	792	1937	0	0	0	0
normalized size	1	1.	1.98	4.84	0.	0.	0.	0.
time (sec)	N/A	0.955	6.916	15.242	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	826	1977	0	0	0	0
normalized size	1	1.	1.93	4.63	0.	0.	0.	0.
time (sec)	N/A	1.057	7.067	17.077	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	871	2195	0	0	0	0
normalized size	1	1.	1.67	4.21	0.	0.	0.	0.
time (sec)	N/A	1.529	7.299	19.167	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	214	0	0	0	0
normalized size	1	1.	0.73	3.34	0.	0.	0.	0.
time (sec)	N/A	0.037	0.068	3.359	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	46	102	0	0	0	0
normalized size	1	1.	0.77	1.7	0.	0.	0.	0.
time (sec)	N/A	0.036	0.044	2.691	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	0	0	0
normalized size	1	1.	1.	3.62	0.	0.	0.	0.
time (sec)	N/A	0.023	0.029	2.184	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	0	0	0
normalized size	1	1.	1.	3.62	0.	0.	0.	0.
time (sec)	N/A	0.023	0.034	1.898	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	50	180	0	0	0	0
normalized size	1	1.	0.78	2.81	0.	0.	0.	0.
time (sec)	N/A	0.039	0.044	2.876	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	203	0	0	0	0
normalized size	1	1.	0.88	3.17	0.	0.	0.	0.
time (sec)	N/A	0.037	0.07	2.808	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	473	473	3321	3436	0	0	0	0
normalized size	1	1.	7.02	7.26	0.	0.	0.	0.
time (sec)	N/A	1.438	24.408	0.706	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	423	2489	0	0	0	0
normalized size	1	1.	1.08	6.38	0.	0.	0.	0.
time (sec)	N/A	1.057	17.922	0.576	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	346	1735	0	0	0	0
normalized size	1	1.	1.07	5.35	0.	0.	0.	0.
time (sec)	N/A	0.675	14.977	0.467	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	639	1361	0	0	0	0
normalized size	1	1.	1.55	3.31	0.	0.	0.	0.
time (sec)	N/A	0.685	17.21	0.57	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	795	1369	0	0	0	0
normalized size	1	1.	1.79	3.08	0.	0.	0.	0.
time (sec)	N/A	0.903	17.423	0.701	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	533	533	1133	2054	0	0	0	0
normalized size	1	1.	2.13	3.85	0.	0.	0.	0.
time (sec)	N/A	1.265	18.629	0.565	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	620	620	1549	2957	0	0	0	0
normalized size	1	1.	2.5	4.77	0.	0.	0.	0.
time (sec)	N/A	1.74	14.671	0.787	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	562	562	3739	4399	0	0	0	0
normalized size	1	1.	6.65	7.83	0.	0.	0.	0.
time (sec)	N/A	2.078	25.675	0.969	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	473	473	3318	3421	0	0	0	0
normalized size	1	1.	7.01	7.23	0.	0.	0.	0.
time (sec)	N/A	1.529	24.336	0.64	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	427	2674	0	0	0	0
normalized size	1	1.	1.09	6.8	0.	0.	0.	0.
time (sec)	N/A	1.091	18.546	0.605	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	479	6011	2326	0	0	0	0
normalized size	1	1.	12.55	4.86	0.	0.	0.	0.
time (sec)	N/A	1.076	24.265	0.694	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	935	2193	0	0	0	0
normalized size	1	1.	1.84	4.31	0.	0.	0.	0.
time (sec)	N/A	1.38	18.395	0.476	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	532	532	1146	2432	0	0	0	0
normalized size	1	1.	2.15	4.57	0.	0.	0.	0.
time (sec)	N/A	1.367	18.572	0.547	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	626	626	1505	3141	0	0	0	0
normalized size	1	1.	2.4	5.02	0.	0.	0.	0.
time (sec)	N/A	1.973	19.217	0.65	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	1907	4056	0	0	0	0
normalized size	1	1.	2.61	5.56	0.	0.	0.	0.
time (sec)	N/A	2.441	21.621	0.964	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	662	662	4198	5381	0	0	0	0
normalized size	1	1.	6.34	8.13	0.	0.	0.	0.
time (sec)	N/A	2.911	26.766	1.276	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	562	562	3755	4400	0	0	0	0
normalized size	1	1.	6.68	7.83	0.	0.	0.	0.
time (sec)	N/A	2.072	25.823	0.887	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	474	474	3348	3636	0	0	0	0
normalized size	1	1.	7.06	7.67	0.	0.	0.	0.
time (sec)	N/A	1.501	24.911	0.774	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	553	553	7062	3282	0	0	0	0
normalized size	1	1.	12.77	5.93	0.	0.	0.	0.
time (sec)	N/A	1.473	25.054	0.713	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	596	596	7752	3212	0	0	0	0
normalized size	1	1.	13.01	5.39	0.	0.	0.	0.
time (sec)	N/A	1.895	25.597	0.612	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	607	607	1290	3278	0	0	0	0
normalized size	1	1.	2.13	5.4	0.	0.	0.	0.
time (sec)	N/A	1.879	19.388	0.735	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	624	1521	3514	0	0	0	0
normalized size	1	1.	2.44	5.63	0.	0.	0.	0.
time (sec)	N/A	1.946	19.368	0.727	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	724	724	1877	4240	0	0	0	0
normalized size	1	1.	2.59	5.86	0.	0.	0.	0.
time (sec)	N/A	2.518	20.152	0.883	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	839	839	705	5172	0	0	0	0
normalized size	1	1.	0.84	6.16	0.	0.	0.	0.
time (sec)	N/A	3.603	15.536	0.967	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	2987	2488	0	0	0	0
normalized size	1	1.	7.41	6.17	0.	0.	0.	0.
time (sec)	N/A	1.026	22.507	0.671	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	355	1544	0	0	0	0
normalized size	1	1.	1.08	4.68	0.	0.	0.	0.
time (sec)	N/A	0.664	16.03	0.769	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	279	812	0	0	0	0
normalized size	1	1.	1.03	3.01	0.	0.	0.	0.
time (sec)	N/A	0.457	13.98	0.708	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	159	199	0	0	0	0
normalized size	1	1.	0.59	0.74	0.	0.	0.	0.
time (sec)	N/A	0.399	2.646	0.674	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	1091	1004	0	0	0	0
normalized size	1	1.	2.24	2.06	0.	0.	0.	0.
time (sec)	N/A	1.265	18.276	0.708	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	539	1169	1878	0	0	0	0
normalized size	1	1.	2.17	3.48	0.	0.	0.	0.
time (sec)	N/A	1.253	19.794	0.648	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	3433	3343	0	0	0	0
normalized size	1	1.	7.93	7.72	0.	0.	0.	0.
time (sec)	N/A	1.159	24.54	0.651	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	345	345	433	2291	0	0	0	0
normalized size	1	1.	1.26	6.64	0.	0.	0.	0.
time (sec)	N/A	0.791	18.571	0.668	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	305	1636	0	0	0	0
normalized size	1	1.	0.94	5.05	0.	0.	0.	0.
time (sec)	N/A	0.685	13.673	0.684	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	1403	2016	0	0	0	0
normalized size	1	1.	2.95	4.24	0.	0.	0.	0.
time (sec)	N/A	0.775	13.995	0.596	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	1567	2890	0	0	0	0
normalized size	1	1.	2.8	5.16	0.	0.	0.	0.
time (sec)	N/A	1.509	19.578	0.549	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	607	607	4316	8101	0	0	0	0
normalized size	1	1.	7.11	13.35	0.	0.	0.	0.
time (sec)	N/A	2.203	26.559	0.757	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	496	3891	6506	0	0	0	0
normalized size	1	1.	7.84	13.12	0.	0.	0.	0.
time (sec)	N/A	1.383	26.087	0.824	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	3493	5205	0	0	0	0
normalized size	1	1.	7.45	11.1	0.	0.	0.	0.
time (sec)	N/A	1.19	24.323	0.641	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	528	4243	0	0	0	0
normalized size	1	1.	1.23	9.84	0.	0.	0.	0.
time (sec)	N/A	1.081	18.992	0.681	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	602	602	1994	5757	0	0	0	0
normalized size	1	1.	3.31	9.56	0.	0.	0.	0.
time (sec)	N/A	1.645	16.147	0.571	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	733	733	2342	8621	0	0	0	0
normalized size	1	1.	3.2	11.76	0.	0.	0.	0.
time (sec)	N/A	2.494	22.491	0.714	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	298	621	0	0	0	0
normalized size	1	1.	1.12	2.33	0.	0.	0.	0.
time (sec)	N/A	0.37	6.1	0.583	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	104	126	0	0	0	0
normalized size	1	1.	0.8	0.97	0.	0.	0.	0.
time (sec)	N/A	0.155	0.135	0.679	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	149	144	0	0	0	0
normalized size	1	1.	1.09	1.05	0.	0.	0.	0.
time (sec)	N/A	0.155	0.178	0.642	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	479	5018	634	0	0	0	0
normalized size	1	1.	10.48	1.32	0.	0.	0.	0.
time (sec)	N/A	0.922	6.117	0.697	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	8.455	3.207	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	644	644	317	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	2.042	3.959	2.862	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	455	455	259	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.145	2.163	2.351	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	205	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.643	0.935	2.099	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	163	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.36	0.362	2.181	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	10630	0	0	0	0	0
normalized size	1	1.	35.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.576	26.5	1.394	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	209	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.721	57.077	0.714	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	12.341	0.773	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	8.204	0.715	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	212	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.686	10.502	0.716	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [317] had the largest ratio of [0.3333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2	A	7	6	1.	29	0.207
3	A	3	3	1.	27	0.111
4	A	1	1	1.	21	0.048
5	A	4	4	1.	27	0.148
6	A	4	4	1.	29	0.138
7	A	6	6	1.	29	0.207
8	A	7	7	1.	29	0.241
9	A	7	6	1.	29	0.207
10	A	9	7	1.	31	0.226
11	A	8	7	1.	31	0.226
12	A	4	4	1.	29	0.138
13	A	2	2	1.	23	0.087
14	A	5	5	1.	29	0.172
15	A	5	5	1.	31	0.161
16	A	5	5	1.	31	0.161
17	A	7	7	1.	31	0.226
18	A	8	8	1.	31	0.258
19	A	9	7	1.	31	0.226
20	A	10	8	1.	29	0.276
21	A	8	6	1.	23	0.261
22	A	6	5	1.	29	0.172
23	A	6	6	1.	31	0.194
24	A	6	5	1.	31	0.161
25	A	6	5	1.	31	0.161
26	A	8	7	1.	31	0.226
27	A	9	8	1.	31	0.258
28	A	10	7	1.	31	0.226
29	A	13	8	1.	29	0.276
30	A	11	6	1.	23	0.261
31	A	7	5	1.	29	0.172
32	A	7	6	1.	31	0.194
33	A	7	6	1.	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
34	A	7	5	1.	31	0.161
35	A	7	5	1.	31	0.161
36	A	9	7	1.	31	0.226
37	A	10	8	1.	31	0.258
38	A	7	5	1.	31	0.161
39	A	6	5	1.	31	0.161
40	A	2	2	1.1	31	0.065
41	A	5	5	1.	29	0.172
42	A	2	2	1.	23	0.087
43	A	3	3	1.	29	0.103
44	A	5	5	1.	31	0.161
45	A	6	6	1.	31	0.194
46	A	6	5	1.	31	0.161
47	A	7	5	1.	31	0.161
48	A	3	2	1.	31	0.065
49	A	6	6	1.	31	0.194
50	A	4	4	1.	29	0.138
51	A	2	2	1.	23	0.087
52	A	4	3	1.	29	0.103
53	A	6	5	1.	31	0.161
54	A	7	6	1.	31	0.194
55	A	7	5	1.	31	0.161
56	A	8	5	1.	31	0.161
57	A	4	2	1.	31	0.065
58	A	7	6	1.	31	0.194
59	A	5	5	1.	31	0.161
60	A	4	4	1.	29	0.138
61	A	3	3	1.	23	0.13
62	A	5	3	1.	29	0.103
63	A	7	5	1.	31	0.161
64	A	8	6	1.	31	0.194
65	A	5	2	1.	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
66	A	8	6	1.	31	0.194
67	A	6	5	1.	31	0.161
68	A	5	5	1.	31	0.161
69	A	5	5	1.	29	0.172
70	A	4	3	1.	23	0.13
71	A	6	3	1.	29	0.103
72	A	8	5	1.	31	0.161
73	A	9	6	1.	31	0.194
74	A	5	5	1.	33	0.152
75	A	4	4	1.	33	0.121
76	A	4	4	1.	31	0.129
77	A	2	2	1.	25	0.08
78	A	3	3	1.	31	0.097
79	A	3	3	1.	33	0.091
80	A	4	4	1.	33	0.121
81	A	5	4	1.	33	0.121
82	A	6	6	1.	33	0.182
83	A	5	5	1.	33	0.152
84	A	5	5	1.	31	0.161
85	A	3	3	1.	25	0.12
86	A	4	4	1.	31	0.129
87	A	4	4	1.	33	0.121
88	A	4	4	1.	33	0.121
89	A	5	5	1.	33	0.152
90	A	6	5	1.	33	0.152
91	A	6	5	1.	33	0.152
92	A	6	5	1.	31	0.161
93	A	4	3	1.	25	0.12
94	A	5	4	1.	31	0.129
95	A	5	5	1.	33	0.152
96	A	5	4	1.	33	0.121
97	A	5	4	1.	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
98	A	6	5	1.	33	0.152
99	A	7	5	1.	33	0.152
100	A	7	6	1.	33	0.182
101	A	6	6	1.	33	0.182
102	A	5	5	1.	31	0.161
103	A	3	3	1.	25	0.12
104	A	5	4	1.	31	0.129
105	A	6	5	1.	33	0.152
106	A	7	5	1.	33	0.152
107	A	8	7	1.	33	0.212
108	A	7	7	1.	33	0.212
109	A	6	6	1.	33	0.182
110	A	5	5	1.	31	0.161
111	A	3	3	1.	25	0.12
112	A	6	5	1.	31	0.161
113	A	7	6	1.	33	0.182
114	A	8	6	1.	33	0.182
115	A	8	7	1.	33	0.212
116	A	7	6	1.	33	0.182
117	A	6	6	1.	33	0.182
118	A	5	5	1.	31	0.161
119	A	4	4	1.	25	0.16
120	A	7	5	1.	31	0.161
121	A	8	6	1.	33	0.182
122	A	9	6	1.	33	0.182
123	A	8	6	1.	31	0.194
124	A	7	6	1.	31	0.194
125	A	6	6	1.	31	0.194
126	A	5	5	1.	31	0.161
127	A	5	5	1.	31	0.161
128	A	6	6	1.	31	0.194
129	A	7	6	1.	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	8	7	1.	33	0.212
131	A	7	7	1.	33	0.212
132	A	6	6	1.	33	0.182
133	A	6	6	1.	33	0.182
134	A	6	6	1.	33	0.182
135	A	7	7	1.	33	0.212
136	A	8	7	1.	33	0.212
137	A	9	7	1.	33	0.212
138	A	8	7	1.	33	0.212
139	A	7	6	1.	33	0.182
140	A	7	7	1.	33	0.212
141	A	7	6	1.	33	0.182
142	A	7	6	1.	33	0.182
143	A	8	7	1.	33	0.212
144	A	9	7	1.	33	0.212
145	A	6	5	1.	33	0.152
146	A	5	5	1.	33	0.152
147	A	4	4	1.	33	0.121
148	A	4	4	1.	33	0.121
149	A	5	5	1.	33	0.152
150	A	6	5	1.	33	0.152
151	A	7	5	1.	33	0.152
152	A	6	5	1.	33	0.152
153	A	5	4	1.	33	0.121
154	A	5	5	1.	33	0.152
155	A	5	4	1.	33	0.121
156	A	6	5	1.	33	0.152
157	A	7	5	1.	33	0.152
158	A	8	5	1.	33	0.152
159	A	7	5	1.	33	0.152
160	A	6	4	1.	33	0.121
161	A	6	5	1.	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
162	A	6	5	1.	33	0.152
163	A	6	4	1.	33	0.121
164	A	7	5	1.	33	0.152
165	A	8	5	1.	33	0.152
166	A	6	4	1.	35	0.114
167	A	5	4	1.	35	0.114
168	A	4	4	1.	35	0.114
169	A	3	3	1.	35	0.086
170	A	3	3	1.	35	0.086
171	A	2	2	1.	35	0.057
172	A	3	3	1.	35	0.086
173	A	4	3	1.	35	0.086
174	A	6	5	1.	35	0.143
175	A	5	5	1.	35	0.143
176	A	4	4	1.	35	0.114
177	A	4	4	1.	35	0.114
178	A	4	4	1.	35	0.114
179	A	3	3	1.	35	0.086
180	A	4	4	1.	35	0.114
181	A	5	4	1.	35	0.114
182	A	7	5	1.	35	0.143
183	A	6	5	1.	35	0.143
184	A	5	4	1.	35	0.114
185	A	5	5	1.	35	0.143
186	A	5	4	1.	35	0.114
187	A	5	4	1.	35	0.114
188	A	4	3	1.	35	0.086
189	A	5	4	1.	35	0.114
190	A	6	4	1.	35	0.114
191	A	7	6	1.	35	0.171
192	A	6	6	1.	35	0.171
193	A	5	5	1.	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
194	A	4	4	1.	35	0.114
195	A	5	4	1.	35	0.114
196	A	6	4	1.	35	0.114
197	A	7	7	1.	35	0.2
198	A	6	6	1.	35	0.171
199	A	4	4	1.	35	0.114
200	A	5	5	1.	35	0.143
201	A	6	5	1.	35	0.143
202	A	8	7	1.	35	0.2
203	A	7	6	1.	35	0.171
204	A	5	5	1.	35	0.143
205	A	5	4	1.	35	0.114
206	A	6	5	1.	35	0.143
207	A	7	5	1.	35	0.143
208	A	9	7	1.	35	0.2
209	A	8	6	1.	35	0.171
210	A	6	5	1.	35	0.143
211	A	6	5	1.	35	0.143
212	A	6	4	1.	35	0.114
213	A	7	5	1.	35	0.143
214	A	8	5	1.	35	0.143
215	A	7	6	1.	29	0.207
216	A	3	3	1.	27	0.111
217	A	1	1	1.	21	0.048
218	A	4	4	1.	27	0.148
219	A	4	4	1.	29	0.138
220	A	6	6	1.	29	0.207
221	A	7	7	1.	29	0.241
222	A	7	6	1.	29	0.207
223	A	7	6	1.	31	0.194
224	A	4	4	1.	29	0.138
225	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	4	4	1.	29	0.138
227	A	4	4	1.	31	0.129
228	A	4	4	1.	31	0.129
229	A	6	6	1.	31	0.194
230	A	7	7	1.	31	0.226
231	A	8	7	1.	31	0.226
232	A	5	4	1.	29	0.138
233	A	3	2	1.	23	0.087
234	A	5	5	1.	29	0.172
235	A	5	5	1.	31	0.161
236	A	5	5	1.	31	0.161
237	A	5	5	1.	31	0.161
238	A	7	7	1.	31	0.226
239	A	8	8	1.	31	0.258
240	A	9	8	1.	31	0.258
241	A	6	4	1.	29	0.138
242	A	4	2	1.	23	0.087
243	A	6	6	1.	29	0.207
244	A	6	6	1.	31	0.194
245	A	6	6	1.	31	0.194
246	A	6	6	1.	31	0.194
247	A	6	6	1.	31	0.194
248	A	8	8	1.	31	0.258
249	A	9	9	1.	31	0.29
250	A	6	6	1.	31	0.194
251	A	5	5	1.	31	0.161
252	A	6	6	1.	29	0.207
253	A	3	3	1.	23	0.13
254	A	4	4	1.	29	0.138
255	A	6	6	1.	31	0.194
256	A	6	6	1.	31	0.194
257	A	7	6	1.	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
258	A	6	6	1.	31	0.194
259	A	5	5	1.	31	0.161
260	A	5	5	1.	29	0.172
261	A	4	4	1.	23	0.174
262	A	5	5	1.	29	0.172
263	A	6	6	1.	31	0.194
264	A	7	6	1.	31	0.194
265	A	7	7	1.	31	0.226
266	A	6	6	1.	31	0.194
267	A	5	5	1.	31	0.161
268	A	6	6	1.	29	0.207
269	A	5	4	1.	23	0.174
270	A	6	6	1.	29	0.207
271	A	7	6	1.	31	0.194
272	A	8	6	1.	31	0.194
273	A	7	7	1.	31	0.226
274	A	6	6	1.	31	0.194
275	A	6	6	1.	31	0.194
276	A	7	6	1.	29	0.207
277	A	6	4	1.	23	0.174
278	A	7	6	1.	29	0.207
279	A	8	6	1.	31	0.194
280	A	9	6	1.	31	0.194
281	A	3	2	1.	34	0.059
282	A	3	3	1.	34	0.088
283	A	2	2	1.	32	0.062
284	A	2	2	1.	26	0.077
285	A	2	2	1.	32	0.062
286	A	3	3	1.	34	0.088
287	A	3	3	1.	34	0.088
288	A	3	2	1.	34	0.059
289	A	6	6	1.	34	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
290	A	6	6	1.	34	0.176
291	A	4	4	1.	32	0.125
292	A	3	3	1.	26	0.115
293	A	5	5	1.	32	0.156
294	A	7	7	1.	34	0.206
295	A	7	7	1.	34	0.206
296	A	9	9	1.	33	0.273
297	A	8	8	1.	33	0.242
298	A	8	8	1.	31	0.258
299	A	6	6	1.	25	0.24
300	A	8	8	1.	31	0.258
301	A	9	9	1.	33	0.273
302	A	10	10	1.	33	0.303
303	A	11	10	1.	33	0.303
304	A	9	8	1.	33	0.242
305	A	9	8	1.	31	0.258
306	A	7	6	1.	25	0.24
307	A	9	9	1.	31	0.29
308	A	9	9	1.	33	0.273
309	A	10	10	1.	33	0.303
310	A	11	10	1.	33	0.303
311	A	10	8	1.	33	0.242
312	A	10	8	1.	31	0.258
313	A	8	6	1.	25	0.24
314	A	10	10	1.	31	0.323
315	A	10	10	1.	33	0.303
316	A	10	10	1.	33	0.303
317	A	11	11	1.	33	0.333
318	A	12	11	1.	33	0.333
319	A	8	8	1.	33	0.242
320	A	7	7	1.	33	0.212
321	A	7	7	1.	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
322	A	5	5	1.	25	0.2
323	A	5	5	1.	31	0.161
324	A	9	9	1.	33	0.273
325	A	10	10	1.	33	0.303
326	A	8	8	1.	33	0.242
327	A	7	7	1.	33	0.212
328	A	7	7	1.	31	0.226
329	A	6	6	1.	25	0.24
330	A	7	7	1.	31	0.226
331	A	10	10	1.	33	0.303
332	A	11	10	1.	33	0.303
333	A	9	9	1.	33	0.273
334	A	8	8	1.	33	0.242
335	A	7	7	1.	33	0.212
336	A	8	8	1.	31	0.258
337	A	7	6	1.	25	0.24
338	A	10	10	1.	31	0.323
339	A	11	11	1.	33	0.333
340	A	12	10	1.	33	0.303
341	A	3	3	1.	28	0.107
342	A	3	3	1.	34	0.088
343	A	5	4	1.	28	0.143
344	A	8	8	1.	34	0.235
345	A	8	6	1.	31	0.194
346	A	7	6	1.	31	0.194
347	A	6	6	1.	31	0.194
348	A	5	5	1.	31	0.161
349	A	5	5	1.	31	0.161
350	A	6	6	1.	31	0.194
351	A	7	6	1.	31	0.194
352	A	8	6	1.	33	0.182
353	A	7	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
354	A	6	6	1.	33	0.182
355	A	5	5	1.	33	0.152
356	A	5	5	1.	33	0.152
357	A	5	5	1.	33	0.152
358	A	6	6	1.	33	0.182
359	A	8	7	1.	33	0.212
360	A	7	7	1.	33	0.212
361	A	6	6	1.	33	0.182
362	A	6	6	1.	33	0.182
363	A	6	6	1.	33	0.182
364	A	6	6	1.	33	0.182
365	A	7	7	1.	33	0.212
366	A	6	6	1.	33	0.182
367	A	5	5	1.	33	0.152
368	A	3	3	1.	33	0.091
369	A	5	5	1.	33	0.152
370	A	7	7	1.	33	0.212
371	A	7	7	1.	33	0.212
372	A	6	6	1.	33	0.182
373	A	6	6	1.	33	0.182
374	A	6	6	1.	33	0.182
375	A	7	7	1.	33	0.212
376	A	8	7	1.	33	0.212
377	A	7	7	1.	33	0.212
378	A	7	7	1.	33	0.212
379	A	7	7	1.	33	0.212
380	A	7	7	1.	33	0.212
381	A	8	7	1.	33	0.212
382	A	9	7	1.	33	0.212
383	A	3	3	1.	36	0.083
384	A	3	3	1.	36	0.083
385	A	2	2	1.	36	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
386	A	2	2	1.	36	0.056
387	A	3	3	1.	36	0.083
388	A	3	3	1.	36	0.083
389	A	7	7	1.	36	0.194
390	A	6	6	1.	36	0.167
391	A	4	4	1.	36	0.111
392	A	2	2	1.	36	0.056
393	A	6	6	1.	36	0.167
394	A	8	8	1.	36	0.222
395	A	8	8	1.	35	0.229
396	A	7	7	1.	35	0.2
397	A	6	6	1.	35	0.171
398	A	5	5	1.	35	0.143
399	A	4	4	1.	35	0.114
400	A	5	5	1.	35	0.143
401	A	6	5	1.	35	0.143
402	A	9	8	1.	35	0.229
403	A	8	8	1.	35	0.229
404	A	7	7	1.	35	0.2
405	A	7	7	1.	35	0.2
406	A	6	6	1.	35	0.171
407	A	5	5	1.	35	0.143
408	A	6	5	1.	35	0.143
409	A	7	5	1.	35	0.143
410	A	10	8	1.	35	0.229
411	A	9	8	1.	35	0.229
412	A	8	8	1.	35	0.229
413	A	8	8	1.	35	0.229
414	A	8	8	1.	35	0.229
415	A	7	7	1.	35	0.2
416	A	6	6	1.	35	0.171
417	A	7	6	1.	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
418	A	8	6	1.	35	0.171
419	A	6	6	1.	43	0.14
420	A	7	7	1.	35	0.2
421	A	7	7	1.	35	0.2
422	A	3	3	1.	35	0.086
423	A	3	3	1.	35	0.086
424	A	4	4	1.	35	0.114
425	A	5	5	1.	35	0.143
426	A	7	7	1.	35	0.2
427	A	6	6	1.	35	0.171
428	A	4	4	1.	35	0.114
429	A	4	4	1.	35	0.114
430	A	5	5	1.	35	0.143
431	A	8	8	1.	35	0.229
432	A	7	7	1.	35	0.2
433	A	5	5	1.	35	0.143
434	A	5	5	1.	35	0.143
435	A	5	5	1.	35	0.143
436	A	6	5	1.	35	0.143
437	A	9	9	1.	38	0.237
438	A	2	2	1.	38	0.053
439	A	2	2	1.	38	0.053
440	A	4	4	1.	38	0.105
441	A	1	1	1.	33	0.03
442	A	1	1	1.	33	0.03
443	A	2	2	1.	33	0.061
444	A	2	2	1.	33	0.061
445	A	1	1	1.	33	0.03
446	A	1	1	1.	33	0.03
447	A	2	2	1.	33	0.061
448	A	2	2	1.	33	0.061
449	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
450	A	7	6	1.	33	0.182
451	A	6	5	1.	33	0.152
452	A	5	4	1.	33	0.121
453	A	5	4	1.	31	0.129
454	A	7	5	1.	33	0.152
455	A	0	0	0.	0	0.
456	A	0	0	0.	0	0.
457	A	0	0	0.	0	0.
458	A	0	0	0.	0	0.
459	A	9	7	1.	31	0.226
460	A	8	7	1.	31	0.226
461	A	7	6	1.	31	0.194
462	A	7	6	1.	31	0.194
463	A	8	7	1.	31	0.226
464	A	9	7	1.	31	0.226
465	A	9	8	1.	33	0.242
466	A	8	7	1.	33	0.212
467	A	8	7	1.	33	0.212
468	A	8	7	1.	33	0.212
469	A	9	8	1.	33	0.242
470	A	10	8	1.	33	0.242
471	A	9	7	1.	33	0.212
472	A	9	8	1.	33	0.242
473	A	9	7	1.	33	0.212
474	A	9	7	1.	33	0.212
475	A	10	8	1.	33	0.242
476	A	9	7	1.	33	0.212
477	A	8	7	1.	33	0.212
478	A	7	6	1.	33	0.182
479	A	7	6	1.	33	0.182
480	A	8	7	1.	33	0.212
481	A	9	7	1.	33	0.212

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
482	A	9	7	1.	33	0.212
483	A	8	6	1.	33	0.182
484	A	8	7	1.	33	0.212
485	A	8	6	1.	33	0.182
486	A	9	7	1.	33	0.212
487	A	10	7	1.	33	0.212
488	A	9	6	1.	33	0.182
489	A	9	7	1.	33	0.212
490	A	9	7	1.	33	0.212
491	A	9	6	1.	33	0.182
492	A	10	7	1.	33	0.212
493	A	6	4	1.	35	0.114
494	A	5	4	1.	35	0.114
495	A	4	4	1.	35	0.114
496	A	3	3	1.	35	0.086
497	A	4	4	1.	35	0.114
498	A	4	4	1.	35	0.114
499	A	5	5	1.	35	0.143
500	A	6	5	1.	35	0.143
501	A	7	5	1.	35	0.143
502	A	6	5	1.	35	0.143
503	A	5	5	1.	35	0.143
504	A	4	4	1.	35	0.114
505	A	5	5	1.	35	0.143
506	A	5	5	1.	35	0.143
507	A	5	5	1.	35	0.143
508	A	6	6	1.	35	0.171
509	A	7	6	1.	35	0.171
510	A	8	5	1.	35	0.143
511	A	7	5	1.	35	0.143
512	A	6	5	1.	35	0.143
513	A	5	4	1.	35	0.114

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
514	A	6	5	1.	35	0.143
515	A	6	5	1.	35	0.143
516	A	6	6	1.	35	0.171
517	A	6	5	1.	35	0.143
518	A	7	6	1.	35	0.171
519	A	8	6	1.	35	0.171
520	A	9	5	1.	35	0.143
521	A	8	5	1.	35	0.143
522	A	7	5	1.	35	0.143
523	A	6	5	1.	35	0.143
524	A	5	5	1.	35	0.143
525	A	6	6	1.	35	0.171
526	A	7	7	1.	35	0.2
527	A	8	7	1.	35	0.2
528	A	7	7	1.	54	0.13
529	A	9	6	1.	35	0.171
530	A	8	6	1.	35	0.171
531	A	7	6	1.	35	0.171
532	A	6	6	1.	35	0.171
533	A	5	5	1.	35	0.143
534	A	7	7	1.	35	0.2
535	A	8	8	1.	35	0.229
536	A	9	6	1.	35	0.171
537	A	8	6	1.	35	0.171
538	A	7	6	1.	35	0.171
539	A	6	5	1.	35	0.143
540	A	6	6	1.	35	0.171
541	A	8	7	1.	35	0.2
542	A	9	8	1.	35	0.229
543	A	9	6	1.	35	0.171
544	A	8	6	1.	35	0.171
545	A	7	5	1.	35	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
546	A	7	6	1.	35	0.171
547	A	7	6	1.	35	0.171
548	A	9	7	1.	35	0.2
549	A	10	8	1.	35	0.229
550	A	9	7	1.	31	0.226
551	A	8	7	1.	31	0.226
552	A	7	6	1.	31	0.194
553	A	7	6	1.	31	0.194
554	A	8	7	1.	31	0.226
555	A	9	7	1.	31	0.226
556	A	9	8	1.	33	0.242
557	A	8	7	1.	33	0.212
558	A	8	7	1.	33	0.212
559	A	8	7	1.	33	0.212
560	A	9	8	1.	33	0.242
561	A	10	9	1.	33	0.273
562	A	9	8	1.	33	0.242
563	A	9	8	1.	33	0.242
564	A	9	8	1.	33	0.242
565	A	9	8	1.	33	0.242
566	A	10	9	1.	33	0.273
567	A	11	10	1.	33	0.303
568	A	8	8	1.	33	0.242
569	A	6	6	1.	33	0.182
570	A	8	8	1.	33	0.242
571	A	10	9	1.	33	0.273
572	A	12	10	1.	33	0.303
573	A	11	10	1.	33	0.303
574	A	10	9	1.	33	0.273
575	A	10	9	1.	33	0.273
576	A	10	9	1.	33	0.273
577	A	11	10	1.	33	0.303

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
578	A	12	11	1.	33	0.333
579	A	11	10	1.	33	0.303
580	A	11	10	1.	33	0.303
581	A	11	10	1.	33	0.303
582	A	11	10	1.	33	0.303
583	A	12	11	1.	33	0.333
584	A	4	4	1.	36	0.111
585	A	4	4	1.	36	0.111
586	A	3	3	1.	36	0.083
587	A	3	3	1.	36	0.083
588	A	4	4	1.	36	0.111
589	A	4	4	1.	36	0.111
590	A	7	6	1.	35	0.171
591	A	6	6	1.	35	0.171
592	A	5	5	1.	35	0.143
593	A	6	6	1.	35	0.171
594	A	7	7	1.	35	0.2
595	A	8	8	1.	35	0.229
596	A	9	9	1.	35	0.257
597	A	8	6	1.	35	0.171
598	A	7	6	1.	35	0.171
599	A	6	6	1.	35	0.171
600	A	7	7	1.	35	0.2
601	A	8	8	1.	35	0.229
602	A	8	8	1.	35	0.229
603	A	9	9	1.	35	0.257
604	A	10	9	1.	35	0.257
605	A	9	7	1.	35	0.2
606	A	8	7	1.	35	0.2
607	A	7	7	1.	35	0.2
608	A	8	8	1.	35	0.229
609	A	9	9	1.	35	0.257

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
610	A	9	9	1.	35	0.257
611	A	9	9	1.	35	0.257
612	A	10	9	1.	35	0.257
613	A	11	9	1.	35	0.257
614	A	6	6	1.	35	0.171
615	A	5	5	1.	35	0.143
616	A	4	4	1.	35	0.114
617	A	4	4	1.	35	0.114
618	A	8	8	1.	35	0.229
619	A	8	8	1.	35	0.229
620	A	6	6	1.	35	0.171
621	A	5	5	1.	35	0.143
622	A	5	5	1.	35	0.143
623	A	7	7	1.	35	0.2
624	A	8	8	1.	35	0.229
625	A	7	6	1.	35	0.171
626	A	6	6	1.	35	0.171
627	A	6	6	1.	35	0.171
628	A	6	6	1.	35	0.171
629	A	8	8	1.	35	0.229
630	A	9	9	1.	35	0.257
631	A	5	5	1.	38	0.132
632	A	3	3	1.	38	0.079
633	A	3	3	1.	38	0.079
634	A	10	10	1.	38	0.263
635	A	0	0	0.	0	0.
636	A	10	8	1.	33	0.242
637	A	9	7	1.	33	0.212
638	A	8	6	1.	33	0.182
639	A	7	5	1.	31	0.161
640	A	10	8	1.	33	0.242
641	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
642	A	0	0	0.	0	0.
643	A	0	0	0.	0	0.
644	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=125

$$-\frac{a(5A+4B)\sin^3(c+dx)}{15d}+\frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)\cos(c+dx)}{8d}$$

```
[Out] (3*a*(A + B)*x)/8 + (a*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (3*a*(A + B)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*B*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A + 4*B)*Sin[c + d*x]^3)
/(15*d)
```

Rubi [A] time = 0.167091, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 2748, 2633, 2635, 8}

$$-\frac{a(5A+4B)\sin^3(c+dx)}{15d}+\frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*a*(A + B)*x)/8 + (a*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (3*a*(A + B)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*B*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A + 4*B)*Sin[c + d*x]^3)
```

```
/(15*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^3(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\right.$$
$$= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\int \cos^3(c+dx)(a(5A+4B$$
$$= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \cos^4(c+dx)\,dx$$
$$= \frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{aB\cos^4(c+dx)\sin(c+$$
$$= \frac{a(5A+4B)\sin(c+dx)}{5d} + \frac{3a(A+B)\cos(c+dx)\sin(c+dx}{8d}$$
$$= \frac{3}{8}a(A+B)x + \frac{a(5A+4B)\sin(c+dx)}{5d} + \frac{3a(A+B)\cos(c+}{8d}$$

Mathematica [A] time = 0.24435, size = 77, normalized size = 0.62

$$\frac{a\left(-160(A+2B)\sin^3(c+dx)+480(A+B)\sin(c+dx)+15(A+B)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))+9\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(480*(A + B)*Sin[c + d*x] - 160*(A + 2*B)*Sin[c + d*x]^3 + 96*B*Sin[c +
d*x]^5 + 15*(A + B)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
)/(480*d)
```

Maple [A] time = 0.05, size = 128, normalized size = 1.

$$\frac{1}{d}\left(\frac{aB\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+aA\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*B*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
```

)

Maxima [A] time = 1.0119, size = 167, normalized size = 1.34

$$-\frac{160\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa-32\left(3\,\sin\left(dx+c\right)^{5}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 15*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 32*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*B*a - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*B*a)/d
```

Fricas [A] time = 1.76045, size = 239, normalized size = 1.91

$$\frac{45\left(A+B\right)adx+\left(24\,Ba\cos\left(dx+c\right)^{4}+30\left(A+B\right)a\cos\left(dx+c\right)^{3}+8\left(5\,A+4\,B\right)a\cos\left(dx+c\right)^{2}+45\left(A+B\right)a\cos\left(dx+\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(45*(A + B)*a*d*x + (24*B*a*cos(d*x + c)^4 + 30*(A + B)*a*cos(d*x + c
)^3 + 8*(5*A + 4*B)*a*cos(d*x + c)^2 + 45*(A + B)*a*cos(d*x + c) + 16*(5*A
+ 4*B)*a)*sin(d*x + c))/d
```

Sympy [A] time = 3.58734, size = 333, normalized size = 2.66

$$\begin{cases}\frac{3Aax\sin^{4}(c+dx)}{8}+\frac{3Aax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{3Aax\cos^{4}(c+dx)}{8}+\frac{3Aa\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{2Aa\sin^{3}(c+dx)}{3d}+\frac{5Aa\sin(c+dx)\cos^{3}(c+dx)}{8d}+ \\ x\left(A+B\cos\left(c\right)\right)\left(a\cos\left(c\right)+a\right)\cos^{3}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a*x*sin(c + d*x)**4/8 + 3*A*a*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*A*a*x*cos(c + d*x)**4/8 + 3*A*a*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 2*A*a*sin(c + d*x)**3/(3*d) + 5*A*a*sin(c + d*x)*cos(c + d*x)**3/(8*d)
+ A*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 8*B*a*sin(c
 + d*x)**5/(15*d) + 4*B*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*
a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c
) + a)*cos(c)**3, True))
```

Giac [A] time = 1.10095, size = 151, normalized size = 1.21

$$\frac{3}{8}(Aa+Ba)x+\frac{Ba\sin(5\,dx+5\,c)}{80\,d}+\frac{(Aa+Ba)\sin(4\,dx+4\,c)}{32\,d}+\frac{(4\,Aa+5\,Ba)\sin(3\,dx+3\,c)}{48\,d}+\frac{(Aa+Ba)\sin(2\,dx\cdot}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 3/8*(A*a + B*a)*x + 1/80*B*a*sin(5*d*x + 5*c)/d + 1/32*(A*a + B*a)*sin(4*d*
x + 4*c)/d + 1/48*(4*A*a + 5*B*a)*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(
2*d*x + 2*c)/d + 1/8*(6*A*a + 5*B*a)*sin(d*x + c)/d
```

3.2 $\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3B)+\frac{aB\sin(c+dx)}{4d}$$

```
[Out] (a*(4*A + 3*B)*x)/8 + (a*(A + B)*Sin[c + d*x])/d + (a*(4*A + 3*B)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(A +
 B)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.149015, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3B)+\frac{aB\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(4*A + 3*B)*x)/8 + (a*(A + B)*Sin[c + d*x])/d + (a*(4*A + 3*B)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(A +
 B)*Sin[c + d*x]^3)/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^2(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\right.$$
$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^2(c+dx)(a(4A+3B$$
$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}+(a(A+B))\int \cos^3(c+dx)\,dx$$
$$= \frac{a(4A+3B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aB\cos^3(c+dx)\sin(c$$
$$= \frac{1}{8}a(4A+3B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\cos(c}{8d}$$

Mathematica [A] time = 0.223432, size = 75, normalized size = 0.77

$$\frac{a\left(-32(A+B)\sin^3(c+dx)+96(A+B)\sin(c+dx)+24(A+B)\sin(2(c+dx))+48Ac+48Adx+3B\sin(4(c+dx))+\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(48*A*c + 36*B*c + 48*A*d*x + 36*B*d*x + 96*(A + B)*Sin[c + d*x] - 32*(A
 + B)*Sin[c + d*x]^3 + 24*(A + B)*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))
/(96*d)
```

Maple [A] time = 0.052, size = 107, normalized size = 1.1

$$\frac{1}{d}\left(aB\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{aA\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{aB\left(2+(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a
*A*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00651, size = 136, normalized size = 1.4

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-24\left(2dx+2c+\sin(2dx+2c)\right)Aa+32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba-3(}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 24*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 3*(12*d*x + 12*
c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a)/d
```

Fricas [A] time = 1.43322, size = 193, normalized size = 1.99

$$\frac{3(4A+3B)adx+\left(6Ba\cos(dx+c)^3+8(A+B)a\cos(dx+c)^2+3(4A+3B)a\cos(dx+c)+16(A+B)a\right)\sin(dx+c}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A + 3*B)*a*d*x + (6*B*a*cos(d*x + c)^3 + 8*(A + B)*a*cos(d*x + c
)^2 + 3*(4*A + 3*B)*a*cos(d*x + c) + 16*(A + B)*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.80525, size = 252, normalized size = 2.6

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{2Aa\sin^3(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^2(c+dx)}{d} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{3Bax\sin^4(c+dx)}{8} + \frac{3Bax\sin^2(c+d \\ x(A+B\cos(c))(a\cos(c)+a)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + 2*A*a*sin(c
+ d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d + A*a*sin(c + d*x)*cos
(c + d*x)/(2*d) + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 3*B*a*sin(c + d*x)**3*cos(c + d*
x)/(8*d) + 2*B*a*sin(c + d*x)**3/(3*d) + 5*B*a*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(
a*cos(c) + a)*cos(c)**2, True))
```

Giac [A] time = 1.14739, size = 120, normalized size = 1.24

$$\frac{1}{8}(4\,Aa + 3\,Ba)x + \frac{Ba\sin(4\,dx + 4\,c)}{32\,d} + \frac{(Aa + Ba)\sin(3\,dx + 3\,c)}{12\,d} + \frac{(Aa + Ba)\sin(2\,dx + 2\,c)}{4\,d} + \frac{3\,(Aa + Ba)\sin(dx +}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + 3*B*a)*x + 1/32*B*a*sin(4*d*x + 4*c)/d + 1/12*(A*a + B*a)*sin(
3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*x + 2*c)/d + 3/4*(A*a + B*a)*sin(d
*x + c)/d
```

3.3 $\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] (a*(A + B)*x)/2 + (a*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a*(A + B)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0783281, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2968, 3023, 2734}

$$\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(A + B)*x)/2 + (a*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a*(A + B)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx\right.$$
$$= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\int\cos(c+dx)(a(3A+2B)$$
$$= \frac{1}{2}a(A+B)x+\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+B)\cos(c+d}{2d}$$

Mathematica [A] time = 0.166033, size = 65, normalized size = 0.84

$$\frac{a(3(4A+3B)\sin(c+dx)+3(A+B)\sin(2(c+dx))+6Ac+6Adx+B\sin(3(c+dx))+6Bc+6Bdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(6*A*c + 6*B*c + 6*A*d*x + 6*B*d*x + 3*(4*A + 3*B)*Sin[c + d*x] + 3*(A +
 B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.053, size = 85, normalized size = 1.1

$$\frac{1}{d}\left(\frac{aB\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+aA\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c))
```

Maxima [A] time = 0.984232, size = 107, normalized size = 1.39

$$\frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa-4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba+3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba+12\,Aa\sin\left(\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 4*(sin(d*x + c)^3 - 3*sin(d*
x + c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 12*A*a*sin(d*x + c))
/d
```

Fricas [A] time = 1.37541, size = 146, normalized size = 1.9

$$\frac{3\,(A+B)adx+\left(2\,Ba\cos\left(dx+c\right)^{2}+3\,(A+B)a\cos\left(dx+c\right)+2\,(3\,A+2\,B)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/6*(3*(A + B)*a*d*x + (2*B*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) + 2
*(3*A + 2*B)*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.928453, size = 168, normalized size = 2.18

$$\begin{cases}\frac{Aax\sin^{2}(c+dx)}{2}+\frac{Aax\cos^{2}(c+dx)}{2}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{Aa\sin(c+dx)}{d}+\frac{Bax\sin^{2}(c+dx)}{2}+\frac{Bax\cos^{2}(c+dx)}{2}+\frac{2Ba\sin^{3}(c+dx)}{3d}+\frac{Ba\sin(c+\right.}{} \\ x\,(A+B\cos(c))\,(a\cos(c)+a)\cos(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(
c + d*x)**2/d + B*a*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*c
os(c))*(a*cos(c) + a)*cos(c), True))
```

Giac [A] time = 1.10459, size = 92, normalized size = 1.19

$$\frac{1}{2}\,(Aa + Ba)x + \frac{Ba\sin\left(3\,dx + 3\,c\right)}{12\,d} + \frac{(Aa + Ba)\sin\left(2\,dx + 2\,c\right)}{4\,d} + \frac{(4\,Aa + 3\,Ba)\sin\left(dx + c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(A*a + B*a)*x + 1/12*B*a*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*x
 + 2*c)/d + 1/4*(4*A*a + 3*B*a)*sin(d*x + c)/d
```

3.4 $\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx$

Optimal. Leaf size=47

$$\frac{a(A + B)\sin(c + dx)}{d} + \frac{1}{2}ax(2A + B) + \frac{aB\sin(c + dx)\cos(c + dx)}{2d}$$

```
[Out] (a*(2*A + B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```

Rubi [A] time = 0.020611, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$\frac{a(A + B)\sin(c + dx)}{d} + \frac{1}{2}ax(2A + B) + \frac{aB\sin(c + dx)\cos(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(2*A + B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx = \frac{1}{2}a(2A + B)x + \frac{a(A + B)\sin(c + dx)}{d} + \frac{aB\cos(c + dx)\sin(c + dx)}{2d}$$

Mathematica [A] time = 0.0979611, size = 44, normalized size = 0.94

$$\frac{a(4(A + B)\sin(c + dx) + 4Adx + B\sin(2(c + dx)) + 2Bc + 2Bdx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(2*B*c + 4*A*d*x + 2*B*d*x + 4*(A + B)*Sin[c + d*x] + B*Sin[2*(c + d*x)]
))/(4*d)
```

Maple [A] time = 0.046, size = 57, normalized size = 1.2

$$\frac{1}{d}\left(aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin(dx+c)+aB\sin(dx+c)+aA(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c)+a*B*sin(d
*x+c)+a*A*(d*x+c))
```

Maxima [A] time = 0.986873, size = 74, normalized size = 1.57

$$\frac{4(dx+c)Aa+(2dx+2c+\sin(2dx+2c))Ba+4Aa\sin(dx+c)+4Ba\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 4*A*a*sin(d*x
 + c) + 4*B*a*sin(d*x + c))/d
```

Fricas [A] time = 1.33308, size = 99, normalized size = 2.11

$$\frac{(2A+B)adx+(Ba\cos(dx+c)+2(A+B)a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*A + B)*a*d*x + (B*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.411819, size = 94, normalized size = 2.

$$\begin{cases} Aax + \frac{Aa\sin(c+dx)}{d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{Ba\sin(c+dx)\cos(c+dx)}{2d} + \frac{Ba\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(A + B\cos(c)\right)\left(a\cos(c) + a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*a*x*cos
(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/d,
Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a), True))
```

Giac [A] time = 1.16775, size = 61, normalized size = 1.3

$$\frac{1}{2}(2\,Aa + Ba)x + \frac{Ba\sin(2\,dx + 2\,c)}{4\,d} + \frac{(Aa + Ba)\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*a)*x + 1/4*B*a*sin(2*d*x + 2*c)/d + (A*a + B*a)*sin(d*x + c)
/d
```

3.5 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=32

$$ax(A+B)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\sin(c+dx)}{d}$$

```
[Out] a*(A + B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Rubi [A] time = 0.0913122, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2968, 3023, 2735, 3770}

$$ax(A+B)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*(A + B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx &= \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\sec(c+dx \\ &= \frac{aB\sin(c+dx)}{d}+\int (aA+a(A+B)\cos(c+dx))\sec(c+dx)\,dx \\ &= a(A+B)x+\frac{aB\sin(c+dx)}{d}+(aA)\int \sec(c+dx)\,dx \\ &= a(A+B)x+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0241613, size = 46, normalized size = 1.44

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{aB\sin(c)\cos(dx)}{d}+\frac{aB\cos(c)\sin(dx)}{d}+aBx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*A*x + a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Cos[d*x]*Sin[c])/d + (
a*B*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.076, size = 56, normalized size = 1.8

$$aAx+aBx+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Aac}{d}+\frac{aB\sin(dx+c)}{d}+\frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*A*x+a*B*x+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*c+a*B*sin(d*x+c)/d+1/
d*B*a*c
```

Maxima [A] time = 0.989942, size = 63, normalized size = 1.97

$$\frac{(dx+c)Aa+(dx+c)Ba+Aa\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+Ba\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
```

```
[Out] ((d*x + c)*A*a + (d*x + c)*B*a + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*a
*sin(d*x + c))/d
```

Fricas [A] time = 1.45065, size = 139, normalized size = 4.34

$$\frac{2\,(A+B)adx+Aa\log\left(\sin\left(dx+c\right)+1\right)-Aa\log\left(-\sin\left(dx+c\right)+1\right)+2\,Ba\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(A + B)*a*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c) +
1) + 2*B*a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^{2}\left(c+dx\right)\sec\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*sec(
c + d*x), x))
```

Giac [B] time = 1.23301, size = 107, normalized size = 3.34

$$\frac{Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Aa+Ba)(dx+c)+\frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (A*a + B*a)*(d*x + c) + 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.6 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=32

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+aBx$$

```
[Out] a*B*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rubi [A] time = 0.103491, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3021, 2735, 3770}

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+aBx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a*B*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\sec^2(c+a\\ &= \frac{aA\tan(c+dx)}{d}+\int (a(A+B)+aB\cos(c+dx))\sec(c+dx)\\ &= aBx+\frac{aA\tan(c+dx)}{d}+(a(A+B))\int \sec(c+dx)\,dx\\ &= aBx+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0190776, size = 43, normalized size = 1.34

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+aBx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
A*Tan[c + d*x])/d
```

Maple [A] time = 0.092, size = 65, normalized size = 2.

$$aBx+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aA\tan(dx+c)}{d}+\frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a*B*x+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+a*A*tan(d*x+c)/d+1/d*a*B*ln(sec(d*x
+c)+tan(d*x+c))+1/d*B*a*c
```

Maxima [B] time = 1.03773, size = 99, normalized size = 3.09

$$\frac{2\,(dx+c)Ba + Aa(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + A*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c))
/d
```

Fricas [B] time = 1.47257, size = 220, normalized size = 6.88

$$\frac{2\,Badx\cos(dx+c) + (A+B)a\cos(dx+c)\log(\sin(dx+c)+1) - (A+B)a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,Aa}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] 1/2*(2*B*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
- (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^2(c+dx)\,dx + \int A\cos(c+dx)\sec^2(c+dx)\,dx + \int B\cos(c+dx)\sec^2(c+dx)\,dx + \int B\cos^2(c+dx)\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**2
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**2, x))
```

Giac [B] time = 1.21314, size = 113, normalized size = 3.53

$$\frac{(dx+c)Ba + (Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] ((d*x + c)*B*a + (A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + B*
a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.7 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*(A + 2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c + d*x])/d + (a
*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.136926, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*(A + 2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c + d*x])/d + (a
*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\sec^3(c+
$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (2a(A+B)+a(A+2B)\,\mathrm{co}$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d} + (a(A+B))\int \sec^2(c+dx)\,dx +$$
$$= \frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$
$$= \frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(A+B)\tan(c+dx)}{d} + \frac{aA}{}$$

Mathematica [A] time = 0.0267637, size = 75, normalized size = 1.34

$$\frac{aA\tan(c+dx)}{d} + \frac{aA\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aA\tan(c+dx)\sec(c+dx)}{2d} + \frac{aB\tan(c+dx)}{d} + \frac{aB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.093, size = 86, normalized size = 1.5

$$\frac{aA\tan(dx+c)}{d}+\frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] a*A*tan(d*x+c)/d+1/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2*a*A*sec(d*x+c)*tan(d
*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B*tan(d*x+c)
```

Maxima [A] time = 0.990203, size = 128, normalized size = 2.29

$$-\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2Ba(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*A*a*tan(d*x + c) - 4*B*a*tan(d*x + c))/d
```

Fricas [A] time = 1.40884, size = 239, normalized size = 4.27

$$\frac{(A+2B)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(A+2B)a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2(A+B)a\cos(dx}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] 1/4*((A + 2*B)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*B)*a*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(A + B)*a*cos(d*x + c) + A*a)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24106, size = 167, normalized size = 2.98

$$\frac{(Aa + 2\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Aa + 2\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3 \cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2} \cdots\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1
/2*d*x + 1/2*c)^3 - 3*A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.8 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{a(2A+3B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] (a*(A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*B)*Tan[c + d*x])/(3*d
) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rubi [A] time = 0.153655, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a(2A+3B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*(A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*B)*Tan[c + d*x])/(3*d
) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\sec^4(c+
$$

$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (3a(A+B)+a(2A+3B)$$

$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+(a(A+B))\int \sec^3(c+dx)\,dx$$

$$= \frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)\tan(c+}{3d}$$

$$= \frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2A+3B)\tan(c+dx)}{3d}+$$

Mathematica [A] time = 0.303337, size = 56, normalized size = 0.65

$$\frac{a\left(3(A+B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(A+B)\sec(c+dx)+6(A+B)+2A\tan^2(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*(3*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B) + 3*(A + B)*S
ec[c + d*x] + 2*A*Tan[c + d*x]^2)))/(6*d)
```

Maple [A] time = 0.101, size = 128, normalized size = 1.5

$$\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aB\tan(dx+c)}{d}+\frac{2aA\tan(dx+c)}{3d}+\frac{aA(\sec(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B
*tan(d*x+c)+2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*a*
B*sec(d*x+c)*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02869, size = 171, normalized size = 1.99

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa-3Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3Ba\left(\frac{2\sin(dx}{\sin(dx+c}\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
```

```
c) - 1)) + 12*B*a*tan(d*x + c))/d
```

Fricas [A] time = 1.42323, size = 288, normalized size = 3.35

$$\frac{3\,(A+B)a\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,(A+B)a\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1)}+2\left(2\,(2\,A+3\,B)a\cos{(d}\right.}{12\,d\cos{(dx+c)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(A + B)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + B)*a*cos(d*
x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A + 3*B)*a*cos(d*x + c)^2 + 3*(A
+ B)*a*cos(d*x + c) + 2*A*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25015, size = 208, normalized size = 2.42

$$\frac{3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*tan(1/2*d*x + 1
/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

3.9 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4B)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (a*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(A + B)*Tan[c + d*x])/d +
(a*(3*A + 4*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(A + B)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.164655, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 3768, 3770}

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4B)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(A + B)*Tan[c + d*x])/d +
(a*(3*A + 4*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(A + B)*Tan[c + d*x]^3)/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\sec^5(c+\\ &= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (4a(A+B)+a(3A+4B\\ &= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+(a(A+B))\int \sec^4(c+dx)\,dx\\ &= \frac{a(3A+4B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec^3(c+dx)\tan(\\ &= \frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan(c+dx)}{d}+\end{aligned}$$

Mathematica [A] time = 0.363648, size = 77, normalized size = 0.73

$$\frac{a\left(3(3A+4B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(A+B)(\cos(2(c+dx))+2)\sec(c+dx)+6A\sec^2(c+\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*A + 12*B + 8*(A +
 B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*A*Sec[c + d*x]^2)*Tan[c + d*x])
)/(24*d)
```

Maple [A] time = 0.151, size = 171, normalized size = 1.6

$$\frac{2aA\tan(dx+c)}{3d}+\frac{aA(\sec(dx+c))^2\tan(dx+c)}{3d}+\frac{aB\sec(dx+c)\tan(dx+c)}{2d}+\frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*a*B*sec(d*x+c)
*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/4*a*A*sec(d*x+c)^3*tan(d*
x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+
2/3/d*a*B*tan(d*x+c)+1/3/d*a*B*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.982084, size = 220, normalized size = 2.08

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba-3Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin \cdots\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.40998, size = 339, normalized size = 3.2

$$\frac{3\,(3\,A+4\,B)a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,A+4\,B)a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(A+B)a\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(3*A + 4*B)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A + 4*B)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(A + B)*a*cos(d*x + c)^3 +
3*(3*A + 4*B)*a*cos(d*x + c)^2 + 8*(A + B)*a*cos(d*x + c) + 6*A*a)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27073, size = 254, normalized size = 2.4

$$\frac{3\,(3\,Aa+4\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa+4\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ba\tan\left(\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*B
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7 +
12*B*a*tan(1/2*d*x + 1/2*c)^7 - 49*A*a*tan(1/2*d*x + 1/2*c)^5 - 28*B*a*tan(
1/2*d*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*x + 1
/2*c)^3 - 39*A*a*tan(1/2*d*x + 1/2*c) - 36*B*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.10 $\int \cos^3(c + dx)(a + a\cos(c + dx))^2(A + B\cos(c + dx))\,dx$

Optimal. Leaf size=191

$$-\frac{a^2(9A+8B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c+dx)}{30d}+\frac{a^2(12A+11B)\sin(c-}{24d}$$

```
[Out] (a^2*(12*A + 11*B)*x)/16 + (a^2*(9*A + 8*B)*Sin[c + d*x])/(5*d) + (a^2*(12*
A + 11*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(12*A + 11*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(24*d) + (a^2*(6*A + 7*B)*Cos[c + d*x]^4*Sin[c + d*x])/(
30*d) + (B*Cos[c + d*x]^4*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(6*d) - (a
^2*(9*A + 8*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.310445, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2633, 2635, 8}

$$-\frac{a^2(9A+8B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c+dx)}{30d}+\frac{a^2(12A+11B)\sin(c-}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```


```
[Out] (a^2*(12*A + 11*B)*x)/16 + (a^2*(9*A + 8*B)*Sin[c + d*x])/(5*d) + (a^2*(12*
A + 11*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(12*A + 11*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(24*d) + (a^2*(6*A + 7*B)*Cos[c + d*x]^4*Sin[c + d*x])/(
30*d) + (B*Cos[c + d*x]^4*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(6*d) - (a
^2*(9*A + 8*B)*Sin[c + d*x]^3)/(15*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
```

```
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B\cos^4(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{6d}+\frac{1}{6}\int \cos$$

$$= \frac{B\cos^4(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{6d}+\frac{1}{6}\int \cos$$

$$= \frac{a^2(6A+7B)\cos^4(c+dx)\sin(c+dx)}{30d}+\frac{B\cos^4(c+dx)\left(a^2\right.}{}$$

$$= \frac{a^2(6A+7B)\cos^4(c+dx)\sin(c+dx)}{30d}+\frac{B\cos^4(c+dx)\left(a^2\right.}{}$$

$$= \frac{a^2(12A+11B)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^2(6A+7B)\cos}{}$$

$$= \frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(12A+11B)\cos(c+dx)\sin(}{16d}$$

$$= \frac{1}{16}a^2(12A+11B)x+\frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(12A+}{}$$

Mathematica [A] time = 0.590142, size = 134, normalized size = 0.7

$$\frac{a^2(120(11A+10B)\sin(c+dx)+15(32A+31B)\sin(2(c+dx))+180A\sin(3(c+dx))+60A\sin(4(c+dx))+12A\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(660*B*c + 720*A*d*x + 660*B*d*x + 120*(11*A + 10*B)*Sin[c + d*x] + 15
*(32*A + 31*B)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 200*B*Sin[3*(c +
 d*x)] + 60*A*Sin[4*(c + d*x)] + 75*B*Sin[4*(c + d*x)] + 12*A*Sin[5*(c + d*
x)] + 24*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.057, size = 217, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba^2\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+2
*a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/5*B*a
^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/3*a^2*A*(2+cos(d*x+c)^2
)*sin(d*x+c)+B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/
8*c))
```

Maxima [A] time = 1.05208, size = 292, normalized size = 1.53

$$\frac{64\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)Aa^2 - 320\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Aa^2 + 60\,(12\,dx + 12\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 60*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 + 128*(3*sin(d*x + c)^5 - 10*sin(d*x +
c)^3 + 15*sin(d*x + c))*B*a^2 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^2 + 30*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

Fricas [A] time = 1.42918, size = 327, normalized size = 1.71

$$\frac{15\,(12\,A + 11\,B)a^2dx + \left(40\,Ba^2\cos(dx+c)^5 + 48\,(A + 2\,B)a^2\cos(dx+c)^4 + 10\,(12\,A + 11\,B)a^2\cos(dx+c)^3 + 16\,(9\,A}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(12*A + 11*B)*a^2*d*x + (40*B*a^2*cos(d*x + c)^5 + 48*(A + 2*B)*a
^2*cos(d*x + c)^4 + 10*(12*A + 11*B)*a^2*cos(d*x + c)^3 + 16*(9*A + 8*B)*a^
2*cos(d*x + c)^2 + 15*(12*A + 11*B)*a^2*cos(d*x + c) + 32*(9*A + 8*B)*a^2)*
sin(d*x + c))/d
```

Sympy [A] time = 6.14203, size = 600, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**2*x*sin(c + d*x)**4/4 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/4 + 8*A*a**2*sin(c + d*x)**5/(15*d
) + 4*A*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + A*a**2*
sin(c + d*x)*cos(c + d*x)**2/d + 5*B*a**2*x*sin(c + d*x)**6/16 + 15*B*a**2*
x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B*a**2*x*sin(c + d*x)**4/8 + 15*B*
a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**2*x*sin(c + d*x)**2*cos(
c + d*x)**2/4 + 5*B*a**2*x*cos(c + d*x)**6/16 + 3*B*a**2*x*cos(c + d*x)**4/
8 + 5*B*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 16*B*a**2*sin(c + d*x)**
5/(15*d) + 5*B*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*B*a**2*sin(c
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*
d) + 11*B*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**2*sin(c + d*x)*
cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)),
 (x*(A + B*cos(c))*(a*cos(c) + a)**2*cos(c)**3, True))
```

Giac [A] time = 1.19672, size = 224, normalized size = 1.17

$$\frac{Ba^2 \sin(6\,dx+6\,c)}{192\,d} + \frac{1}{16}\left(12\,Aa^2 + 11\,Ba^2\right)x + \frac{\left(Aa^2 + 2\,Ba^2\right)\sin(5\,dx+5\,c)}{80\,d} + \frac{\left(4\,Aa^2 + 5\,Ba^2\right)\sin(4\,dx+4\,c)}{64\,d} + \frac{(9\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^2*sin(6*d*x + 6*c)/d + 1/16*(12*A*a^2 + 11*B*a^2)*x + 1/80*(A*a^2
 + 2*B*a^2)*sin(5*d*x + 5*c)/d + 1/64*(4*A*a^2 + 5*B*a^2)*sin(4*d*x + 4*c)/
d + 1/48*(9*A*a^2 + 10*B*a^2)*sin(3*d*x + 3*c)/d + 1/64*(32*A*a^2 + 31*B*a^
2)*sin(2*d*x + 2*c)/d + 1/8*(11*A*a^2 + 10*B*a^2)*sin(d*x + c)/d
```

3.11 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=160

$$-\frac{a^2(10A+9B)\sin^3(c+dx)}{15d}+\frac{a^2(10A+9B)\sin(c+dx)}{5d}+\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(7A+6B)\sin(c+\ldots}{8d}$$

```
[Out] (a^2*(7*A + 6*B)*x)/8 + (a^2*(10*A + 9*B)*Sin[c + d*x])/(5*d) + (a^2*(7*A +
 6*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*A + 6*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (B*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(10*A + 9*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.277828, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^2(10A+9B)\sin^3(c+dx)}{15d}+\frac{a^2(10A+9B)\sin(c+dx)}{5d}+\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(7A+6B)\sin(c+\ldots}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 6*B)*x)/8 + (a^2*(10*A + 9*B)*Sin[c + d*x])/(5*d) + (a^2*(7*A +
 6*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*A + 6*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (B*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(10*A + 9*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{1}{5}\int \cos^2$$
$$= \frac{B\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{1}{5}\int \cos^2$$
$$= \frac{a^2(5A+6B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\cos^3(c+dx)\left(a^2+\right.}{}$$
$$= \frac{a^2(5A+6B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\cos^3(c+dx)\left(a^2+\right.}{}$$
$$= \frac{a^2(7A+6B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2(5A+6B)\cos^3(c-}{20d}$$
$$= \frac{1}{8}a^2(7A+6B)x + \frac{a^2(10A+9B)\sin(c+dx)}{5d} + \frac{a^2(7A+6B)}{}$$

Mathematica [A] time = 0.39821, size = 108, normalized size = 0.68

$$\frac{a^2(60(12A+11B)\sin(c+dx)+240(A+B)\sin(2(c+dx))+80A\sin(3(c+dx))+15A\sin(4(c+dx))+420Adx+90B\,\mathrm{s}}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(360*B*c + 420*A*d*x + 360*B*d*x + 60*(12*A + 11*B)*Sin[c + d*x] + 240
*(A + B)*Sin[2*(c + d*x)] + 80*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] +
 15*A*Sin[4*(c + d*x)] + 30*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(48
0*d)
```

Maple [A] time = 0.076, size = 186, normalized size = 1.2

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ba^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5
*B*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2/3*a^2*A*(2+cos(d*x+
c)^2)*sin(d*x+c)+2*B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*
d*x+3/8*c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*B*a^2*(2+cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.975302, size = 240, normalized size = 1.5

$$-\frac{320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa^2-120(2dx+2c\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/480*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 120*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^2 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^2 + 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 30*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

Fricas [A] time = 1.39993, size = 271, normalized size = 1.69

$$\frac{15(7A+6B)a^2dx+\left(24Ba^2\cos(dx+c)^4+30(A+2B)a^2\cos(dx+c)^3+8(10A+9B)a^2\cos(dx+c)^2+15(7A+6\cdots}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(7*A + 6*B)*a^2*d*x + (24*B*a^2*cos(d*x + c)^4 + 30*(A + 2*B)*a^2
*cos(d*x + c)^3 + 8*(10*A + 9*B)*a^2*cos(d*x + c)^2 + 15*(7*A + 6*B)*a^2*co
s(d*x + c) + 16*(10*A + 9*B)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 3.41671, size = 459, normalized size = 2.87

$$\begin{cases} \frac{3Aa^2x\sin^4(c+dx)}{8} + \frac{3Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Aa^2x\sin^2(c+dx)}{2} + \frac{3Aa^2x\cos^4(c+dx)}{8} + \frac{Aa^2x\cos^2(c+dx)}{2} + \frac{3Aa^2\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{4A \\ x\,(A+B\cos(c))\,(a\cos(c)+a)^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/8 + A
*a**2*x*cos(c + d*x)**2/2 + 3*A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4
*A*a**2*sin(c + d*x)**3/(3*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 2*A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 3*B*a**2*x*sin(c + d*x)**4/4 + 3*B*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*B*a**2*x*cos(c + d*x)**4/4 + 8*B*a**2*sin(c + d*x)**5/(15*d
) + 4*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + B*a**2*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)
**2*cos(c)**2, True))
```

Giac [A] time = 1.22221, size = 185, normalized size = 1.16

$$\frac{Ba^2\sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\left(7\,Aa^2+6\,Ba^2\right)x + \frac{\left(Aa^2+2\,Ba^2\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{\left(8\,Aa^2+9\,Ba^2\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(Aa^2+B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*a^2*sin(5*d*x + 5*c)/d + 1/8*(7*A*a^2 + 6*B*a^2)*x + 1/32*(A*a^2 + 2
*B*a^2)*sin(4*d*x + 4*c)/d + 1/48*(8*A*a^2 + 9*B*a^2)*sin(3*d*x + 3*c)/d +
1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/8*(12*A*a^2 + 11*B*a^2)*sin(d*x
+ c)/d
```

3.12 $\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a^2(8A+7B)\sin(c+dx)}{6d}+\frac{a^2(8A+7B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(8A+7B)+\frac{(4A-B)\sin(c+dx)(a\cos(c+dx}{12d}$$

```
[Out] (a^2*(8*A + 7*B)*x)/8 + (a^2*(8*A + 7*B)*Sin[c + d*x])/(6*d) + (a^2*(8*A +
7*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A - B)*(a + a*Cos[c + d*x])^2*
Sin[c + d*x])/(12*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.176065, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2751, 2644}

$$\frac{a^2(8A+7B)\sin(c+dx)}{6d}+\frac{a^2(8A+7B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(8A+7B)+\frac{(4A-B)\sin(c+dx)(a\cos(c+dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(8*A + 7*B)*x)/8 + (a^2*(8*A + 7*B)*Sin[c + d*x])/(6*d) + (a^2*(8*A +
7*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A - B)*(a + a*Cos[c + d*x])^2*
Sin[c + d*x])/(12*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^2\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$
$$= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{\int (a+a\cos(c+dx))^2(3a$$
$$= \frac{(4A-B)(a+a\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{B(a+a\cos(c+d}{4a}$$
$$= \frac{1}{8}a^2(8A+7B)x + \frac{a^2(8A+7B)\sin(c+dx)}{6d} + \frac{a^2(8A+7B)\,\mathrm{co}}{}$$

Mathematica [A] time = 0.331178, size = 86, normalized size = 0.67

$$\frac{a^2(24(7A+6B)\sin(c+dx)+48(A+B)\sin(2(c+dx))+8A\sin(3(c+dx))+96Adx+16B\sin(3(c+dx))+3B\sin(4(c+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(84*B*c + 96*A*d*x + 84*B*d*x + 24*(7*A + 6*B)*Sin[c + d*x] + 48*(A +
B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 16*B*Sin[3*(c + d*x)] + 3*B*Si
n[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.051, size = 154, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^2A\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ba^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2A\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos
(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d
*x+1/2*c)+2/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*A*sin(d*x+c)+B*a^2*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.990614, size = 194, normalized size = 1.5

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-48\left(2dx+2c+\sin(2dx+2c)\right)Aa^2+64\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^2}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 48*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^2 + 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 3*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 24*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^2 - 96*A*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.43896, size = 213, normalized size = 1.65

$$\frac{3(8A+7B)a^2dx+\left(6Ba^2\cos(dx+c)^3+8(A+2B)a^2\cos(dx+c)^2+3(8A+7B)a^2\cos(dx+c)+8(5A+4B)a^2\right)s}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(8*A + 7*B)*a^2*d*x + (6*B*a^2*cos(d*x + c)^3 + 8*(A + 2*B)*a^2*cos
(d*x + c)^2 + 3*(8*A + 7*B)*a^2*cos(d*x + c) + 8*(5*A + 4*B)*a^2)*sin(d*x +
 c))/d
```

Sympy [A] time = 2.19487, size = 338, normalized size = 2.62

$$\begin{cases} Aa^2x\sin^2(c+dx) + Aa^2x\cos^2(c+dx) + \frac{2Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{d} + \frac{Aa^2\sin(c+dx)}{d} + \\ x(A+B\cos(c))(a\cos(c)+a)^2\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2 + A*a**2*x*cos(c + d*x)**2 + 2*A*a**2*s
in(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c
 + d*x)*cos(c + d*x)/d + A*a**2*sin(c + d*x)/d + 3*B*a**2*x*sin(c + d*x)**4
/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**2*x*sin(c + d*x)**
2/2 + 3*B*a**2*x*cos(c + d*x)**4/8 + B*a**2*x*cos(c + d*x)**2/2 + 3*B*a**2*
sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*B*a**2*sin(c + d*x)**3/(3*d) + 5*B*a
**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*B*a**2*sin(c + d*x)*cos(c + d*x)
**2/d + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*cos(c)
)*(a*cos(c) + a)**2*cos(c), True))
```

Giac [A] time = 1.18655, size = 149, normalized size = 1.16

$$\frac{Ba^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(8\,Aa^2 + 7\,Ba^2\right)x + \frac{\left(Aa^2 + 2\,Ba^2\right)\sin(3\,dx+3\,c)}{12\,d} + \frac{\left(Aa^2 + Ba^2\right)\sin(2\,dx+2\,c)}{2\,d} + \frac{\left(7\,Aa^2 + 6\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*B*a^2*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^2 + 7*B*a^2)*x + 1/12*(A*a^2 + 2
*B*a^2)*sin(3*d*x + 3*c)/d + 1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/4*(
7*A*a^2 + 6*B*a^2)*sin(d*x + c)/d
```

3.13 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=94

$$\frac{2a^2(3A+2B)\sin(c+dx)}{3d}+\frac{a^2(3A+2B)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(3A+2B)+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

```
[Out] (a^2*(3*A + 2*B)*x)/2 + (2*a^2*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a^2*(3*A
+ 2*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + a*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d)
```

Rubi [A] time = 0.0586307, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2751, 2644}

$$\frac{2a^2(3A+2B)\sin(c+dx)}{3d}+\frac{a^2(3A+2B)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(3A+2B)+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*x)/2 + (2*a^2*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a^2*(3*A
+ 2*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + a*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}(3A+2B)\int (a+a\cos(c+dx))^2\,dx$$
$$=\frac{1}{2}a^2(3A+2B)x+\frac{2a^2(3A+2B)\sin(c+dx)}{3d}+\frac{a^2(3A+2B)\cos(c+dx)\,\mathrm{s}}{6d}$$

Mathematica [A] time = 0.179524, size = 61, normalized size = 0.65

$$\frac{a^2(3(8A+7B)\sin(c+dx)+3(A+2B)\sin(2(c+dx))+18Adx+B\sin(3(c+dx))+12Bdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(18*A*d*x + 12*B*d*x + 3*(8*A + 7*B)*Sin[c + d*x] + 3*(A + 2*B)*Sin[2*
(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.046, size = 116, normalized size = 1.2

$$\frac{1}{d}\left(\frac{Ba^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,Ba^2\,(1/2\,\cos(dx+c)\sin(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*A*s
in(d*x+c)+B*a^2*sin(d*x+c)+a^2*A*(d*x+c))
```

Maxima [A] time = 1.00034, size = 149, normalized size = 1.59

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+12\,(dx+c)Aa^2-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2+6\,(2\,dx+2\,c+\sin(2\,dx+2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 12*(d*x + c)*A*a^2 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B
*a^2 + 24*A*a^2*sin(d*x + c) + 12*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.40573, size = 165, normalized size = 1.76

$$\frac{3\,(3\,A+2\,B)a^2dx+\left(2\,Ba^2\cos\left(dx+c\right)^2+3\,(A+2\,B)a^2\cos\left(dx+c\right)+2\,(6\,A+5\,B)a^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(3*A + 2*B)*a^2*d*x + (2*B*a^2*cos(d*x + c)^2 + 3*(A + 2*B)*a^2*cos(
d*x + c) + 2*(6*A + 5*B)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 0.792427, size = 199, normalized size = 2.12

$$\begin{cases} \frac{Aa^2x\sin^2\left(c+dx\right)}{2}+\frac{Aa^2x\cos^2\left(c+dx\right)}{2}+Aa^2x+\frac{Aa^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Aa^2\sin\left(c+dx\right)}{d}+Ba^2x\sin^2\left(c+dx\right)+Ba^2x\cos^2\left(c+dx\right) \\ x\left(A+B\cos\left(c\right)\right)\left(a\cos\left(c\right)+a\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*x + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*sin(c + d*x)/d + B*a
**2*x*sin(c + d*x)**2 + B*a**2*x*cos(c + d*x)**2 + 2*B*a**2*sin(c + d*x)**3
/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*sin(c + d*x)*cos(c
+ d*x)/d + B*a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) +
a)**2, True))
```

Giac [A] time = 1.19751, size = 115, normalized size = 1.22

$$\frac{Ba^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{1}{2}\left(3\,Aa^2+2\,Ba^2\right)x+\frac{\left(Aa^2+2\,Ba^2\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(8\,Aa^2+7\,Ba^2\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*sin(3*d*x + 3*c)/d + 1/2*(3*A*a^2 + 2*B*a^2)*x + 1/4*(A*a^2 + 2*
B*a^2)*sin(2*d*x + 2*c)/d + 1/4*(8*A*a^2 + 7*B*a^2)*sin(d*x + c)/d
```

3.14 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=82

$$\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(4A+3B)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

```
[Out] (a^2*(4*A + 3*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A + 3*B)*
Sin[c + d*x])/(2*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.192693, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2976, 2968, 3023, 2735, 3770}

$$\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(4A+3B)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*(4*A + 3*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A + 3*B)*
Sin[c + d*x])/(2*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d}+\frac{1}{2}\int (a+a\cos(c+dx))\\ &= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2a^2A+\left(2a^2A+\right.\right.\\ &= \frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx}{2d}\\ &= \frac{1}{2}a^2(4A+3B)x+\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B\left(a^2+a^2\cos(\right.}{}\\ &= \frac{1}{2}a^2(4A+3B)x+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(2A+3B)\,\mathrm{si}}{2d}\end{aligned}$$

Mathematica [A] time = 0.171159, size = 96, normalized size = 1.17

$$\frac{a^2\left(4(A+2B)\sin(c+dx)-4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+8\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*(8*A*d*x + 6*B*d*x - 4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(A + 2*B)*Sin[c + d*x] + B*S
in[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.081, size = 108, normalized size = 1.3

$$\frac{a^2 A \sin(dx+c)}{d} + \frac{Ba^2 \cos(dx+c)\sin(dx+c)}{2d} + \frac{3\,a^2 Bx}{2} + \frac{3\,Ba^2 c}{2\,d} + 2\,a^2 Ax + 2\,\frac{Aa^2 c}{d} + 2\,\frac{Ba^2 \sin(dx+c)}{d} + \frac{a^2 A \ln(s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*a^2*A*sin(d*x+c)+1/2/d*B*a^2*cos(d*x+c)*sin(d*x+c)+3/2*a^2*B*x+3/2/d*B*
a^2*c+2*a^2*A*x+2/d*A*a^2*c+2/d*B*a^2*sin(d*x+c)+1/d*a^2*A*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 0.970566, size = 127, normalized size = 1.55

$$\frac{8\,(dx+c)Aa^2 + (2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ba^2 + 4\,(dx+c)Ba^2 + 4\,Aa^2 \log(\sec(dx+c) + \tan(dx+c)) + 4\,Aa^2 \sin(}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(8*(d*x + c)*A*a^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 4*(d*x +
c)*B*a^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 4*A*a^2*sin(d*x + c)
+ 8*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.44351, size = 194, normalized size = 2.37

$$\frac{(4\,A + 3\,B)a^2 dx + Aa^2 \log(\sin(dx+c)+1) - Aa^2 \log(-\sin(dx+c)+1) + \left(Ba^2 \cos(dx+c) + 2\,(A+2\,B)a^2\right)\sin(dx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*((4*A + 3*B)*a^2*d*x + A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*x
 + c) + 1) + (B*a^2*cos(d*x + c) + 2*(A + 2*B)*a^2)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int A \sec(c+dx)\, dx + \int 2A\cos(c+dx)\sec(c+dx)\, dx + \int A\cos^2(c+dx)\sec(c+dx)\, dx + \int B\cos(c+dx)\sec(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*x)
*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x), x) + Integra
l(B*cos(c + d*x)**3*sec(c + d*x), x))
```

Giac [A] time = 1.22461, size = 196, normalized size = 2.39

$$\frac{2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \left(4\,Aa^2+3\,Ba^2\right)(dx+c) + \frac{2\left(2\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (4*A*a^2 + 3*B*a^2)*(d*x + c) + 2*(2*A*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 3*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^2*tan(1/2*d*x + 1/2*c)
+ 5*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.15 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

```
[Out] a^2*(A + 2*B)*x + (a^2*(2*A + B)*ArcTanh[Sin[c + d*x]])/d - (a^2*(A - B)*Si
n[c + d*x])/d + (A*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d
```

Rubi [A] time = 0.209642, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3023, 2735, 3770}

$$-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a^2*(A + 2*B)*x + (a^2*(2*A + B)*ArcTanh[Sin[c + d*x]])/d - (a^2*(A - B)*Si
n[c + d*x])/d + (A*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{A\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d}+\int (a+a\cos(c+dx))($$
$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d}+\int \left(a^2(2A+B)+\left(-a\right.\right.$$
$$= -\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx}{d}$$
$$= a^2(A+2B)x-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A\left(a^2+a^2\cos(c+d\right.}{d}$$
$$= a^2(A+2B)x+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2(A-B)}{}$$

Mathematica [A] time = 0.311407, size = 143, normalized size = 1.93

$$\frac{a^2\left(A\tan(c+dx)-2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+Ac+Adx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*(A*c + 2*B*c + A*d*x + 2*B*d*x - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + B*S
in[c + d*x] + A*Tan[c + d*x]))/d
```

Maple [A] time = 0.092, size = 107, normalized size = 1.5

$$a^2Ax + 2\,a^2Bx + \frac{a^2A\tan(dx+c)}{d} + 2\,\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Aa^2c}{d} + \frac{Ba^2\sin(dx+c)}{d} + \frac{Ba^2\ln(\sec(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a^2*A*x+2*a^2*B*x+a^2*A*tan(d*x+c)/d+2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/
d*A*a^2*c+1/d*B*a^2*sin(d*x+c)+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a^
2*c
```

Maxima [A] time = 1.00087, size = 142, normalized size = 1.92

$$\frac{2\,(dx+c)Aa^2 + 4\,(dx+c)Ba^2 + 2\,Aa^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Ba^2(\log(\sin(dx+c)+1) - \log(}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*(d*x + c)*A*a^2 + 4*(d*x + c)*B*a^2 + 2*A*a^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*B*a^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.42364, size = 278, normalized size = 3.76

$$\frac{2\,(A+2\,B)a^2dx\cos(dx+c)+(2\,A+B)a^2\cos(dx+c)\log(\sin(dx+c)+1)-(2\,A+B)a^2\cos(dx+c)\log(-\sin(dx+c}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*(A + 2*B)*a^2*d*x*cos(d*x + c) + (2*A + B)*a^2*cos(d*x + c)*log(sin(
d*x + c) + 1) - (2*A + B)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^
2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24293, size = 209, normalized size = 2.82

$$\frac{\left(Aa^2+2\,Ba^2\right)(dx+c)+\left(2\,Aa^2+Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Aa^2+Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Aa^2\,\mathrm{t}\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((A*a^2 + 2*B*a^2)*(d*x + c) + (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x +
 1/2*c) + B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.16 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+a^2Bx$$

```
[Out] a^2*B*x + (a^2*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(3*A + 2*B)*
Tan[c + d*x])/(2*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rubi [A] time = 0.219088, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3021, 2735, 3770}

$$\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+a^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] a^2*B*x + (a^2*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(3*A + 2*B)*
Tan[c + d*x])/(2*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a+a\\ &= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(a^2(3\\ &= \frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+}{2d}\\ &= a^2Bx+\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{A\left(a^2+a^2\cos(c+dx)\right)}{2d}\\ &= a^2Bx+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3A+2B)\tan}{2d}\end{aligned}$$

Mathematica [B] time = 1.1834, size = 277, normalized size = 3.15

$$\frac{1}{16}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(2A+B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{d\left(\sin\left(\frac{c}{2}\right)+\cos\right(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(4*B*x - (2*(3*A + 4*B)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(3*A + 4*B)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]])/d + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
4*(2*A + B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) - A/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*A +
B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))))/16
```

Maple [A] time = 0.106, size = 113, normalized size = 1.3

$$\frac{3\,a^2A\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+a^2Bx+\frac{Ba^2c}{d}+2\,\frac{a^2A\tan(dx+c)}{d}+2\,\frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 3/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*x+1/d*B*a^2*c+2*a^2*A*tan(d*x+c
)/d+2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2*a^2*A*sec(d*x+c)*tan(d*x+c)/d+1
/d*B*a^2*tan(d*x+c)
```

Maxima [A] time = 0.980099, size = 192, normalized size = 2.18

$$\frac{4\,(dx+c)Ba^2-Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+2\,Aa^2(\log(\sin(dx+c)+1)-\log(\mathrm{si}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 4*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 8*A*a^2*tan(d*x + c) + 4*B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.39228, size = 297, normalized size = 3.38

$$\frac{4\,Ba^2dx\cos(dx+c)^2 + (3\,A+4\,B)a^2\cos(dx+c)^2\log(\sin(dx+c)+1) - (3\,A+4\,B)a^2\cos(dx+c)^2\log(-\sin(dx+c)+\cdots}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*B*a^2*d*x*cos(d*x + c)^2 + (3*A + 4*B)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (3*A + 4*B)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(2*A + B)*a^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32333, size = 208, normalized size = 2.36

$$\frac{2\,(dx+c)Ba^2 + \left(3\,Aa^2+4\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(3\,Aa^2+4\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(3\,Aa^2\tan\left(\frac{1}{2}\cdots\right.\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^2*
tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*A*a^2*tan(1/2*d
*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)
/d
```

3.17 $\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$

Optimal. Leaf size=113

$$\frac{a^2(5A + 6B)\tan(c + dx)}{3d} + \frac{a^2(2A + 3B)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(4A + 3B)\tan(c + dx)\sec(c + dx)}{6d} + \frac{A\tan(c + dx)\,\mathrm{se}}{}$$

```
[Out] (a^2*(2*A + 3*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(5*A + 6*B)*Tan[c + d*
x])/(3*d) + (a^2*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a^2 + a
^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.270142, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^2(5A + 6B)\tan(c + dx)}{3d} + \frac{a^2(2A + 3B)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(4A + 3B)\tan(c + dx)\sec(c + dx)}{6d} + \frac{A\tan(c + dx)\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*(2*A + 3*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(5*A + 6*B)*Tan[c + d*
x])/(3*d) + (a^2*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a^2 + a
^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (a+$$

$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \left(a^2(\right.$$

$$= \frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d}+\frac{A\left(a^2+a^2\cos(c+d\right.}{}$$

$$= \frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d}+\frac{A\left(a^2+a^2\cos(c+d\right.}{}$$

$$= \frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4A+3B)\sec(c+dx}{6d}$$

$$= \frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(5A+6B)\tan(c+dx}{3d}$$

Mathematica [B] time = 5.64151, size = 451, normalized size = 3.99

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(5A+6B)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{4(5A+6B)\sin\left(\frac{dx}{2}\right)}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{(\mathrm{c}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*(2*A + 3*B)*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] + 6*(2*A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (2*A*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^3) + ((7*A + 3*B)*Cos[c/2] - (5*A + 3*B)*Sin[c/2])/((Cos[c/
2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(5*A + 6*B)*Si
n[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) +
 (2*A*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3) - ((7*A + 3*B)*Cos[c/2] + (5*A + 3*B)*Sin[c/2])/((Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*A + 6*B)*Sin[(d*x)/2]
)/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(48*d)
```

Maple [A] time = 0.099, size = 141, normalized size = 1.3

$$\frac{5\,a^2A\tan(dx+c)}{3\,d}+\frac{3\,Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{a^2A\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2A\ln(\sec(dx+c)+\tan(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 5/3*a^2*A*tan(d*x+c)/d+3/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*A*sec(d*x+
c)*tan(d*x+c)/d+1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a^2*tan(d*x+c)+1/
3*a^2*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*B*a^2*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.01928, size = 235, normalized size = 2.08

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{2}-6\,Aa^{2}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ba^{2}\left(\frac{2\,\mathrm{si}}{\sin(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 6*A*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
12*A*a^2*tan(d*x + c) + 24*B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.37897, size = 315, normalized size = 2.79

$$\frac{3\,(2\,A+3\,B)a^{2}\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A+3\,B)a^{2}\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(5\,A+6\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(2*A + 3*B)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A + 3*B
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(5*A + 6*B)*a^2*cos(d*x
+ c)^2 + 3*(2*A + B)*a^2*cos(d*x + c) + 2*A*a^2)*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23298, size = 240, normalized size = 2.12

$$\frac{3\left(2\,Aa^2+3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Aa^2+3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Ba^2}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(2*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a^2
+ 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 -
 24*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) + 15*B*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.18 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{a^2(4A+5B)\tan(c+dx)}{3d}+\frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+4B)\tan(c+dx)\sec^2(c+dx)}{12d}+\frac{a^2(7A+8B)\mathrm{t}}{}$$

```
[Out] (a^2*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*A + 5*B)*Tan[c + d*
x])/(3*d) + (a^2*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*A +
 4*B)*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.30362, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^2(4A+5B)\tan(c+dx)}{3d}+\frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+4B)\tan(c+dx)\sec^2(c+dx)}{12d}+\frac{a^2(7A+8B)\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```



```
[Out] (a^2*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*A + 5*B)*Tan[c + d*
x])/(3*d) + (a^2*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*A +
 4*B)*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (a$$
$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int \left(a\right.$$
$$= \frac{a^2(5A+4B)\sec^2(c+dx)\tan(c+dx)}{12d} + \frac{A\left(a^2+a^2\cos(c+\right.}{}$$
$$= \frac{a^2(5A+4B)\sec^2(c+dx)\tan(c+dx)}{12d} + \frac{A\left(a^2+a^2\cos(c+\right.}{}$$
$$= \frac{a^2(7A+8B)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2(5A+4B)\sec^2(c}{12}$$
$$= \frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(4A+5B)\tan(c+d}{3d}$$

Mathematica [A] time = 1.13358, size = 262, normalized size = 1.82

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(24(7A+8B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(7*A + 8*B
)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(4*A + 5*B)*Sin[c] + 3*(15*A + 8*
B)*Sin[d*x] + 45*A*Sin[2*c + d*x] + 24*B*Sin[2*c + d*x] + 128*A*Sin[c + 2*d
*x] + 136*B*Sin[c + 2*d*x] - 24*B*Sin[3*c + 2*d*x] + 21*A*Sin[2*c + 3*d*x]
+ 24*B*Sin[2*c + 3*d*x] + 21*A*Sin[4*c + 3*d*x] + 24*B*Sin[4*c + 3*d*x] + 3
2*A*Sin[3*c + 4*d*x] + 40*B*Sin[3*c + 4*d*x])))/(768*d)
```

Maple [A] time = 0.115, size = 187, normalized size = 1.3

$$\frac{7a^2A\sec(dx+c)\tan(dx+c)}{8d} + \frac{7a^2A\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{5Ba^2\tan(dx+c)}{3d} + \frac{4a^2A\tan(dx+c)}{3d} + \frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 7/8*a^2*A*sec(d*x+c)*tan(d*x+c)/d+7/8/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+5/3
/d*B*a^2*tan(d*x+c)+4/3*a^2*A*tan(d*x+c)/d+2/3*a^2*A*sec(d*x+c)^2*tan(d*x+c
)/d+1/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4
*a^2*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.02096, size = 311, normalized size = 2.16

$$\frac{32\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^2+16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^2-3\,Aa^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^2 - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^2*tan(d*x +
 c))/d
```

Fricas [A] time = 1.48132, size = 362, normalized size = 2.51

$$\frac{3\left(7\,A+8\,B\right)a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(7\,A+8\,B\right)a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(4\,A+5\,B\right.\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(7*A + 8*B)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(7*A + 8*B
)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(4*A + 5*B)*a^2*cos(d*x
+ c)^3 + 3*(7*A + 8*B)*a^2*cos(d*x + c)^2 + 8*(2*A + B)*a^2*cos(d*x + c) +
```

```
6*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24173, size = 286, normalized size = 1.99

$$\frac{3\left(7\,Aa^2+8\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Aa^2+8\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(7*A*a^2
 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*a^2*tan(1/2*d*x +
1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*A*a^2*tan(1/2*d*x + 1/2*c)^
5 - 88*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 136
*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*A*a^2*tan(1/2*d*x + 1/2*c) - 72*B*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.19 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=201

$$-\frac{a^3(19A+17B)\sin^3(c+dx)}{15d}+\frac{a^3(19A+17B)\sin(c+dx)}{5d}+\frac{a^3(22A+21B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{(3A+4B)\sin(c}{}$$

```
[Out] (a^3*(26*A + 23*B)*x)/16 + (a^3*(19*A + 17*B)*Sin[c + d*x])/(5*d) + (a^3*(2
6*A + 23*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(22*A + 21*B)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((3*A + 4*B)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(19*A + 17*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.432123, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^3(19A+17B)\sin^3(c+dx)}{15d}+\frac{a^3(19A+17B)\sin(c+dx)}{5d}+\frac{a^3(22A+21B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{(3A+4B)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```



```
[Out] (a^3*(26*A + 23*B)*x)/16 + (a^3*(19*A + 17*B)*Sin[c + d*x])/(5*d) + (a^3*(2
6*A + 23*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(22*A + 21*B)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((3*A + 4*B)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(19*A + 17*B)*Sin[c + d*x]^3)/(15*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
```

```
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^2($$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{(3A+4B)}{}$$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{(3A+4B)}{}$$

$$= \frac{a^3(22A+21B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aB\cos^3(c+dx)(a}{}$$

$$= \frac{a^3(22A+21B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aB\cos^3(c+dx)(a}{}$$

$$= \frac{a^3(26A+23B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^3(22A+21B)\cos}{4}$$

$$= \frac{1}{16}a^3(26A+23B)x + \frac{a^3(19A+17B)\sin(c+dx)}{5d} + \frac{a^3(26A}{}$$

Mathematica [A] time = 0.545822, size = 134, normalized size = 0.67

$$\frac{a^3(120(23A+21B)\sin(c+dx)+15(64A+63B)\sin(2(c+dx))+340A\sin(3(c+dx))+90A\sin(4(c+dx))+12A\sin(5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(1380*B*c + 1560*A*d*x + 1380*B*d*x + 120*(23*A + 21*B)*Sin[c + d*x] +
 15*(64*A + 63*B)*Sin[2*(c + d*x)] + 340*A*Sin[3*(c + d*x)] + 380*B*Sin[3*(
c + d*x)] + 90*A*Sin[4*(c + d*x)] + 135*B*Sin[4*(c + d*x)] + 12*A*Sin[5*(c
+ d*x)] + 36*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.073, size = 266, normalized size = 1.3

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a^3B\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+3
*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*a^3
*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*
c)+A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(2+cos(d*x+c)^
2)*sin(d*x+c))
```

Maxima [A] time = 0.998033, size = 354, normalized size = 1.76

$$\frac{64\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Aa^3-960\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^3+90\left(12\,dx+12\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^3 + 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3
- 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*B*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 90*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3)/d
```

Fricas [A] time = 1.45942, size = 332, normalized size = 1.65

$$\frac{15\,(26\,A+23\,B)a^3dx+\left(40\,Ba^3\cos\left(dx+c\right)^5+48\,(A+3\,B)a^3\cos\left(dx+c\right)^4+10\,(18\,A+23\,B)a^3\cos\left(dx+c\right)^3+16\,(19\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(26*A + 23*B)*a^3*d*x + (40*B*a^3*cos(d*x + c)^5 + 48*(A + 3*B)*a
^3*cos(d*x + c)^4 + 10*(18*A + 23*B)*a^3*cos(d*x + c)^3 + 16*(19*A + 17*B)*
a^3*cos(d*x + c)^2 + 15*(26*A + 23*B)*a^3*cos(d*x + c) + 32*(19*A + 17*B)*a
```

```
^3)*sin(d*x + c))/d
```

Sympy [A] time = 8.22508, size = 695, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((9*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 9*A*a**3*x*cos(c + d*x)**4/8 + A
*a**3*x*cos(c + d*x)**2/2 + 8*A*a**3*sin(c + d*x)**5/(15*d) + 4*A*a**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 2*A*a**3*sin(c + d*x)**3/d + A*a**3*sin(c + d*x)*cos(c + d*x)**4/d +
 15*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*A*a**3*sin(c + d*x)*cos(c
 + d*x)**2/d + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a**3*x*sin(c +
d*x)**6/16 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*B*a**3*x*si
n(c + d*x)**4/8 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*B*a**3
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*B*a**3*x*cos(c + d*x)**6/16 + 9*B*
a**3*x*cos(c + d*x)**4/8 + 5*B*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8
*B*a**3*sin(c + d*x)**5/(5*d) + 5*B*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6
*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*B*a**3*sin(c + d*x)**3
*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/(3*d) + 11*B*a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) + 3*B*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*B*
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c)**2, True))
```

Giac [A] time = 1.25192, size = 224, normalized size = 1.11

$$\frac{Ba^3 \sin(6\,dx + 6\,c)}{192\,d} + \frac{1}{16}\left(26\,Aa^3 + 23\,Ba^3\right)x + \frac{\left(Aa^3 + 3\,Ba^3\right)\sin(5\,dx + 5\,c)}{80\,d} + \frac{3\left(2\,Aa^3 + 3\,Ba^3\right)\sin(4\,dx + 4\,c)}{64\,d} + \frac{(17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^3*sin(6*d*x + 6*c)/d + 1/16*(26*A*a^3 + 23*B*a^3)*x + 1/80*(A*a^3
 + 3*B*a^3)*sin(5*d*x + 5*c)/d + 3/64*(2*A*a^3 + 3*B*a^3)*sin(4*d*x + 4*c)/
```

```
d + 1/48*(17*A*a^3 + 19*B*a^3)*sin(3*d*x + 3*c)/d + 1/64*(64*A*a^3 + 63*B*a
^3)*sin(2*d*x + 2*c)/d + 1/8*(23*A*a^3 + 21*B*a^3)*sin(d*x + c)/d
```

3.20 $\int \cos(c + dx)(a + a\cos(c + dx))^3(A + B\cos(c + dx))\,dx$

Optimal. Leaf size=154

$$-\frac{a^3(15A+13B)\sin^3(c+dx)}{60d}+\frac{a^3(15A+13B)\sin(c+dx)}{5d}+\frac{3a^3(15A+13B)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(15A+13B$$

```
[Out] (a^3*(15*A + 13*B)*x)/8 + (a^3*(15*A + 13*B)*Sin[c + d*x])/(5*d) + (3*a^3*(
15*A + 13*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*A - B)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a
*d) - (a^3*(15*A + 13*B)*Sin[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.229204, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(15A+13B)\sin^3(c+dx)}{60d}+\frac{a^3(15A+13B)\sin(c+dx)}{5d}+\frac{3a^3(15A+13B)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(15A+13B$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(15*A + 13*B)*x)/8 + (a^3*(15*A + 13*B)*Sin[c + d*x])/(5*d) + (3*a^3*(
15*A + 13*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*A - B)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a
*d) - (a^3*(15*A + 13*B)*Sin[c + d*x]^3)/(60*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^3\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$
$$= \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5ad} + \frac{\int (a+a\cos(c+dx))^3(4a$$
$$= \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B(a+a\cos(c+d}{5a}$$
$$= \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B(a+a\cos(c+d}{5a}$$
$$= \frac{1}{20}a^3(15A+13B)x + \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx}{20d}$$
$$= \frac{1}{20}a^3(15A+13B)x + \frac{3a^3(15A+13B)\sin(c+dx)}{20d} + \frac{3a^3(15A}{}$$
$$= \frac{1}{8}a^3(15A+13B)x + \frac{a^3(15A+13B)\sin(c+dx)}{5d} + \frac{3a^3(15A+}{}$$

Mathematica [A] time = 0.409067, size = 108, normalized size = 0.7

$$\frac{a^3(60(26A+23B)\sin(c+dx)+480(A+B)\sin(2(c+dx))+120A\sin(3(c+dx))+15A\sin(4(c+dx))+900Adx+170}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(780*B*c + 900*A*d*x + 780*B*d*x + 60*(26*A + 23*B)*Sin[c + d*x] + 480
*(A + B)*Sin[2*(c + d*x)] + 120*A*Sin[3*(c + d*x)] + 170*B*Sin[3*(c + d*x)]
 + 15*A*Sin[4*(c + d*x)] + 45*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(
480*d)
```

Maple [A] time = 0.052, size = 223, normalized size = 1.5

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^3B\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5
*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(2+cos(d*x+c)^2
)*sin(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+3*A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+A*a^3*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c))
```

Maxima [A] time = 0.975773, size = 288, normalized size = 1.87

$$-\frac{480\left(\sin(dx+c)^{3}-3\sin(dx+c)\right)Aa^{3}-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Aa^{3}-360\left(2\,dx+2\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/480*(480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^3 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^3 + 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 45*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 120*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a^3 - 480*A*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.4533, size = 278, normalized size = 1.81

$$\frac{15\,(15\,A+13\,B)a^{3}dx+\left(24\,Ba^{3}\cos(dx+c)^{4}+30\,(A+3\,B)a^{3}\cos(dx+c)^{3}+8\,(15\,A+19\,B)a^{3}\cos(dx+c)^{2}+15\,(15\,A\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(15*(15*A + 13*B)*a^3*d*x + (24*B*a^3*cos(d*x + c)^4 + 30*(A + 3*B)*a
^3*cos(d*x + c)^3 + 8*(15*A + 19*B)*a^3*cos(d*x + c)^2 + 15*(15*A + 13*B)*a
^3*cos(d*x + c) + 8*(45*A + 38*B)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 4.32785, size = 530, normalized size = 3.44

$$\begin{cases} \frac{3Aa^3x\sin^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Aa^3x\sin^2(c+dx)}{2} + \frac{3Aa^3x\cos^4(c+dx)}{8} + \frac{3Aa^3x\cos^2(c+dx)}{2} + \frac{3Aa^3\sin^3(c+dx)\cos(c+dx)}{8d} + \\ x(A+B\cos(c))(a\cos(c)+a)^3\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 +
 3*A*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 2*A*a**3*sin(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**3*sin(c + d*x)*cos(c +
d*x)/(2*d) + A*a**3*sin(c + d*x)/d + 9*B*a**3*x*sin(c + d*x)**4/8 + 9*B*a**
3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**3*x*sin(c + d*x)**2/2 + 9*B*a*
*3*x*cos(c + d*x)**4/8 + B*a**3*x*cos(c + d*x)**2/2 + 8*B*a**3*sin(c + d*x)
**5/(15*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**3*sin(
c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/d + B*a**3*sin(c
+ d*x)*cos(c + d*x)**4/d + 15*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3
*B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + B*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c), True))
```

Giac [A] time = 1.21036, size = 184, normalized size = 1.19

$$\frac{Ba^3\sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\left(15\,Aa^3 + 13\,Ba^3\right)x + \frac{\left(Aa^3 + 3\,Ba^3\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{\left(12\,Aa^3 + 17\,Ba^3\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(Aa^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/80*B*a^3*sin(5*d*x + 5*c)/d + 1/8*(15*A*a^3 + 13*B*a^3)*x + 1/32*(A*a^3 +
 3*B*a^3)*sin(4*d*x + 4*c)/d + 1/48*(12*A*a^3 + 17*B*a^3)*sin(3*d*x + 3*c)/
d + (A*a^3 + B*a^3)*sin(2*d*x + 2*c)/d + 1/8*(26*A*a^3 + 23*B*a^3)*sin(d*x
+ c)/d
```

3.21 $\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx))\,dx$

Optimal. Leaf size=116

$$-\frac{a^3(4A+3B)\sin^3(c+dx)}{12d}+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(4A+3B)+\frac{B\,\mathrm{s}}{}$$

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 + (a^3*(4*A + 3*B)*Sin[c + d*x])/d + (3*a^3*(4*A +
3*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(4*d) - (a^3*(4*A + 3*B)*Sin[c + d*x]^3)/(12*d)
```

Rubi [A] time = 0.0984662, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(4A+3B)\sin^3(c+dx)}{12d}+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(4A+3B)+\frac{B\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 + (a^3*(4*A + 3*B)*Sin[c + d*x])/d + (3*a^3*(4*A +
3*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(4*d) - (a^3*(4*A + 3*B)*Sin[c + d*x]^3)/(12*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx &= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}(4A+3B)\int (a+a\cos(c+dx))^3\,dx\\ &= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}(4A+3B)\int \left(a^3+3a^3\cos(c+dx)\right.\\ &= \frac{1}{4}a^3(4A+3B)x+\frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\left(a^3(4A+3B)\right)\int\\ &= \frac{1}{4}a^3(4A+3B)x+\frac{3a^3(4A+3B)\sin(c+dx)}{4d}+\frac{3a^3(4A+3B)\cos(c+dx)}{8d}\\ &= \frac{5}{8}a^3(4A+3B)x+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\cos(c+dx)\,\mathrm{s}}{8d}\end{aligned}$$

Mathematica [A] time = 0.303321, size = 86, normalized size = 0.74

$$\frac{a^3(24(15A+13B)\sin(c+dx)+24(3A+4B)\sin(2(c+dx))+8A\sin(3(c+dx))+240Adx+24B\sin(3(c+dx))+3B\sin}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(240*A*d*x + 180*B*d*x + 24*(15*A + 13*B)*Sin[c + d*x] + 24*(3*A + 4*B
)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 24*B*Sin[3*(c + d*x)] + 3*B*Sin
[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.047, size = 176, normalized size = 1.5

$$\frac{1}{d}\left(a^3B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^3B\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a^
3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*B*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c)+3*A*a^3*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c))
```

Maxima [A] time = 1.02594, size = 225, normalized size = 1.94

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-72\left(2dx+2c+\sin(2dx+2c)\right)Aa^3-96(dx+c)Aa^3+96\left(\sin(dx+c)^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 72*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 96*(sin(d*x + c)^3 - 3*sin(d*x +
c))*B*a^3 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3
 - 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 288*A*a^3*sin(d*x + c) - 96*
B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.33383, size = 216, normalized size = 1.86

$$\frac{15(4A+3B)a^3dx+\left(6Ba^3\cos(dx+c)^3+8(A+3B)a^3\cos(dx+c)^2+9(4A+5B)a^3\cos(dx+c)+8(11A+9B)a^3\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(15*(4*A + 3*B)*a^3*d*x + (6*B*a^3*cos(d*x + c)^3 + 8*(A + 3*B)*a^3*co
s(d*x + c)^2 + 9*(4*A + 5*B)*a^3*cos(d*x + c) + 8*(11*A + 9*B)*a^3)*sin(d*x
 + c))/d
```

Sympy [A] time = 1.6103, size = 371, normalized size = 3.2

$$\begin{cases} \frac{3Aa^3x\sin^2(c+dx)}{2} + \frac{3Aa^3x\cos^2(c+dx)}{2} + Aa^3x + \frac{2Aa^3\sin^3(c+dx)}{3d} + \frac{Aa^3\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Aa^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{3Aa^3\sin(c+dx)}{d} \\ x(A+B\cos(c))(a\cos(c)+a)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**2/2 + A*
a**3*x + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*A*a**3*sin(c + d*x)/d +
 3*B*a**3*x*sin(c + d*x)**4/8 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*B*a**3*x*sin(c + d*x)**2/2 + 3*B*a**3*x*cos(c + d*x)**4/8 + 3*B*a**3*
x*cos(c + d*x)**2/2 + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**
3*sin(c + d*x)**3/d + 5*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*B*a**
3*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d)
 + B*a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3, T
rue))
```

Giac [A] time = 1.1839, size = 151, normalized size = 1.3

$$\frac{Ba^3\sin(4\,dx+4\,c)}{32\,d} + \frac{5}{8}\left(4\,Aa^3+3\,Ba^3\right)x + \frac{\left(Aa^3+3\,Ba^3\right)\sin(3\,dx+3\,c)}{12\,d} + \frac{\left(3\,Aa^3+4\,Ba^3\right)\sin(2\,dx+2\,c)}{4\,d} + \frac{\left(15\,Aa^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*B*a^3*sin(4*d*x + 4*c)/d + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3
*B*a^3)*sin(3*d*x + 3*c)/d + 1/4*(3*A*a^3 + 4*B*a^3)*sin(2*d*x + 2*c)/d + 1
```

```
/4*(15*A*a^3 + 13*B*a^3)*sin(d*x + c)/d
```

3.22 $\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec(c + dx)\, dx$

Optimal. Leaf size=111

$$\frac{5a^3(A+B)\sin(c+dx)}{2d} + \frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d} + \frac{1}{2}a^3x(7A+5B) + \frac{a^3A\tanh^{-1}(\sin(c+dx))}{d} + \cdots$$

```
[Out] (a^3*(7*A + 5*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Si
n[c + d*x])/(2*d) + (a*B*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*A
 + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.30435, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2976, 2968, 3023, 2735, 3770}

$$\frac{5a^3(A+B)\sin(c+dx)}{2d} + \frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d} + \frac{1}{2}a^3x(7A+5B) + \frac{a^3A\tanh^{-1}(\sin(c+dx))}{d} + \cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] (a^3*(7*A + 5*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Si
n[c + d*x])/(2*d) + (a*B*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*A
 + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+a\cos(c+dx)$$

$$= \frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3A+5B)\left(a^3+a^3\cos\right.}{6a}$$

$$= \frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3A+5B)\left(a^3+a^3\cos\right.}{6a}$$

$$= \frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{aB(a+a\cos(c+dx))^2\sin(c+dx}{3d}$$

$$= \frac{1}{2}a^3(7A+5B)x+\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{aB(a+a\cos(c}{}$$

$$= \frac{1}{2}a^3(7A+5B)x+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3(A+B)\sin}{2d}$$

Mathematica [A] time = 0.24898, size = 113, normalized size = 1.02

$$\frac{a^3\left(9(4A+5B)\sin(c+dx)+3(A+3B)\sin(2(c+dx))-12A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12A\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^3*(42*A*d*x + 30*B*d*x - 12*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(4*A + 5*B)*Sin[c + d*x]
 + 3*(A + 3*B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.111, size = 153, normalized size = 1.4

$$\frac{Aa^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{7\,Aa^3x}{2}+\frac{7\,Aa^3c}{2d}+\frac{B\sin(dx+c)(\cos(dx+c))^2a^3}{3d}+\frac{11\,a^3B\sin(dx+c)}{3d}+3\,\frac{Aa^3\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/2/d*A*a^3*cos(d*x+c)*sin(d*x+c)+7/2*A*a^3*x+7/2/d*A*a^3*c+1/3/d*B*sin(d*x
+c)*cos(d*x+c)^2*a^3+11/3*a^3*B*sin(d*x+c)/d+3*a^3*A*sin(d*x+c)/d+3/2/d*a^3
*B*cos(d*x+c)*sin(d*x+c)+5/2*a^3*B*x+5/2/d*a^3*B*c+1/d*A*a^3*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 1.00012, size = 190, normalized size = 1.71

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3+36\,(dx+c)Aa^3-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^3+9\,(2\,dx+2\,c+\sin(2\,dx+2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 36*(d*x + c)*A*a^3 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*B
```

```
*a^3 + 12*(d*x + c)*B*a^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*
A*a^3*sin(d*x + c) + 36*B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.49719, size = 254, normalized size = 2.29

$$\frac{3\,(7\,A+5\,B)a^3dx+3\,Aa^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Aa^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Ba^3\cos\left(dx+c\right)^2+3\,(A+3\,B)a^3\,c\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(7*A + 5*B)*a^3*d*x + 3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-s
in(d*x + c) + 1) + (2*B*a^3*cos(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c) +
 2*(9*A + 11*B)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28875, size = 243, normalized size = 2.19

$$\frac{6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(7\,Aa^3+5\,Ba^3\right)(dx+c)+\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(7*A*a^3 + 5*B*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^3*tan(1/2*d*x + 1/
2*c)^3 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^3*tan(1/2*d*x + 1/2*c) +
33*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.23 $\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx))\sec^2(c + dx)\,dx$

Optimal. Leaf size=110

$$\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d} - \frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d} + \frac{1}{2}a^3x(6A+7B) + \frac{5a^3B\sin(c+dx)}{2d} + \cdots$$

```
[Out] (a^3*(6*A + 7*B)*x)/2 + (a^3*(3*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*B*
Sin[c + d*x])/(2*d) - ((2*A - B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(2*
d) + (a*A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.308204, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2975, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d} - \frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d} + \frac{1}{2}a^3x(6A+7B) + \frac{5a^3B\sin(c+dx)}{2d} + \cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(6*A + 7*B)*x)/2 + (a^3*(3*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*B*
Sin[c + d*x])/(2*d) - ((2*A - B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(2*
d) + (a*A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+a\cos(c+dx)$$

$$= -\frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{aA(a+a\,\mathrm{co}}{}$$

$$= -\frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{aA(a+a\,\mathrm{co}}{}$$

$$= \frac{5a^3B\sin(c+dx)}{2d} - \frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+}{2d}$$

$$= \frac{1}{2}a^3(6A+7B)x + \frac{5a^3B\sin(c+dx)}{2d} - \frac{(2A-B)\left(a^3+a^3\,\mathrm{co}\right.}{2}$$

$$= \frac{1}{2}a^3(6A+7B)x + \frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3B}{}$$

Mathematica [B] time = 1.69006, size = 272, normalized size = 2.47

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(A+3B)\sin(c)\cos(dx)}{d} + \frac{4(A+3B)\cos(c)\sin(dx)}{d} - \frac{4(3A+B)\log\left(\cos\left(\frac{1}{2}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*(6*A + 7*B)*x - (4*(3*A + B
)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (4*(3*A + B)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 3*B)*Cos[d*x]*Sin[c])/d + (B*Cos[2*
d*x]*Sin[2*c])/d + (4*(A + 3*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin[2*d*x]
)/d + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))))/32
```

Maple [A] time = 0.1, size = 145, normalized size = 1.3

$$\frac{Aa^3\sin(dx+c)}{d} + \frac{a^3B\cos(dx+c)\sin(dx+c)}{2d} + \frac{7a^3Bx}{2} + \frac{7a^3Bc}{2d} + 3Aa^3x + 3\frac{Aa^3c}{d} + 3\frac{a^3B\sin(dx+c)}{d} + 3\frac{Aa^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a^3*A*sin(d*x+c)/d+1/2/d*a^3*B*cos(d*x+c)*sin(d*x+c)+7/2*a^3*B*x+7/2/d*a^3*
B*c+3*A*a^3*x+3/d*A*a^3*c+3*a^3*B*sin(d*x+c)/d+3/d*A*a^3*ln(sec(d*x+c)+tan(
d*x+c))+1/d*A*a^3*tan(d*x+c)+1/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02568, size = 189, normalized size = 1.72

$$\frac{12\,(dx+c)Aa^3+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3+12\,(dx+c)Ba^3+6\,Aa^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*A*a^3 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 12*(d*x
 + c)*B*a^3 + 6*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*B*
a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x + c)
 + 12*B*a^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.41616, size = 323, normalized size = 2.94

$$\frac{(6\,A+7\,B)a^3dx\cos(dx+c)+(3\,A+B)a^3\cos(dx+c)\log(\sin(dx+c)+1)-(3\,A+B)a^3\cos(dx+c)\log(-\sin(dx+c}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*((6*A + 7*B)*a^3*d*x*cos(d*x + c) + (3*A + B)*a^3*cos(d*x + c)*log(sin(
d*x + c) + 1) - (3*A + B)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (B*a^3*
cos(d*x + c)^2 + 2*(A + 3*B)*a^3*cos(d*x + c) + 2*A*a^3)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32327, size = 259, normalized size = 2.35

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\left(6\,Aa^3+7\,Ba^3\right)(dx+c)-2\left(3\,Aa^3+Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\left(3\,Aa^3+Ba^3\right)\log\left(\left|\tan\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*A*a^3
+ 7*B*a^3)*(d*x + c) - 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan(1/2*d*x
 + 1/2*c) + 7*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.24 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+a^3x(A+3B)-\frac{5a^3A\sin(c+dx)}{2d}+\frac{aA}{}$$

```
[Out] a^3*(A + 3*B)*x + (a^3*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*A*
Sin[c + d*x])/(2*d) + ((2*A + B)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/d +
 (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.337916, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3023, 2735, 3770}

$$\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+a^3x(A+3B)-\frac{5a^3A\sin(c+dx)}{2d}+\frac{aA}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```


```
[Out] a^3*(A + 3*B)*x + (a^3*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*A*
Sin[c + d*x])/(2*d) + ((2*A + B)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/d +
 (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a+$$

$$= \frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d}+\frac{aA(a+a\cos$$

$$= \frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d}+\frac{aA(a+a\cos$$

$$= -\frac{5a^3A\sin(c+dx)}{2d}+\frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c$$

$$= a^3(A+3B)x-\frac{5a^3A\sin(c+dx)}{2d}+\frac{(2A+B)\left(a^3+a^3\cos(c\right.}{d}$$

$$= a^3(A+3B)x+\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^3A\,\mathrm{s}}{}$$

Mathematica [A] time = 1.83876, size = 208, normalized size = 1.82

$$a^3\left(4(3A+B)\tan(c+dx)+\frac{A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{A}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}-14A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^3*(4*A*c + 12*B*c + 4*A*d*x + 12*B*d*x - 14*A*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 14*A*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*B*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + A/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 4*B*Sin[c + d*x] + 4*(3*A + B)*Tan[c + d*x]))/(4*d)
```

Maple [A] time = 0.111, size = 144, normalized size = 1.3

$$Aa^3x+\frac{Aa^3c}{d}+\frac{a^3B\sin(dx+c)}{d}+\frac{7\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+3\,a^3Bx+3\,\frac{Ba^3c}{d}+3\,\frac{Aa^3\tan(dx+c)}{d}+3\,\frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] A*a^3*x+1/d*A*a^3*c+a^3*B*sin(d*x+c)/d+7/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+3*a^3*B*x+3/d*a^3*B*c+3/d*A*a^3*tan(d*x+c)+3/d*a^3*B*ln(sec(d*x+c)+tan(d*
x+c))+1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/d*a^3*B*tan(d*x+c)
```

Maxima [A] time = 1.01014, size = 223, normalized size = 1.96

$$4\,(dx+c)Aa^3+12\,(dx+c)Ba^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6\,Aa^3(\log(\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^3 + 12*(d*x + c)*B*a^3 - A*a^3*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c)
 + 4*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.47112, size = 342, normalized size = 3.

$$\frac{4\,(A+3\,B)a^3dx\cos\left(dx+c\right)^2+(7\,A+6\,B)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(7\,A+6\,B)a^3\cos\left(dx+c\right)^2\log\left(-\sin\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*(A + 3*B)*a^3*d*x*cos(d*x + c)^2 + (7*A + 6*B)*a^3*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (7*A + 6*B)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
 + 2*(2*B*a^3*cos(d*x + c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26239, size = 259, normalized size = 2.27

$$\frac{\frac{4\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(Aa^3+3\,Ba^3\right)(dx+c)+\left(7\,Aa^3+6\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(7\,Aa^3+6\,Ba^3\right)\log\left(\left|\tan\left(\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(A*a^3 +
 3*B*a^3)*(d*x + c) + (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*tan
(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^3*tan(1/2*d*x
+ 1/2*c) - 2*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.25 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=125

$$\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+3B)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}$$

```
[Out] a^3*B*x + (a^3*(5*A + 7*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(A + B)*Ta
n[c + d*x])/(2*d) + ((5*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]*Tan[
c + d*x])/(6*d) + (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/
(3*d)
```

Rubi [A] time = 0.336653, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3021, 2735, 3770}

$$\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+3B)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^3*B*x + (a^3*(5*A + 7*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(A + B)*Ta
n[c + d*x])/(2*d) + ((5*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]*Tan[
c + d*x])/(6*d) + (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/
(3*d)
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int(a$$

$$=\frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{6d}+$$

$$=\frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{6d}+$$

$$=\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)}{6d}$$

$$=a^3Bx+\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{(5A+3B)\left(a^3+a^3\cos(\right.}{}$$

$$=a^3Bx+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^3(A+B)\tan}{2d}$$

Mathematica [B] time = 6.34731, size = 786, normalized size = 6.29

$$\frac{\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(11A\sin\left(\frac{dx}{2}\right)+9B\sin\left(\frac{dx}{2}\right)\right)}{24d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(11A\sin\left(\frac{dx}{2}\right)+9B\right.}{24d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (B*x*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/8 + ((-5*A - 7*B)*(a + a*
Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x
)/2]^6)/(16*d) + ((5*A + 7*B)*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(16*d) + (A*(a + a*Cos[c + d*x
])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (
d*x)/2]^6*(10*A*Cos[c/2] + 3*B*Cos[c/2] - 8*A*Sin[c/2] - 3*B*Sin[c/2]))/(96
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a
 + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*A*Sin[(d*x)/2] + 9*B*Sin[(d*x
)/2]))/(24*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)) + (A*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Co
s[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*C
os[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-10*A*Cos[c/2] - 3*B*Cos[c/2] - 8*A*Si
n[c/2] - 3*B*Sin[c/2]))/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*A*
Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d
```

```
*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.152, size = 158, normalized size = 1.3

$$\frac{5\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+a^3Bx+\frac{Ba^3c}{d}+\frac{11\,Aa^3\tan(dx+c)}{3\,d}+\frac{7\,a^3B\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{3\,\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*x+1/d*a^3*B*c+11/3/d*A*a^3*tan(
d*x+c)+7/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a^3*sec(d*x+c)*tan(d*x
+c)+3/d*a^3*B*tan(d*x+c)+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^3*B*se
c(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.00377, size = 286, normalized size = 2.29

$$\frac{4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3+12\,(dx+c)Ba^3-9\,Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*a^3 - 9*A*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 3*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 18*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 36*A*a^3*tan(d*x + c) + 36*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.42093, size = 356, normalized size = 2.85

$$\frac{12\,Ba^3dx\cos(dx+c)^3+3\,(5\,A+7\,B)a^3\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(5\,A+7\,B)a^3\cos(dx+c)^3\log(-\sin(dx\ldots}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*B*a^3*d*x*cos(d*x + c)^3 + 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(2*(11*A + 9*B)*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + 2*A*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32854, size = 255, normalized size = 2.04

$$\frac{6\,(dx+c)Ba^3+3\left(5\,Aa^3+7\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(5\,Aa^3+7\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,Aa^3\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*B*a^3 + 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A
*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d
*x + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3
)/d
```

3.26 $\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx$

Optimal. Leaf size=154

$$\frac{a^3(9A+11B)\tan(c+dx)}{3d}+\frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27A+28B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3A+2B)\,\mathrm{t}}{}$$

```
[Out] (5*a^3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(9*A + 11*B)*Tan[c +
 d*x])/(3*d) + (a^3*(27*A + 28*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A
 + 2*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*
(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.418148, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^3(9A+11B)\tan(c+dx)}{3d}+\frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27A+28B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3A+2B)\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```


```
[Out] (5*a^3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(9*A + 11*B)*Tan[c +
 d*x])/(3*d) + (a^3*(27*A + 28*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A
 + 2*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*
(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (a+$$

$$= \frac{(3A+2B)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{6d}+$$

$$= \frac{(3A+2B)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{6d}+$$

$$= \frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(3A+2B)\left(a^3+a\right.}{}$$

$$= \frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(3A+2B)\left(a^3+a\right.}{}$$

$$= \frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27A+28B)\sec(c+}{24d}$$

$$= \frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(9A+11B)\tan(c+}{3d}$$

Mathematica [A] time = 1.23551, size = 273, normalized size = 1.77

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(120(3A+4B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^4*(120*(3*A + 4*
B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(9*A + 11*B)*Sin[c] + (69*A + 36
*B)*Sin[d*x] + 69*A*Sin[2*c + d*x] + 36*B*Sin[2*c + d*x] + 264*A*Sin[c + 2*
d*x] + 280*B*Sin[c + 2*d*x] - 24*A*Sin[3*c + 2*d*x] - 72*B*Sin[3*c + 2*d*x]
 + 45*A*Sin[2*c + 3*d*x] + 36*B*Sin[2*c + 3*d*x] + 45*A*Sin[4*c + 3*d*x] +
36*B*Sin[4*c + 3*d*x] + 72*A*Sin[3*c + 4*d*x] + 88*B*Sin[3*c + 4*d*x])))/(1
536*d)
```

Maple [A] time = 0.121, size = 188, normalized size = 1.2

$$3\,\frac{Aa^3\tan(dx+c)}{d}+\frac{5\,a^3B\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{15\,Aa^3\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{15\,Aa^3\ln(\sec(dx+c)+}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 3/d*A*a^3*tan(d*x+c)+5/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+15/8/d*A*a^3*sec
(d*x+c)*tan(d*x+c)+15/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3/d*a^3*B*tan(
d*x+c)+1/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a^3*B*sec(d*x+c)*tan(d*x+c)+
1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+1/3/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.00729, size = 363, normalized size = 2.36

$$\frac{48\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{3}-3\,Aa^{3}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\mathrm{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^3 - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*a^3*(log(sin(
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A*a^3*tan(d*x + c) + 144*B*a^3*
tan(d*x + c))/d
```

Fricas [A] time = 1.40401, size = 366, normalized size = 2.38

$$\frac{15\,(3\,A+4\,B)a^{3}\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-15\,(3\,A+4\,B)a^{3}\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(9\,A+\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(15*(3*A + 4*B)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(3*A + 4
*B)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(9*A + 11*B)*a^3*cos(d
*x + c)^3 + 9*(5*A + 4*B)*a^3*cos(d*x + c)^2 + 8*(3*A + B)*a^3*cos(d*x + c)
 + 6*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33884, size = 286, normalized size = 1.86

$$\frac{15\left(3\,Aa^{3}+4\,Ba^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(3\,Aa^{3}+4\,Ba^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+60\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(15*(3*A*a^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*a
^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a^3*tan(1/2*d*x
+ 1/2*c)^7 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 165*A*a^3*tan(1/2*d*x + 1/2*
c)^5 - 220*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*A*a^3*tan(1/2*d*x + 1/2*c)^3
+ 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 147*A*a^3*tan(1/2*d*x + 1/2*c) - 132*B
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.27 $\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx))\sec^6(c + dx)\,dx$

Optimal. Leaf size=185

$$\frac{a^3(38A+45B)\tan(c+dx)}{15d}+\frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(43A+45B)\tan(c+dx)\sec^2(c+dx)}{60d}+\frac{a^3(13A}{}$$

```
[Out] (a^3*(13*A + 15*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 45*B)*Tan[c
+ d*x])/(15*d) + (a^3*(13*A + 15*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3
*(43*A + 45*B)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((7*A + 5*B)*(a^3 + a^
3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + a*Cos[c + d
*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.44741, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^3(38A+45B)\tan(c+dx)}{15d}+\frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(43A+45B)\tan(c+dx)\sec^2(c+dx)}{60d}+\frac{a^3(13A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```


```
[Out] (a^3*(13*A + 15*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 45*B)*Tan[c
+ d*x])/(15*d) + (a^3*(13*A + 15*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3
*(43*A + 45*B)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((7*A + 5*B)*(a^3 + a^
3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + a*Cos[c + d
*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int(a$$
$$= \frac{(7A+5B)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{20d}+$$
$$= \frac{(7A+5B)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{20d}+$$
$$= \frac{a^3(43A+45B)\sec^2(c+dx)\tan(c+dx)}{60d}+\frac{(7A+5B)\left(a^3\right.}{}$$
$$= \frac{a^3(43A+45B)\sec^2(c+dx)\tan(c+dx)}{60d}+\frac{(7A+5B)\left(a^3\right.}{}$$
$$= \frac{a^3(13A+15B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3(43A+45B)\,\mathrm{s}}{}$$
$$= \frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(38A+45B)\tan(}{15d}$$

Mathematica [A] time = 1.41489, size = 294, normalized size = 1.59

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(13A+15B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(13*A + 1
5*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(29*A + 30*B)*Sin[d*x] - 240*(3
*A + 5*B)*Sin[2*c + d*x] + 750*A*Sin[c + 2*d*x] + 570*B*Sin[c + 2*d*x] + 75
0*A*Sin[3*c + 2*d*x] + 570*B*Sin[3*c + 2*d*x] + 1520*A*Sin[2*c + 3*d*x] + 1
680*B*Sin[2*c + 3*d*x] - 120*B*Sin[4*c + 3*d*x] + 195*A*Sin[3*c + 4*d*x] +
225*B*Sin[3*c + 4*d*x] + 195*A*Sin[5*c + 4*d*x] + 225*B*Sin[5*c + 4*d*x] +
304*A*Sin[4*c + 5*d*x] + 360*B*Sin[4*c + 5*d*x])))/(15360*d)
```

Maple [A] time = 0.114, size = 234, normalized size = 1.3

$$\frac{13\,Aa^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{13\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+3\,\frac{a^3B\tan(dx+c)}{d}+\frac{38\,Aa^3\tan(dx+c)}{15d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 13/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3
/d*a^3*B*tan(d*x+c)+38/15/d*A*a^3*tan(d*x+c)+19/15/d*A*a^3*tan(d*x+c)*sec(d
*x+c)^2+15/8/d*a^3*B*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*B*ln(sec(d*x+c)+tan(d
*x+c))+3/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+1/d*a^3*B*tan(d*x+c)*sec(d*x+c)^
2+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+1/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [A] time = 1.02777, size = 455, normalized size = 2.46

$$\frac{16\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^3+240\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3+240\left(\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^3 - 45*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 60*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 180*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.47847, size = 431, normalized size = 2.33

$$\frac{15\,(13\,A+15\,B)a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(13\,A+15\,B)a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(8\,(38\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(13*A + 15*B)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(13*A
 + 15*B)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(38*A + 45*B)*a^3
*cos(d*x + c)^4 + 15*(13*A + 15*B)*a^3*cos(d*x + c)^3 + 8*(19*A + 15*B)*a^3
*cos(d*x + c)^2 + 30*(3*A + B)*a^3*cos(d*x + c) + 24*A*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27076, size = 332, normalized size = 1.79

$$15\left(13\,Aa^3+15\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(13\,Aa^3+15\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,Aa^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(13
*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(195*A*a^3*tan(1/
2*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*A*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1050*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x +
1/2*c)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1830*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c)
```

```
+ 735*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.28 $\int \cos^2(c + dx)(a + a\cos(c + dx))^4(A + B\cos(c + dx))\,dx$

Optimal. Leaf size=241

$$-\frac{a^4(252A+227B)\sin^3(c+dx)}{105d}+\frac{a^4(252A+227B)\sin(c+dx)}{35d}+\frac{a^4(301A+276B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{(7A+}{}$$

```
[Out] (a^4*(49*A + 44*B)*x)/16 + (a^4*(252*A + 227*B)*Sin[c + d*x])/(35*d) + (a^4
*(49*A + 44*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(301*A + 276*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x]
)^3*Sin[c + d*x])/(7*d) + ((7*A + 10*B)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d
*x])^2*Sin[c + d*x])/(42*d) + (7*(A + B)*Cos[c + d*x]^3*(a^4 + a^4*Cos[c +
d*x])*Sin[c + d*x])/(15*d) - (a^4*(252*A + 227*B)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.592742, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^4(252A+227B)\sin^3(c+dx)}{105d}+\frac{a^4(252A+227B)\sin(c+dx)}{35d}+\frac{a^4(301A+276B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{(7A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```


```
[Out] (a^4*(49*A + 44*B)*x)/16 + (a^4*(252*A + 227*B)*Sin[c + d*x])/(35*d) + (a^4
*(49*A + 44*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(301*A + 276*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x]
)^3*Sin[c + d*x])/(7*d) + ((7*A + 10*B)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d
*x])^2*Sin[c + d*x])/(42*d) + (7*(A + B)*Cos[c + d*x]^3*(a^4 + a^4*Cos[c +
d*x])*Sin[c + d*x])/(15*d) - (a^4*(252*A + 227*B)*Sin[c + d*x]^3)/(105*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
```

```
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{1}{7}\int \cos$$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{(7A+1}{}$$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{(7A+1}{}$$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{(7A+1}{}$$

$$= \frac{a^4(301A+276B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{aB\cos^3(c+d}{}$$

$$= \frac{a^4(301A+276B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{aB\cos^3(c+d}{}$$

$$= \frac{a^4(49A+44B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^4(301A+276B}{}$$

$$= \frac{1}{16}a^4(49A+44B)x + \frac{a^4(252A+227B)\sin(c+dx)}{35d} + \frac{a^4(4}{}$$

Mathematica [A] time = 0.793152, size = 156, normalized size = 0.65

$$\frac{a^4(105(352A+323B)\sin(c+dx)+105(127A+124B)\sin(2(c+dx))+5040A\sin(3(c+dx))+1575A\sin(4(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(18480*B*c + 20580*A*d*x + 18480*B*d*x + 105*(352*A + 323*B)*Sin[c + d
*x] + 105*(127*A + 124*B)*Sin[2*(c + d*x)] + 5040*A*Sin[3*(c + d*x)] + 5495
*B*Sin[3*(c + d*x)] + 1575*A*Sin[4*(c + d*x)] + 2100*B*Sin[4*(c + d*x)] + 3
36*A*Sin[5*(c + d*x)] + 651*B*Sin[5*(c + d*x)] + 35*A*Sin[6*(c + d*x)] + 14
0*B*Sin[6*(c + d*x)] + 15*B*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.067, size = 358, normalized size = 1.5

$$\frac{1}{d}\left(Aa^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5\,(\cos(dx+c))^3}{4}+\frac{15\,\cos(dx+c)}{8}\right)+\frac{5\,dx}{16}+\frac{5\,c}{16}\right)+\frac{a^4B\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/7*a^4*B*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+
c)^2)*sin(d*x+c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
*a^4*B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+6*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+6/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/3*A*a^4*(
2+cos(d*x+c)^2)*sin(d*x+c)+4*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c)+A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^
4*B*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.01811, size = 481, normalized size = 2.

$$\frac{1792\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-35\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6720*(1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4
 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2
*d*x + 2*c))*A*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 1260*(1
2*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 1680*(2*d*x +
 2*c + sin(2*d*x + 2*c))*A*a^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5
+ 35*sin(d*x + c)^3 - 35*sin(d*x + c))*B*a^4 + 2688*(3*sin(d*x + c)^5 - 10*
sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 140*(4*sin(2*d*x + 2*c)^3 - 60*d*
x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^4 - 2240*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*B*a^4 + 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*
sin(2*d*x + 2*c))*B*a^4)/d
```

Fricas [A] time = 1.4319, size = 396, normalized size = 1.64

$$\frac{105(49A+44B)a^4dx+\left(240Ba^4\cos(dx+c)^6+280(A+4B)a^4\cos(dx+c)^5+192(7A+12B)a^4\cos(dx+c)^4+70(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(49*A + 44*B)*a^4*d*x + (240*B*a^4*cos(d*x + c)^6 + 280*(A + 4*
B)*a^4*cos(d*x + c)^5 + 192*(7*A + 12*B)*a^4*cos(d*x + c)^4 + 70*(41*A + 44
*B)*a^4*cos(d*x + c)^3 + 16*(252*A + 227*B)*a^4*cos(d*x + c)^2 + 105*(49*A
+ 44*B)*a^4*cos(d*x + c) + 32*(252*A + 227*B)*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 11.627, size = 960, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a**4*x*sin(c + d*x)**6/16 + 15*A*a**4*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 9*A*a**4*x*sin(c + d*x)**4/4 + 15*A*a**4*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 9*A*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + A*a**4
*x*sin(c + d*x)**2/2 + 5*A*a**4*x*cos(c + d*x)**6/16 + 9*A*a**4*x*cos(c + d
*x)**4/4 + A*a**4*x*cos(c + d*x)**2/2 + 5*A*a**4*sin(c + d*x)**5*cos(c + d*
x)/(16*d) + 32*A*a**4*sin(c + d*x)**5/(15*d) + 5*A*a**4*sin(c + d*x)**3*cos
(c + d*x)**3/(6*d) + 16*A*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*
a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) +
11*A*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*A*a**4*sin(c + d*x)*cos(c
 + d*x)**4/d + 15*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*A*a**4*sin(
c + d*x)*cos(c + d*x)**2/d + A*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a
**4*x*sin(c + d*x)**6/4 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 3
*B*a**4*x*sin(c + d*x)**4/2 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/4
 + 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 5*B*a**4*x*cos(c + d*x)**6/
4 + 3*B*a**4*x*cos(c + d*x)**4/2 + 16*B*a**4*sin(c + d*x)**7/(35*d) + 8*B*a
**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 5*B*a**4*sin(c + d*x)**5*cos(c
+ d*x)/(4*d) + 16*B*a**4*sin(c + d*x)**5/(5*d) + 2*B*a**4*sin(c + d*x)**3*c
os(c + d*x)**4/d + 10*B*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 8*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a**4*sin(c + d*x)**3*cos(c + d*x
)/(2*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*sin(c + d*x)*cos(c + d*x)
**6/d + 11*B*a**4*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 6*B*a**4*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + B*a**4*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)
```

```
**4*cos(c)**2, True))
```

Giac [A] time = 1.31712, size = 261, normalized size = 1.08

$$\frac{Ba^4 \sin(7\,dx + 7\,c)}{448\,d} + \frac{1}{16}\left(49\,Aa^4 + 44\,Ba^4\right)x + \frac{\left(Aa^4 + 4\,Ba^4\right)\sin(6\,dx + 6\,c)}{192\,d} + \frac{\left(16\,Aa^4 + 31\,Ba^4\right)\sin(5\,dx + 5\,c)}{320\,d} + \frac{5\,(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*a^4*sin(7*d*x + 7*c)/d + 1/16*(49*A*a^4 + 44*B*a^4)*x + 1/192*(A*a^
4 + 4*B*a^4)*sin(6*d*x + 6*c)/d + 1/320*(16*A*a^4 + 31*B*a^4)*sin(5*d*x + 5
*c)/d + 5/64*(3*A*a^4 + 4*B*a^4)*sin(4*d*x + 4*c)/d + 1/192*(144*A*a^4 + 15
7*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(127*A*a^4 + 124*B*a^4)*sin(2*d*x + 2*c)
/d + 1/64*(352*A*a^4 + 323*B*a^4)*sin(d*x + c)/d
```

3.29 $\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=185

$$-\frac{2a^4(8A+7B)\sin^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\sin(c+dx)}{5d}+\frac{a^4(8A+7B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{27a^4(8A+7B)\sin\cdots}{8\cdots}$$

```
[Out] (7*a^4*(8*A + 7*B)*x)/16 + (4*a^4*(8*A + 7*B)*Sin[c + d*x])/(5*d) + (27*a^4
*(8*A + 7*B)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(8*A + 7*B)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + ((6*A - B)*(a + a*Cos[c + d*x])^4*Sin[c + d*x]
)/(30*d) + (B*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(6*a*d) - (2*a^4*(8*A +
7*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.303942, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{2a^4(8A+7B)\sin^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\sin(c+dx)}{5d}+\frac{a^4(8A+7B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{27a^4(8A+7B)\sin\cdots}{8\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```


```
[Out] (7*a^4*(8*A + 7*B)*x)/16 + (4*a^4*(8*A + 7*B)*Sin[c + d*x])/(5*d) + (27*a^4
*(8*A + 7*B)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(8*A + 7*B)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + ((6*A - B)*(a + a*Cos[c + d*x])^4*Sin[c + d*x]
)/(30*d) + (B*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(6*a*d) - (2*a^4*(8*A +
7*B)*Sin[c + d*x]^3)/(15*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^4\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$
$$= \frac{B(a+a\cos(c+dx))^5\sin(c+dx)}{6ad} + \frac{\int (a+a\cos(c+dx))^4(5}{}$$
$$= \frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{B(a+a\cos(c+}{6}$$
$$= \frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{B(a+a\cos(c+}{6}$$
$$= \frac{1}{10}a^4(8A+7B)x + \frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx}{30d}$$
$$= \frac{1}{10}a^4(8A+7B)x + \frac{2a^4(8A+7B)\sin(c+dx)}{5d} + \frac{3a^4(8A+7}{}$$
$$= \frac{2}{5}a^4(8A+7B)x + \frac{4a^4(8A+7B)\sin(c+dx)}{5d} + \frac{27a^4(8A+7}{}$$
$$= \frac{7}{16}a^4(8A+7B)x + \frac{4a^4(8A+7B)\sin(c+dx)}{5d} + \frac{27a^4(8A+}{}$$

Mathematica [A] time = 0.490434, size = 134, normalized size = 0.72

$$\frac{a^4(120(49A+44B)\sin(c+dx)+15(128A+127B)\sin(2(c+dx))+580A\sin(3(c+dx))+120A\sin(4(c+dx))+12A\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(2940*B*c + 3360*A*d*x + 2940*B*d*x + 120*(49*A + 44*B)*Sin[c + d*x] +
 15*(128*A + 127*B)*Sin[2*(c + d*x)] + 580*A*Sin[3*(c + d*x)] + 720*B*Sin[3
*(c + d*x)] + 120*A*Sin[4*(c + d*x)] + 225*B*Sin[4*(c + d*x)] + 12*A*Sin[5*
(c + d*x)] + 48*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.059, size = 306, normalized size = 1.7

$$\frac{1}{d}\left(\frac{Aa^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a^4B\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^4*B*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+4
*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*a^4
*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*A*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+6*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/
8*c)+4*A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4/3*a^4*B*(2+cos(d*x
+c)^2)*sin(d*x+c)+A*a^4*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.01919, size = 401, normalized size = 2.17

$$64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-1920\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4+120\,(12\,dx+12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 120*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 960*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*A*a^4 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^
4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2
*d*x + 2*c))*B*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 180*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 240*(2*d*x + 2
*c + sin(2*d*x + 2*c))*B*a^4 + 960*A*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.46569, size = 329, normalized size = 1.78

$$\frac{105\,(8\,A+7\,B)a^4dx+\left(40\,Ba^4\cos(dx+c)^5+48\,(A+4\,B)a^4\cos(dx+c)^4+10\,(24\,A+41\,B)a^4\cos(dx+c)^3+32\,(17\,A}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(105*(8*A + 7*B)*a^4*d*x + (40*B*a^4*cos(d*x + c)^5 + 48*(A + 4*B)*a^
4*cos(d*x + c)^4 + 10*(24*A + 41*B)*a^4*cos(d*x + c)^3 + 32*(17*A + 18*B)*a
^4*cos(d*x + c)^2 + 105*(8*A + 7*B)*a^4*cos(d*x + c) + 16*(83*A + 72*B)*a^4
)*sin(d*x + c))/d
```

Sympy [A] time = 6.80233, size = 765, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/2 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2 + 2*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/2 + 2*A
*a**4*x*cos(c + d*x)**2 + 8*A*a**4*sin(c + d*x)**5/(15*d) + 4*A*a**4*sin(c
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 4*A*a**4*sin(c + d*x)**3/d + A*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5
*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 6*A*a**4*sin(c + d*x)*cos(c +
d*x)**2/d + 2*A*a**4*sin(c + d*x)*cos(c + d*x)/d + A*a**4*sin(c + d*x)/d +
5*B*a**4*x*sin(c + d*x)**6/16 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2
/16 + 9*B*a**4*x*sin(c + d*x)**4/4 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**4/16 + 9*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + B*a**4*x*sin(c +
d*x)**2/2 + 5*B*a**4*x*cos(c + d*x)**6/16 + 9*B*a**4*x*cos(c + d*x)**4/4 +
B*a**4*x*cos(c + d*x)**2/2 + 5*B*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) +
 32*B*a**4*sin(c + d*x)**5/(15*d) + 5*B*a**4*sin(c + d*x)**3*cos(c + d*x)**
3/(6*d) + 16*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**4*sin(c
+ d*x)**3*cos(c + d*x)/(4*d) + 8*B*a**4*sin(c + d*x)**3/(3*d) + 11*B*a**4*s
in(c + d*x)*cos(c + d*x)**5/(16*d) + 4*B*a**4*sin(c + d*x)*cos(c + d*x)**4/
d + 15*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*B*a**4*sin(c + d*x)*co
s(c + d*x)**2/d + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A
+ B*cos(c))*(a*cos(c) + a)**4*cos(c), True))
```

Giac [A] time = 1.23271, size = 224, normalized size = 1.21

$$\frac{Ba^4 \sin(6\,dx + 6\,c)}{192\,d} + \frac{7}{16}\left(8\,Aa^4 + 7\,Ba^4\right)x + \frac{\left(Aa^4 + 4\,Ba^4\right)\sin(5\,dx + 5\,c)}{80\,d} + \frac{\left(8\,Aa^4 + 15\,Ba^4\right)\sin(4\,dx + 4\,c)}{64\,d} + \frac{(29\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*B*a^4*sin(6*d*x + 6*c)/d + 7/16*(8*A*a^4 + 7*B*a^4)*x + 1/80*(A*a^4 +
 4*B*a^4)*sin(5*d*x + 5*c)/d + 1/64*(8*A*a^4 + 15*B*a^4)*sin(4*d*x + 4*c)/d
 + 1/48*(29*A*a^4 + 36*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(128*A*a^4 + 127*B*
a^4)*sin(2*d*x + 2*c)/d + 1/8*(49*A*a^4 + 44*B*a^4)*sin(d*x + c)/d
```

3.30 $\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\, dx$

Optimal. Leaf size=150

$$-\frac{4a^4(5A+4B)\sin^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{a^4(5A+4B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(5A+4B)\sin}{4}$$

```
[Out] (7*a^4*(5*A + 4*B)*x)/8 + (8*a^4*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (27*a^4*
(5*A + 4*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(5*A + 4*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) -
 (4*a^4*(5*A + 4*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.139084, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{4a^4(5A+4B)\sin^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{a^4(5A+4B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(5A+4B)\sin}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```


```
[Out] (7*a^4*(5*A + 4*B)*x)/8 + (8*a^4*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (27*a^4*
(5*A + 4*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(5*A + 4*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) -
 (4*a^4*(5*A + 4*B)*Sin[c + d*x]^3)/(15*d)
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx &= \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}(5A+4B)\int (a+a\cos(c+dx))^4\,dx \\
&= \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}(5A+4B)\int \left(a^4+4a^4\cos(c+dx)\right. \\
&= \frac{1}{5}a^4(5A+4B)x+\frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}\left(a^4(5A+4B)\right)\int \\
&= \frac{1}{5}a^4(5A+4B)x+\frac{4a^4(5A+4B)\sin(c+dx)}{5d}+\frac{3a^4(5A+4B)\cos(c+dx)}{5d} \\
&= \frac{4}{5}a^4(5A+4B)x+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{27a^4(5A+4B)\cos(c+dx}{40d} \\
&= \frac{7}{8}a^4(5A+4B)x+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{27a^4(5A+4B)\cos(c+dx}{40d}
\end{aligned}$$

Mathematica [A] time = 0.335601, size = 108, normalized size = 0.72

$$\frac{a^4(420(8A+7B)\sin(c+dx)+120(7A+8B)\sin(2(c+dx))+160A\sin(3(c+dx))+15A\sin(4(c+dx))+2100Adx+29}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(2100*A*d*x + 1680*B*d*x + 420*(8*A + 7*B)*Sin[c + d*x] + 120*(7*A + 8
*B)*Sin[2*(c + d*x)] + 160*A*Sin[3*(c + d*x)] + 290*B*Sin[3*(c + d*x)] + 15
*A*Sin[4*(c + d*x)] + 60*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(480*d
)
```

Maple [A] time = 0.051, size = 248, normalized size = 1.7

$$\frac{1}{d}\left(\frac{a^4 B\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+Aa^4\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\,\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^4*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a^4*B*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+2*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+6*A*a^4*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c)+4*a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*A
*a^4*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [A] time = 1.0301, size = 319, normalized size = 2.13

$$-\frac{640\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^4-15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^4-720\,(2\,dx+2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/480*(640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^4 - 480*(d*x + c)*A*a^4 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c
)^3 + 15*sin(d*x + c))*B*a^4 + 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4
- 60*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 480*(2
```

```
*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 1920*A*a^4*sin(d*x + c) - 480*B*a^4*
sin(d*x + c))/d
```

Fricas [A] time = 1.31348, size = 279, normalized size = 1.86

$$\frac{105\,(5\,A+4\,B)a^4dx+\Big(24\,Ba^4\cos{(dx+c)}^4+30\,(A+4\,B)a^4\cos{(dx+c)}^3+16\,(10\,A+17\,B)a^4\cos{(dx+c)}^2+15\,(27\,A}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(105*(5*A + 4*B)*a^4*d*x + (24*B*a^4*cos(d*x + c)^4 + 30*(A + 4*B)*a^
4*cos(d*x + c)^3 + 16*(10*A + 17*B)*a^4*cos(d*x + c)^2 + 15*(27*A + 28*B)*a
^4*cos(d*x + c) + 8*(100*A + 83*B)*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 4.46919, size = 544, normalized size = 3.63

$$\begin{cases}\frac{3Aa^4x\sin^4{(c+dx)}}{8}+\frac{3Aa^4x\sin^2{(c+dx)}\cos^2{(c+dx)}}{4}+3Aa^4x\sin^2{(c+dx)}+\frac{3Aa^4x\cos^4{(c+dx)}}{8}+3Aa^4x\cos^2{(c+dx)}+Aa^4x+\frac{3Aa^4\sin}{} \\ x\,(A+B\cos{(c)})\,(a\cos{(c)}+a)^4\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/8 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/8 + 3
*A*a**4*x*cos(c + d*x)**2 + A*a**4*x + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) + 5*A*a**4*sin(c + d*x)*cos(c + d*
x)**3/(8*d) + 4*A*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**4*sin(c + d*
x)*cos(c + d*x)/d + 4*A*a**4*sin(c + d*x)/d + 3*B*a**4*x*sin(c + d*x)**4/2
+ 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 2*B*a**4*x*sin(c + d*x)**2 +
 3*B*a**4*x*cos(c + d*x)**4/2 + 2*B*a**4*x*cos(c + d*x)**2 + 8*B*a**4*sin(c
 + d*x)**5/(15*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*B*a**4*sin(c + d*x)**3/d + B*a**4
*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*
d) + 6*B*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*B*a**4*sin(c + d*x)*cos(c
+ d*x)/d + B*a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) +
```

```
a)**4, True))
```

Giac [A] time = 1.23771, size = 188, normalized size = 1.25

$$\frac{Ba^4 \sin(5\,dx+5\,c)}{80\,d} + \frac{7}{8}\left(5\,Aa^4 + 4\,Ba^4\right)x + \frac{\left(Aa^4 + 4\,Ba^4\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{\left(16\,Aa^4 + 29\,Ba^4\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{(7\,A\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*a^4*sin(5*d*x + 5*c)/d + 7/8*(5*A*a^4 + 4*B*a^4)*x + 1/32*(A*a^4 + 4
*B*a^4)*sin(4*d*x + 4*c)/d + 1/48*(16*A*a^4 + 29*B*a^4)*sin(3*d*x + 3*c)/d
+ 1/4*(7*A*a^4 + 8*B*a^4)*sin(2*d*x + 2*c)/d + 7/8*(8*A*a^4 + 7*B*a^4)*sin(
d*x + c)/d
```

3.31 $\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec(c + dx)\,dx$

Optimal. Leaf size=151

$$\frac{5a^4(8A+7B)\sin(c+dx)}{8d} + \frac{(4A+7B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{12d} + \frac{(32A+35B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{24d}$$

```
[Out] (a^4*(48*A + 35*B)*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(8*A + 7
*B)*Sin[c + d*x])/(8*d) + (a*B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) +
 ((4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + ((32*A + 35
*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 0.412612, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2976, 2968, 3023, 2735, 3770}

$$\frac{5a^4(8A+7B)\sin(c+dx)}{8d} + \frac{(4A+7B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{12d} + \frac{(32A+35B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^4*(48*A + 35*B)*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(8*A + 7
*B)*Sin[c + d*x])/(8*d) + (a*B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) +
 ((4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + ((32*A + 35
*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\sec(c + dx)\,dx = \frac{aB(a + a\cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{1}{4}\int (a + a\cos(c + dx))^3$$

$$= \frac{aB(a + a\cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{(4A + 7B)\left(a^2 + a^2\cos(\right.}{12d}$$

$$= \frac{aB(a + a\cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{(4A + 7B)\left(a^2 + a^2\cos(\right.}{12d}$$

$$= \frac{aB(a + a\cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{(4A + 7B)\left(a^2 + a^2\cos(\right.}{12d}$$

$$= \frac{5a^4(8A + 7B)\sin(c + dx)}{8d} + \frac{aB(a + a\cos(c + dx))^3 \sin(c + dx}{4d}$$

$$= \frac{1}{8}a^4(48A + 35B)x + \frac{5a^4(8A + 7B)\sin(c + dx)}{8d} + \frac{aB(a + a\cos}{}$$

$$= \frac{1}{8}a^4(48A + 35B)x + \frac{a^4 A\tanh^{-1}(\sin(c + dx))}{d} + \frac{5a^4(8A + 7B}{8}$$

Mathematica [A] time = 0.372112, size = 138, normalized size = 0.91

$$\frac{a^4\left(24(27A + 28B)\sin(c + dx) + 24(4A + 7B)\sin(2(c + dx)) + 8A\sin(3(c + dx)) - 96A\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c\right.\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^4*(576*A*d*x + 420*B*d*x - 96*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 96*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(27*A + 28*B)*Sin[c +
 d*x] + 24*(4*A + 7*B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 32*B*Sin[3
*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.105, size = 199, normalized size = 1.3

$$\frac{A\sin(dx + c)(\cos(dx + c))^2 a^4}{3d} + \frac{20Aa^4\sin(dx + c)}{3d} + \frac{a^4 B\sin(dx + c)(\cos(dx + c))^3}{4d} + \frac{27a^4 B\cos(dx + c)\sin(dx + c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*A*a^4*sin(d*x+c)+1/4/d*a^4*B*sin
(d*x+c)*cos(d*x+c)^3+27/8/d*a^4*B*cos(d*x+c)*sin(d*x+c)+35/8*a^4*B*x+35/8/d
*a^4*B*c+2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+6*A*a^4*x+6/d*A*a^4*c+4/3/d*B*sin(
d*x+c)*cos(d*x+c)^2*a^4+20/3/d*a^4*B*sin(d*x+c)+1/d*A*a^4*ln(sec(d*x+c)+tan
(d*x+c))
```

Maxima [A] time = 1.01195, size = 267, normalized size = 1.77

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Aa^{4}-96\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{4}-384\left(dx+c\right)Aa^{4}+128\left(\sin\left(dx+c\right)^{3}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 96*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^4 - 384*(d*x + c)*A*a^4 + 128*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*B*a^4 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a
^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 96*(d*x + c)*B*a^4 - 96*A
*a^4*log(sec(d*x + c) + tan(d*x + c)) - 576*A*a^4*sin(d*x + c) - 384*B*a^4*
sin(d*x + c))/d
```

Fricas [A] time = 1.81413, size = 306, normalized size = 2.03

$$\frac{3\left(48\,A+35\,B\right)a^{4}dx+12\,Aa^{4}\log\left(\sin\left(dx+c\right)+1\right)-12\,Aa^{4}\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Ba^{4}\cos\left(dx+c\right)^{3}+8\left(A+4\,B\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(48*A + 35*B)*a^4*d*x + 12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*l
og(-sin(d*x + c) + 1) + (6*B*a^4*cos(d*x + c)^3 + 8*(A + 4*B)*a^4*cos(d*x +
 c)^2 + 3*(16*A + 27*B)*a^4*cos(d*x + c) + 160*(A + B)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2994, size = 289, normalized size = 1.91

$$24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(48\,Aa^4+35\,Ba^4\right)(dx+c)+\frac{2\left(120\,Aa^4\tan\left(\frac{1}{2}\,d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(48*A*a^4 + 35*B*a^4)*(d*x + c) + 2*(120*A*a^4*tan
(1/2*d*x + 1/2*c)^7 + 105*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 424*A*a^4*tan(1/2*
d*x + 1/2*c)^5 + 385*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 520*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 511*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 216*A*a^4*tan(1/2*d*x + 1/2*
c) + 279*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.32 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=150

$$\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}-\frac{(3A-8B)}{}$$

```
[Out] (a^4*(13*A + 12*B)*x)/2 + (a^4*(4*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*
(A + 2*B)*Sin[c + d*x])/(2*d) - ((3*A - B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c
 + d*x])/(3*d) - ((3*A - 8*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d)
+ (a*A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.453221, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2975, 2976, 2968, 3023, 2735, 3770}

$$\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}-\frac{(3A-8B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```


```
[Out] (a^4*(13*A + 12*B)*x)/2 + (a^4*(4*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*
(A + 2*B)*Sin[c + d*x])/(2*d) - ((3*A - B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c
 + d*x])/(3*d) - ((3*A - 8*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d)
+ (a*A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\tan(c+dx)}{d} + \int (a+a\cos(c+dx)$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d} + \frac{aA(a+a\,c}{}$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d} - \frac{(3A-8B)}{}$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d} - \frac{(3A-8B)}{}$$

$$= \frac{5a^4(A+2B)\sin(c+dx)}{2d} - \frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2}{3d}$$

$$= \frac{1}{2}a^4(13A+12B)x + \frac{5a^4(A+2B)\sin(c+dx)}{2d} - \frac{(3A-B)\left(\right.}{}$$

$$= \frac{1}{2}a^4(13A+12B)x + \frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a}{}$$

Mathematica [B] time = 1.54755, size = 312, normalized size = 2.08

$$\frac{1}{192}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(16A+27B)\sin(c)\cos(dx)}{d} + \frac{3(A+4B)\sin(2c)\cos(2dx)}{d} + \frac{3(16A+27B)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(78*A*x + 72*B*x - (12*(4*A +
B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (12*(4*A + B)*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]])/d + (3*(16*A + 27*B)*Cos[d*x]*Sin[c])/d + (3*
(A + 4*B)*Cos[2*d*x]*Sin[2*c])/d + (B*Cos[3*d*x]*Sin[3*c])/d + (3*(16*A + 2
7*B)*Cos[c]*Sin[d*x])/d + (3*(A + 4*B)*Cos[2*c]*Sin[2*d*x])/d + (B*Cos[3*c]
*Sin[3*d*x])/d + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))))/192
```

Maple [A] time = 0.148, size = 190, normalized size = 1.3

$$\frac{Aa^4\cos(dx+c)\sin(dx+c)}{2d} + \frac{13\,Aa^4x}{2} + \frac{13\,Aa^4c}{2d} + \frac{B\sin(dx+c)(\cos(dx+c))^2a^4}{3d} + \frac{20\,a^4B\sin(dx+c)}{3d} + 4\,\frac{Aa^4\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+13/2*A*a^4*x+13/2/d*A*a^4*c+1/3/d*B*sin(d
*x+c)*cos(d*x+c)^2*a^4+20/3/d*a^4*B*sin(d*x+c)+4/d*A*a^4*sin(d*x+c)+2/d*a^4
*B*cos(d*x+c)*sin(d*x+c)+6*a^4*B*x+6/d*a^4*B*c+4/d*A*a^4*ln(sec(d*x+c)+tan(
d*x+c))+1/d*A*a^4*tan(d*x+c)+1/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01106, size = 252, normalized size = 1.68

$$3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^4 + 72\,(dx + c)Aa^4 - 4\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Ba^4 + 12\,(2\,dx + 2\,c + \sin(2\,dx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 72*(d*x + c)*A*a^4 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 12*(2*d*x + 2*c + sin(2*d*x + 2*c))*
B*a^4 + 48*(d*x + c)*B*a^4 + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 6*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A
*a^4*sin(d*x + c) + 72*B*a^4*sin(d*x + c) + 12*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.67625, size = 383, normalized size = 2.55

$$\frac{3\,(13\,A + 12\,B)a^4dx\cos(dx + c) + 3\,(4\,A + B)a^4\cos(dx + c)\log(\sin(dx + c) + 1) - 3\,(4\,A + B)a^4\cos(dx + c)\log(-\sin}{6\,d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(13*A + 12*B)*a^4*d*x*cos(d*x + c) + 3*(4*A + B)*a^4*cos(d*x + c)*lo
g(sin(d*x + c) + 1) - 3*(4*A + B)*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 (2*B*a^4*cos(d*x + c)^3 + 3*(A + 4*B)*a^4*cos(d*x + c)^2 + 8*(3*A + 5*B)*a
```

```
^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34447, size = 305, normalized size = 2.03

$$-\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\left(13\,Aa^4 + 12\,Ba^4\right)(dx+c) - 6\left(4\,Aa^4 + Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 6\left(4\,Aa^4 + Ba^4\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(13*A*
a^4 + 12*B*a^4)*(d*x + c) - 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) + 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*
a^4*tan(1/2*d*x + 1/2*c)^5 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 76*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*
x + 1/2*c) + 54*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)
/d
```

3.33 $\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^3(c + dx)\,dx$

Optimal. Leaf size=162

$$-\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(13A+8B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{2d}+\frac{(5A+2B)}{}$$

```
[Out] (a^4*(8*A + 13*B)*x)/2 + (a^4*(13*A + 8*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (
5*a^4*(A - B)*Sin[c + d*x])/(2*d) - ((6*A + B)*(a^4 + a^4*Cos[c + d*x])*Sin
[c + d*x])/(2*d) + ((5*A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/(2
*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.475574, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2975, 2976, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(13A+8B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{2d}+\frac{(5A+2B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```



```
[Out] (a^4*(8*A + 13*B)*x)/2 + (a^4*(13*A + 8*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (
5*a^4*(A - B)*Sin[c + d*x])/(2*d) - ((6*A + B)*(a^4 + a^4*Cos[c + d*x])*Sin
[c + d*x])/(2*d) + ((5*A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/(2
*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^4(A + B\cos(c + dx))\sec^3(c + dx)\,dx = \frac{aA(a + a\cos(c + dx))^3\sec(c + dx)\tan(c + dx)}{2d} + \frac{1}{2}\int (a + a$$

$$= \frac{(5A + 2B)\left(a^2 + a^2\cos(c + dx)\right)^2\tan(c + dx)}{2d} + \frac{aA(a + a\,\mathrm{co}}{}$$

$$= -\frac{(6A + B)\left(a^4 + a^4\cos(c + dx)\right)\sin(c + dx)}{2d} + \frac{(5A + 2B)\left(a\right.}{}$$

$$= -\frac{(6A + B)\left(a^4 + a^4\cos(c + dx)\right)\sin(c + dx)}{2d} + \frac{(5A + 2B)\left(a\right.}{}$$

$$= -\frac{5a^4(A - B)\sin(c + dx)}{2d} - \frac{(6A + B)\left(a^4 + a^4\cos(c + dx)\right)\mathrm{si}}{2d}$$

$$= \frac{1}{2}a^4(8A + 13B)x - \frac{5a^4(A - B)\sin(c + dx)}{2d} - \frac{(6A + B)\left(a^4 +\right.}{}$$

$$= \frac{1}{2}a^4(8A + 13B)x + \frac{a^4(13A + 8B)\tanh^{-1}(\sin(c + dx))}{2d} - \frac{5a^4}{}$$

Mathematica [B] time = 4.28311, size = 343, normalized size = 2.12

$$\frac{1}{64}a^4(\cos(c + dx) + 1)^4\sec^8\left(\frac{1}{2}(c + dx)\right)\left(\frac{4(A + 4B)\sin(c)\cos(dx)}{d} + \frac{4(A + 4B)\cos(c)\sin(dx)}{d} + \frac{4(4A}{d\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right)\left(\mathrm{c}\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(2*(8*A + 13*B)*x - (2*(13*A +
 8*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(13*A + 8*B)*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 4*B)*Cos[d*x]*Sin[c])/d + (B
*Cos[2*d*x]*Sin[2*c])/d + (4*(A + 4*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin
[2*d*x])/d + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(4*A + B)*S
in[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) - A/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(4*A + B)*Sin[(d*x)/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

Maple [A] time = 0.127, size = 182, normalized size = 1.1

$$\frac{Aa^4\sin(dx + c)}{d} + \frac{a^4B\cos(dx + c)\sin(dx + c)}{2\,d} + \frac{13\,a^4Bx}{2} + \frac{13\,a^4Bc}{2\,d} + 4\,Aa^4x + 4\,\frac{Aa^4c}{d} + 4\,\frac{a^4B\sin(dx + c)}{d} + \frac{13\,Aa^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+1/2/d*a^4*B*cos(d*x+c)*sin(d*x+c)+13/2*a^4*B*x+13/2/d*
a^4*B*c+4*A*a^4*x+4/d*A*a^4*c+4/d*a^4*B*sin(d*x+c)+13/2/d*A*a^4*ln(sec(d*x+
c)+tan(d*x+c))+4/d*A*a^4*tan(d*x+c)+4/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+1/2
/d*A*a^4*sec(d*x+c)*tan(d*x+c)+1/d*a^4*B*tan(d*x+c)
```

Maxima [A] time = 1.02769, size = 269, normalized size = 1.66

$$16\,(dx+c)Aa^4 + (2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ba^4 + 24\,(dx+c)Ba^4 - Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(16*(d*x + c)*A*a^4 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 24*(d*x
+ c)*B*a^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
+ 1) + log(sin(d*x + c) - 1)) + 12*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 8*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4
*A*a^4*sin(d*x + c) + 16*B*a^4*sin(d*x + c) + 16*A*a^4*tan(d*x + c) + 4*B*a
^4*tan(d*x + c))/d
```

Fricas [A] time = 1.45169, size = 390, normalized size = 2.41

$$\frac{2\,(8\,A+13\,B)a^4dx\cos(dx+c)^2 + (13\,A+8\,B)a^4\cos(dx+c)^2\log(\sin(dx+c)+1) - (13\,A+8\,B)a^4\cos(dx+c)^2\log(}{4\,d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(8*A + 13*B)*a^4*d*x*cos(d*x + c)^2 + (13*A + 8*B)*a^4*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (13*A + 8*B)*a^4*cos(d*x + c)^2*log(-sin(d*x + c)
```

```
+ 1) + 2*(B*a^4*cos(d*x + c)^3 + 2*(A + 4*B)*a^4*cos(d*x + c)^2 + 2*(4*A +
B)*a^4*cos(d*x + c) + A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33526, size = 311, normalized size = 1.92

$$\frac{\left(8\,Aa^4+13\,Ba^4\right)(dx+c)+\left(13\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(13\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((8*A*a^4 + 13*B*a^4)*(d*x + c) + (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*(5*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 5*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 7*
A*a^4*tan(1/2*d*x + 1/2*c)^5 + 7*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 11*A*a^4*tan(1/2*d*x
 + 1/2*c) - 11*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/
d
```

3.34 $\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\sec^4(c + dx)\,dx$

Optimal. Leaf size=165

$$-\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(11A+9B)\tan(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{3d}+\frac{(2A}{}$$

```
[Out] a^4*(A + 4*B)*x + (a^4*(12*A + 13*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^4*
(2*A + B)*Sin[c + d*x])/(2*d) + ((11*A + 9*B)*(a^4 + a^4*Cos[c + d*x])*Tan[
c + d*x])/(3*d) + ((2*A + B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(2*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3
*d)
```

Rubi [A] time = 0.514103, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(11A+9B)\tan(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{3d}+\frac{(2A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] a^4*(A + 4*B)*x + (a^4*(12*A + 13*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^4*
(2*A + B)*Sin[c + d*x])/(2*d) + ((11*A + 9*B)*(a^4 + a^4*Cos[c + d*x])*Tan[
c + d*x])/(3*d) + ((2*A + B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(2*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3
*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int (a$$

$$= \frac{(2A+B)\left(a^2+a^2\cos(c+dx)\right)^2\sec(c+dx)\tan(c+dx)}{2d} +$$

$$= \frac{(11A+9B)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{3d} + \frac{(2A+B)\left(\right.}{}$$

$$= \frac{(11A+9B)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{3d} + \frac{(2A+B)\left(\right.}{}$$

$$= -\frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{(11A+9B)\left(a^4+a^4\cos(c+d\right.}{3d}$$

$$= a^4(A+4B)x - \frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{(11A+9B)\left(a^4\right.}{}$$

$$= a^4(A+4B)x + \frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^4(}{}$$

Mathematica [B] time = 6.20254, size = 380, normalized size = 2.3

$$a^4\left(\frac{(A+4B)(c+dx)}{d} + \frac{-13A-3B}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4\left(5A\sin\left(\frac{1}{2}(c+dx)\right)+3B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4(5}{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(((A + 4*B)*(c + d*x))/d + ((-12*A - 13*B)*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]])/(2*d) + ((12*A + 13*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]])/(2*d) + (13*A + 3*B)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
A*Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (A*Sin[
(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (-13*A - 3*B)
/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*A*Sin[(c + d*x)/2]
+ 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*(
5*A*Sin[(c + d*x)/2] + 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])) + (B*Sin[c + d*x])/d)
```

Maple [A] time = 0.124, size = 189, normalized size = 1.2

$$Aa^4x+\frac{Aa^4c}{d}+\frac{a^4B\sin(dx+c)}{d}+6\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+4a^4Bx+4\frac{Ba^4c}{d}+\frac{20Aa^4\tan(dx+c)}{3d}+\frac{13}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] A*a^4*x+1/d*A*a^4*c+1/d*a^4*B*sin(d*x+c)+6/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c)
)+4*a^4*B*x+4/d*a^4*B*c+20/3/d*A*a^4*tan(d*x+c)+13/2/d*a^4*B*ln(sec(d*x+c)+
tan(d*x+c))+2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+4/d*a^4*B*tan(d*x+c)+1/3/d*A*a^
4*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.01896, size = 317, normalized size = 1.92

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+12(dx+c)Aa^4+48(dx+c)Ba^4-12Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 12*(d*x + c)*A*a^4 + 48*(
d*x + c)*B*a^4 - 12*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*B*a^4*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + 12*B*a^4*sin(d*x + c) + 72*A*a^4*tan(d*x + c) +
 48*B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.54502, size = 405, normalized size = 2.45

$$\frac{12(A+4B)a^4dx\cos(dx+c)^3+3(12A+13B)a^4\cos(dx+c)^3\log(\sin(dx+c)+1)-3(12A+13B)a^4\cos(dx+c)^3\log}{12d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*(A + 4*B)*a^4*d*x*cos(d*x + c)^3 + 3*(12*A + 13*B)*a^4*cos(d*x + c
)^3*log(sin(d*x + c) + 1) - 3*(12*A + 13*B)*a^4*cos(d*x + c)^3*log(-sin(d*x
 + c) + 1) + 2*(6*B*a^4*cos(d*x + c)^3 + 8*(5*A + 3*B)*a^4*cos(d*x + c)^2 +
 3*(4*A + B)*a^4*cos(d*x + c) + 2*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30296, size = 306, normalized size = 1.85

$$\frac{\frac{12\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(Aa^4+4\,Ba^4\right)(dx+c)+3\left(12\,Aa^4+13\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(12\,Aa^4+13\,Ba^4\right)\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*B*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(A*a^4
+ 4*B*a^4)*(d*x + c) + 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30
*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 76*A*a^4*
tan(1/2*d*x + 1/2*c)^3 - 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2
*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3)/d
```

3.35 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=173

$$\frac{5a^4(7A+8B)\tan(c+dx)}{8d}+\frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(7A+4B)\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+\right.}{12d}$$

```
[Out] a^4*B*x + (a^4*(35*A + 48*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*(7*A + 8
*B)*Tan[c + d*x])/(8*d) + ((35*A + 32*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + ((7*A + 4*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c +
d*x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d)
```

Rubi [A] time = 0.522871, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3021, 2735, 3770}

$$\frac{5a^4(7A+8B)\tan(c+dx)}{8d}+\frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(7A+4B)\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```


```
[Out] a^4*B*x + (a^4*(35*A + 48*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*(7*A + 8
*B)*Tan[c + d*x])/(8*d) + ((35*A + 32*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + ((7*A + 4*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c +
d*x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (a+$$

$$= \frac{(7A+4B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^2(c+dx)\tan(c+dx)}{12d} +$$

$$= \frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{24d} +$$

$$= \frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{24d} +$$

$$= \frac{5a^4(7A+8B)\tan(c+dx)}{8d} + \frac{(35A+32B)\left(a^4+a^4\cos(c+d\right.}{24d}$$

$$= a^4Bx + \frac{5a^4(7A+8B)\tan(c+dx)}{8d} + \frac{(35A+32B)\left(a^4+a^4\,\mathrm{c}\right.}{}$$

$$= a^4Bx + \frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{5a^4(7A+8B)}{8d}$$

Mathematica [A] time = 1.71883, size = 326, normalized size = 1.88

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\sec(c)(105A\sin(2c+dx)+544A\sin(c+2dx)-96A\sin(3c+2dx)+81A\sin(2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(35*A + 48*B)*Cos[c + d*x
]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + Sec[c]*(72*B*d*x*Cos[c] + 48*B*d*x*Cos[c + 2*d*x] + 48*B*d
*x*Cos[3*c + 2*d*x] + 12*B*d*x*Cos[3*c + 4*d*x] + 12*B*d*x*Cos[5*c + 4*d*x]
 - 480*A*Sin[c] - 480*B*Sin[c] + 105*A*Sin[d*x] + 48*B*Sin[d*x] + 105*A*Sin
[2*c + d*x] + 48*B*Sin[2*c + d*x] + 544*A*Sin[c + 2*d*x] + 496*B*Sin[c + 2*
d*x] - 96*A*Sin[3*c + 2*d*x] - 144*B*Sin[3*c + 2*d*x] + 81*A*Sin[2*c + 3*d*
x] + 48*B*Sin[2*c + 3*d*x] + 81*A*Sin[4*c + 3*d*x] + 48*B*Sin[4*c + 3*d*x]
+ 160*A*Sin[3*c + 4*d*x] + 160*B*Sin[3*c + 4*d*x])))/(3072*d)
```

Maple [A] time = 0.122, size = 204, normalized size = 1.2

$$\frac{35\,Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{8\,d} + a^4Bx + \frac{Ba^4c}{d} + \frac{20\,Aa^4\tan(dx+c)}{3\,d} + 6\,\frac{a^4B\ln(\sec(dx+c)+\tan(dx+c))}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 35/8/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*x+1/d*a^4*B*c+20/3/d*A*a^4*tan
(d*x+c)+6/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+27/8/d*A*a^4*sec(d*x+c)*tan(d*x
+c)+20/3/d*a^4*B*tan(d*x+c)+4/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+2/d*a^4*B*s
ec(d*x+c)*tan(d*x+c)+1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+1/3/d*a^4*B*tan(d*
x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.25626, size = 414, normalized size = 2.39

$$\frac{64\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{4}+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{4}+48\,(dx+c)Ba^{4}-3\,Aa^{4}\left(\frac{2\left(3\,\sin(dx+c)^{3}}{\sin(dx+c)^{4}-2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^4 + 48*(d*x + c)*B*a^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 72*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 48*B*a^4*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*B*a^4*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*A*a^4*tan(d*x + c) + 288*
B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.43609, size = 408, normalized size = 2.36

$$\frac{48\,Ba^{4}dx\cos\left(dx+c\right)^{4}+3\,(35\,A+48\,B)a^{4}\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(35\,A+48\,B)a^{4}\cos\left(dx+c\right)^{4}\log\left(-\sin\right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(48*B*a^4*d*x*cos(d*x + c)^4 + 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log(
sin(d*x + c) + 1) - 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 2*(160*(A + B)*a^4*cos(d*x + c)^3 + 3*(27*A + 16*B)*a^4*cos(d*x + c)^2
 + 8*(4*A + B)*a^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2737, size = 301, normalized size = 1.74

$$24\,(dx+c)Ba^4 + 3\left(35\,Aa^4 + 48\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(35\,Aa^4 + 48\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*(d*x + c)*B*a^4 + 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
*(105*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 385
*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^
4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*A*a^4*tan
(1/2*d*x + 1/2*c) - 216*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^4)/d
```

3.36 $\int (a + a\cos(c + dx))^4(A + B\cos(c + dx))\sec^6(c + dx)\,dx$

Optimal. Leaf size=198

$$\frac{a^4(83A+100B)\tan(c+dx)}{15d}+\frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(244A+275B)\tan(c+dx)\sec(c+dx)}{120d}+\frac{(8A\cdots}{}$$

```
[Out] (7*a^4*(4*A + 5*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(83*A + 100*B)*Tan[c
 + d*x])/(15*d) + (a^4*(244*A + 275*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) +
 ((26*A + 25*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(30*d
) + ((8*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(2
0*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.587073, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^4(83A+100B)\tan(c+dx)}{15d}+\frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(244A+275B)\tan(c+dx)\sec(c+dx)}{120d}+\frac{(8A\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```


```
[Out] (7*a^4*(4*A + 5*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(83*A + 100*B)*Tan[c
 + d*x])/(15*d) + (a^4*(244*A + 275*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) +
 ((26*A + 25*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(30*d
) + ((8*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(2
0*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{5}\int (a$$

$$= \frac{(8A+5B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^3(c+dx)\tan(c+dx)}{20d}$$

$$= \frac{(26A+25B)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx}{30d}$$

$$= \frac{(26A+25B)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx}{30d}$$

$$= \frac{a^4(244A+275B)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(26A+25B)(}{}$$

$$= \frac{a^4(244A+275B)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(26A+25B)(}{}$$

$$= \frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4(244A+275B)\mathrm{sec}}{12}$$

$$= \frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4(83A+100B)\tan(}{15d}$$

Mathematica [A] time = 1.5713, size = 306, normalized size = 1.55

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(1680(4A+5B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^5*(1680*(4*A + 5
*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(59*A + 64*B)*Sin[d*x] - 960*(2*
A + 3*B)*Sin[2*c + d*x] + 1320*A*Sin[c + 2*d*x] + 930*B*Sin[c + 2*d*x] + 13
20*A*Sin[3*c + 2*d*x] + 930*B*Sin[3*c + 2*d*x] + 3200*A*Sin[2*c + 3*d*x] +
3520*B*Sin[2*c + 3*d*x] - 120*A*Sin[4*c + 3*d*x] - 480*B*Sin[4*c + 3*d*x] +
 420*A*Sin[3*c + 4*d*x] + 405*B*Sin[3*c + 4*d*x] + 420*A*Sin[5*c + 4*d*x] +
 405*B*Sin[5*c + 4*d*x] + 664*A*Sin[4*c + 5*d*x] + 800*B*Sin[4*c + 5*d*x]))
)/(30720*d)
```

Maple [A] time = 0.152, size = 234, normalized size = 1.2

$$\frac{83\,Aa^4\tan(dx+c)}{15\,d}+\frac{35\,a^4B\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{7\,Aa^4\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{7\,Aa^4\ln(\sec(dx+c)+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 83/15/d*A*a^4*tan(d*x+c)+35/8/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*A*a^4
*sec(d*x+c)*tan(d*x+c)+7/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*a^4*B*t
an(d*x+c)+34/15/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+27/8/d*a^4*B*sec(d*x+c)*tan
(d*x+c)+1/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+4/3/d*a^4*B*tan(d*x+c)*sec(d*x+c)
^2+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+1/4/d*a^4*B*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [B] time = 1.17355, size = 508, normalized size = 2.57

$$\frac{16\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^4+480\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+320\left(\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 320*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^4 - 60*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 240*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 360*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*a^4*(log(sin(d*x + c
) + 1) - log(sin(d*x + c) - 1)) + 240*A*a^4*tan(d*x + c) + 960*B*a^4*tan(d*
x + c))/d
```

Fricas [A] time = 1.3947, size = 431, normalized size = 2.18

$$\frac{105\,(4\,A+5\,B)a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-105\,(4\,A+5\,B)a^4\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(8\,(83\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(105*(4*A + 5*B)*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*(4*A
+ 5*B)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(83*A + 100*B)*a^4*
cos(d*x + c)^4 + 15*(28*A + 27*B)*a^4*cos(d*x + c)^3 + 16*(17*A + 10*B)*a^4
*cos(d*x + c)^2 + 30*(4*A + B)*a^4*cos(d*x + c) + 24*A*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30743, size = 332, normalized size = 1.68

$$105\left(4\,Aa^4+5\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(4\,Aa^4+5\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(420\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(105*(4*A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(4*
A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(420*A*a^4*tan(1/2*
d*x + 1/2*c)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 1960*A*a^4*tan(1/2*d*x
+ 1/2*c)^7 - 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 4480*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 3160*A*a^4*tan(1/2*d*x + 1/2*
c)^3 - 3950*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*d*x + 1/2*c)
```

```
+ 1395*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.37 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal. Leaf size=229

$$\frac{a^4(72A+83B)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(159A+176B)\tan(c+dx)\sec^2(c+dx)}{120d}+\frac{7a^4(7$$

```
[Out] (7*a^4*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(72*A + 83*B)*Tan[c
 + d*x])/(15*d) + (7*a^4*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a
^4*(159*A + 176*B)*Sec[c + d*x]^2*Tan[c + d*x])/(120*d) + ((73*A + 72*B)*(a
^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + ((3*A + 2*B)*
(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a*A*(a +
a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.649648, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^4(72A+83B)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(159A+176B)\tan(c+dx)\sec^2(c+dx)}{120d}+\frac{7a^4(7$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```


```
[Out] (7*a^4*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(72*A + 83*B)*Tan[c
 + d*x])/(15*d) + (7*a^4*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a
^4*(159*A + 176*B)*Sec[c + d*x]^2*Tan[c + d*x])/(120*d) + ((73*A + 72*B)*(a
^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + ((3*A + 2*B)*
(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a*A*(a +
a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{6}\int (a$$

$$= \frac{(3A+2B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^4(c+dx)\tan(c+dx)}{10d}$$

$$= \frac{(73A+72B)\left(a^4+a^4\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx}{120d}$$

$$= \frac{(73A+72B)\left(a^4+a^4\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx}{120d}$$

$$= \frac{a^4(159A+176B)\sec^2(c+dx)\tan(c+dx)}{120d}+\frac{(73A+72B)}{}$$

$$= \frac{a^4(159A+176B)\sec^2(c+dx)\tan(c+dx)}{120d}+\frac{(73A+72B)}{}$$

$$= \frac{7a^4(7A+8B)\sec(c+dx)\tan(c+dx)}{16d}+\frac{a^4(159A+176B)}{}$$

$$= \frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(72A+83B)\tan(c}{15d}$$

Mathematica [A] time = 2.07869, size = 358, normalized size = 1.56

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(3360(7A+8B)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(3360*(7*A + 8
*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-160*(72*A + 83*B)*Sin[c] + 30*(125
*A + 88*B)*Sin[d*x] + 3750*A*Sin[2*c + d*x] + 2640*B*Sin[2*c + d*x] + 15360
*A*Sin[c + 2*d*x] + 15840*B*Sin[c + 2*d*x] - 1920*A*Sin[3*c + 2*d*x] - 4080
*B*Sin[3*c + 2*d*x] + 3845*A*Sin[2*c + 3*d*x] + 3480*B*Sin[2*c + 3*d*x] + 3
```

```
845*A*Sin[4*c + 3*d*x] + 3480*B*Sin[4*c + 3*d*x] + 6912*A*Sin[3*c + 4*d*x]
+ 7728*B*Sin[3*c + 4*d*x] - 240*B*Sin[5*c + 4*d*x] + 735*A*Sin[4*c + 5*d*x]
 + 840*B*Sin[4*c + 5*d*x] + 735*A*Sin[6*c + 5*d*x] + 840*B*Sin[6*c + 5*d*x]
 + 1152*A*Sin[5*c + 6*d*x] + 1328*B*Sin[5*c + 6*d*x])))/(122880*d)
```

Maple [A] time = 0.125, size = 280, normalized size = 1.2

$$\frac{49\,Aa^4\sec(dx+c)\tan(dx+c)}{16\,d}+\frac{49\,Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{16\,d}+\frac{83\,a^4B\tan(dx+c)}{15\,d}+\frac{24\,Aa^4\tan(dx+c)}{5\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x)
```

```
[Out] 49/16/d*A*a^4*sec(d*x+c)*tan(d*x+c)+49/16/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))
+83/15/d*a^4*B*tan(d*x+c)+24/5/d*A*a^4*tan(d*x+c)+12/5/d*A*a^4*tan(d*x+c)*s
ec(d*x+c)^2+7/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)+7/2/d*a^4*B*ln(sec(d*x+c)+tan
(d*x+c))+41/24/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+34/15/d*a^4*B*tan(d*x+c)*sec
(d*x+c)^2+4/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+1/d*a^4*B*tan(d*x+c)*sec(d*x+
c)^3+1/6/d*A*a^4*tan(d*x+c)*sec(d*x+c)^5+1/5/d*a^4*B*tan(d*x+c)*sec(d*x+c)^
4
```

Maxima [B] time = 1.0665, size = 626, normalized size = 2.73

$$\frac{128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+32\left(3\tan(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="ma
xima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 32*(3*tan(d*x + c)^5 + 10*ta
n(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 960*(tan(d*x + c)^3 + 3*tan(d*x + c
))*B*a^4 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
 c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(si
n(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*A*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
```

```
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 480*B*a^4*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
)) + 480*B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.41561, size = 481, normalized size = 2.1

$$\frac{105\,(7\,A+8\,B)a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-105\,(7\,A+8\,B)a^4\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\,\big(16\,(72\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*(7*A + 8*B)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(7*A
+ 8*B)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(72*A + 83*B)*a^4*
cos(d*x + c)^5 + 105*(7*A + 8*B)*a^4*cos(d*x + c)^4 + 32*(18*A + 17*B)*a^4*
cos(d*x + c)^3 + 10*(41*A + 24*B)*a^4*cos(d*x + c)^2 + 48*(4*A + B)*a^4*cos
(d*x + c) + 40*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31649, size = 378, normalized size = 1.65

$$\frac{105\left(7\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(7\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(735\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\ldots\right.\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(7*
A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(735*A*a^4*tan(1/2*
d*x + 1/2*c)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*A*a^4*tan(1/2*d*
x + 1/2*c)^9 - 4760*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*x +
 1/2*c)^7 + 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*A*a^4*tan(1/2*d*x +
1/2*c)^5 - 13488*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x + 1/
2*c)^3 + 9320*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*A*a^4*tan(1/2*d*x + 1/2*c
) - 3000*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.38 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=153

$$-\frac{4(A-B)\sin^3(c+dx)}{3ad}+\frac{4(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(4A-5B)\sin(c+dx)\cos^3(c+dx}{4ad}$$

```
[Out] (-3*(4*A - 5*B)*x)/(8*a) + (4*(A - B)*Sin[c + d*x])/(a*d) - (3*(4*A - 5*B)*
Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - ((4*A - 5*B)*Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])
) - (4*(A - B)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.206986, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2633, 2635, 8}

$$-\frac{4(A-B)\sin^3(c+dx)}{3ad}+\frac{4(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(4A-5B)\sin(c+dx)\cos^3(c+dx}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*(4*A - 5*B)*x)/(8*a) + (4*(A - B)*Sin[c + d*x])/(a*d) - (3*(4*A - 5*B)*
Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - ((4*A - 5*B)*Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])
) - (4*(A - B)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^3(c+dx)(4a(A-B)-a(4A-5B)\cos}{a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(4A-5B)\int \cos^4(c+dx)\,dx}{a} + \frac{(4(A-B))\int}{}$$

$$= -\frac{(4A-5B)\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3(4}{}$$

$$= \frac{4(A-B)\sin(c+dx)}{ad} - \frac{3(4A-5B)\cos(c+dx)\sin(c+dx)}{8ad} - \frac{(4A-5B)\cos^3(c}{4a}$$

$$= -\frac{3(4A-5B)x}{8a} + \frac{4(A-B)\sin(c+dx)}{ad} - \frac{3(4A-5B)\cos(c+dx)\sin(c+dx)}{8ad} -$$

Mathematica [B] time = 0.607227, size = 311, normalized size = 2.03

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-72dx(4A-5B)\cos\left(c+\frac{dx}{2}\right)-72dx(4A-5B)\cos\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{dx}{2}\right)+144A\sin\left(c+\frac{3dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-72*(4*A - 5*B)*d*x*Cos[(d*x)/2] - 72*(4*A - 5*
B)*d*x*Cos[c + (d*x)/2] + 552*A*Sin[(d*x)/2] - 552*B*Sin[(d*x)/2] + 168*A*S
in[c + (d*x)/2] - 168*B*Sin[c + (d*x)/2] + 144*A*Sin[c + (3*d*x)/2] - 120*B
*Sin[c + (3*d*x)/2] + 144*A*Sin[2*c + (3*d*x)/2] - 120*B*Sin[2*c + (3*d*x)/
2] - 16*A*Sin[2*c + (5*d*x)/2] + 40*B*Sin[2*c + (5*d*x)/2] - 16*A*Sin[3*c +
 (5*d*x)/2] + 40*B*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*d*x)/2] - 5*B*Si
n[3*c + (7*d*x)/2] + 8*A*Sin[4*c + (7*d*x)/2] - 5*B*Sin[4*c + (7*d*x)/2] +
3*B*Sin[4*c + (9*d*x)/2] + 3*B*Sin[5*c + (9*d*x)/2]))/(192*a*d*(1 + Cos[c +
 d*x]))
```

Maple [B] time = 0.069, size = 351, normalized size = 2.3

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{25\,B}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-4}+5\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{7}A}{da\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*A*tan(1/2*d*x+1/2*c)-1/d/a*B*tan(1/2*d*x+1/2*c)-25/4/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*B+5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^7*A-115/12/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
^5*B+31/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A-109/12/d/a/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*B+25/3/d/a/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A-7/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*B*tan(
1/2*d*x+1/2*c)+3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*A*tan(1/2*d*x+1/2*c)-3/d/a*
arctan(tan(1/2*d*x+1/2*c))*A+15/4/d/a*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.53736, size = 532, normalized size = 3.48

$$-\frac{B\left(\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{115\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{75\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a+\frac{4\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}-\frac{45\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\,\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-4\,A\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\,\sin(dx+c)}{(\cos(dx+c)+1}}{a+\frac{3\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,a\sin(dx}{(\cos(dx+c}}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(B*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 4*A*((9*sin(d*x +
 c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3*sin(d*x + c)/(a*
(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.37627, size = 301, normalized size = 1.97

$$-\frac{9\,(4\,A-5\,B)dx\cos\left(dx+c\right)+9\,(4\,A-5\,B)dx-\left(6\,B\cos\left(dx+c\right)^{4}+2\,(4\,A-B)\cos\left(dx+c\right)^{3}-(4\,A-13\,B)\cos\left(dx+c\right.}{24\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/24*(9*(4*A - 5*B)*d*x*cos(d*x + c) + 9*(4*A - 5*B)*d*x - (6*B*cos(d*x +
c)^4 + 2*(4*A - B)*cos(d*x + c)^3 - (4*A - 13*B)*cos(d*x + c)^2 + (28*A - 1
9*B)*cos(d*x + c) + 64*A - 64*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 13.353, size = 1794, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-36*A*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
```

```
/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**
8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c
/2 + d*x/2)**2 + 24*a*d) - 216*A*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 +
d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a
*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**2/(24*a*d*ta
n(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**
4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 36*A*d*x/(24*a*d*tan(c/2 + d*x/2
)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*ta
n(c/2 + d*x/2)**2 + 24*a*d) + 24*A*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*
x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d
*tan(c/2 + d*x/2)**2 + 24*a*d) + 216*A*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2
+ d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96
*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 392*A*tan(c/2 + d*x/2)**5/(24*a*d*tan(
c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4
+ 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 296*A*tan(c/2 + d*x/2)**3/(24*a*d*
tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)
**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 96*A*tan(c/2 + d*x/2)/(24*a*d*
tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)
**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*B*d*x*tan(c/2 + d*x/2)**8/(
24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 +
 d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*B*d*x*tan(c/2 + d*x
/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*t
an(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 270*B*d*x*tan(c
/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 1
44*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*B*d
*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2
)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) +
 45*B*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*
d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 24*B*tan(c/2
 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144
*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 246*B*tan
(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 +
 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 374*B
*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)*
*6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 3
14*B*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x
/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
 - 66*B*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x
/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
, Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a), True))
```

<u>Giac</u> [A] time = 1.18388, size = 244, normalized size = 1.59

$$-\frac{\frac{9\,(dx+c)(4\,A-5\,B)}{a}-\frac{24\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-75\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+124\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-115\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,d\cdots\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/24*(9*(d*x + c)*(4*A - 5*B)/a - 24*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d
*x + 1/2*c))/a - 2*(60*A*tan(1/2*d*x + 1/2*c)^7 - 75*B*tan(1/2*d*x + 1/2*c)
^7 + 124*A*tan(1/2*d*x + 1/2*c)^5 - 115*B*tan(1/2*d*x + 1/2*c)^5 + 100*A*ta
n(1/2*d*x + 1/2*c)^3 - 109*B*tan(1/2*d*x + 1/2*c)^3 + 36*A*tan(1/2*d*x + 1/
2*c) - 21*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.39 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{(3A-4B)\sin^3(c+dx)}{3ad}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] (3*(A - B)*x)/(2*a) - ((3*A - 4*B)*Sin[c + d*x])/(a*d) + (3*(A - B)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a +
 a*Cos[c + d*x])) + ((3*A - 4*B)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.171539, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2635, 8, 2633}

$$\frac{(3A-4B)\sin^3(c+dx)}{3ad}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```


```
[Out] (3*(A - B)*x)/(2*a) - ((3*A - 4*B)*Sin[c + d*x])/(a*d) + (3*(A - B)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a +
 a*Cos[c + d*x])) + ((3*A - 4*B)*Sin[c + d*x]^3)/(3*a*d)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx &= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos^2(c+dx)(3a(A-B)-a(3A-4B)\cos\\ &= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A-4B)\int\cos^3(c+dx)\,dx}{a}+\frac{(3(A-B))\int\\ &= \frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad}+\frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{(3(A-\\ &= \frac{3(A-B)x}{2a}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad}+\frac{(A-\end{aligned}$$

Mathematica [B] time = 0.534345, size = 249, normalized size = 2.04

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(A-B)\cos\left(c+\frac{dx}{2}\right)+36dx(A-B)\cos\left(\frac{dx}{2}\right)-12A\sin\left(c+\frac{dx}{2}\right)-9A\sin\left(c+\frac{3dx}{2}\right)-9A\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(A - B)*d*x*Cos[(d*x)/2] + 36*(A - B)*d*x*Co
s[c + (d*x)/2] - 60*A*Sin[(d*x)/2] + 69*B*Sin[(d*x)/2] - 12*A*Sin[c + (d*x)
/2] + 21*B*Sin[c + (d*x)/2] - 9*A*Sin[c + (3*d*x)/2] + 18*B*Sin[c + (3*d*x)
/2] - 9*A*Sin[2*c + (3*d*x)/2] + 18*B*Sin[2*c + (3*d*x)/2] + 3*A*Sin[2*c +
(5*d*x)/2] - 2*B*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2] - 2*B*Sin[
3*c + (5*d*x)/2] + B*Sin[3*c + (7*d*x)/2] + B*Sin[4*c + (7*d*x)/2]))/(24*a*
d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.066, size = 281, normalized size = 2.3

$$-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{5}A}{da\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{3}}+5\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{5}B}{da\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{3}}+\frac{16\,B}{3\,da}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*A*tan(1/2*d*x+1/2*c)+1/d/a*B*tan(1/2*d*x+1/2*c)-3/d/a/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/
2*d*x+1/2*c)^5*B+16/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B
-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A-1/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(
1/2*d*x+1/2*c)+3/d/a*arctan(tan(1/2*d*x+1/2*c))*A-3/d/a*arctan(tan(1/2*d*x+
1/2*c))*B
```

Maxima [B] time = 1.52061, size = 419, normalized size = 3.43

$$\frac{B\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a+\frac{3\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\,\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3\,A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a+\frac{2\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\arctan\left(\right.}{}\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
```

```
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*A*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.46638, size = 242, normalized size = 1.98

$$\frac{9\,(A-B)dx\cos\left(dx+c\right)+9\,(A-B)dx+\left(2\,B\cos\left(dx+c\right)^{3}+(3\,A-B)\cos\left(dx+c\right)^{2}-(3\,A-7\,B)\cos\left(dx+c\right)-12\,A+1}{6\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(9*(A - B)*d*x*cos(d*x + c) + 9*(A - B)*d*x + (2*B*cos(d*x + c)^3 + (3*
A - B)*cos(d*x + c)^2 - (3*A - 7*B)*cos(d*x + c) - 12*A + 16*B)*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 7.63455, size = 1161, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((9*A*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*
tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(c/
2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*
a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) + 9*A*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*A*tan(c/2 + d*x/2)**7/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) - 36*A*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c
/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 42*A*tan(c/2 + d*x/2
```

```
)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c
/2 + d*x/2)**2 + 6*a*d) - 12*A*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6
+ 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*B*d*
x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)*
*4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2 + d*x/2)**4/(6*
a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x
/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 +
 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*B*d*x
/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 +
 d*x/2)**2 + 6*a*d) + 6*B*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 +
18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 48*B*tan
(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*B*tan(c/2 + d*x/2)**3/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) + 24*B*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
+ d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*(A + B*cos
(c))*cos(c)**3/(a*cos(c) + a), True))
```

Giac [A] time = 1.24128, size = 204, normalized size = 1.67

$$\frac{\frac{9\,(dx+c)(A-B)}{a} - \frac{6\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a} - \frac{2\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-16\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(9*(d*x + c)*(A - B)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/
2*c))/a - 2*(9*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 + 12*
A*tan(1/2*d*x + 1/2*c)^3 - 16*B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x +
1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.40 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)}{ad(\cos(c+dx)+1)}-\frac{x(A-B)}{a}+\frac{B\sin(c+dx)\cos(c+dx)}{2ad}+\frac{Bx}{2a}$$

```
[Out] -(((A - B)*x)/a) + (B*x)/(2*a) + ((A - B)*Sin[c + d*x])/(a*d) + (B*Cos[c +
d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Sin[c + d*x])/(a*d*(1 + Cos[c + d*x])
)
```

Rubi [A] time = 0.12299, antiderivative size = 99, normalized size of antiderivative = 1.1, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2A-3B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x(2A-3B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((2*A - 3*B)*x)/(2*a) + (2*(A - B)*Sin[c + d*x])/(a*d) - ((2*A - 3*B)*Cos[
c + d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos(c+dx)(2a(A-B)-a(2A-3B)\cos$$

$$= -\frac{(2A-3B)x}{2a} + \frac{2(A-B)\sin(c+dx)}{ad} - \frac{(2A-3B)\cos(c+dx)\sin(c+dx)}{2ad} + $$

Mathematica [B] time = 0.437828, size = 197, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-4dx(2A-3B)\cos\left(c+\frac{dx}{2}\right)-4dx(2A-3B)\cos\left(\frac{dx}{2}\right)+4A\sin\left(c+\frac{dx}{2}\right)+4A\sin\left(c+\frac{3dx}{2}\right)+4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-4*(2*A - 3*B)*d*x*Cos[(d*x)/2] - 4*(2*A - 3*B)
*d*x*Cos[c + (d*x)/2] + 20*A*Sin[(d*x)/2] - 20*B*Sin[(d*x)/2] + 4*A*Sin[c +
 (d*x)/2] - 4*B*Sin[c + (d*x)/2] + 4*A*Sin[c + (3*d*x)/2] - 3*B*Sin[c + (3*
d*x)/2] + 4*A*Sin[2*c + (3*d*x)/2] - 3*B*Sin[2*c + (3*d*x)/2] + B*Sin[2*c +
 (5*d*x)/2] + B*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.069, size = 211, normalized size = 2.3

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{(\tan(1/2\,dx+c/2))^3B}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+2\,\frac{(\tan(1/2\,dx+c/2))^3A}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{B}{da}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*A*tan(1/2*d*x+1/2*c)-1/d/a*B*tan(1/2*d*x+1/2*c)-3/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)^3*A-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+2/d/a
```

```
/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-2/d/a*arctan(tan(1/2*d*x+1
/2*c))*A+3/d/a*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.50592, size = 304, normalized size = 3.38

$$-\frac{B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(B*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.42564, size = 204, normalized size = 2.27

$$-\frac{(2A-3B)dx\cos(dx+c)+(2A-3B)dx-\left(B\cos(dx+c)^2+(2A-B)\cos(dx+c)+4A-4B\right)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*((2*A - 3*B)*d*x*cos(d*x + c) + (2*A - 3*B)*d*x - (B*cos(d*x + c)^2 +
(2*A - B)*cos(d*x + c) + 4*A - 4*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 4.28966, size = 665, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-2*A*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*
tan(c/2 + d*x/2)**2 + 2*a*d) - 4*A*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 +
 d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*A*d*x/(2*a*d*tan(c/2 +
d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*A*tan(c/2 + d*x/2)**5/(2
*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 8*A*tan(c/2
 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d
) + 6*A*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2
)**2 + 2*a*d) + 3*B*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*
a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*B*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c
/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*B*d*x/(2*a*d*tan(c/
2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*B*tan(c/2 + d*x/2)**
5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*B*ta
n(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 +
2*a*d) - 4*B*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 +
d*x/2)**2 + 2*a*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a),
True))
```

Giac [A] time = 1.17866, size = 167, normalized size = 1.86

$$-\frac{\frac{(dx+c)(2A-3B)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*((d*x + c)*(2*A - 3*B)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
 1/2*c))/a - 2*(2*A*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c)^3 + 2
*A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 + 1)^2*a))/d
```

3.41 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=54

$$-\frac{(A-B)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(A-B)}{a}+\frac{B\sin(c+dx)}{ad}$$

```
[Out] ((A - B)*x)/a + (B*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x])/(a*d*(1 + C
os[c + d*x]))
```

Rubi [A] time = 0.138921, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2968, 3023, 12, 2735, 2648}

$$-\frac{(A-B)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(A-B)}{a}+\frac{B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((A - B)*x)/a + (B*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x])/(a*d*(1 + C
os[c + d*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx\\
&= \frac{B\sin(c+dx)}{ad}+\frac{\int \frac{a(A-B)\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\
&= \frac{B\sin(c+dx)}{ad}+(A-B)\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx\\
&= \frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}+(-A+B)\int \frac{1}{a+a\cos(c+dx)}\,dx\\
&= \frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [B] time = 0.231352, size = 126, normalized size = 2.33

$$
\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2dx(A-B)\cos\left(c+\frac{dx}{2}\right)+2dx(A-B)\cos\left(\frac{dx}{2}\right)-4A\sin\left(\frac{dx}{2}\right)+B\sin\left(c+\frac{dx}{2}\right)+B\sin\left(c+\frac{3dx}{2}\right.\right.}{2ad(\cos(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(2*(A - B)*d*x*Cos[(d*x)/2] + 2*(A - B)*d*x*Cos[
c + (d*x)/2] - 4*A*Sin[(d*x)/2] + 5*B*Sin[(d*x)/2] + B*Sin[c + (d*x)/2] + B
```

```
*Sin[c + (3*d*x)/2] + B*Sin[2*c + (3*d*x)/2]))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.061, size = 108, normalized size = 2.

$$-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{B\tan\left(1/2\,dx+c/2\right)}{da\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)}+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)A}{da}-2\,\frac{\arctan\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*A*tan(1/2*d*x+1/2*c)+1/d/a*B*tan(1/2*d*x+1/2*c)+2/d/a*B*tan(1/2*d*x+
1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/a*arctan(tan(1/2*d*x+1/2*c))*A-2/d/a*ar
ctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.61438, size = 193, normalized size = 3.57

$$-\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -(B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.33714, size = 147, normalized size = 2.72

$$\frac{(A-B)dx\cos\left(dx+c\right)+(A-B)dx+\left(B\cos\left(dx+c\right)-A+2\,B\right)\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] ((A - B)*d*x*cos(d*x + c) + (A - B)*d*x + (B*cos(d*x + c) - A + 2*B)*sin(d*
x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 2.18575, size = 264, normalized size = 4.89

$$\begin{cases} \frac{Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{Adx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{Bdx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{B\tan^3}{ad\tan^2(} \\ \frac{x(A+B\cos(c))\cos(c)}{a\cos(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + A*d*
x/(a*d*tan(c/2 + d*x/2)**2 + a*d) - A*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*
x/2)**2 + a*d) - A*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*d*x
*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*d*x/(a*d*tan(c/2 +
 d*x/2)**2 + a*d) + B*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) +
 3*B*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*(A + B
*cos(c))*cos(c)/(a*cos(c) + a), True))
```

Giac [A] time = 1.21337, size = 105, normalized size = 1.94

$$\frac{\frac{(dx+c)(A-B)}{a} - \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} + \frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a
+ 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.42 $\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Bx}{a}$$

```
[Out] (B*x)/a + ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0498775, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2735, 2648}

$$\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Bx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (B*x)/a + ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{Bx}{a}-(-A+B)\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= \frac{Bx}{a}+\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.11739, size = 72, normalized size = 2.12

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\sin\left(\frac{dx}{2}\right)+Bdx\cos\left(c+\frac{dx}{2}\right)+Bdx\cos\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(B*d*x*Cos[(d*x)/2] + B*d*x*Cos[c + (d*x)/2] + 2
*(A - B)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.053, size = 56, normalized size = 1.7

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)B}{da}-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*A*tan(1/2*d*x+1/2*c)+2/d/a*arctan(tan(1/2*d*x+1/2*c))*B-1/d/a*B*tan(1
/2*d*x+1/2*c)
```

Maxima [B] time = 1.50053, size = 99, normalized size = 2.91

$$\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] (B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + A*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.31193, size = 105, normalized size = 3.09

$$\frac{Bdx\cos(dx+c)+Bdx+(A-B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] (B*d*x*cos(d*x + c) + B*d*x + (A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [A] time = 1.10818, size = 49, normalized size = 1.44

$$\begin{cases} \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad}+\frac{Bx}{a}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise(((A*tan(c/2 + d*x/2)/(a*d) + B*x/a - B*tan(c/2 + d*x/2)/(a*d), Ne(
d, 0)), (x*(A + B*cos(c))/(a*cos(c) + a), True))
```

Giac [A] time = 1.16723, size = 58, normalized size = 1.71

$$\frac{\frac{(dx+c)B}{a}+\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

3.43 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rubi [A] time = 0.0783777, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int aA\sec(c+dx)\,dx}{a^2}\\ &= -\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{A\int\sec(c+dx)\,dx}{a}\\ &= \frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.238312, size = 109, normalized size = 2.48

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((B-A)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+A\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(A*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-A + B)*Sec[c/2]*Sin
[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.082, size = 78, normalized size = 1.8

$$-\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*A*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a*A*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a*A*
tan(1/2*d*x+1/2*c)+1/d/a*B*tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 1.01454, size = 134, normalized size = 3.05

$$\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] (A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.38954, size = 197, normalized size = 4.48

$$\frac{(A\cos(dx+c)+A)\log(\sin(dx+c)+1)-(A\cos(dx+c)+A)\log(-\sin(dx+c)+1)-2(A-B)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - (A*cos(d*x + c) + A)*log(
-sin(d*x + c) + 1) - 2*(A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec(c+dx)}{\cos(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*s
ec(c + d*x)/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.22654, size = 96, normalized size = 2.18

$$\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

3.44 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] -(((A - B)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A - B)*Tan[c + d*x])/(a*d) -
 ((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.156232, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((A - B)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A - B)*Tan[c + d*x])/(a*d) -
 ((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int (a(2A-B)-a(A-B)\cos(c+dx))\sec^2(c+dx)\,dx}{a^2}\\ &= -\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-B)\int \sec(c+dx)\,dx}{a}+\frac{(2A-B)\int \sec^2(c+dx)\,dx}{a}\\ &= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A-B)\,\text{Subst}(\int 1\,dx}{ad}\\ &= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 1.10028, size = 201, normalized size = 2.91

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\right.\right.\right.\right.}{ad(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((A -
 B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
```

```
(c + d*x)/2]]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
))))/(a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.106, size = 163, normalized size = 2.4

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{B}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*A*tan(1/2*d*x+1/2*c)-1/d/a*B*tan(1/2*d*x+1/2*c)-1/d/a*A/(tan(1/2*d*x+
1/2*c)-1)+1/d/a*A*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a*ln(tan(1/2*d*x+1/2*c)-1)*B
-1/d/a*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a*ln(t
an(1/2*d*x+1/2*c)+1)*B
```

Maxima [B] time = 1.01823, size = 265, normalized size = 3.84

$$-\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\right.}{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.39676, size = 320, normalized size = 4.64

$$-\frac{\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*(((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(sin(d*x + c) + 1)
 - ((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
 2*((2*A - B)*cos(d*x + c) + A)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**2/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.26591, size = 149, normalized size = 2.16

$$-\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (A - B)*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a +
2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.45 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{2(A-B)\tan(c+dx)}{ad}+\frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-2B)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B)\tan(c+dx)\sec}{d(a\cos(c+dx)+}$$

```
[Out] ((3*A - 2*B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*(A - B)*Tan[c + d*x])/(a*d
) + ((3*A - 2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.169136, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(A-B)\tan(c+dx)}{ad}+\frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-2B)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B)\tan(c+dx)\sec}{d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((3*A - 2*B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*(A - B)*Tan[c + d*x])/(a*d
) + ((3*A - 2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int (a(3A-2B)-2a(A-B)\cos(c+dx))\,\mathrm{s}}{a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(3A-2B)\int \sec^3(c+dx)\,dx}{a} - \frac{(2(A-B)}{}$$

$$= \frac{(3A-2B)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(3A-}{}$$

$$= \frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{2(A-B)\tan(c+dx)}{ad} + \frac{(3A-2B)\sec(c+}{2ad}$$

Mathematica [B] time = 3.0809, size = 289, normalized size = 2.7

$$\cos\left(\frac{1}{2}(c+dx)\right)\left(4(B-A)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4(A-B)\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(\right.\right.}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(4*(-A + B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-6
*A + 4*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*A*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] - 4*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + A/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2 - (4*(A - B)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)))))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.115, size = 252, normalized size = 2.4

$$-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{3\,A}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{B}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*A*tan(1/2*d*x+1/2*c)+1/d/a*B*tan(1/2*d*x+1/2*c)+1/2/d/a*A/(tan(1/2*d
*x+1/2*c)-1)^2+3/2/d/a*A/(tan(1/2*d*x+1/2*c)-1)-1/d/a/(tan(1/2*d*x+1/2*c)-1
)*B-3/2/d/a*A*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a*ln(tan(1/2*d*x+1/2*c)-1)*B-1/2
/d/a*A/(tan(1/2*d*x+1/2*c)+1)^2+3/2/d/a*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a/(tan
(1/2*d*x+1/2*c)+1)*B+3/2/d/a*A*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a*ln(tan(1/2*d*
x+1/2*c)+1)*B
```

Maxima [B] time = 1.02482, size = 381, normalized size = 3.56

$$-\frac{A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.37279, size = 385, normalized size = 3.6

$$\frac{\left((3\,A-2\,B)\cos\left(dx+c\right)^{3}+(3\,A-2\,B)\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((3\,A-2\,B)\cos\left(dx+c\right)^{3}+(3\,A-2\,B)\,\mathrm{co}\right.}{4\left(ad\cos\left(dx+c\right)^{3}+ad\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - ((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(4*(A - B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c) -
A)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{3}(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^{3}(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**3/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.25401, size = 212, normalized size = 1.98

$$\frac{\frac{(3\,A-2\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{(3\,A-2\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}+\frac{2\left(3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,B\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((3*A - 2*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (3*A - 2*B)*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
 1/2*c))/a + 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - A
*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
- 1)^2*a))/d
```

3.46 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{(4A-3B)\tan^3(c+dx)}{3ad}+\frac{(4A-3B)\tan(c+dx)}{ad}-\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3(A-B)\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] (-3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A - 3*B)*Tan[c + d*x])/(a*
d) - (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]^
2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A - 3*B)*Tan[c + d*x]^3)/(3*
a*d)
```

Rubi [A] time = 0.17956, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 3768, 3770}

$$\frac{(4A-3B)\tan^3(c+dx)}{3ad}+\frac{(4A-3B)\tan(c+dx)}{ad}-\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3(A-B)\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```


```
[Out] (-3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A - 3*B)*Tan[c + d*x])/(a*
d) - (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]^
2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A - 3*B)*Tan[c + d*x]^3)/(3*
a*d)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int (a(4A-3B)-3a(A-B)\cos(c+dx))\,\mathrm{se}}{a^2}$$

$$= -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{(4A-3B)\int \sec^4(c+dx)\,dx}{a}-\frac{(3(A-B))}{}$$

$$= -\frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3(A}{}$$

$$= -\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(4A-3B)\tan(c+dx)}{ad}-\frac{3(A-B)\sec(c+d}{2ad}$$

Mathematica [B] time = 4.11443, size = 490, normalized size = 3.74

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^3(c+dx)\left(6(A+B)\sin\left(\frac{dx}{2}\right)+3(13A-9B)\sin\left(\frac{3dx}{2}\right)-24A\sin\left(c-\frac{dx}{2}\right)-6A\sin\left(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(144*(A - B)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[c
]*Sec[c + d*x]^3*(6*(A + B)*Sin[(d*x)/2] + 3*(13*A - 9*B)*Sin[(3*d*x)/2] -
24*A*Sin[c - (d*x)/2] + 12*B*Sin[c - (d*x)/2] - 6*A*Sin[c + (d*x)/2] + 6*B*
Sin[c + (d*x)/2] - 24*A*Sin[2*c + (d*x)/2] + 24*B*Sin[2*c + (d*x)/2] + 21*A
*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3*d*x)/2] - 9
*B*Sin[2*c + (3*d*x)/2] - 9*A*Sin[3*c + (3*d*x)/2] + 9*B*Sin[3*c + (3*d*x)/
2] + 7*A*Sin[c + (5*d*x)/2] - 3*B*Sin[c + (5*d*x)/2] + A*Sin[2*c + (5*d*x)/
2] + 3*B*Sin[2*c + (5*d*x)/2] - 3*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin[3*c + (5
*d*x)/2] - 9*A*Sin[4*c + (5*d*x)/2] + 9*B*Sin[4*c + (5*d*x)/2] + 16*A*Sin[2
*c + (7*d*x)/2] - 12*B*Sin[2*c + (7*d*x)/2] + 10*A*Sin[3*c + (7*d*x)/2] - 6
*B*Sin[3*c + (7*d*x)/2] + 6*A*Sin[4*c + (7*d*x)/2] - 6*B*Sin[4*c + (7*d*x)/
2])))/(48*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.104, size = 340, normalized size = 2.6

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}+\frac{B}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*A*tan(1/2*d*x+1/2*c)-1/d/a*B*tan(1/2*d*x+1/2*c)-1/3/d/a*A/(tan(1/2*d*
x+1/2*c)-1)^3+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2*B-1/d/a*A/(tan(1/2*d*x+1/2*c
)-1)^2+3/2/d/a*A*ln(tan(1/2*d*x+1/2*c)-1)-3/2/d/a*ln(tan(1/2*d*x+1/2*c)-1)*
B-5/2/d/a*A/(tan(1/2*d*x+1/2*c)-1)+3/2/d/a/(tan(1/2*d*x+1/2*c)-1)*B-1/3/d/a
*A/(tan(1/2*d*x+1/2*c)+1)^3+1/d/a*A/(tan(1/2*d*x+1/2*c)+1)^2-1/2/d/a/(tan(1
/2*d*x+1/2*c)+1)^2*B-3/2/d/a*A*ln(tan(1/2*d*x+1/2*c)+1)+3/2/d/a*ln(tan(1/2*
d*x+1/2*c)+1)*B-5/2/d/a*A/(tan(1/2*d*x+1/2*c)+1)+3/2/d/a/(tan(1/2*d*x+1/2*c
)+1)*B
```

Maxima [B] time = 1.32503, size = 497, normalized size = 3.79

$$\frac{A\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\cdots\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\cdots}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(A*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.45431, size = 419, normalized size = 3.2

$$-\frac{9\left((A-B)\cos(dx+c)^4+(A-B)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-9\left((A-B)\cos(dx+c)^4+(A-B)\cos(dx+c)^3\right.}{12\left(ad\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/12*(9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(sin(d*x + c)
 + 1) - 9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(-sin(d*x +
c) + 1) - 2*(4*(4*A - 3*B)*cos(d*x + c)^3 + (7*A - 3*B)*cos(d*x + c)^2 - (A
 - 3*B)*cos(d*x + c) + 2*A)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x
 + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

[Out] Timed out

Giac [A] time = 1.18941, size = 246, normalized size = 1.88

$$-\frac{\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-9B\tan\left(\frac{1}{2}d\right.\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(A - B)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/
2*c))/a + 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 - 16*
A*tan(1/2*d*x + 1/2*c)^3 + 12*B*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/2*d*x +
1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.47 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$\frac{4(2A-3B)\sin^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(7A-10B)\sin(c+dx)\cos(c}{2a^2d}$$

```
[Out] ((7*A - 10*B)*x)/(2*a^2) - (4*(2*A - 3*B)*Sin[c + d*x])/(a^2*d) + ((7*A - 1
0*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((7*A - 10*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(2*A - 3*B)*Sin[c + d*x]^3)/(3*a^2*
d)
```

Rubi [A] time = 0.322422, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2635, 8, 2633}

$$\frac{4(2A-3B)\sin^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(7A-10B)\sin(c+dx)\cos(c}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] ((7*A - 10*B)*x)/(2*a^2) - (4*(2*A - 3*B)*Sin[c + d*x])/(a^2*d) + ((7*A - 1
0*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((7*A - 10*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(2*A - 3*B)*Sin[c + d*x]^3)/(3*a^2*
d)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-3a(A-2B)\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= \frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \mathrm{c}}{}$$

$$= \frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(7A}{}$$

$$= \frac{(7A-10B)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \cdots$$

$$= \frac{(7A-10B)x}{2a^2} - \frac{4(2A-3B)\sin(c+dx)}{a^2d} + \frac{(7A-10B)\cos(c+dx)\sin(c+dx)}{2a^2d}$$

Mathematica [B] time = 0.617073, size = 369, normalized size = 2.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(7A-10B)\cos\left(c+\frac{dx}{2}\right)+36dx(7A-10B)\cos\left(\frac{dx}{2}\right)+147A\sin\left(c+\frac{dx}{2}\right)-239A\sin\left(c+\frac{3d}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*A - 10*B)*d*x*Cos[(d*x)/2] + 36*(7*A - 10
*B)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 120*B*d*x*Cos[c +
(3*d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] - 120*B*d*x*Cos[2*c + (3*d*x)/2]
 - 381*A*Sin[(d*x)/2] + 516*B*Sin[(d*x)/2] + 147*A*Sin[c + (d*x)/2] - 156*B
*Sin[c + (d*x)/2] - 239*A*Sin[c + (3*d*x)/2] + 342*B*Sin[c + (3*d*x)/2] - 6
3*A*Sin[2*c + (3*d*x)/2] + 118*B*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c + (5*d
*x)/2] + 30*B*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*c + (5*d*x)/2] + 30*B*Sin[3
*c + (5*d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] - 3*B*Sin[3*c + (7*d*x)/2] + 3*A
*Sin[4*c + (7*d*x)/2] - 3*B*Sin[4*c + (7*d*x)/2] + B*Sin[4*c + (9*d*x)/2] +
 B*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.066, size = 322, normalized size = 1.9

$$\frac{A}{6\,a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6\,a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{2\,a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{9\,B}{2\,a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\,\frac{(\tan(1/2\,dx+c/2}{a^2d\left(1+(\tan(1/2\,dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+9/2/d/a^2*B*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+10/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)^5*B-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3
*A+40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^3-3/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+6/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^3*B*tan(1/2*d*x+1/2*c)+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A-10/d/a^2*arc
tan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 2.01015, size = 502, normalized size = 2.95

$$\frac{B\left(\frac{4\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{6\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 1.38919, size = 387, normalized size = 2.28

$$\frac{3\,(7\,A-10\,B)dx\cos(dx+c)^2+6\,(7\,A-10\,B)dx\cos(dx+c)+3\,(7\,A-10\,B)dx+\left(2\,B\cos(dx+c)^4+(3\,A-2\,B)\cos(\right.}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(7*A - 10*B)*d*x*cos(d*x + c)^2 + 6*(7*A - 10*B)*d*x*cos(d*x + c) +
3*(7*A - 10*B)*d*x + (2*B*cos(d*x + c)^4 + (3*A - 2*B)*cos(d*x + c)^3 - 6*(
A - 2*B)*cos(d*x + c)^2 - (43*A - 66*B)*cos(d*x + c) - 32*A + 48*B)*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 21.0575, size = 1425, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((21*A*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*
a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63
*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/
2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*A*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4
 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*A*d*x/(6*a**2*d*tan(c/2 +
 d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
+ 6*a**2*d) + A*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2
*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 18*A*t
an(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/
2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*A*tan(c/2 + d*x/2)**
5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 110*A*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(
c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2
)**2 + 6*a**2*d) - 39*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 3
0*B*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*d*x*tan(c
/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*d*x*tan(c/2 + d*x/2)**
2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 +
 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
- B*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*B*tan(c/2 + d*x
/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*B*tan(c/2 + d*x/2)**5/(6*a**2*d
*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 +
d*x/2)**2 + 6*a**2*d) + 160*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2
)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 63*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan
(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x
*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.19495, size = 259, normalized size = 1.52

$$\frac{\frac{3(dx+c)(7A-10B)}{a^2} - \frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 30B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 24A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 40B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 9A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 18B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*A - 10*B)/a^2 - 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 30*B*t
an(1/2*d*x + 1/2*c)^5 + 24*A*tan(1/2*d*x + 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/
2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) - 18*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*
x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c
))/a^6)/d
```

3.48 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$\frac{2(5A-8B)\sin(c+dx)}{3a^2d}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(4A-7B)}{2a^2}+\frac{(A-}{}$$

```
[Out] -((4*A - 7*B)*x)/(2*a^2) + (2*(5*A - 8*B)*Sin[c + d*x])/(3*a^2*d) - ((4*A -
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((5*A - 8*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^3*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.341251, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{2(5A-8B)\sin(c+dx)}{3a^2d}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(4A-7B)}{2a^2}+\frac{(A-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] -((4*A - 7*B)*x)/(2*a^2) + (2*(5*A - 8*B)*Sin[c + d*x])/(3*a^2*d) - ((4*A -
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((5*A - 8*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^3*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(2A-5B)\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= \frac{(5A-8B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \text{co}$$

$$= -\frac{(4A-7B)x}{2a^2} + \frac{2(5A-8B)\sin(c+dx)}{3a^2d} - \frac{(4A-7B)\cos(c+dx)\sin(c+dx)}{2a^2d}$$

Mathematica [B] time = 0.756869, size = 315, normalized size = 2.14

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(4A-7B)\cos\left(c+\frac{dx}{2}\right)-36dx(4A-7B)\cos\left(\frac{dx}{2}\right)-120A\sin\left(c+\frac{dx}{2}\right)+164A\sin\left(c+\frac{3}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(4*A - 7*B)*d*x*Cos[(d*x)/2] - 36*(4*A - 7*
B)*d*x*Cos[c + (d*x)/2] - 48*A*d*x*Cos[c + (3*d*x)/2] + 84*B*d*x*Cos[c + (3
*d*x)/2] - 48*A*d*x*Cos[2*c + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] +
264*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] - 120*A*Sin[c + (d*x)/2] + 147*B*Si
n[c + (d*x)/2] + 164*A*Sin[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 36*A
*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 12*A*Sin[2*c + (5*d*x)/
2] - 15*B*Sin[2*c + (5*d*x)/2] + 12*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[3*c +
 (5*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2]))/(48*a^2
*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.061, size = 252, normalized size = 1.7

$$-\frac{A}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5A}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7B}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\frac{B(\tan(1/2\,dx}{a^2d\left(1+(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+5/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)^3+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan
(1/2*d*x+1/2*c)^3*A-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)
+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-4/d/a^2*arctan(tan
(1/2*d*x+1/2*c))*A+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.95323, size = 382, normalized size = 2.6

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))))/d
```

Fricas [A] time = 1.38781, size = 343, normalized size = 2.33

$$-\frac{3(4A-7B)dx\cos(dx+c)^2+6(4A-7B)dx\cos(dx+c)+3(4A-7B)dx-\left(3B\cos(dx+c)^3+6(A-B)\cos(dx+\right.}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(4*A - 7*B)*d*x*cos(d*x + c)^2 + 6*(4*A - 7*B)*d*x*cos(d*x + c) + 3
*(4*A - 7*B)*d*x - (3*B*cos(d*x + c)^3 + 6*(A - B)*cos(d*x + c)^2 + (28*A -
 43*B)*cos(d*x + c) + 20*A - 32*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 12.3027, size = 843, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12
*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 24*A*d*x*tan(c/2 + d*x/2)**2/(6*a
**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*
A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) - A*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 13*A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 +
d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 41*A*tan(c/2 + d*x/
2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) + 27*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 21*B*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/
2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*B*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) + 21*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) + B*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*B*tan(c/2 + d*x/2)**5/(6
*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 7
1*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) - 39*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(A + B*cos(c))
*cos(c)**3/(a*cos(c) + a)**2, True))
```

<u>Giac</u> [A] time = 1.24459, size = 221, normalized size = 1.5

$$-\frac{\frac{3\,(dx+c)(4\,A-7\,B)}{a^2}-\frac{6\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(d*x + c)*(4*A - 7*B)/a^2 - 6*(2*A*tan(1/2*d*x + 1/2*c)^3 - 5*B*tan
(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/
((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^
4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2
*d*x + 1/2*c))/a^6)/d
```

3.49 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=99

$$-\frac{(A-4B)\sin(c+dx)}{3a^2d}-\frac{(A-2B)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(A-2B)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] ((A - 2*B)*x)/a^2 - ((A - 4*B)*Sin[c + d*x])/(3*a^2*d) - ((A - 2*B)*Sin[c +
 d*x])/(a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.275746, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(A-4B)\sin(c+dx)}{3a^2d}-\frac{(A-2B)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(A-2B)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] ((A - 2*B)*x)/a^2 - ((A - 4*B)*Sin[c + d*x])/(3*a^2*d) - ((A - 2*B)*Sin[c +
 d*x])/(a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(2a(A-B)-a(A-4B)\cos(c+dx))}{a+a\cos(c+dx)}dx}{3a^2}$$
$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{2a(A-B)\cos(c+dx)-a(A-4B)\cos^2(c+dx)}{a+a\cos(c+dx)}dx}{3a^2}$$
$$= -\frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{3a^2(A-2B)\cos(c+}{a+a\cos(c+dx)}}{3a^3}$$
$$= -\frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A-2B)\int \frac{\cos(}{a+a\,\mathrm{co}}}{a}$$
$$= \frac{(A-2B)x}{a^2} - \frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(A-}{}$$
$$= \frac{(A-2B)x}{a^2} - \frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(A-}{d\left(a^2\right.}$$

Mathematica [A] time = 0.677156, size = 137, normalized size = 1.38

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6\cos^3\left(\frac{1}{2}(c+dx)\right)(dx(A-2B)+B\sin(c+dx))+(A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\sec\left(\frac{c}{2}\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] - 2*(5*A - 8*B)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((A - 2*B)*d*x + B*
Sin[c + d*x]) + (A - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d
*x])^2)
```

Maple [A] time = 0.079, size = 149, normalized size = 1.5

$$\frac{A}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{B}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{3A}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{5B}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\frac{B\tan(1/2\,dx+}{a^2d\left(1+(\tan(1/2\,dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*B*tan(1/2*d*x+
1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A-4/d/a^
2*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.89457, size = 258, normalized size = 2.61

$$\frac{B\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-A\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 1.36591, size = 294, normalized size = 2.97

$$\frac{3\,(A-2\,B)dx\cos{(dx+c)}^2+6\,(A-2\,B)dx\cos{(dx+c)}+3\,(A-2\,B)dx+\left(3\,B\cos{(dx+c)}^2-(5\,A-14\,B)\cos{(dx+c)}-\right.}{3\left(a^2d\cos{(dx+c)}^2+2\,a^2d\cos{(dx+c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*(A - 2*B)*d*x*cos(d*x + c)^2 + 6*(A - 2*B)*d*x*cos(d*x + c) + 3*(A -
 2*B)*d*x + (3*B*cos(d*x + c)^2 - (5*A - 14*B)*cos(d*x + c) - 4*A + 10*B)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 7.01014, size = 411, normalized size = 4.15

$$\begin{cases} \frac{6Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{6Adx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{8A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{9A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{12Bdx\tan}{6a^2d\tan^2} \\ \frac{x(A+B\cos(c))\cos^2(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((6*A*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 6*A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + A*tan(c/2 + d*x
/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 8*A*tan(c/2 + d*x/2)**3/
(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 9*A*tan(c/2 + d*x/2)/(6*a**2*d*
tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*ta
n(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*
a**2*d) - B*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) +
 14*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*B*
tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(
A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.26666, size = 161, normalized size = 1.63

$$\frac{\frac{6(dx+c)(A-2B)}{a^2} + \frac{12B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^2} + \frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 9Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 15Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*(A - 2*B)/a^2 + 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/
2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)
/d
```

3.50 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{(2A-5B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Bx}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (B*x)/a^2 + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A -
 B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.155169, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3019, 2735, 2648}

$$\frac{(2A-5B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Bx}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/a^2 + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A -
 B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx \\
&= -\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{-2a(A-B)-3aB\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2} \\
&= \frac{Bx}{a^2} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2A-5B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a} \\
&= \frac{Bx}{a^2} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2A-5B)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}
\end{aligned}
$$

Mathematica [B] time = 0.333466, size = 153, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-6A\sin\left(c+\frac{dx}{2}\right)+4A\sin\left(c+\frac{3dx}{2}\right)+6A\sin\left(\frac{dx}{2}\right)+12B\sin\left(c+\frac{dx}{2}\right)-10B\sin\left(c+\frac{3dx}{2}\right)+9\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*B*d*x*Cos[(d*x)/2] + 9*B*d*x*Cos[c + (d*x)/
2] + 3*B*d*x*Cos[c + (3*d*x)/2] + 3*B*d*x*Cos[2*c + (3*d*x)/2] + 6*A*Sin[(d
*x)/2] - 18*B*Sin[(d*x)/2] - 6*A*Sin[c + (d*x)/2] + 12*B*Sin[c + (d*x)/2] +
 4*A*Sin[c + (3*d*x)/2] - 10*B*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.052, size = 97, normalized size = 1.4

$$-\frac{A}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3B}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{\arctan(\tan(1/2\,d}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+1/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/
2*d*x+1/2*c))*B
```

Maxima [A] time = 1.8467, size = 162, normalized size = 2.31

$$-\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - A*(3*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.35865, size = 228, normalized size = 3.26

$$\frac{3\,Bdx\cos(dx+c)^2+6\,Bdx\cos(dx+c)+3\,Bdx+((2\,A-5\,B)\cos(dx+c)+A-4\,B)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(3*B*d*x*cos(d*x + c)^2 + 6*B*d*x*cos(d*x + c) + 3*B*d*x + ((2*A - 5*B)
*cos(d*x + c) + A - 4*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [A] time = 3.51063, size = 105, normalized size = 1.5

$$\begin{cases} -\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}+\frac{Bx}{a^2}+\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}-\frac{3B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d
) + B*x/a**2 + B*tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*B*tan(c/2 + d*x/2)/(2*a
**2*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.2418, size = 116, normalized size = 1.66

$$\frac{\frac{6(dx+c)B}{a^2}-\frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+9Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(6*(d*x + c)*B/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 3*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.51 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(A+2B)\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((A + 2*B)*Sin[c + d*
x])/(3*d*(a^2 + a^2*Cos[c + d*x]))
```

Rubi [A] time = 0.0537473, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2750, 2648}

$$\frac{(A+2B)\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((A + 2*B)*Sin[c + d*
x])/(3*d*(a^2 + a^2*Cos[c + d*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A+2B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$= \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A+2B)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [A] time = 0.168725, size = 76, normalized size = 1.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((A+2B)\sin\left(c+\frac{3dx}{2}\right)+3(A+B)\sin\left(\frac{dx}{2}\right)-3B\sin\left(c+\frac{dx}{2}\right)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(3*(A + B)*Sin[(d*x)/2] - 3*B*Sin[c + (d*x)/2] +
 (A + 2*B)*Sin[c + (3*d*x)/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.045, size = 60, normalized size = 0.9

$$\frac{1}{2a^2d}\left(\frac{A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3*A-1/3*B*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*
d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.2343, size = 126, normalized size = 1.94

$$\frac{\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}+\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(A*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 + B*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.36321, size = 144, normalized size = 2.22

$$\frac{((A+2B)\cos(dx+c)+2A+B)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((A + 2*B)*cos(d*x + c) + 2*A + B)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 2.85249, size = 94, normalized size = 1.45

$$\begin{cases} \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d)
 - B*tan(c/2 + d*x/2)**3/(6*a**2*d) + B*tan(c/2 + d*x/2)/(2*a**2*d), Ne(d,
0)), (x*(A + B*cos(c))/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.21363, size = 81, normalized size = 1.25

$$\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(A*tan(1/2*d*x + 1/2*c)^3 - B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x
+ 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.52 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{(4A-B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*A - B)*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.179628, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$-\frac{(4A-B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*A - B)*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(3aA-a(A-B)\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int 3a^2A\sec(c+dx)\,dx}{3a^4}$$
$$= -\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{A\int \sec(c+dx)\,dx}{a^2}$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

Mathematica [B] time = 0.487366, size = 170, normalized size = 2.15

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(4A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^2\left(\frac{1}{2}(c+\right.\right.}{3a^2d(\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*A*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A - B)*Sec[c/2]*
Sin[(d*x)/2] + 2*(4*A - B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (A -
B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.082, size = 119, normalized size = 1.5

$$-\frac{A}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3A}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-3/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*B*tan(1/2*d*x+1/2*c)-1/d/a^2*A*ln(tan(1/2*
d*x+1/2*c)-1)+1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.26967, size = 196, normalized size = 2.48

$$-\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.37993, size = 338, normalized size = 4.28

$$\frac{3\left(A\cos(dx+c)^2+2\,A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-3\left(A\cos(dx+c)^2+2\,A\cos(dx+c)+A\right)\log(-\sin(d}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 3*
(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*((4*A
- B)*cos(d*x + c) + 5*A - 2*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + Integ
ral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x))
/a**2
```

Giac [A] time = 1.23498, size = 153, normalized size = 1.94

$$\frac{\frac{6A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{6A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} - \frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 9Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 3Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*A*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2
*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 3*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.53 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2} dx$

Optimal. Leaf size=107

$$\frac{2(5A-2B)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -(((2*A - B)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + (2*(5*A - 2*B)*Tan[c + d*x])
/(3*a^2*d) - ((2*A - B)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A - B)
*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.295281, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{2(5A-2B)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```



```
[Out] -(((2*A - B)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + (2*(5*A - 2*B)*Tan[c + d*x])
/(3*a^2*d) - ((2*A - B)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A - B)
*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(4A-B)-2a(A-B)\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(2a^2(5A-2B)-3a^2(2A\right.}{}$$

$$= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2(5A-2B))\int \sec^2(c+d}{3a^2}$$

$$= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\tan(c+d}{3d(a+a\cos(c+d}$$

$$= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{2(5A-2B)\tan(c+dx)}{3a^2d} - \frac{(2A-B)\tan(c}{a^2d(1+\cos(c+}$$

Mathematica [B] time = 1.51164, size = 264, normalized size = 2.47

$$2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left((2A-B)\left(\log\left(\cos\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + 2*(7*A - 4*B)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((2*A - B)*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (A - B)
*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.095, size = 205, normalized size = 1.9

$$\frac{A}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5A}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3B}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{A\ln(\tan(1/2\,dx+c/2}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*A*ln(tan(1/2*d
*x+1/2*c)-1)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^2*A/(tan(1/2*d*x+1/2*
c)-1)-2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B
-1/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.04919, size = 329, normalized size = 3.07

$$\frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
```

```
c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2))/d
```

Fricas [B] time = 1.46332, size = 502, normalized size = 4.69

$$-\frac{3\left((2A-B)\cos(dx+c)^3+2(2A-B)\cos(dx+c)^2+(2A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left((2A-B)\cos\right.}{6\left(a^2d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A - B)*cos(d*x + c)^2 + (2*A - B)*
cos(d*x + c))*log(sin(d*x + c) + 1) - 3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A
- B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2
*(5*A - 2*B)*cos(d*x + c)^2 + (14*A - 5*B)*cos(d*x + c) + 3*A)*sin(d*x + c)
)/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x))/a**2
```

Giac [A] time = 1.21405, size = 209, normalized size = 1.95

$$-\frac{\frac{6(2A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{6(2A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{12A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}-\frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15Aa}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(2*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(2*A - B)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 - 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) - 9*B*a^4*tan(1/2*d*x + 1/2*c))/
a^6)/d
```

3.54 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{2(8A-5B)\tan(c+dx)}{3a^2d}+\frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-4B)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8A-5B)\tan(c+dx)}{3a^2d(\cos(c+dx)+1)}$$

```
[Out] ((7*A - 4*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*A - 5*B)*Tan[c + d*x]
)/(3*a^2*d) + ((7*A - 4*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((8*A - 5
*B)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*Sec[
c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.313663, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(8A-5B)\tan(c+dx)}{3a^2d}+\frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-4B)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8A-5B)\tan(c+dx)}{3a^2d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] ((7*A - 4*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*A - 5*B)*Tan[c + d*x]
)/(3*a^2*d) + ((7*A - 4*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((8*A - 5
*B)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*Sec[
c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(5A-2B)-3a(A-B)\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int (3a\cdots}{}$$

$$= -\frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(2(8A\cdots}{}$$

$$= \frac{(7A-4B)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-\cdots}{}$$

$$= \frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{2(8A-5B)\tan(c+dx)}{3a^2d} + \frac{(7A-4B)\sec(c+\cdots}{2a^2\cdots}$$

Mathematica [B] time = 3.10649, size = 496, normalized size = 3.26

$$-\frac{96(7A-4B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(96*(7*A - 4*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]^2*(-14*(A - B)*Sin[(d*x)/2] + (97*A - 64*B)*Sin[(3*d*x)
/2] - 126*A*Sin[c - (d*x)/2] + 84*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (d*x)/2
] - 42*B*Sin[c + (d*x)/2] - 98*A*Sin[2*c + (d*x)/2] + 56*B*Sin[2*c + (d*x)/
2] - 3*A*Sin[c + (3*d*x)/2] + 6*B*Sin[c + (3*d*x)/2] + 37*A*Sin[2*c + (3*d*
x)/2] - 34*B*Sin[2*c + (3*d*x)/2] - 63*A*Sin[3*c + (3*d*x)/2] + 36*B*Sin[3*
c + (3*d*x)/2] + 75*A*Sin[c + (5*d*x)/2] - 48*B*Sin[c + (5*d*x)/2] + 15*A*S
in[2*c + (5*d*x)/2] - 6*B*Sin[2*c + (5*d*x)/2] + 39*A*Sin[3*c + (5*d*x)/2]
- 30*B*Sin[3*c + (5*d*x)/2] - 21*A*Sin[4*c + (5*d*x)/2] + 12*B*Sin[4*c + (5
*d*x)/2] + 32*A*Sin[2*c + (7*d*x)/2] - 20*B*Sin[2*c + (7*d*x)/2] + 12*A*Sin
[3*c + (7*d*x)/2] - 6*B*Sin[3*c + (7*d*x)/2] + 20*A*Sin[4*c + (7*d*x)/2] -
14*B*Sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.109, size = 294, normalized size = 1.9

$$-\frac{A}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7A}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5B}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7A}{2a^2d}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-7/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*d*x+1/2*c)-7/2/d/a^2*A*ln(tan(1/
2*d*x+1/2*c)-1)+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B+5/2/d/a^2*A/(tan(1/2*d*x
+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)
-1)^2+5/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B+7
/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/
d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.04438, size = 454, normalized size = 2.99

$$-\frac{A\left(\frac{6\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-B\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{\cdots}}{(\cos(dx+c)+\cdots}}{a^2}\cdots\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.44739, size = 564, normalized size = 3.71

$$\frac{3\left((7\,A-4\,B)\cos(dx+c)^4+2\,(7\,A-4\,B)\cos(dx+c)^3+(7\,A-4\,B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((7\,A-4\,B\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((7*A - 4*B)*cos(d*x + c)^4 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A -
 4*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((7*A - 4*B)*cos(d*x + c)^4
 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*log(-sin(d*x
+ c) + 1) - 2*(4*(8*A - 5*B)*cos(d*x + c)^3 + (43*A - 28*B)*cos(d*x + c)^2
+ 6*(A - B)*cos(d*x + c) - 3*A)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2
*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24595, size = 267, normalized size = 1.76

$$\frac{\frac{3(7A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{3(7A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2B\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(7*A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A - 4*B)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2*B
*tan(1/2*d*x + 1/2*c)^3 - 3*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 -
B*a^4*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^4*tan
(1/2*d*x + 1/2*c))/a^6)/d
```

3.55 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{4(3A-2B)\tan^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(10A-7B)\tan(c+dx)\sec(c+$$

```
[Out] -((10*A - 7*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (4*(3*A - 2*B)*Tan[c + d*
x])/(a^2*d) - ((10*A - 7*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((10*A -
 7*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*
Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(3*A - 2*B)*
Tan[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.364972, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 3768, 3770}

$$\frac{4(3A-2B)\tan^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(10A-7B)\tan(c+dx)\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] -((10*A - 7*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (4*(3*A - 2*B)*Tan[c + d*
x])/(a^2*d) - ((10*A - 7*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((10*A -
 7*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*
Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(3*A - 2*B)*
Tan[c + d*x]^3)/(3*a^2*d)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(3a(2A-B)-4a(A-B)\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\
&= -\frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\\
&= -\frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}-\\
&= -\frac{(10A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}\\
&= -\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-7B)\mathrm{se}}{}\end{aligned}$$

Mathematica [B] time = 4.84746, size = 609, normalized size = 3.4

$$\frac{192(10A-7B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (192*(10*A - 7*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]
*Sec[c]*Sec[c + d*x]^3*((-6*A + 45*B)*Sin[(d*x)/2] + (310*A - 201*B)*Sin[(3
*d*x)/2] - 306*A*Sin[c - (d*x)/2] + 195*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (
d*x)/2] - 51*B*Sin[c + (d*x)/2] - 270*A*Sin[2*c + (d*x)/2] + 189*B*Sin[2*c
+ (d*x)/2] + 50*A*Sin[c + (3*d*x)/2] - B*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c
+ (3*d*x)/2] - 81*B*Sin[2*c + (3*d*x)/2] - 170*A*Sin[3*c + (3*d*x)/2] + 119
*B*Sin[3*c + (3*d*x)/2] + 198*A*Sin[c + (5*d*x)/2] - 129*B*Sin[c + (5*d*x)/
2] + 42*A*Sin[2*c + (5*d*x)/2] - 9*B*Sin[2*c + (5*d*x)/2] + 66*A*Sin[3*c +
(5*d*x)/2] - 57*B*Sin[3*c + (5*d*x)/2] - 90*A*Sin[4*c + (5*d*x)/2] + 63*B*S
in[4*c + (5*d*x)/2] + 114*A*Sin[2*c + (7*d*x)/2] - 75*B*Sin[2*c + (7*d*x)/2
] + 36*A*Sin[3*c + (7*d*x)/2] - 15*B*Sin[3*c + (7*d*x)/2] + 48*A*Sin[4*c +
(7*d*x)/2] - 39*B*Sin[4*c + (7*d*x)/2] - 30*A*Sin[5*c + (7*d*x)/2] + 21*B*S
in[5*c + (7*d*x)/2] + 48*A*Sin[3*c + (9*d*x)/2] - 32*B*Sin[3*c + (9*d*x)/2]
 + 22*A*Sin[4*c + (9*d*x)/2] - 12*B*Sin[4*c + (9*d*x)/2] + 26*A*Sin[5*c + (
9*d*x)/2] - 20*B*Sin[5*c + (9*d*x)/2]))/(96*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.125, size = 382, normalized size = 2.1

$$\frac{A}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{9A}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7B}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3A}{2a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*d*x+1/2*c)-3/2/d/a^2*A/(tan(1/2*d
*x+1/2*c)-1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2*B+5/d/a^2*A*ln(tan(1/2*d*
x+1/2*c)-1)-7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B-5/d/a^2*A/(tan(1/2*d*x+1/2
*c)-1)+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B-1/3/d/a^2*A/(tan(1/2*d*x+1/2*c)-1
)^3-5/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B
```

```
+3/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2*B-
5/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)+5/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B-1/3/d/a
^2*A/(tan(1/2*d*x+1/2*c)+1)^3
```

Maxima [B] time = 1.04846, size = 574, normalized size = 3.21

$$\frac{A\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}\cdots\right.}{a^2-\frac{2a^2\sin\cdots}{(\cos\cdots}}\cdots\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*
(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3
/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

Fricas [A] time = 1.47871, size = 617, normalized size = 3.45

$$-\frac{3\left((10A-7B)\cos(dx+c)^5+2(10A-7B)\cos(dx+c)^4+(10A-7B)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-3\left((10\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/12*(3*((10*A - 7*B)*cos(d*x + c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (10
*A - 7*B)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((10*A - 7*B)*cos(d*x +
```

```
 c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (10*A - 7*B)*cos(d*x + c)^3)*log(-s
in(d*x + c) + 1) - 2*(16*(3*A - 2*B)*cos(d*x + c)^4 + (66*A - 43*B)*cos(d*x
 + c)^3 + 6*(2*A - B)*cos(d*x + c)^2 - (2*A - 3*B)*cos(d*x + c) + 2*A)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x +
c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20286, size = 305, normalized size = 1.7

$$-\frac{\frac{3\,(10\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\,(10\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\left(30\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,B \cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\cdots\right.}}{6\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(10*A - 7*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(10*A - 7*B
)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(30*A*tan(1/2*d*x + 1/2*c)^5 -
 15*B*tan(1/2*d*x + 1/2*c)^5 - 40*A*tan(1/2*d*x + 1/2*c)^3 + 24*B*tan(1/2*d
*x + 1/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan
(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*tan(1/2*d*x
+ 1/2*c))/a^6)/d
```

3.56 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$\frac{4(19A-34B)\sin^3(c+dx)}{15a^3d}-\frac{4(19A-34B)\sin(c+dx)}{5a^3d}+\frac{(13A-23B)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(13A-23B)\sin(c+}{2a^3d}$$

```
[Out] ((13*A - 23*B)*x)/(2*a^3) - (4*(19*A - 34*B)*Sin[c + d*x])/(5*a^3*d) + ((13
*A - 23*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^5*S
in[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((8*A - 13*B)*Cos[c + d*x]^4*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((13*A - 23*B)*Cos[c + d*x]^3
*Sin[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) + (4*(19*A - 34*B)*Sin[c + d*
x]^3)/(15*a^3*d)
```

Rubi [A] time = 0.515435, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2635, 8, 2633}

$$\frac{4(19A-34B)\sin^3(c+dx)}{15a^3d}-\frac{4(19A-34B)\sin(c+dx)}{5a^3d}+\frac{(13A-23B)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(13A-23B)\sin(c+}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] ((13*A - 23*B)*x)/(2*a^3) - (4*(19*A - 34*B)*Sin[c + d*x])/(5*a^3*d) + ((13
*A - 23*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^5*S
in[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((8*A - 13*B)*Cos[c + d*x]^4*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((13*A - 23*B)*Cos[c + d*x]^3
*Sin[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) + (4*(19*A - 34*B)*Sin[c + d*
x]^3)/(15*a^3*d)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(3A-8B)\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \text{c}}{}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(13}{}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(13}{}$$

$$= \frac{(13A-23B)\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A}{}$$

$$= \frac{(13A-23B)x}{2a^3} - \frac{4(19A-34B)\sin(c+dx)}{5a^3d} + \frac{(13A-23B)\cos(c+dx)\sin(c+}{2a^3d}$$

<u>Mathematica</u> [B] time = 0.908189, size = 491, normalized size = 2.25

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(600dx(13A-23B)\cos\left(c+\frac{dx}{2}\right)+600dx(13A-23B)\cos\left(\frac{dx}{2}\right)+7560A\sin\left(c+\frac{dx}{2}\right)-9230A\,\text{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*A - 23*B)*d*x*Cos[(d*x)/2] + 600*(13*A
- 23*B)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] - 6900*B*d*x*C
os[c + (3*d*x)/2] + 3900*A*d*x*Cos[2*c + (3*d*x)/2] - 6900*B*d*x*Cos[2*c +
(3*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] - 1380*B*d*x*Cos[2*c + (5*d*x)/
2] + 780*A*d*x*Cos[3*c + (5*d*x)/2] - 1380*B*d*x*Cos[3*c + (5*d*x)/2] - 127
60*A*Sin[(d*x)/2] + 20410*B*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/2] - 11110*
B*Sin[c + (d*x)/2] - 9230*A*Sin[c + (3*d*x)/2] + 15380*B*Sin[c + (3*d*x)/2]
 + 930*A*Sin[2*c + (3*d*x)/2] - 380*B*Sin[2*c + (3*d*x)/2] - 2782*A*Sin[2*c
 + (5*d*x)/2] + 4777*B*Sin[2*c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d*x)/2] +
1625*B*Sin[3*c + (5*d*x)/2] - 105*A*Sin[3*c + (7*d*x)/2] + 230*B*Sin[3*c +
(7*d*x)/2] - 105*A*Sin[4*c + (7*d*x)/2] + 230*B*Sin[4*c + (7*d*x)/2] + 15*A
*Sin[4*c + (9*d*x)/2] - 20*B*Sin[4*c + (9*d*x)/2] + 15*A*Sin[5*c + (9*d*x)/
2] - 20*B*Sin[5*c + (9*d*x)/2] + 5*B*Sin[5*c + (11*d*x)/2] + 5*B*Sin[6*c +
(11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.065, size = 362, normalized size = 1.7

$$-\frac{A}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,A}{3\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{5\,B}{6\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31\,A}{4\,d a^3}\tan\left(\frac{d\ldots}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+2/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A-5/6/d/a^3*B*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*ta
n(1/2*d*x+1/2*c)+49/4/d/a^3*B*tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2
*c)^2)^3*A*tan(1/2*d*x+1/2*c)^5+17/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1
/2*d*x+1/2*c)^5-12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+
76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^3-5/d/a^3/(1+tan
(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+11/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^3*B*tan(1/2*d*x+1/2*c)+13/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A-23/d/a^3*arct
an(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.51081, size = 556, normalized size = 2.55

$$\frac{B\left(\frac{20\left(\frac{33\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{51\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{735\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{1380\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\ldots\right.}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\ldots}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(B*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*(6
0*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(c
```

```
os(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 1.40411, size = 539, normalized size = 2.47

$$\frac{15\,(13\,A-23\,B)dx\cos{(dx+c)}^{3}+45\,(13\,A-23\,B)dx\cos{(dx+c)}^{2}+45\,(13\,A-23\,B)dx\cos{(dx+c)}+15\,(13\,A-23\,B}{30\left(a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(13*A - 23*B)*d*x*cos(d*x + c)^3 + 45*(13*A - 23*B)*d*x*cos(d*x +
c)^2 + 45*(13*A - 23*B)*d*x*cos(d*x + c) + 15*(13*A - 23*B)*d*x + (10*B*cos
(d*x + c)^5 + 15*(A - B)*cos(d*x + c)^4 - 5*(9*A - 19*B)*cos(d*x + c)^3 - (
479*A - 869*B)*cos(d*x + c)^2 - 3*(239*A - 429*B)*cos(d*x + c) - 304*A + 54
4*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 45.6154, size = 1584, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((390*A*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 + 1
80*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
 + 1170*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3
*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 1170
*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(
c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 390*A*d*x/(
60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan(c/2 + d*x/2)**11/(60*a**3*d*tan
(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 31*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)
```

```
**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*
a**3*d) - 354*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a*
*3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 16
98*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/
2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 2075*A*tan(c/
2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)*
*4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*A*tan(c/2 + d*x/2)/(
60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 690*B*d*x*tan(c/2 + d*x/2)**6/(60*a**3*
d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2
 + d*x/2)**2 + 60*a**3*d) - 2070*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c
/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/
2)**2 + 60*a**3*d) - 2070*B*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*
x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 +
 60*a**3*d) - 690*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2
 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*B*tan(c/2 +
d*x/2)**11/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4
+ 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 41*B*tan(c/2 + d*x/2)**9/(6
0*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*
tan(c/2 + d*x/2)**2 + 60*a**3*d) + 594*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan
(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 3078*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/
2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 6
0*a**3*d) + 3675*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180
*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 1395*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/
2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (x*
(A + B*cos(c))*cos(c)**5/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.21456, size = 308, normalized size = 1.41

$$\frac{\frac{30\,(dx+c)(13\,A-23\,B)}{a^3}-\frac{20\left(21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-51\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-76\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-33\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\ldots\right.\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{60\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(d*x + c)*(13*A - 23*B)/a^3 - 20*(21*A*tan(1/2*d*x + 1/2*c)^5 - 51
*B*tan(1/2*d*x + 1/2*c)^5 + 36*A*tan(1/2*d*x + 1/2*c)^3 - 76*B*tan(1/2*d*x
```

```
+ 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) - 33*B*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12
*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 50*B*a^12*tan(
1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 735*B*a^12*tan(1/2*d
*x + 1/2*c))/a^15)/d
```

3.57 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{8(9A-19B)\sin(c+dx)}{15a^3d}+\frac{4(9A-19B)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{x(6A-13B)}{2a^3}+$$

```
[Out] -((6*A - 13*B)*x)/(2*a^3) + (8*(9*A - 19*B)*Sin[c + d*x])/(15*a^3*d) - ((6*
A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((6*A - 11*B)*Cos[c + d*x]^3*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (4*(9*A - 19*B)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.467678, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{8(9A-19B)\sin(c+dx)}{15a^3d}+\frac{4(9A-19B)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{x(6A-13B)}{2a^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] -((6*A - 13*B)*x)/(2*a^3) + (8*(9*A - 19*B)*Sin[c + d*x])/(15*a^3*d) - ((6*
A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((6*A - 11*B)*Cos[c + d*x]^3*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (4*(9*A - 19*B)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(2A-7B)\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6A-11B)\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6A-11B)\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + 4(9$$

$$= -\frac{(6A-13B)x}{2a^3} + \frac{8(9A-19B)\sin(c+dx)}{15a^3d} - \frac{(6A-13B)\cos(c+dx)\sin(c+d}{2a^3d}$$

Mathematica [B] time = 0.791297, size = 435, normalized size = 2.25

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-600dx(6A-13B)\cos\left(c+\frac{dx}{2}\right)-600dx(6A-13B)\cos\left(\frac{dx}{2}\right)-4500A\sin\left(c+\frac{dx}{2}\right)+4860A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(6*A - 13*B)*d*x*Cos[(d*x)/2] - 600*(6*A -
 13*B)*d*x*Cos[c + (d*x)/2] - 1800*A*d*x*Cos[c + (3*d*x)/2] + 3900*B*d*x*Co
s[c + (3*d*x)/2] - 1800*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (
3*d*x)/2] - 360*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x)/2]
 - 360*A*d*x*Cos[3*c + (5*d*x)/2] + 780*B*d*x*Cos[3*c + (5*d*x)/2] + 7020*A
*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] - 4500*A*Sin[c + (d*x)/2] + 7560*B*Sin
[c + (d*x)/2] + 4860*A*Sin[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] - 900
*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] + 1452*A*Sin[2*c + (5*
d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 300*A*Sin[3*c + (5*d*x)/2] - 750*B*
```

```
Sin[3*c + (5*d*x)/2] + 60*A*Sin[3*c + (7*d*x)/2] - 105*B*Sin[3*c + (7*d*x)/
2] + 60*A*Sin[4*c + (7*d*x)/2] - 105*B*Sin[4*c + (7*d*x)/2] + 15*B*Sin[4*c
+ (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.066, size = 292, normalized size = 1.5

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{2\,B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*A+2/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1/2*
c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2
*d*x+1/2*c)^3+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-5/d/a
^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)-6/d/a^3*arctan(tan(1/2*d
*x+1/2*c))*A+13/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.5081, size = 435, normalized size = 2.25

$$-\frac{B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3\,A\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)\right.}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*A*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
```

(85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d

---

Fricas [A] time = 1.47336, size = 497, normalized size = 2.58

$$-\frac{15\,(6\,A-13\,B)dx\cos\left(dx+c\right)^{3}+45\,(6\,A-13\,B)dx\cos\left(dx+c\right)^{2}+45\,(6\,A-13\,B)dx\cos\left(dx+c\right)+15\,(6\,A-13\,B)dx}{30\,\left(a^{3}d\cos\left(dx+c\right)^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(6*A - 13*B)*d*x*cos(d*x + c)^3 + 45*(6*A - 13*B)*d*x*cos(d*x + c
)^2 + 45*(6*A - 13*B)*d*x*cos(d*x + c) + 15*(6*A - 13*B)*d*x - (15*B*cos(d*
x + c)^4 + 15*(2*A - 3*B)*cos(d*x + c)^3 + (234*A - 479*B)*cos(d*x + c)^2 +
 3*(114*A - 239*B)*cos(d*x + c) + 144*A - 304*B)*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 27.8623, size = 966, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-180*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 360*A*d*x*tan(c/2 + d*x/2)**2
/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*
d) - 180*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)
**2 + 60*a**3*d) + 3*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 24*A*tan(c/2 + d*x/2)**7/(60
*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 198*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(
c/2 + d*x/2)**2 + 60*a**3*d) + 600*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2
 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*A*tan(c/2
+ d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 +
```

```
60*a**3*d) + 390*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*B*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3
*d) + 390*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) - 3*B*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4
+ 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*B*tan(c/2 + d*x/2)**7/(6
0*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
- 388*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) - 1310*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c
/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*B*tan(c/
2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
+ 60*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a)**3, Tru
e))
```

Giac [A] time = 1.17815, size = 270, normalized size = 1.4

$$-\frac{\frac{30\,(dx+c)(6\,A-13\,B)}{a^3}-\frac{60\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(30*(d*x + c)*(6*A - 13*B)/a^3 - 60*(2*A*tan(1/2*d*x + 1/2*c)^3 - 7*B
*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 5*B*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5
 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*
B*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 465*B*a^1
2*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.58 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=147

$$-\frac{(7A-27B)\sin(c+dx)}{15a^3d}-\frac{(A-3B)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(A-3B)}{a^3}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(4A-9B)\sin(c}{15ad(a\cos(}$$

```
[Out] ((A - 3*B)*x)/a^3 - ((7*A - 27*B)*Sin[c + d*x])/(15*a^3*d) + ((A - B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A - 9*B)*Cos[c +
 d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((A - 3*B)*Sin[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.457031, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(7A-27B)\sin(c+dx)}{15a^3d}-\frac{(A-3B)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(A-3B)}{a^3}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(4A-9B)\sin(c}{15ad(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - 3*B)*x)/a^3 - ((7*A - 27*B)*Sin[c + d*x])/(15*a^3*d) + ((A - B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A - 9*B)*Cos[c +
 d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((A - 3*B)*Sin[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(A-6B)\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\mathrm{co}}{}}{}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{2a}{}}{}$$

$$= -\frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos}{15ad(a+}$$

$$= -\frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos}{15ad(a+}$$

$$= \frac{(A-3B)x}{a^3} - \frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4.}{}$$

$$= \frac{(A-3B)x}{a^3} - \frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4.}{}$$

Mathematica [B] time = 0.832577, size = 361, normalized size = 2.46

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(300dx(A-3B)\cos\left(c+\frac{dx}{2}\right)+300dx(A-3B)\cos\left(\frac{dx}{2}\right)+540A\sin\left(c+\frac{dx}{2}\right)-460A\sin\left(c+\frac{3d}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(300*(A - 3*B)*d*x*Cos[(d*x)/2] + 300*(A - 3*B)*
d*x*Cos[c + (d*x)/2] + 150*A*d*x*Cos[c + (3*d*x)/2] - 450*B*d*x*Cos[c + (3*
d*x)/2] + 150*A*d*x*Cos[2*c + (3*d*x)/2] - 450*B*d*x*Cos[2*c + (3*d*x)/2] +
 30*A*d*x*Cos[2*c + (5*d*x)/2] - 90*B*d*x*Cos[2*c + (5*d*x)/2] + 30*A*d*x*C
os[3*c + (5*d*x)/2] - 90*B*d*x*Cos[3*c + (5*d*x)/2] - 740*A*Sin[(d*x)/2] +
1755*B*Sin[(d*x)/2] + 540*A*Sin[c + (d*x)/2] - 1125*B*Sin[c + (d*x)/2] - 46
0*A*Sin[c + (3*d*x)/2] + 1215*B*Sin[c + (3*d*x)/2] + 180*A*Sin[2*c + (3*d*x
)/2] - 225*B*Sin[2*c + (3*d*x)/2] - 128*A*Sin[2*c + (5*d*x)/2] + 363*B*Sin[
2*c + (5*d*x)/2] + 75*B*Sin[3*c + (5*d*x)/2] + 15*B*Sin[3*c + (7*d*x)/2] +
15*B*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.065, size = 189, normalized size = 1.3

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{4\,da^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A-1/2/d/a^3*B*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan
(1/2*d*x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3*B*tan(1/2*d*x+1/2*c
)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A-6/d/a^3*arc
tan(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.56786, size = 312, normalized size = 2.12

$$\frac{3\,B\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3}{(c}}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*B*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 1.36359, size = 429, normalized size = 2.92

$$\frac{15\,(A-3\,B)dx\cos(dx+c)^3+45\,(A-3\,B)dx\cos(dx+c)^2+45\,(A-3\,B)dx\cos(dx+c)+15\,(A-3\,B)dx+\left(15\,B\cos(d\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*(A - 3*B)*d*x*cos(d*x + c)^3 + 45*(A - 3*B)*d*x*cos(d*x + c)^2 + 4
5*(A - 3*B)*d*x*cos(d*x + c) + 15*(A - 3*B)*d*x + (15*B*cos(d*x + c)^3 - (3
2*A - 117*B)*cos(d*x + c)^2 - 3*(17*A - 57*B)*cos(d*x + c) - 22*A + 72*B)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [A] time = 16.5648, size = 496, normalized size = 3.37

$$\begin{cases} \frac{60Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{60Adx}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{3A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{17A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{85A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \\ \frac{x(A+B\cos(c))\cos^3(c)}{(a\cos(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d) + 60*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan(c
/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 17*A*tan(c/2 +
 d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 85*A*tan(c/2 + d*x
/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 105*A*tan(c/2 + d*x/2)
/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x/(60*a**3*d*tan(c/
2 + d*x/2)**2 + 60*a**3*d) + 3*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d
*x/2)**2 + 60*a**3*d) - 27*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) + 225*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**
2 + 60*a**3*d) + 375*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.2116, size = 209, normalized size = 1.42

$$\frac{\frac{60(dx+c)(A-3B)}{a^3} + \frac{120B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^3} - \frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 20Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 30Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 105Aa}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(d*x + c)*(A - 3*B)/a^3 + 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2
*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^12*tan(1/2*d*x
+ 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12*tan(1/2*d*x + 1/2
*c))/a^15)/d
```

3.59 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$\frac{(4A-29B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Bx}{a^3}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] (B*x)/a^3 + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((2*A - 7*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((4*A -
29*B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.321075, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2968, 3019, 2735, 2648}

$$\frac{(4A-29B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Bx}{a^3}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (B*x)/a^3 + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((2*A - 7*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((4*A -
29*B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(2a(A-B)+5aB\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{2a(A-B)\cos(c+dx)+5aB\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{-2a^2(2A-7B)-15a\ldots}{a+a\cos(c\ldots}}{15a\ldots}$$

$$= \frac{Bx}{a^3} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(4A-29B\ldots}{\ldots}$$

$$= \frac{Bx}{a^3} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(4A-29\ldots}{15d\left(a^3+\ldots\right.}$$

Mathematica [B] time = 0.5373, size = 241, normalized size = 2.08

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-60A\sin\left(c+\frac{dx}{2}\right)+40A\sin\left(c+\frac{3dx}{2}\right)-30A\sin\left(2c+\frac{3dx}{2}\right)+14A\sin\left(2c+\frac{5dx}{2}\right)+80A\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*B*d*x*Cos[(d*x)/2] + 150*B*d*x*Cos[c + (d
*x)/2] + 75*B*d*x*Cos[c + (3*d*x)/2] + 75*B*d*x*Cos[2*c + (3*d*x)/2] + 15*B
*d*x*Cos[2*c + (5*d*x)/2] + 15*B*d*x*Cos[3*c + (5*d*x)/2] + 80*A*Sin[(d*x)/
2] - 370*B*Sin[(d*x)/2] - 60*A*Sin[c + (d*x)/2] + 270*B*Sin[c + (d*x)/2] +
40*A*Sin[c + (3*d*x)/2] - 230*B*Sin[c + (3*d*x)/2] - 30*A*Sin[2*c + (3*d*x)
/2] + 90*B*Sin[2*c + (3*d*x)/2] + 14*A*Sin[2*c + (5*d*x)/2] - 64*B*Sin[2*c
+ (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.056, size = 137, normalized size = 1.2

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{4\,da^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/6/d/a
^3*tan(1/2*d*x+1/2*c)^3*A+1/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(
1/2*d*x+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/
2*c))*B
```

Maxima [A] time = 1.48969, size = 216, normalized size = 1.86

$$-\frac{B\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{A\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - A*(15*sin(d*x + c)/(cos(d*x + c) + 1) -
 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3)/d
```

Fricas [A] time = 1.37787, size = 351, normalized size = 3.03

$$\frac{15\,Bdx\cos\left(dx+c\right)^{3}+45\,Bdx\cos\left(dx+c\right)^{2}+45\,Bdx\cos\left(dx+c\right)+15\,Bdx+\left(\left(7\,A-32\,B\right)\cos\left(dx+c\right)^{2}+3\left(2\,A-17\,B\right)\right.}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*B*d*x*cos(d*x + c)^3 + 45*B*d*x*cos(d*x + c)^2 + 45*B*d*x*cos(d*x
+ c) + 15*B*d*x + ((7*A - 32*B)*cos(d*x + c)^2 + 3*(2*A - 17*B)*cos(d*x + c
) + 2*A - 22*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 9.22754, size = 148, normalized size = 1.28

$$\begin{cases} \frac{A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^{3}d}-\frac{A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{3}d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^{3}d}+\frac{Bx}{a^{3}}-\frac{B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^{3}d}+\frac{B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{3}d}-\frac{7B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^{3}d} & \text{for}\: d \neq 0 \\ \frac{x(A+B\cos\left(c\right))\cos^{2}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) - A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) + B*x/a**3 - B*tan(c/2 + d*x/2)**5/(20
*a**3*d) + B*tan(c/2 + d*x/2)**3/(3*a**3*d) - 7*B*tan(c/2 + d*x/2)/(4*a**3*
```

```
d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.23817, size = 162, normalized size = 1.4

$$\frac{\frac{60\,(dx+c)B}{a^3} + \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 105\,Ba^{12}\tan\left(\frac{1}{2}\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(d*x + c)*B/a^3 + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(
1/2*d*x + 1/2*c)^5 - 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*B*a^12*tan(1/2*d
*x + 1/2*c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

3.60 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(3A+7B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A-8B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] -((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Sin[c +
 d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A + 7*B)*Sin[c + d*x])/(15*d*(
a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.188036, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3019, 2750, 2648}

$$\frac{(3A+7B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A-8B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Sin[c +
 d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A + 7*B)*Sin[c + d*x])/(15*d*(
a^3 + a^3*Cos[c + d*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{-3a(A-B)-5aB\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A+7B)\int \frac{1}{a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A+7B)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx\right)}$$

Mathematica [A] time = 0.319653, size = 135, normalized size = 1.32

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-15(A+2B)\sin\left(c+\frac{dx}{2}\right)+5(3A+8B)\sin\left(\frac{dx}{2}\right)+15A\sin\left(c+\frac{3dx}{2}\right)+3A\sin\left(2c+\frac{5dx}{2}\right)+20B\right.}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 8*B)*Sin[(d*x)/2] - 15*(A + 2*B)*Sin[c
 + (d*x)/2] + 15*A*Sin[c + (3*d*x)/2] + 20*B*Sin[c + (3*d*x)/2] - 15*B*Sin[
2*c + (3*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 7*B*Sin[2*c + (5*d*x)/2]))/(3
```

```
0*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.05, size = 64, normalized size = 0.6

$$\frac{1}{4\,d{a}^{3}}\left(\frac{-A+B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{2\,B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(-A+B)*tan(1/2*d*x+1/2*c)^5-2/3*B*tan(1/2*d*x+1/2*c)^3+A*tan
(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.01729, size = 155, normalized size = 1.52

$$\frac{\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{3}}+\frac{3\,A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{3}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*A*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.31214, size = 227, normalized size = 2.23

$$\frac{\left((3\,A+7\,B)\cos\left(dx+c\right)^{2}+3\,(3\,A+2\,B)\cos\left(dx+c\right)+3\,A+2\,B\right)\sin\left(dx+c\right)}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((3*A + 7*B)*cos(d*x + c)^2 + 3*(3*A + 2*B)*cos(d*x + c) + 3*A + 2*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [A] time = 5.70016, size = 117, normalized size = 1.15

$$\begin{cases} -\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)/(4*a**3*
d) + B*tan(c/2 + d*x/2)**5/(20*a**3*d) - B*tan(c/2 + d*x/2)**3/(6*a**3*d) +
 B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(
c) + a)**3, True))
```

Giac [A] time = 1.21715, size = 101, normalized size = 0.99

$$-\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 10\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*B*tan(1
/2*d*x + 1/2*c)^3 - 15*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/
(a^3*d)
```

3.61 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+3B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 3*B)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.0788386, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2750, 2650, 2648}

$$\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+3B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 3*B)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}\\ &= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(2A+3B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2}\\ &= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.259017, size = 96, normalized size = 0.94

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((2A+3B)\left(5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)+5(4A+3B)\sin\left(\frac{dx}{2}\right)-15B\sin\left(c+\frac{dx}{2}\right)\right)}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(4*A + 3*B)*Sin[(d*x)/2] - 15*B*Sin[c + (d*x)
/2] + (2*A + 3*B)*(5*Sin[c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2])))/(30*a^3*d
*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.044, size = 64, normalized size = 0.6

$$\frac{1}{4\,da^3}\left(\frac{A-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(A-B)*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3*A+A*tan(
1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.0223, size = 155, normalized size = 1.52

$$\frac{\frac{A\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,B\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.33754, size = 227, normalized size = 2.23

$$\frac{\left((2\,A+3\,B)\cos\left(dx+c\right)^2+3\,(2\,A+3\,B)\cos\left(dx+c\right)+7\,A+3\,B\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*A + 3*B)*cos(d*x + c)^2 + 3*(2*A + 3*B)*cos(d*x + c) + 7*A + 3*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [A] time = 3.66219, size = 114, normalized size = 1.12

$$\begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for}\ d\neq 0 \\ \frac{x(A+B\cos\left(c\right))}{\left(a\cos\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) - B*tan(c/2 + d*x/2)**5/(20*a**3*d) +
B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))/(a*cos(c) + a)*
*3, True))
```

Giac [A] time = 1.20609, size = 101, normalized size = 0.99

$$\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+10\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*A*tan(1/
2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c))/(
a^3*d)
```

3.62 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{2(11A-B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-2B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*A - 2*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 (2*(11*A - B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.310994, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$-\frac{2(11A-B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-2B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*A - 2*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 (2*(11*A - B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(5aA-2a(A-B)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(15a^2A-a^2(7A-2B)\cos(c+a}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11A-B)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11A-B)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx\right.}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

Mathematica [A] time = 0.912192, size = 197, normalized size = 1.68

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-5(29A-4B)\sin\left(\frac{dx}{2}\right)+75A\sin\left(c+\frac{dx}{2}\right)-95A\sin\left(c+\frac{3dx}{2}\right)+15A\sin\left(2c+\frac{3dx}{2}\right)-22A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-240*A*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-5*(29*A
 - 4*B)*Sin[(d*x)/2] + 75*A*Sin[c + (d*x)/2] - 95*A*Sin[c + (3*d*x)/2] + 10
*B*Sin[c + (3*d*x)/2] + 15*A*Sin[2*c + (3*d*x)/2] - 22*A*Sin[2*c + (5*d*x)/
2] + 2*B*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.094, size = 159, normalized size = 1.4

$$\frac{B}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{A}{da^3}\ln\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/4/d/a^3*B*tan(1/2*d*x+1/2*c)+1/6/d/a^3*B*tan(1/2*d*x+1/2*c)^3+1/d/a^3*A*l
n(tan(1/2*d*x+1/2*c)+1)-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A-1/d/a^3*A*ln(tan(1
/2*d*x+1/2*c)-1)-7/4/d/a^3*A*tan(1/2*d*x+1/2*c)-1/20/d/a^3*A*tan(1/2*d*x+1/
2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5
```

Maxima [A] time = 1.00668, size = 252, normalized size = 2.15

$$-\frac{A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.36406, size = 481, normalized size = 4.11

$$\frac{15\left(A\cos\left(dx+c\right)^3+3\,A\cos\left(dx+c\right)^2+3\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(A\cos\left(dx+c\right)^3+3\,A\cos\left(dx\right.\right.}{30\left(a^3d\cos\left(dx+c\right)^3+3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/30*(15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*log
(sin(d*x + c) + 1) - 15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*
x + c) + A)*log(-sin(d*x + c) + 1) - 2*(2*(11*A - B)*cos(d*x + c)^2 + 3*(17
*A - 2*B)*cos(d*x + c) + 32*A - 7*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**3 + 3*co
s(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.26929, size = 200, normalized size = 1.71

$$\frac{\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,Ba^{12}\tan\left(\frac{1}{2}\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(60*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*A*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*
d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^12*tan(1/2*d*x +
 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 15*B*a^12*tan(1/2*d*x + 1/2*c
))/a^15)/d
```

3.63 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{2(36A-11B)\tan(c+dx)}{15a^3d}-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3A-B)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-4B)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-}{5d(a\,c}$$

```
[Out] -(((3*A - B)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*A - 11*B)*Tan[c + d*x
])/(15*a^3*d) - ((A - B)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*A
 - 4*B)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*A - B)*Tan[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.469333, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{2(36A-11B)\tan(c+dx)}{15a^3d}-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3A-B)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-4B)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-}{5d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] -(((3*A - B)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*A - 11*B)*Tan[c + d*x
])/(15*a^3*d) - ((A - B)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*A
 - 4*B)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*A - B)*Tan[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(6A-B)-3a(A-B)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(a^2(27A-7B)-2a^2(9A-4B)}{a+a\cos(}}{15a}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3A-B)\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3A-B)\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)}$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c}{15ad(a+a\cos(c}$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{2(36A-11B)\tan(c+dx)}{15a^3d} - \frac{(A-B)\tan(}{5d(a+a\cos(c}$$

Mathematica [B] time = 2.9235, size = 482, normalized size = 3.32

$$\frac{960(3A-B)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (960*(3*A - B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Se
c[c]*Sec[c + d*x]*(-5*(51*A - 32*B)*Sin[(d*x)/2] + (567*A - 167*B)*Sin[(3*d
*x)/2] - 600*A*Sin[c - (d*x)/2] + 170*B*Sin[c - (d*x)/2] + 375*A*Sin[c + (d
*x)/2] - 170*B*Sin[c + (d*x)/2] - 480*A*Sin[2*c + (d*x)/2] + 160*B*Sin[2*c
+ (d*x)/2] - 60*A*Sin[c + (3*d*x)/2] + 75*B*Sin[c + (3*d*x)/2] + 402*A*Sin[
2*c + (3*d*x)/2] - 167*B*Sin[2*c + (3*d*x)/2] - 225*A*Sin[3*c + (3*d*x)/2]
+ 75*B*Sin[3*c + (3*d*x)/2] + 315*A*Sin[c + (5*d*x)/2] - 95*B*Sin[c + (5*d*
x)/2] + 30*A*Sin[2*c + (5*d*x)/2] + 15*B*Sin[2*c + (5*d*x)/2] + 240*A*Sin[3
*c + (5*d*x)/2] - 95*B*Sin[3*c + (5*d*x)/2] - 45*A*Sin[4*c + (5*d*x)/2] + 1
5*B*Sin[4*c + (5*d*x)/2] + 72*A*Sin[2*c + (7*d*x)/2] - 22*B*Sin[2*c + (7*d*
x)/2] + 15*A*Sin[3*c + (7*d*x)/2] + 57*A*Sin[4*c + (7*d*x)/2] - 22*B*Sin[4*
c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.108, size = 245, normalized size = 1.7

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*A-1/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+3/d/a^3*A*ln(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)-
3/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a
^3*A/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.02875, size = 386, normalized size = 2.66

$$\frac{3\,A\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-B\left(\frac{105\,\mathrm{si}}{\cos(}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 1.4683, size = 670, normalized size = 4.62

$$-\frac{15\left((3\,A-B)\cos(dx+c)^4+3\,(3\,A-B)\cos(dx+c)^3+3\,(3\,A-B)\cos(dx+c)^2+(3\,A-B)\cos(dx+c)\right)\log(\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*((3*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A -
 B)*cos(d*x + c)^2 + (3*A - B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((3
*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A - B)*cos(d*x +
 c)^2 + (3*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(36*A - 11*B)
*cos(d*x + c)^3 + 3*(57*A - 17*B)*cos(d*x + c)^2 + (117*A - 32*B)*cos(d*x +
 c) + 15*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**3
+ 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.26267, size = 257, normalized size = 1.77

$$-\frac{\frac{60\,(3\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,(3\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(3*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(3*A - B)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*ta
n(1/2*d*x + 1/2*c)^5 + 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*B*a^12*tan(1/2
*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x
+ 1/2*c))/a^15)/d
```

3.64 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=196

$$-\frac{8(19A-9B)\tan(c+dx)}{15a^3d}+\frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A-6B)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{4(19A-9B)\tan}{15d\left(a^3\cos\right.}$$

```
[Out] ((13*A - 6*B)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (8*(19*A - 9*B)*Tan[c + d*
x])/(15*a^3*d) + ((13*A - 6*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A -
 B)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*A - 6*B)
*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (4*(19*A - 9*
B)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.541149, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{8(19A-9B)\tan(c+dx)}{15a^3d}+\frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A-6B)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{4(19A-9B)\tan}{15d\left(a^3\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] ((13*A - 6*B)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (8*(19*A - 9*B)*Tan[c + d*
x])/(15*a^3*d) + ((13*A - 6*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A -
 B)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*A - 6*B)
*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (4*(19*A - 9*
B)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(7A-2B)-4a(A-B)\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - 4(1$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - 4(1$$

$$= \frac{(13A-6B)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - (11A$$

$$= \frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{8(19A-9B)\tan(c+dx)}{15a^3d} + (13A-6B)\sec$$

Mathematica [B] time = 4.67301, size = 610, normalized size = 3.11

$$-1920(13A-6B)\cos^6\left(\tfrac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(1920*(13*A - 6*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/
2]*Sec[c]*Sec[c + d*x]^2*((-1235*A + 870*B)*Sin[(d*x)/2] + 5*(761*A - 366*B
)*Sin[(3*d*x)/2] - 4329*A*Sin[c - (d*x)/2] + 2094*B*Sin[c - (d*x)/2] + 1989
*A*Sin[c + (d*x)/2] - 1314*B*Sin[c + (d*x)/2] - 3575*A*Sin[2*c + (d*x)/2] +
 1650*B*Sin[2*c + (d*x)/2] - 475*A*Sin[c + (3*d*x)/2] + 450*B*Sin[c + (3*d*
x)/2] + 2005*A*Sin[2*c + (3*d*x)/2] - 1230*B*Sin[2*c + (3*d*x)/2] - 2275*A*
Sin[3*c + (3*d*x)/2] + 1050*B*Sin[3*c + (3*d*x)/2] + 2673*A*Sin[c + (5*d*x)
/2] - 1278*B*Sin[c + (5*d*x)/2] + 105*A*Sin[2*c + (5*d*x)/2] + 90*B*Sin[2*c
 + (5*d*x)/2] + 1593*A*Sin[3*c + (5*d*x)/2] - 918*B*Sin[3*c + (5*d*x)/2] -
975*A*Sin[4*c + (5*d*x)/2] + 450*B*Sin[4*c + (5*d*x)/2] + 1325*A*Sin[2*c +
(7*d*x)/2] - 630*B*Sin[2*c + (7*d*x)/2] + 255*A*Sin[3*c + (7*d*x)/2] - 60*B
*Sin[3*c + (7*d*x)/2] + 875*A*Sin[4*c + (7*d*x)/2] - 480*B*Sin[4*c + (7*d*x
)/2] - 195*A*Sin[5*c + (7*d*x)/2] + 90*B*Sin[5*c + (7*d*x)/2] + 304*A*Sin[3
```

```
*c + (9*d*x)/2] - 144*B*Sin[3*c + (9*d*x)/2] + 90*A*Sin[4*c + (9*d*x)/2] -
30*B*Sin[4*c + (9*d*x)/2] + 214*A*Sin[5*c + (9*d*x)/2] - 114*B*Sin[5*c + (9
*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.112, size = 334, normalized size = 1.7

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2\,A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31\,A}{4\,da^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-2/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*B*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*ta
n(1/2*d*x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)-13/2/d/a^3*A*ln(tan(1/2*d*
x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B+7/2/d/a^3*A/(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^
2+7/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*B+13/2/
d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a
^3*A/(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.00176, size = 509, normalized size = 2.6

$$-\frac{A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-3B\left(\cdots\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
```

```
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 1.44425, size = 756, normalized size = 3.86

$$\frac{15\left((13\,A-6\,B)\cos\left(dx+c\right)^{5}+3\,(13\,A-6\,B)\cos\left(dx+c\right)^{4}+3\,(13\,A-6\,B)\cos\left(dx+c\right)^{3}+(13\,A-6\,B)\cos\left(dx+c\right)^{2}\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 + 3*(
13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - 15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 + 3*
(13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(-sin(d*x + c
) + 1) - 2*(16*(19*A - 9*B)*cos(d*x + c)^4 + 3*(239*A - 114*B)*cos(d*x + c)
^3 + (479*A - 234*B)*cos(d*x + c)^2 + 15*(3*A - 2*B)*cos(d*x + c) - 15*A)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22871, size = 315, normalized size = 1.61

$$\frac{\frac{30\,(13\,A-6\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(13\,A-6\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{60\left(7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,B\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(13*A - 6*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(13*A - 6
*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*A*tan(1/2*d*x + 1/2*c)^3
 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/
2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^12*tan(1/2*d*x + 1/2*c)^3
 - 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 255
*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.65 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=229

$$\frac{8(83A-216B)\sin(c+dx)}{105a^4d}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4(83A-216B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(8A\cdots}{}$$

```
[Out] -((8*A - 21*B)*x)/(2*a^4) + (8*(83*A - 216*B)*Sin[c + d*x])/(105*a^4*d) - (
(8*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) + ((52*A - 129*B)*Cos[c +
 d*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*(83*A - 216*B)*
Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[
c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((A - 2*B)*Cos[c +
d*x]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.672043, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{8(83A-216B)\sin(c+dx)}{105a^4d}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4(83A-216B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(8A\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] -((8*A - 21*B)*x)/(2*a^4) + (8*(83*A - 216*B)*Sin[c + d*x])/(105*a^4*d) - (
(8*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) + ((52*A - 129*B)*Cos[c +
 d*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*(83*A - 216*B)*
Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[
c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((A - 2*B)*Cos[c +
d*x]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(2A-9B)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(A-2B)\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c}{}}{}$$

$$= \frac{(52A-129B)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(A}{}$$

$$= \frac{(52A-129B)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(A}{}$$

$$= -\frac{(8A-21B)x}{2a^4} + \frac{8(83A-216B)\sin(c+dx)}{105a^4d} - \frac{(8A-21B)\cos(c+dx)\sin(c+d}{2a^4d}$$

Mathematica [B] time = 1.25512, size = 555, normalized size = 2.42

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-14700dx(8A-21B)\cos\left(c+\frac{dx}{2}\right)-14700dx(8A-21B)\cos\left(\frac{dx}{2}\right)-184520A\sin\left(c+\frac{dx}{2}\right)+184\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-14700*(8*A - 21*B)*d*x*Cos[(d*x)/2] - 14700*(8
*A - 21*B)*d*x*Cos[c + (d*x)/2] - 70560*A*d*x*Cos[c + (3*d*x)/2] + 185220*B
*d*x*Cos[c + (3*d*x)/2] - 70560*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*B*d*x*C
os[2*c + (3*d*x)/2] - 23520*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*B*d*x*Cos[2*
c + (5*d*x)/2] - 23520*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*B*d*x*Cos[3*c + (
```

```
5*d*x)/2] - 3360*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*B*d*x*Cos[3*c + (7*d*x)/
2] - 3360*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*B*d*x*Cos[4*c + (7*d*x)/2] + 24
3320*A*Sin[(d*x)/2] - 539490*B*Sin[(d*x)/2] - 184520*A*Sin[c + (d*x)/2] + 3
86190*B*Sin[c + (d*x)/2] + 184464*A*Sin[c + (3*d*x)/2] - 422478*B*Sin[c + (
3*d*x)/2] - 72240*A*Sin[2*c + (3*d*x)/2] + 132930*B*Sin[2*c + (3*d*x)/2] +
77168*A*Sin[2*c + (5*d*x)/2] - 181461*B*Sin[2*c + (5*d*x)/2] - 8400*A*Sin[3
*c + (5*d*x)/2] + 3675*B*Sin[3*c + (5*d*x)/2] + 15164*A*Sin[3*c + (7*d*x)/2
] - 36003*B*Sin[3*c + (7*d*x)/2] + 2940*A*Sin[4*c + (7*d*x)/2] - 9555*B*Sin
[4*c + (7*d*x)/2] + 420*A*Sin[4*c + (9*d*x)/2] - 945*B*Sin[4*c + (9*d*x)/2]
 + 420*A*Sin[5*c + (9*d*x)/2] - 945*B*Sin[5*c + (9*d*x)/2] + 105*B*Sin[5*c
+ (11*d*x)/2] + 105*B*Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x]
)^4)
```

Maple [A] time = 0.064, size = 332, normalized size = 1.5

$$-\frac{A}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{9\,B}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{23\,A}{24\,d a^4}\Big(\mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7+7/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5-9/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5-23/24/d/a^4*t
an(1/2*d*x+1/2*c)^3*A+13/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/
2*d*x+1/2*c)-111/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4/(1+tan(1/2*d*x+1/2*c)
^2)^2*tan(1/2*d*x+1/2*c)^3*A-9/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d
*x+1/2*c)^3+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-7/d/a^4
/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)-8/d/a^4*arctan(tan(1/2*d*x
+1/2*c))*A+21/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.48977, size = 491, normalized size = 2.14

$$-\frac{3\,B\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-A\Bigg(\frac{1\cdots}{\left(a^4+\frac{a^4\sin\cdots}{(\cos(d\cdots}\right.}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*B*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) - A*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*
x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 1.42473, size = 647, normalized size = 2.83

$$-\frac{105\,(8\,A-21\,B)dx\cos{(dx+c)}^{4}+420\,(8\,A-21\,B)dx\cos{(dx+c)}^{3}+630\,(8\,A-21\,B)dx\cos{(dx+c)}^{2}+420\,(8\,A-21\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*(8*A - 21*B)*d*x*cos(d*x + c)^4 + 420*(8*A - 21*B)*d*x*cos(d*x
+ c)^3 + 630*(8*A - 21*B)*d*x*cos(d*x + c)^2 + 420*(8*A - 21*B)*d*x*cos(d*x
 + c) + 105*(8*A - 21*B)*d*x - (105*B*cos(d*x + c)^5 + 210*(A - 2*B)*cos(d*
x + c)^4 + 4*(592*A - 1509*B)*cos(d*x + c)^3 + 4*(1318*A - 3411*B)*cos(d*x
+ c)^2 + (4472*A - 11619*B)*cos(d*x + c) + 1328*A - 3456*B)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 91.6891, size = 1085, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-3360*A*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4
+ 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 6720*A*d*x*tan(c/2 + d*x/
2)**2/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 8
40*a**4*d) - 3360*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c
/2 + d*x/2)**2 + 840*a**4*d) - 15*A*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/
2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 117*A*tan(c
/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/
2)**2 + 840*a**4*d) - 526*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2
)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 3682*A*tan(c/2 + d*x
/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 +
840*a**4*d) + 11165*A*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 +
 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*A*tan(c/2 + d*x/2)/(8
40*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*
d) + 8820*B*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*
a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 17640*B*d*x*tan(c/2 + d*x/2)**2/
(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**
4*d) + 8820*B*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d
*x/2)**2 + 840*a**4*d) + 15*B*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 + d*
x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 159*B*tan(c/2 + d
*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2
+ 840*a**4*d) + 1002*B*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4
+ 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 9114*B*tan(c/2 + d*x/2)**
5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a
**4*d) - 29505*B*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680
*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 17535*B*tan(c/2 + d*x/2)/(840*a
**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d),
Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**5/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.28697, size = 315, normalized size = 1.38

$$-\frac{\frac{420\,(dx+c)(8\,A-21\,B)}{a^4}-\frac{840\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(420*(d*x + c)*(8*A - 21*B)/a^4 - 840*(2*A*tan(1/2*d*x + 1/2*c)^3 -
9*B*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 7*B*tan(1/2*d*x + 1
```

```
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*
c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*A*a^24*tan(1/2*d*x + 1/2*c)^5
 + 189*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 -
1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*A*a^24*tan(1/2*d*x + 1/2*c) + 116
55*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.66 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=185

$$-\frac{(55A-244B)\sin(c+dx)}{105a^4d}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A-4B)\sin(c+dx)}{a^4d(\cos(c+dx)+1)}+\frac{x(A-4B)}{a^4}+\frac{(A-B)\sin}{7d(a\,c}$$

```
[Out] ((A - 4*B)*x)/a^4 - ((55*A - 244*B)*Sin[c + d*x])/(105*a^4*d) + ((25*A - 88
*B)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - ((A - 4
*B)*Sin[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[
c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((5*A - 12*B)*Cos[c + d*x]^3*Sin[c
 + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.679183, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(55A-244B)\sin(c+dx)}{105a^4d}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A-4B)\sin(c+dx)}{a^4d(\cos(c+dx)+1)}+\frac{x(A-4B)}{a^4}+\frac{(A-B)\sin}{7d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] ((A - 4*B)*x)/a^4 - ((55*A - 244*B)*Sin[c + d*x])/(105*a^4*d) + ((25*A - 88
*B)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - ((A - 4
*B)*Sin[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[
c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((5*A - 12*B)*Cos[c + d*x]^3*Sin[c
 + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(A-8B)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(5A-12B)\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int}{}$$

$$= \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(5}{}$$

$$= \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(5}{}$$

$$= -\frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B}{7a}$$

$$= -\frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B}{7a}$$

$$= \frac{(A-4B)x}{a^4} - \frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+d}{105a^4d(1+\cos(c+dx))^2}$$

$$= \frac{(A-4B)x}{a^4} - \frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+d}{105a^4d(1+\cos(c+dx))^2}$$

Mathematica [B] time = 0.854941, size = 481, normalized size = 2.6

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(7350dx(A-4B)\cos\left(c+\frac{dx}{2}\right)+7350dx(A-4B)\cos\left(\frac{dx}{2}\right)+16520A\sin\left(c+\frac{dx}{2}\right)-14280A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(7350*(A - 4*B)*d*x*Cos[(d*x)/2] + 7350*(A - 4*B
)*d*x*Cos[c + (d*x)/2] + 4410*A*d*x*Cos[c + (3*d*x)/2] - 17640*B*d*x*Cos[c
+ (3*d*x)/2] + 4410*A*d*x*Cos[2*c + (3*d*x)/2] - 17640*B*d*x*Cos[2*c + (3*d
*x)/2] + 1470*A*d*x*Cos[2*c + (5*d*x)/2] - 5880*B*d*x*Cos[2*c + (5*d*x)/2]
+ 1470*A*d*x*Cos[3*c + (5*d*x)/2] - 5880*B*d*x*Cos[3*c + (5*d*x)/2] + 210*A
*d*x*Cos[3*c + (7*d*x)/2] - 840*B*d*x*Cos[3*c + (7*d*x)/2] + 210*A*d*x*Cos[
4*c + (7*d*x)/2] - 840*B*d*x*Cos[4*c + (7*d*x)/2] - 19880*A*Sin[(d*x)/2] +
60830*B*Sin[(d*x)/2] + 16520*A*Sin[c + (d*x)/2] - 46130*B*Sin[c + (d*x)/2]
- 14280*A*Sin[c + (3*d*x)/2] + 46116*B*Sin[c + (3*d*x)/2] + 7560*A*Sin[2*c
```

```
+ (3*d*x)/2] - 18060*B*Sin[2*c + (3*d*x)/2] - 5600*A*Sin[2*c + (5*d*x)/2] +
 19292*B*Sin[2*c + (5*d*x)/2] + 1680*A*Sin[3*c + (5*d*x)/2] - 2100*B*Sin[3*
c + (5*d*x)/2] - 1040*A*Sin[3*c + (7*d*x)/2] + 3791*B*Sin[3*c + (7*d*x)/2]
+ 735*B*Sin[4*c + (7*d*x)/2] + 105*B*Sin[4*c + (9*d*x)/2] + 105*B*Sin[5*c +
 (9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.058, size = 229, normalized size = 1.2

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,B}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{11\,A}{24\,da^4}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-1/8/d/a
^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*tan
(1/2*d*x+1/2*c)^3*A-23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2
*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4*B*tan(1/2*d*x+1/2*c)/(1
+tan(1/2*d*x+1/2*c)^2)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*A-8/d/a^4*arctan(
tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.47777, size = 366, normalized size = 1.98

$$\frac{B\left(\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\cdots}{}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) - 5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x
```

```
 + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4))/d
```

Fricas [A] time = 1.31615, size = 572, normalized size = 3.09

$$\frac{105\,(A-4\,B)dx\cos{(dx+c)}^4+420\,(A-4\,B)dx\cos{(dx+c)}^3+630\,(A-4\,B)dx\cos{(dx+c)}^2+420\,(A-4\,B)dx\cos{(dx}{105\left(a^4d\cos{(dx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*(A - 4*B)*d*x*cos(d*x + c)^4 + 420*(A - 4*B)*d*x*cos(d*x + c)^3
+ 630*(A - 4*B)*d*x*cos(d*x + c)^2 + 420*(A - 4*B)*d*x*cos(d*x + c) + 105*(
A - 4*B)*d*x + (105*B*cos(d*x + c)^4 - 4*(65*A - 296*B)*cos(d*x + c)^3 - 4*
(155*A - 659*B)*cos(d*x + c)^2 - (535*A - 2236*B)*cos(d*x + c) - 160*A + 66
4*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d
*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 42.4924, size = 578, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((840*A*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 +
840*a**4*d) + 840*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 15*
A*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 90*A*
tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 280*A*t
an(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1190*A*t
an(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1575*A*t
an(c/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*B*d*x*
tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*B*
d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*B*tan(c/2 + d*x/2)**
9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*B*tan(c/2 + d*x/2)**7
```

```
/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*B*tan(c/2 + d*x/2)**5/
(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*B*tan(c/2 + d*x/2)**3/
(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*B*tan(c/2 + d*x/2)/(84
0*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*(A + B*cos(c))*co
s(c)**4/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.28351, size = 254, normalized size = 1.37

$$\frac{\frac{840\,(dx+c)(A-4\,B)}{a^4}+\frac{1680\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+147\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+385}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(d*x + c)*(A - 4*B)/a^4 + 1680*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*ta
n(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*B*a^24*tan(1
/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 805*B*a^24*tan(1/2*
d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

3.67 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$\frac{(12A-215B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(6A-55B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Bx}{a^4}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(3A-10B)\sin(c+d}{35ad(a\cos(c+}$$

```
[Out] (B*x)/a^4 - ((6*A - 55*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) +
((12*A - 215*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A - 10*B)*Cos[
c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.498107, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2968, 3019, 2735, 2648}

$$\frac{(12A-215B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(6A-55B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Bx}{a^4}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(3A-10B)\sin(c+d}{35ad(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (B*x)/a^4 - ((6*A - 55*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) +
((12*A - 215*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A - 10*B)*Cos[
c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)+7aB\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)\cos^2(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int}{}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)\cos^2(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int 2}{}$$

$$= -\frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)}{35ad(a}$$

$$= \frac{Bx}{a^4} - \frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-1}{35}$$

$$= \frac{Bx}{a^4} - \frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-1}{35}$$

Mathematica [B] time = 0.741084, size = 329, normalized size = 2.14

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-1260A\sin\left(c+\frac{dx}{2}\right)+882A\sin\left(c+\frac{3dx}{2}\right)-630A\sin\left(2c+\frac{3dx}{2}\right)+294A\sin\left(2c+\frac{5dx}{2}\right)-210\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*B*d*x*Cos[(d*x)/2] + 3675*B*d*x*Cos[c +
(d*x)/2] + 2205*B*d*x*Cos[c + (3*d*x)/2] + 2205*B*d*x*Cos[2*c + (3*d*x)/2]
+ 735*B*d*x*Cos[2*c + (5*d*x)/2] + 735*B*d*x*Cos[3*c + (5*d*x)/2] + 105*B*d
*x*Cos[3*c + (7*d*x)/2] + 105*B*d*x*Cos[4*c + (7*d*x)/2] + 1260*A*Sin[(d*x)
/2] - 9940*B*Sin[(d*x)/2] - 1260*A*Sin[c + (d*x)/2] + 8260*B*Sin[c + (d*x)/
2] + 882*A*Sin[c + (3*d*x)/2] - 7140*B*Sin[c + (3*d*x)/2] - 630*A*Sin[2*c +
 (3*d*x)/2] + 3780*B*Sin[2*c + (3*d*x)/2] + 294*A*Sin[2*c + (5*d*x)/2] - 28
00*B*Sin[2*c + (5*d*x)/2] - 210*A*Sin[3*c + (5*d*x)/2] + 840*B*Sin[3*c + (5
*d*x)/2] + 72*A*Sin[3*c + (7*d*x)/2] - 520*B*Sin[3*c + (7*d*x)/2]))/(13440*
a^4*d)
```

Maple [A] time = 0.059, size = 177, normalized size = 1.2

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{8\,da^4}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7+3/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*tan(
1/2*d*x+1/2*c)^3*A+11/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3+1/8/d/a^4*A*tan(1/2*d
*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c)
)*B
```

Maxima [A] time = 1.47903, size = 271, normalized size = 1.76

$$-\frac{5\,B\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(d}{(\cos(dx+}\right.}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.3857, size = 475, normalized size = 3.08

$$\frac{105\,Bdx\cos(dx+c)^4+420\,Bdx\cos(dx+c)^3+630\,Bdx\cos(dx+c)^2+420\,Bdx\cos(dx+c)+105\,Bdx+\left(4\,(9\,A-65\,B\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*B*d*x*cos(d*x + c)^4 + 420*B*d*x*cos(d*x + c)^3 + 630*B*d*x*cos(
d*x + c)^2 + 420*B*d*x*cos(d*x + c) + 105*B*d*x + (4*(9*A - 65*B)*cos(d*x +
 c)^3 + (39*A - 620*B)*cos(d*x + c)^2 + (24*A - 535*B)*cos(d*x + c) + 6*A -
 160*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 24.3107, size = 192, normalized size = 1.25

$$\begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{Bx}{a^4} + \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{11B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} - \frac{15B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} \\ \frac{x(A+B\cos(c))\cos^3(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40
*a**4*d) - A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
 + B*x/a**4 + B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(8*
a**4*d) + 11*B*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*B*tan(c/2 + d*x/2)/(8*a
**4*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.21546, size = 209, normalized size = 1.36

$$\frac{\frac{840\,(dx+c)B}{a^4} - \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 385\,Ba^{24}}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(d*x + c)*B/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 - 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2
*d*x + 1/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/2*c) + 1575*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

3.68 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4} dx$

Optimal. Leaf size=136

$$\frac{(13A+36B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)} - \frac{2(A+27B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2} + \frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4} - \frac{(A-8B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] (-2*(A + 27*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + ((13*A + 36
*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*
Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((A - 8*B)*Sin[c + d*x])/(35*a
*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.347609, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2968, 3019, 2750, 2648}

$$\frac{(13A+36B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)} - \frac{2(A+27B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2} + \frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4} - \frac{(A-8B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-2*(A + 27*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + ((13*A + 36
*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*
Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((A - 8*B)*Sin[c + d*x])/(35*a
*d*(a + a*Cos[c + d*x])^3)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(2a(A-B)+a(A+6B)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{2a(A-B)\cos(c+dx)+a(A+6B)\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(A-8B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{-3a^2(A-8B)-5a}{(a+a\,c}}{3}$$

$$= -\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(A-8B)\,\mathrm{s}}{35ad(a+a\,\mathrm{c}}$$

$$= -\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(A-8B)\,\mathrm{s}}{35ad(a+a\,\mathrm{c}}$$

Mathematica [A] time = 0.43541, size = 193, normalized size = 1.42

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-35(5A+18B)\sin\left(c+\frac{dx}{2}\right)+70(4A+9B)\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)-105A\sin\left(2c+\frac{3dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(4*A + 9*B)*Sin[(d*x)/2] - 35*(5*A + 18*B)*S
in[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 441*B*Sin[c + (3*d*x)/2] - 105
*A*Sin[2*c + (3*d*x)/2] - 315*B*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c + (5*d*
x)/2] + 147*B*Sin[2*c + (5*d*x)/2] - 105*B*Sin[3*c + (5*d*x)/2] + 13*A*Sin[
3*c + (7*d*x)/2] + 36*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x]
)^4)
```

Maple [A] time = 0.053, size = 90, normalized size = 0.7

$$\frac{1}{8\,da^4}\left(\frac{A-B}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{-A+3\,B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{-A-3\,B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*B)*tan(1/2*d*x+1/2*c)^5
+1/3*(-A-3*B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.00878, size = 236, normalized size = 1.74

$$\frac{\frac{A\left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1}-\frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3B\left(\frac{35 \sin(dx+c)}{\cos(dx+c)+1}-\frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.36699, size = 308, normalized size = 2.26

$$\frac{\left((13\,A+36\,B)\cos(dx+c)^3+13\,(4\,A+3\,B)\cos(dx+c)^2+8\,(4\,A+3\,B)\cos(dx+c)+8\,A+6\,B\right)\sin(dx+c)}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*((13*A + 36*B)*cos(d*x + c)^3 + 13*(4*A + 3*B)*cos(d*x + c)^2 + 8*(4*
A + 3*B)*cos(d*x + c) + 8*A + 6*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d
)
```

Sympy [A] time = 16.207, size = 182, normalized size = 1.34

$$\begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} - \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} \\ \frac{x(A+B\cos(c))\cos^2(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a*
*4*d) - A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d) -
 B*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*B*tan(c/2 + d*x/2)**5/(40*a**4*d) -
B*tan(c/2 + d*x/2)**3/(8*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.18538, size = 158, normalized size = 1.16

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 63\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 21*A*tan
(1/2*d*x + 1/2*c)^5 + 63*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*
c)^3 - 105*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

3.69 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{(8A+13B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(8A+13B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(4A-11B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] -((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((4*A - 11*B)*Sin[c
+ d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((8*A + 13*B)*Sin[c + d*x])/(105*
d*(a^2 + a^2*Cos[c + d*x])^2) + ((8*A + 13*B)*Sin[c + d*x])/(105*d*(a^4 + a
^4*Cos[c + d*x]))
```

Rubi [A] time = 0.21469, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2968, 3019, 2750, 2650, 2648}

$$\frac{(8A+13B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(8A+13B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(4A-11B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((4*A - 11*B)*Sin[c
+ d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((8*A + 13*B)*Sin[c + d*x])/(105*
d*(a^2 + a^2*Cos[c + d*x])^2) + ((8*A + 13*B)*Sin[c + d*x])/(105*d*(a^4 + a
^4*Cos[c + d*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
```

```
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{-4a(A-B)-7aB\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)\int \frac{1}{(a+a\cos(c+}}{35a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)\sin(c+d}{105d\left(a^2+a^2\cos(c+d\right.}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)\sin(c+d}{105d\left(a^2+a^2\cos(c+d\right.}$$

Mathematica [A] time = 0.358649, size = 163, normalized size = 1.18

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-35(4A+5B)\sin\left(c+\frac{dx}{2}\right)+140(A+2B)\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+56A\sin\left(2c+\frac{5dx}{2}\right)+\right.}{420a^4d(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(140*(A + 2*B)*Sin[(d*x)/2] - 35*(4*A + 5*B)*Sin
[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 105*B
*Sin[2*c + (3*d*x)/2] + 56*A*Sin[2*c + (5*d*x)/2] + 91*B*Sin[2*c + (5*d*x)/
2] + 8*A*Sin[3*c + (7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 +
 Cos[c + d*x])^4)
```

Maple [A] time = 0.053, size = 88, normalized size = 0.6

$$\frac{1}{8\,da^4}\left(\frac{-A+B}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{-A-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A-B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A-B)*tan(1/2*d*x+1/2*c)^5+
1/3*(A-B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00994, size = 235, normalized size = 1.7

$$\frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.297, size = 309, normalized size = 2.24

$$\frac{\left((8\,A+13\,B)\cos\left(dx+c\right)^{3}+4\,(8\,A+13\,B)\cos\left(dx+c\right)^{2}+4\,(13\,A+8\,B)\cos\left(dx+c\right)+13\,A+8\,B\right)\sin\left(dx+c\right)}{105\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*((8*A + 13*B)*cos(d*x + c)^3 + 4*(8*A + 13*B)*cos(d*x + c)^2 + 4*(13*
A + 8*B)*cos(d*x + c) + 13*A + 8*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [A] time = 11.4166, size = 178, normalized size = 1.29

$$\begin{cases} -\frac{A\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}+\frac{A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+\frac{B\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}-\frac{B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d} \\ \frac{x(A+B\cos\left(c\right))\cos\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a
**4*d) + A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
+ B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) - B
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.17708, size = 158, normalized size = 1.14

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 21*A*ta
n(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2
*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

3.70 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A+4B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B)*Sin[c +
d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d
*(a^2 + a^2*Cos[c + d*x])^2) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d*(a^4 + a
^4*Cos[c + d*x]))
```

Rubi [A] time = 0.137988, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2750, 2650, 2648}

$$\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A+4B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B)*Sin[c +
d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d
*(a^2 + a^2*Cos[c + d*x])^2) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d*(a^4 + a
^4*Cos[c + d*x]))
```


Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{7a}$$

$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(2(3A+4B))\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} + \frac{(2(3A+}{}$$

$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} + \frac{2(3A}{105d\left(a\right.}$$

Mathematica [A] time = 0.324019, size = 109, normalized size = 0.79

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((3A+4B)\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35(3A+2B)\sin\left(\frac{dx}{2}\right)-70B\,\mathrm{s}\right.}{210a^4d(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(35*(3*A + 2*B)*Sin[(d*x)/2] - 70*B*Sin[c + (d*x
)/2] + (3*A + 4*B)*(21*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*
c + (7*d*x)/2])))/(210*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.049, size = 88, normalized size = 0.6

$$\frac{1}{8\,da^4}\left(\frac{A-B}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3\,A+B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-B)*tan(1/2*d*x+1/2*c)^5+
1/3*(3*A+B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00371, size = 236, normalized size = 1.71

$$\frac{\frac{B\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.40174, size = 311, normalized size = 2.25

$$\frac{\left(2\,(3\,A+4\,B)\cos\left(dx+c\right)^3+8\,(3\,A+4\,B)\cos\left(dx+c\right)^2+13\,(3\,A+4\,B)\cos\left(dx+c\right)+36\,A+13\,B\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*A + 4*B)*cos(d*x + c)^3 + 8*(3*A + 4*B)*cos(d*x + c)^2 + 13*(3*
A + 4*B)*cos(d*x + c) + 36*A + 13*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

Sympy [A] time = 8.08858, size = 177, normalized size = 1.28

$$\begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} - \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{f} \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^4} & \text{c} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40*
a**4*d) + A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
- B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) + B
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.18987, size = 158, normalized size = 1.14

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 63*A*tan
(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/2
*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

3.71 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=147

$$-\frac{2(80A-3B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-6B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(10A-3B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B}{7d(a\cos}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 6*B)*Sin[c + d*x])/(105*a^4*d*
(1 + Cos[c + d*x])^2) - (2*(80*A - 3*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c
 + d*x])) - ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((10*A -
3*B)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.465687, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$-\frac{2(80A-3B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-6B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(10A-3B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B}{7d(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 6*B)*Sin[c + d*x])/(105*a^4*d*
(1 + Cos[c + d*x])^2) - (2*(80*A - 3*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c
 + d*x])) - ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((10*A -
3*B)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(7aA-3a(A-B)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(35a^2A-2a^2(10A-3B)\cos(c+}{(a+a\cos(c+dx))^2}}{35a^4}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

Mathematica [A] time = 1.4191, size = 239, normalized size = 1.63

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-70(49A-3B)\sin\left(\frac{dx}{2}\right)+2170A\sin\left(c+\frac{dx}{2}\right)-2625A\sin\left(c+\frac{3dx}{2}\right)+735A\sin\left(2c+\frac{3dx}{2}\right)-10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-6720*A*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-70*(49
*A - 3*B)*Sin[(d*x)/2] + 2170*A*Sin[c + (d*x)/2] - 2625*A*Sin[c + (3*d*x)/2
] + 126*B*Sin[c + (3*d*x)/2] + 735*A*Sin[2*c + (3*d*x)/2] - 1015*A*Sin[2*c
```

```
+ (5*d*x)/2] + 42*B*Sin[2*c + (5*d*x)/2] + 105*A*Sin[3*c + (5*d*x)/2] - 160
*A*Sin[3*c + (7*d*x)/2] + 6*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c
+ d*x])^4)
```

Maple [A] time = 0.095, size = 199, normalized size = 1.4

$$\frac{B}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{da^4}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{11\,A}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{A}{da^4}\ln\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*B*tan(1/2*d*x+1/2*c)+1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3+1/d/a^4*A*l
n(tan(1/2*d*x+1/2*c)+1)-11/24/d/a^4*tan(1/2*d*x+1/2*c)^3*A-1/d/a^4*A*ln(tan
(1/2*d*x+1/2*c)-1)-15/8/d/a^4*A*tan(1/2*d*x+1/2*c)-1/56/d/a^4*tan(1/2*d*x+1
/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*A*tan(1/2*d*x+1/2*c)^
5+3/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5
```

Maxima [A] time = 1.03875, size = 308, normalized size = 2.1

$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-\frac{3\,B\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx}{(\cos(dx+c}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 3*B*(35*sin(d*x +
c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/
d
```

Fricas [A] time = 1.48182, size = 624, normalized size = 4.24

$$\frac{105\left(A\cos\left(dx+c\right)^4+4\,A\cos\left(dx+c\right)^3+6\,A\cos\left(dx+c\right)^2+4\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-105\left(A\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/210*(105*(A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*
A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 105*(A*cos(d*x + c)^4 + 4*A*cos
(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*A*cos(d*x + c) + A)*log(-sin(d*x + c)
+ 1) - 2*(2*(80*A - 3*B)*cos(d*x + c)^3 + (535*A - 24*B)*cos(d*x + c)^2 + (
620*A - 39*B)*cos(d*x + c) + 260*A - 36*B)*sin(d*x + c))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25902, size = 246, normalized size = 1.67

$$\frac{\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-63\,Ba^{24}\tan}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] 1/840*(840*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan
(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 63*B*a^24*tan(1/2
*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*
x + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 105*B*a^24*tan(1/2*d*x +
1/2*c))/a^28)/d
```

3.72 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=175

$$\frac{8(83A-20B)\tan(c+dx)}{105a^4d}-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(4A-B)\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{(88A-25B)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(12A}{35ad(a}$$

```
[Out] -(((4*A - B)*ArcTanh[Sin[c + d*x]])/(a^4*d)) + (8*(83*A - 20*B)*Tan[c + d*x
])/(105*a^4*d) - ((88*A - 25*B)*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
^2) - ((4*A - B)*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A - B)*Tan[c
+ d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((12*A - 5*B)*Tan[c + d*x])/(35*a*d*
(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.671646, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{8(83A-20B)\tan(c+dx)}{105a^4d}-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(4A-B)\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{(88A-25B)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(12A}{35ad(a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] -(((4*A - B)*ArcTanh[Sin[c + d*x]])/(a^4*d)) + (8*(83*A - 20*B)*Tan[c + d*x
])/(105*a^4*d) - ((88*A - 25*B)*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
^2) - ((4*A - B)*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A - B)*Tan[c
+ d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((12*A - 5*B)*Tan[c + d*x])/(35*a*d*
(a + a*Cos[c + d*x])^3)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(8A-B)-4a(A-B)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(2a^2(26A-5B)-3a^2(12A-5B)}{(a+a\cos(c-}}{35a}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\tan(c+dx)}{35ad(a+a\cos(c+dx))}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\tan(c+dx)}{35ad(a+a\cos(c+dx))}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\tan(c+dx)}{35ad(a+a\cos(c+dx))}$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+}{7d(a+a\cos(c+}$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{8(83A-20B)\tan(c+dx)}{105a^4d} - \frac{(88A-25B)\,\mathrm{ta}}{105a^4d(1+\cos}$$

Mathematica [B] time = 4.87917, size = 595, normalized size = 3.4

$$26880(4A-B)\cos^8\left(\tfrac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\right)+\mathrm{se}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (26880*(4*A - B)*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]*(-245*(44*A - 17*B)*Sin[(d*x)/2] + 7*(2684*A - 635*B)*S
in[(3*d*x)/2] - 20524*A*Sin[c - (d*x)/2] + 4795*B*Sin[c - (d*x)/2] + 14644*
A*Sin[c + (d*x)/2] - 4795*B*Sin[c + (d*x)/2] - 16660*A*Sin[2*c + (d*x)/2] +
 4165*B*Sin[2*c + (d*x)/2] - 4690*A*Sin[c + (3*d*x)/2] + 2275*B*Sin[c + (3*
d*x)/2] + 14378*A*Sin[2*c + (3*d*x)/2] - 4445*B*Sin[2*c + (3*d*x)/2] - 9100
*A*Sin[3*c + (3*d*x)/2] + 2275*B*Sin[3*c + (3*d*x)/2] + 11668*A*Sin[c + (5*
d*x)/2] - 2785*B*Sin[c + (5*d*x)/2] - 630*A*Sin[2*c + (5*d*x)/2] + 735*B*Si
n[2*c + (5*d*x)/2] + 9358*A*Sin[3*c + (5*d*x)/2] - 2785*B*Sin[3*c + (5*d*x)
/2] - 2940*A*Sin[4*c + (5*d*x)/2] + 735*B*Sin[4*c + (5*d*x)/2] + 4228*A*Sin
```

```
[2*c + (7*d*x)/2] - 1015*B*Sin[2*c + (7*d*x)/2] + 315*A*Sin[3*c + (7*d*x)/2
] + 105*B*Sin[3*c + (7*d*x)/2] + 3493*A*Sin[4*c + (7*d*x)/2] - 1015*B*Sin[4
*c + (7*d*x)/2] - 420*A*Sin[5*c + (7*d*x)/2] + 105*B*Sin[5*c + (7*d*x)/2] +
 664*A*Sin[3*c + (9*d*x)/2] - 160*B*Sin[3*c + (9*d*x)/2] + 105*A*Sin[4*c +
(9*d*x)/2] + 559*A*Sin[5*c + (9*d*x)/2] - 160*B*Sin[5*c + (9*d*x)/2]))/(168
0*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.113, size = 285, normalized size = 1.6

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{23\,A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7+7/40/d/
a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4*tan
(1/2*d*x+1/2*c)^3*A-11/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/2
*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+4/d/a^4*A*ln(tan(1/2*d*x+1/2*c)
-1)-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)-4/d
/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4*
A/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.03907, size = 440, normalized size = 2.51

$$\frac{A\left(\frac{1680\,\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin(dx+c}{\cos(dx+c)\cdot}\right.}{a^4}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
```

```
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 5
*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [B] time = 1.41286, size = 838, normalized size = 4.79

$$-\frac{105\left((4\,A-B)\cos\left(dx+c\right)^{5}+4\,(4\,A-B)\cos\left(dx+c\right)^{4}+6\,(4\,A-B)\cos\left(dx+c\right)^{3}+4\,(4\,A-B)\cos\left(dx+c\right)^{2}+(4\,A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*x + c)^4 + 6*(4*A
 - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A - B)*cos(d*x + c))
*log(sin(d*x + c) + 1) - 105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*
x + c)^4 + 6*(4*A - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A -
 B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(8*(83*A - 20*B)*cos(d*x + c)^
4 + (2236*A - 535*B)*cos(d*x + c)^3 + 4*(659*A - 155*B)*cos(d*x + c)^2 + 4*
(296*A - 65*B)*cos(d*x + c) + 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 +
4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 +
a^4*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2903, size = 302, normalized size = 1.73

$$-\frac{\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(840*(4*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(4*A - B
)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 1680*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a
^24*tan(1/2*d*x + 1/2*c)^7 + 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*B*a^24
*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*ta
n(1/2*d*x + 1/2*c)^3 + 5145*A*a^24*tan(1/2*d*x + 1/2*c) - 1575*B*a^24*tan(1
/2*d*x + 1/2*c))/a^28)/d
```

3.73 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$-\frac{8(216A-83B)\tan(c+dx)}{105a^4d}+\frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A-8B)\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{4(216A-83B)}{105a^4d(}$$

```
[Out] ((21*A - 8*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(216*A - 83*B)*Tan[c +
d*x])/(105*a^4*d) + ((21*A - 8*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((
129*A - 52*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) -
 (4*(216*A - 83*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((2*
A - B)*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.687616, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{8(216A-83B)\tan(c+dx)}{105a^4d}+\frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A-8B)\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{4(216A-83B)}{105a^4d(}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] ((21*A - 8*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(216*A - 83*B)*Tan[c +
d*x])/(105*a^4*d) + ((21*A - 8*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((
129*A - 52*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) -
 (4*(216*A - 83*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((2*
A - B)*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(9A-2B)-5a(A-B)\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2A-B)\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(a^2(7\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2\cdots}{\cdots}$$

$$= \frac{(21A-8B)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \cdots$$

$$= \frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{8(216A-83B)\tan(c+dx)}{105a^4d} + \frac{(21A-8B)\text{se}\cdots}{\cdots}$$

Mathematica [B] time = 6.44624, size = 798, normalized size = 3.44

$$-\frac{8(21A-8B)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(\cos(c+dx)a+a)^4} + \frac{8(21A-8B)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\cos^8\left(\frac{c}{2}+\cdots\right.}{d(\cos(c+dx)a+a)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^8*
Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) +
(Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(73206*A*Sin[(d*x)/2] -
38668*B*Sin[(d*x)/2] - 166668*A*Sin[(3*d*x)/2] + 64384*B*Sin[(3*d*x)/2] + 1
83162*A*Sin[c - (d*x)/2] - 70896*B*Sin[c - (d*x)/2] - 100842*A*Sin[c + (d*x
)/2] + 50316*B*Sin[c + (d*x)/2] + 155526*A*Sin[2*c + (d*x)/2] - 59248*B*Sin
[2*c + (d*x)/2] + 37380*A*Sin[c + (3*d*x)/2] - 22820*B*Sin[c + (3*d*x)/2] -
 101148*A*Sin[2*c + (3*d*x)/2] + 48004*B*Sin[2*c + (3*d*x)/2] + 102900*A*Si
n[3*c + (3*d*x)/2] - 39200*B*Sin[3*c + (3*d*x)/2] - 119364*A*Sin[c + (5*d*x
)/2] + 46032*B*Sin[c + (5*d*x)/2] + 8820*A*Sin[2*c + (5*d*x)/2] - 8750*B*Si
```

```
n[2*c + (5*d*x)/2] - 78204*A*Sin[3*c + (5*d*x)/2] + 35742*B*Sin[3*c + (5*d*
x)/2] + 49980*A*Sin[4*c + (5*d*x)/2] - 19040*B*Sin[4*c + (5*d*x)/2] - 64053
*A*Sin[2*c + (7*d*x)/2] + 24664*B*Sin[2*c + (7*d*x)/2] - 3885*A*Sin[3*c + (
7*d*x)/2] - 1050*B*Sin[3*c + (7*d*x)/2] - 44733*A*Sin[4*c + (7*d*x)/2] + 19
834*B*Sin[4*c + (7*d*x)/2] + 15435*A*Sin[5*c + (7*d*x)/2] - 5880*B*Sin[5*c
+ (7*d*x)/2] - 21987*A*Sin[3*c + (9*d*x)/2] + 8456*B*Sin[3*c + (9*d*x)/2] -
 3675*A*Sin[4*c + (9*d*x)/2] + 630*B*Sin[4*c + (9*d*x)/2] - 16107*A*Sin[5*c
 + (9*d*x)/2] + 6986*B*Sin[5*c + (9*d*x)/2] + 2205*A*Sin[6*c + (9*d*x)/2] -
 840*B*Sin[6*c + (9*d*x)/2] - 3456*A*Sin[4*c + (11*d*x)/2] + 1328*B*Sin[4*c
 + (11*d*x)/2] - 840*A*Sin[5*c + (11*d*x)/2] + 210*B*Sin[5*c + (11*d*x)/2]
- 2616*A*Sin[6*c + (11*d*x)/2] + 1118*B*Sin[6*c + (11*d*x)/2]))/(6720*d*(a
+ a*Cos[c + d*x])^4)
```

Maple [A] time = 0.115, size = 374, normalized size = 1.6

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,B}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{13\,A}{8\,da^4}\Big(\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-9/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4*ta
n(1/2*d*x+1/2*c)^3*A+23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*A*tan(1
/2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-21/2/d/a^4*A*ln(tan(1/2*d*x+1
/2*c)-1)+4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B+9/2/d/a^4*A/(tan(1/2*d*x+1/2*c)
-1)-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^2+9
/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)*B+21/2/d/a
^4*A*ln(tan(1/2*d*x+1/2*c)+1)-4/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^4*
A/(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [A] time = 1.03788, size = 566, normalized size = 2.44

$$-\frac{3\,A\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(d}{\cos(dx}\right.}{a^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) - B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))
/d
```

Fricas [A] time = 1.51293, size = 938, normalized size = 4.04

$$\frac{105\left((21\,A-8\,B)\cos\left(dx+c\right)^{6}+4\,(21\,A-8\,B)\cos\left(dx+c\right)^{5}+6\,(21\,A-8\,B)\cos\left(dx+c\right)^{4}+4\,(21\,A-8\,B)\cos\left(dx+c\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/420*(105*((21*A - 8*B)*cos(d*x + c)^6 + 4*(21*A - 8*B)*cos(d*x + c)^5 + 6
*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*A - 8*B)*cos(d*x + c)^3 + (21*A - 8*B)
*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((21*A - 8*B)*cos(d*x + c)^6 +
 4*(21*A - 8*B)*cos(d*x + c)^5 + 6*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*A -
8*B)*cos(d*x + c)^3 + (21*A - 8*B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) -
 2*(16*(216*A - 83*B)*cos(d*x + c)^5 + (11619*A - 4472*B)*cos(d*x + c)^4 +
4*(3411*A - 1318*B)*cos(d*x + c)^3 + 4*(1509*A - 592*B)*cos(d*x + c)^2 + 21
0*(2*A - B)*cos(d*x + c) - 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a
^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4
*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28189, size = 360, normalized size = 1.55

$$\frac{\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(420*(21*A - 8*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(21*A
- 8*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(9*A*tan(1/2*d*x + 1/2*
c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 7*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24*tan(1/2*d*x
 + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 189*A*a^24*tan(1/2*d*x + 1
/2*c)^5 - 147*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x + 1/2
*c)^3 - 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 11655*A*a^24*tan(1/2*d*x + 1/2*
c) - 5145*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.74 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=187

$$\frac{2a(9A+8B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4(9A+8B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\sin(c+dx)\sqrt{a\cos(c}}{315d}$$

```
[Out] (4*a*(9*A + 8*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(9*A
+ 8*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
B*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (8*(9*A + 8
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.303999, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2981, 2770, 2759, 2751, 2646}

$$\frac{2a(9A+8B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4(9A+8B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\sin(c+dx)\sqrt{a\cos(c}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a*(9*A + 8*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(9*A
+ 8*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
B*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (8*(9*A + 8
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}(9A+8B)\int \cos^3(c+dx)\sqrt{}$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^4(c+dx)\sin}{9d\sqrt{a+a\cos(c-}}$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^4(c+dx)\sin}{9d\sqrt{a+a\cos(c-}}$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^4(c+dx)\sin}{9d\sqrt{a+a\cos(c-}}$$

$$= \frac{4a(9A+8B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+}{63d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.628738, size = 103, normalized size = 0.55

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(94(9A+8B)\cos(c+dx)+4(54A+83B)\cos(2(c+dx))+90A\cos(3(c+dx))+13}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(1368*A + 1321*B + 94*(9*A + 8*B)*Cos[c + d*x]
+ 4*(54*A + 83*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 80*B*Cos[3*(c
+ d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 1.147, size = 121, normalized size = 0.7

$$\frac{2a\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560B(\sin(1/2dx+c/2))^8+(-360A-1440B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+(756A+1512B)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*B*sin(1/2*d*x+1/2*c)^8+(
-360*A-1440*B)*sin(1/2*d*x+1/2*c)^6+(756*A+1512*B)*sin(1/2*d*x+1/2*c)^4+(-6
```

```
30*A-840*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2
*a)^(1/2)/d
```

Maxima [A] time = 1.90054, size = 196, normalized size = 1.05

$$\frac{18\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(35\sqrt{2}\sin\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/2520*(18*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c)
 + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sq
rt(a) + (35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c)
+ 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 189
0*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.33308, size = 263, normalized size = 1.41

$$\frac{2\left(35\,B\cos\left(dx+c\right)^{4}+5\left(9\,A+8\,B\right)\cos\left(dx+c\right)^{3}+6\left(9\,A+8\,B\right)\cos\left(dx+c\right)^{2}+8\left(9\,A+8\,B\right)\cos\left(dx+c\right)+144\,A+12\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*B*cos(d*x + c)^4 + 5*(9*A + 8*B)*cos(d*x + c)^3 + 6*(9*A + 8*B)*c
os(d*x + c)^2 + 8*(9*A + 8*B)*cos(d*x + c) + 144*A + 128*B)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

3.75 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=144

$$\frac{2(7A+6B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a*(7*A + 6*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (4*(7*A + 6*B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.264805, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2981, 2759, 2751, 2646}

$$\frac{2(7A+6B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*a*(7*A + 6*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (4*(7*A + 6*B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```


Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
```

```
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(7A+6B)\int \cos^2(c+dx)\sqrt{}$$

$$= \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A+6B)(a+a\cos(c+dx)}{35ad}$$

$$= \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{4(7A+6B)\sqrt{a+a\cos(c+dx}}{105d}$$

$$= \frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{4}{}$$

Mathematica [A] time = 0.334778, size = 80, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((112A+141B)\cos(c+dx)+6(7A+6B)\cos(2(c+dx))+266A+15B\cos(3(c+dx}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(266*A + 228*B + (112*A + 141*B)*Cos[c + d*x] +
 6*(7*A + 6*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/
```

```
(210*d)
```

Maple [A] time = 0.998, size = 102, normalized size = 0.7

$$\frac{2\,a\sqrt{2}}{105\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-120\,B\,(\sin(1/2\,dx+c/2))^{6}+(84\,A+252\,B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}+(-140\,A-210\,B)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/105*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-120*B*sin(1/2*d*x+1/2*c)^6+
(84*A+252*B)*sin(1/2*d*x+1/2*c)^4+(-140*A-210*B)*sin(1/2*d*x+1/2*c)^2+105*A
+105*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.93533, size = 159, normalized size = 1.1

$$\frac{14\left(3\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+3\left(5\sqrt{2}\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+7\sqrt{2}\sin\left(\right.\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/420*(14*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 3*(5*sqrt(2)*sin(7/2*d*x + 7
/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) +
105*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.22406, size = 219, normalized size = 1.52

$$\frac{2\left(15\,B\cos(dx+c)^{3}+3\,(7\,A+6\,B)\cos(dx+c)^{2}+4\,(7\,A+6\,B)\cos(dx+c)+56\,A+48\,B\right)\sqrt{a\cos(dx+c)+a}\sin(dx}{105\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/105*(15*B*cos(d*x + c)^3 + 3*(7*A + 6*B)*cos(d*x + c)^2 + 4*(7*A + 6*B)*c
os(d*x + c) + 56*A + 48*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.76 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

```
[Out] (2*a*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(5*A -
2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.201873, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2968, 3023, 2751, 2646}

$$\frac{2(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*a*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(5*A -
2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*a*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int \sqrt{a+a\cos(c+dx)}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)\,dx$$

$$= \frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{2\int \sqrt{a+a\cos(c+dx)}$$

$$= \frac{2(5A-2B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2B(a+a\cos(c+$$

$$= \frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-2B)\sqrt{a+a\cos(c+dx)}\sin}{15d}$$

Mathematica [A] time = 0.185763, size = 64, normalized size = 0.63

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(5A+4B)\cos(c+dx)+20A+3B\cos(2(c+dx))+19B)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(20*A + 19*B + 2*(5*A + 4*B)*Cos[c + d*x] + 3*B
*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 1.095, size = 83, normalized size = 0.8

$$\frac{2a\sqrt{2}}{15d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12\,B\,(\sin(1/2\,dx+c/2))^4+(-10\,A-20\,B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+15\,A+15\,B\right)\frac{}{\sqrt{\left(\cos\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*B*sin(1/2*d*x+1/2*c)^4+(-1
0*A-20*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^
(1/2)/d
```

Maxima [A] time = 1.82406, size = 119, normalized size = 1.18

$$\frac{10\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(3\,\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\,\sqrt{2}\sin\left(\frac{1}{2}\,dx\right.\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*
sqrt(a) + (3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.34265, size = 170, normalized size = 1.68

$$\frac{2\left(3\,B\cos(dx+c)^2+(5\,A+4\,B)\cos(dx+c)+10\,A+8\,B\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{15\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/15*(3*B*cos(d*x + c)^2 + (5*A + 4*B)*cos(d*x + c) + 10*A + 8*B)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.77 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] (2*a*(3*A + B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*B*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0586005, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2751, 2646}

$$\frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(3*A + B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*B*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2B\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(3A+B)\int \sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.0749198, size = 46, normalized size = 0.74

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(3A+B\cos(c+dx)+2B)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(3*A + 2*B + B*Cos[c + d*x])*Tan[(c + d*x)/2]
)/(3*d)
```

Maple [A] time = 1.125, size = 62, normalized size = 1.

$$\frac{2\,a\sqrt{2}}{3\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\,B\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+3\,A+B\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(2*B*cos(1/2*d*x+1/2*c)^2+3*A+B
)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.79246, size = 77, normalized size = 1.24

$$\frac{6\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(6*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*sin(3/2*d*x + 3/2*
c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.33251, size = 126, normalized size = 2.03

$$\frac{2\,(B\cos\left(dx+c\right)+3\,A+2\,B)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{3\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/3*(B*cos(d*x + c) + 3*A + 2*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*c
os(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(B\cos\left(dx+c\right)+A)\sqrt{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a), x)
```

3.78 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.137595, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2981, 2773, 206}

$$\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+A\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx$$
$$= \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(2aA)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.085383, size = 66, normalized size = 1.

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*B*Sin[(c + d*x)/2]))/d
```

Maple [B] time = 3.296, size = 210, normalized size = 3.2

$$\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}-a\sqrt{2}\cos(1/2\,dx+c/2)+2\,a}{-2\,\cos(1/2\,dx+c/2)+\sqrt{2}}\right)a+A\ln\left(4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(-4/(-2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/
2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.63519, size = 28, normalized size = 0.42

$$\frac{2\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 2*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [B] time = 1.52075, size = 344, normalized size = 5.21

$$\frac{(A\cos\left(dx+c\right)+A)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos\left(dx+c\right)+a}B\sin}{2\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*B*sin(d*
x + c))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 3.98457, size = 201, normalized size = 3.05

$$\frac{\frac{2\sqrt{2}Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{Aa^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] (2*sqrt(2)*B*a*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + A*
a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a))/
d
```

3.79 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a}(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (a*A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.162592, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2980, 2773, 206}

$$\frac{\sqrt{a}(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (a*A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{1}{2}(A+2B)\int \sqrt{a+a\cos(c+dx)}\sec$$
$$= \frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(a(A+2B))\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a}{\sqrt{a}}\right.}{d}$$
$$= \frac{\sqrt{a}(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.199585, size = 85, normalized size = 1.25

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(A+2B)\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(A + 2*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*A*Sin[(c + d*x)/2]))/(
2*d)
```

Maple [B] time = 3.951, size = 642, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(A*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))+A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+2*B*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*B*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^
(1/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2
*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 1.8696, size = 959, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d
*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2
*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*lo
g(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqr
```

```
t(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)
*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))*A*sqrt(a)/(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] time = 1.56449, size = 406, normalized size = 5.97

$$\frac{\left((A+2\,B)\cos\left(dx+c\right)^2+(A+2\,B)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)\cdots}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/4*(((A + 2*B)*cos(d*x + c)^2 + (A + 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(
d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sq
rt(a*cos(d*x + c) + a)*A*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.85137, size = 351, normalized size = 5.16

$$\frac{\left(A\sqrt{a}+2\,B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)-\left(A\sqrt{a}+2\,B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\,t\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] 1/2*((A*sqrt(a) + 2*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (A*sqrt(a) + 2*B*
sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(3/2) - A*a^(5/2))/((sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.80 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(3*A + 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a*(3*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.220356, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2980, 2772, 2773, 206}

$$\frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a]*(3*A + 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a*(3*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(3A+4B)\int \sqrt{a+a\cos(c$$
$$= \frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}$$
$$= \frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(a}{}$$
$$= \frac{\sqrt{a}(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a(3A+4B)\tan(c-}{4d\sqrt{a+a\cos(c+}}$$

Mathematica [A] time = 0.781155, size = 101, normalized size = 0.86

$$\frac{\sqrt{a(\cos(c+dx)+1)}\left(6\tan\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)((3A+4B)\cos(c+dx)+2A)+3\sqrt{2}(3A+4B)\sec\left(\frac{1}{2}(c+dx)\right)\tan}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(3*Sqrt[2]*(3*A + 4*B)*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]]*Sec[(c + d*x)/2] + 6*(2*A + (3*A + 4*B)*Cos[c + d*x])*Sec[c + d*x
]^2*Tan[(c + d*x)/2]))/(24*d)
```

Maple [B] time = 3.84, size = 991, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(3*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+4*B*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(3*A*a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+4*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*A*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+
1/2*c)^2+10*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+8*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 20.6011, size = 4525, normalized size = 38.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] 1/16*((3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
cos(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)
*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin
(2*d*x + 2*c) + 2*(6*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x +
```

```
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2))*cos(2*d*x + 2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*s
in(5/2*d*x + 5/2*c) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(4*d*x + 4*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)
*sin(1/2*d*x + 1/2*c) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d
*x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) +
 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*
cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 1
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1) - 4*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(
2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3
```

```
/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2
*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)
^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))
*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1))/d
```

Fricas [A] time = 1.51406, size = 460, normalized size = 3.93

$$\frac{\left((3\,A+4\,B)\cos\left(dx+c\right)^{3}+(3\,A+4\,B)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*((((3*A + 4*B)*cos(d*x + c)^3 + (3*A + 4*B)*cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
 + 4*((3*A + 4*B)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.76518, size = 637, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] 1/8*((3*A*sqrt(a) + 4*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(a) +
4*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(3/2) - 12*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(3/2) + 19*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(5/
2) + 76*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^4*B*a^(5/2) - 17*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^2*A*a^(7/2) - 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*B*a^(7/2) + A*a^(9/2) + 4*B*a^(9/2))/((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.81 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+d}{3d\sqrt{a\cos(}}$$

```
[Out] (Sqrt[a]*(5*A + 6*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*d) + (a*(5*A + 6*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*(5*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
(a*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.292719, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2980, 2772, 2773, 206}

$$\frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+d}{3d\sqrt{a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a]*(5*A + 6*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*d) + (a*(5*A + 6*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*(5*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
(a*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(5A+6B)\int \sqrt{a+a\cos(c-$$

$$= \frac{a(5A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec^2(c+dx)\tan(c}{3d\sqrt{a+a\cos(c+}$$

$$= \frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(5A+6B)\sec(c+dx)\tan(c+d}{12d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(5A+6B)\sec(c+dx)\tan(c+d}{12d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{a}(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a(5A+6B)\tan(c+}{8d\sqrt{a+a\cos(c+d}}$$

Mathematica [A] time = 1.78713, size = 129, normalized size = 0.81

$$\frac{\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\tan\left(\frac{1}{2}(c+dx)\right)(4(5A+6B)\cos(c+dx)+3(5A+6B)\cos(2(c+dx))+31A+18B)+\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[c + d*x]^3*(3*Sqrt[2]*(5*A + 6*B)*ArcTanh[S
qrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3*Sec[(c + d*x)/2] + (31*A + 18*B + 4
*(5*A + 6*B)*Cos[c + d*x] + 3*(5*A + 6*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2
]))/(48*d)
```

Maple [B] time = 3.984, size = 1311, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(5*A*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+5*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))+6*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+6*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(10*A*a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+12*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+15*A*ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+18*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+18*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin
(1/2*d*x+1/2*c)^4-2*(80*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+96
*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+45*A*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a+45*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+54*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+54*B*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+66*A*a^(1/2)*2^(1/2)
```

```
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+60*B*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+18*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a+18*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(1/2)
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/sin(1/2*
d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 21.461, size = 6778, normalized size = 42.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] -1/96*((120*(sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*co
s(13/2*d*x + 13/2*c) - 8*(15*sin(11/2*d*x + 11/2*c) + 50*sin(9/2*d*x + 9/2*
c) + 42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x + 3/2
*c))*cos(6*d*x + 6*c) + 360*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(11/2*
d*x + 11/2*c) + 1200*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(9/2*d*x + 9/
2*c) - 24*(42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x
 + 3/2*c))*cos(4*d*x + 4*c) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*co
s(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)
^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2
*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
```

```
6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2
 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*si
n(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x
 + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x +
4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) + 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^
2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*s
in(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2
)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 120*(cos(6*d*x
+ 6*c) + 3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(13/2*d*x + 13/2*c
) + 8*(15*cos(11/2*d*x + 11/2*c) + 50*cos(9/2*d*x + 9/2*c) + 42*cos(7/2*d*x
 + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*
c) - 120*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(11/2*d*x + 11/2*
c) - 400*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(9/2*d*x + 9/2*c)
 + 24*(42*cos(7/2*d*x + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3
/2*c))*sin(4*d*x + 4*c) - 336*(3*cos(2*d*x + 2*c) + 1)*sin(7/2*d*x + 7/2*c)
 - 24*(3*cos(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c) + 1008*cos(7/2*d*x + 7/
2*c)*sin(2*d*x + 2*c) + 72*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 120*cos(
3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 120*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + 120*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 40*sin(3/2*d*x + 3/2*c))
*A*sqrt(a)/(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*s
qrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*
x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
```

6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)) - 6*(3*(log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 12*
(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x
 + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*cos(7/2*d*x + 7/2*c)*si
n(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 2*(6*(lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x +
2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*sin(5/2*d*x + 5/2*c) - 2*
sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)

```
+ 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4
*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*sin(1/2*d*x + 1/2*c) -
3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*co
s(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - sqrt(2)*co
s(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*cos(1/2*d*x +
 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7
/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/
2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*si
n(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*sin(1/2*d*x +
1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.61382, size = 508, normalized size = 3.18

$$\frac{3\left((5A+6B)\cos(dx+c)^4+(5A+6B)\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos(dx+c)^4+d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
```

```
="fricas")
```

```
[Out] 1/96*(3*((5*A + 6*B)*cos(d*x + c)^4 + (5*A + 6*B)*cos(d*x + c)^3)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)) + 4*(3*(5*A + 6*B)*cos(d*x + c)^2 + 2*(5*A + 6*B)*cos(d*x + c) + 8*A)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.7927, size = 861, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] 1/48*(3*(5*A*sqrt(a) + 6*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(
a) + 6*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(63*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(3/2) - 30*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(3/2)
 - 369*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
8*A*a^(5/2) + 66*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^8*B*a^(5/2) + 1638*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^6*A*a^(7/2) + 756*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(7/2) - 1074*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(9/2) - 732*(sqrt(a)*tan
```

```
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(9/2) + 171*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(1
1/2) + 138*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*B*a^(11/2) - 13*A*a^(13/2) - 6*B*a^(13/2))/((sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.82 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=234

$$\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(187A+168B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (4*a^2*(187*A + 168*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*(187*A + 168*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(11*A + 12*B)*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a + a
*Cos[c + d*x]]) - (8*a*(187*A + 168*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(3465*d) + (2*a*B*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(11*d) + (4*(187*A + 168*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1155*
d)
```

Rubi [A] time = 0.529324, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2976, 2981, 2770, 2759, 2751, 2646}

$$\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(187A+168B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (4*a^2*(187*A + 168*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*(187*A + 168*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(11*A + 12*B)*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a + a
*Cos[c + d*x]]) - (8*a*(187*A + 168*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(3465*d) + (2*a*B*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(11*d) + (4*(187*A + 168*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1155*
d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

Q[a^2 - b^2, 0]

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{11d} + \frac{2}{11}\int \mathrm{co}$$

$$= \frac{2a^2(11A+12B)\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^4(c+d}{}$$

$$= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(11A+1}{99d}$$

$$= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(11A+1}{99d}$$

$$= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(11A+1}{99d}$$

$$= \frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(187A+168B)\cos^3(c}{693d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.956185, size = 125, normalized size = 0.53

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((35156A+34734B)\cos(c+dx)+8(1507A+1743B)\cos(2(c+dx))+3740A\cos}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(59158*A + 55482*B + (35156*A + 34734*B)*Cos[
c + d*x] + 8*(1507*A + 1743*B)*Cos[2*(c + d*x)] + 3740*A*Cos[3*(c + d*x)] +
 4935*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 1470*B*Cos[4*(c + d*x)]
 + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 1.046, size = 142, normalized size = 0.6

$$\frac{4a^2\sqrt{2}}{3465d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-5040B(\sin(1/2dx+c/2))^{10}+(3080A+18480B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8+(-9900A-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/3465*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-5040*B*sin(1/2*d*x+1/2*c
)^10+(3080*A+18480*B)*sin(1/2*d*x+1/2*c)^8+(-9900*A-27720*B)*sin(1/2*d*x+1/
2*c)^6+(12474*A+22176*B)*sin(1/2*d*x+1/2*c)^4+(-8085*A-10395*B)*sin(1/2*d*x
+1/2*c)^2+3465*A+3465*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 2.00005, size = 250, normalized size = 1.07

$$\frac{22\left(35\sqrt{2}a\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+135\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+378\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+1050\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3780\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/55440*(22*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x
+ 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 21*(15*sqrt(2)*
a*sin(11/2*d*x + 11/2*c) + 55*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 165*sqrt(2)*
a*sin(7/2*d*x + 7/2*c) + 429*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 990*sqrt(2)*a
*sin(3/2*d*x + 3/2*c) + 3630*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.33085, size = 351, normalized size = 1.5

$$\frac{2\left(315\,Ba\cos\left(dx+c\right)^{5}+35\left(11\,A+21\,B\right)a\cos\left(dx+c\right)^{4}+5\left(187\,A+168\,B\right)a\cos\left(dx+c\right)^{3}+6\left(187\,A+168\,B\right)a\cos\left(d\right.\right.}{3465\left(d\cos\left(dx+c\right)+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(315*B*a*cos(d*x + c)^5 + 35*(11*A + 21*B)*a*cos(d*x + c)^4 + 5*(187
*A + 168*B)*a*cos(d*x + c)^3 + 6*(187*A + 168*B)*a*cos(d*x + c)^2 + 8*(187*
A + 168*B)*a*cos(d*x + c) + 16*(187*A + 168*B)*a)*sqrt(a*cos(d*x + c) + a)*
```

```
sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x
)
```

3.83 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{2(39A+34B)\sin(c+dx)(a\cos(c+dx)+a)^3}{105d}$$

```
[Out] (2*a^2*(39*A + 34*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(9*A + 10*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]])
- (4*a*(39*A + 34*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*
B*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(39*A +
34*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d)
```

Rubi [A] time = 0.446453, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2981, 2759, 2751, 2646}

$$\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{2(39A+34B)\sin(c+dx)(a\cos(c+dx)+a)^3}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a^2*(39*A + 34*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(9*A + 10*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]])
- (4*a*(39*A + 34*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*
B*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(39*A +
34*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
```

```
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d} + \frac{2}{9}\int \mathrm{c}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^3(c+}{}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^3(c+}{}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} - \frac{4a(39A+34B}{}$$

$$= \frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(9A+10B)\cos^3(c+d}{63d\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.528046, size = 103, normalized size = 0.54

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(759A+799B)\cos(c+dx)+(468A+548B)\cos(2(c+dx))+90A\cos(3(c+d}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(2964*A + 2689*B + 2*(759*A + 799*B)*Cos[c +
d*x] + (468*A + 548*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 170*B*Cos
[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 1.463, size = 123, normalized size = 0.7

$$\frac{4a^2\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280B(\sin(1/2\,dx+c/2))^8+(-180A-900B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+(504A+1134B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*B*sin(1/2*d*x+1/2*c)^8
+(-180*A-900*B)*sin(1/2*d*x+1/2*c)^6+(504*A+1134*B)*sin(1/2*d*x+1/2*c)^4+(-
```

```
525*A-735*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^
2*a)^(1/2)/d
```

Maxima [A] time = 1.96898, size = 208, normalized size = 1.1

$$\frac{6\left(15\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+175\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+735\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(35\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/2520*(6*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5
/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/
2*c))*A*sqrt(a) + (35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/
2*d*x + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.40363, size = 286, normalized size = 1.51

$$\frac{2\left(35\,Ba\cos\left(dx+c\right)^{4}+5\left(9\,A+17\,B\right)a\cos\left(dx+c\right)^{3}+3\left(39\,A+34\,B\right)a\cos\left(dx+c\right)^{2}+4\left(39\,A+34\,B\right)a\cos\left(dx+c\right)+8\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*B*a*cos(d*x + c)^4 + 5*(9*A + 17*B)*a*cos(d*x + c)^3 + 3*(39*A +
34*B)*a*cos(d*x + c)^2 + 4*(39*A + 34*B)*a*cos(d*x + c) + 8*(39*A + 34*B)*a
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

3.84 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

```
[Out] (8*a^2*(21*A + 19*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(21*A + 19*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A - 2*
B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.250352, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2751, 2647, 2646}

$$\frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (8*a^2*(21*A + 19*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(21*A + 19*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A - 2*
B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*a*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^{3/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)$$

$$= \frac{2B(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{2\int (a+a\cos(c+d}{}$$

$$= \frac{2(7A-2B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+a\,c}{}$$

$$= \frac{2a(21A+19B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2(7A-2}{}$$

$$= \frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(21A+19B)\sqrt{a+a\cos(}}{105d}$$

Mathematica [A] time = 0.356695, size = 81, normalized size = 0.59

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((252A+253B)\cos(c+dx)+6(7A+13B)\cos(2(c+dx))+546A+15B\cos(3(c}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(546*A + 494*B + (252*A + 253*B)*Cos[c + d*x]
 + 6*(7*A + 13*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2
])/(210*d)
```

Maple [A] time = 1.29, size = 104, normalized size = 0.8

$$\frac{4\,a^2\sqrt{2}}{105\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-60\,B\,(\sin(1/2\,dx+c/2))^6+(42\,A+168\,B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+(-105\,A-175\,B)\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-60*B*sin(1/2*d*x+1/2*c)^6
+(42*A+168*B)*sin(1/2*d*x+1/2*c)^4+(-105*A-175*B)*sin(1/2*d*x+1/2*c)^2+105*
A+105*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.85982, size = 166, normalized size = 1.2

$$\frac{42\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(15\,\sqrt{2}a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,\sqrt{2}a\,\mathrm{si}\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/420*(42*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c
) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (15*sqrt(2)*a*sin(7/2*d*
x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.61644, size = 236, normalized size = 1.71

$$\frac{2\left(15\,Ba\cos\left(dx+c\right)^{3}+3\,(7\,A+13\,B)a\cos\left(dx+c\right)^{2}+(63\,A+52\,B)a\cos\left(dx+c\right)+2\,(63\,A+52\,B)a\right)\sqrt{a\cos\left(dx+c\right)+}}{105\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/105*(15*B*a*cos(d*x + c)^3 + 3*(7*A + 13*B)*a*cos(d*x + c)^2 + (63*A + 52
*B)*a*cos(d*x + c) + 2*(63*A + 52*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.85 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a(5A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (8*a^2*(5*A + 3*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(5*
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0866019, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2647, 2646}

$$\frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a(5A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(5*
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{3/2}(A + B\cos(c+dx))\,dx = \frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}(5A+3B)\int (a + a\cos(c+d$$

$$= \frac{2a(5A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2B(a+a\cos(c+dx))^{3/}}{5d}$$

$$= \frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a(5A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+d}{15d}$$

Mathematica [A] time = 0.179546, size = 65, normalized size = 0.64

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(5A+9B)\cos(c+dx)+50A+3B\cos(2(c+dx))+39B)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cos[c + d*x] + 3
*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 1.117, size = 85, normalized size = 0.8

$$\frac{4\,a^2\sqrt{2}}{15\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6\,B\,(\sin(1/2\,dx+c/2))^4+(-5\,A-15\,B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+15\,A+15\,B\right)\frac{}{\sqrt{\left(\cos\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/15*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(6*B*sin(1/2*d*x+1/2*c)^4+(-
5*A-15*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^
```

```
(1/2)/d
```

Maxima [A] time = 1.81095, size = 126, normalized size = 1.25

$$\frac{10\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+3\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\right.\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*A*sqrt(a) + 3*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x +
 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.61242, size = 182, normalized size = 1.8

$$\frac{2\left(3\,Ba\cos\left(dx+c\right)^{2}+\left(5\,A+9\,B\right)a\cos\left(dx+c\right)+\left(25\,A+18\,B\right)a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*a*cos(d*x + c)^2 + (5*A + 9*B)*a*cos(d*x + c) + (25*A + 18*B)*a)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.86 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=105

$$\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(3*A + 4*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.265283, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2976, 2981, 2773, 206}

$$\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(3*A + 4*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2}{3}\int \sqrt{a+a\cos(c+$$

$$=\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\sqrt{a+a\cos(c+dx)}\sin(c-$$

$$=\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\sqrt{a+a\cos(c+dx)}\sin(c-$$

$$=\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2(3A+4B)\sin(c+dx}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.191837, size = 85, normalized size = 0.81

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(3A+B\cos(c+dx)+5B)+3\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right.\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*A*ArcTanh[Sqrt[2]
*Sin[(c + d*x)/2]] + 2*(3*A + 5*B + B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d
)
```

Maple [B] time = 3.507, size = 272, normalized size = 2.6

$$\frac{1}{3d}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4B\sqrt{2}\sqrt{a(\sin(1/2\,dx+c/2))^2}\sqrt{a}(\sin(1/2\,dx+c/2))^2+6A\sqrt{a}\sqrt{2}\sqrt{a(\sin(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/3*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+6*A*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+12*B*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1
/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.6593, size = 53, normalized size = 0.5

$$\frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*s
qrt(a)/d
```

Fricas [A] time = 2.15792, size = 397, normalized size = 3.78

$$\frac{3\left(Aa\cos\left(dx+c\right)+Aa\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left(Ba\cos\left(dx+c\right)+(3\right.}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(A*a*cos(d*x + c) + A*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
 + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(B*a*cos(d*x + c) + (3*A +
5*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 5.90329, size = 265, normalized size = 2.52

$$\frac{\frac{3\,Aa^{\frac{5}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(3\,\sqrt{2}Aa^{3}+6\,\sqrt{2}Ba^{3}+\left(3\,\sqrt{2}Aa^{3}+4\,\sqrt{2}Ba^{3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] 1/3*(3*A*a^(5/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))
/abs(a) + 2*(3*sqrt(2)*A*a^3 + 6*sqrt(2)*B*a^3 + (3*sqrt(2)*A*a^3 + 4*sqrt(
2)*B*a^3)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2
*c)^2 + a)^(3/2))/d
```

3.87 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=103

$$-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (a*A*Sq
rt[a + a*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.284083, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2981, 2773, 206}

$$-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (a*A*Sq
rt[a + a*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \frac{aA\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d}+\int \sqrt{a+a\cos(c+dx)}\\ &= -\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(c+dx)}\tan(c+\\ &= -\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(c+dx)}\tan(c+\\ &= \frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a^2(A-2B)\sin(c+}{d\sqrt{a+a\cos(c+}\end{aligned}$$

Mathematica [A] time = 0.301411, size = 98, normalized size = 0.95

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(A+2B\cos(c+dx))+\sqrt{2}(3A+2B)\cos(c+dx)\tan}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(3*A +
 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 2*B*Cos[c + d
*x])*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 3.605, size = 696, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-6*A*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-6*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a-4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-4*B*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-8*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*A*a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)
^(1/2)/d
```

Maxima [B] time = 1.90662, size = 1775, normalized size = 17.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(5
/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sq
rt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt(2)*a*sin(
3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c
)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) -
 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin
(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/2*d*x
+ 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/2*d*x + 1/
2*c))*sin(2*d*x + 2*c))*A*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 2.38678, size = 451, normalized size = 4.38

$$\frac{\left((3\,A+2\,B)a\cos\left(dx+c\right)^2+(3\,A+2\,B)a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+c)-2)\sin(dx+}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/4*(((3*A + 2*B)*a*cos(d*x + c)^2 + (3*A + 2*B)*a*cos(d*x + c))*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) + 4*(2*B*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*
cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.12875, size = 401, normalized size = 3.89

$$\frac{\frac{4\,\sqrt{2}Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\left(3\,Aa^{\frac{3}{2}}+2\,Ba^{\frac{3}{2}}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\,\sqrt{2}+3\right)\right|\right)-\left(3\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] 1/2*(4*sqrt(2)*B*a^2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
) + (3*A*a^(3/2) + 2*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (3*A*a^(3/2) + 2
*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/2) - A*a^(7/2))/((sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/
d
```

3.88 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] (a^(3/2)*(7*A + 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) + (a^2*(5*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
+ (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.325217, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2980, 2773, 206}

$$\frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^(3/2)*(7*A + 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) + (a^2*(5*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
+ (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \sqrt{a+$$

$$= \frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(c+dx)}\sec(c+}{2d}$$

$$= \frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(c+dx)}\sec(c+}{2d}$$

$$= \frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{a^2(5A+4B)\tan(}{4d\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.51095, size = 109, normalized size = 0.92

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((7A+4B)\cos(c+dx)+2A)+\sqrt{2}(7A+12B)\cos^2}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(7*A
 + 12*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*A + (7*A +
 4*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 3.779, size = 991, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(7*A*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+12*B*ln(-4
/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(7*A*a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+4*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+7*A*ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+12*B*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+12*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*
sin(1/2*d*x+1/2*c)^2+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+7*
A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+18*A*a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+12*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*a+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*B*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 2.49179, size = 4508, normalized size = 37.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] -1/16*((12*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48*a*cos(2*d*x + 2*c
)^2*sin(3/2*d*x + 3/2*c) + 12*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 4
8*a*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 160*a*cos(7/2*d*x + 7/2*c)*si
n(2*d*x + 2*c) + 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 72*a*cos(3/2
*d*x + 3/2*c)*sin(2*d*x + 2*c) - 24*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
 - 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 1
2*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 48*(
a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 4*(12*a*c
os(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 20*a*sin(7/2*d*x + 7/2*c) - 21*a*sin
(5/2*d*x + 5/2*c) - 3*a*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 7*(sqrt(2)
*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*si
n(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*
sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(
2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*
d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 2) - 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)
^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*
(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*
cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x +
```

```
 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2
*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x +
 2*c) + a)*sin(13/2*d*x + 13/2*c) - 12*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x
+ 2*c) + a)*sin(11/2*d*x + 11/2*c) - 48*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x
 + 2*c) + a)*sin(9/2*d*x + 9/2*c) + 4*(12*a*sin(2*d*x + 2*c)*sin(3/2*d*x +
3/2*c) + 20*a*cos(7/2*d*x + 7/2*c) + 21*a*cos(5/2*d*x + 5/2*c) + 9*a*cos(3/
2*d*x + 3/2*c))*sin(4*d*x + 4*c) - 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*
x + 7/2*c) - 84*(2*a*cos(2*d*x + 2*c) + a)*sin(5/2*d*x + 5/2*c) - 24*a*sin(
3/2*d*x + 3/2*c) - 4*(a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin
(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2
*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c
) + a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 56*(a
*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*
x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(sqrt(2)*cos(4*d*x
+ 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sq
rt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*
(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2)) + 4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c)
 + 6*sqrt(2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2
 + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3
*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
- 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2))*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*si
n(1/2*d*x + 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/
2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
```

```
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*sin(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqr
t(2)*a)*sin(5/2*d*x + 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2
)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.97786, size = 474, normalized size = 3.98

$$\frac{\left((7A+12B)a\cos(dx+c)^3+(7A+12B)a\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(d}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((7*A + 12*B)*a*cos(d*x + c)^3 + (7*A + 12*B)*a*cos(d*x + c)^2)*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
c)^2)) + 4*((7*A + 4*B)*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.97151, size = 639, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] 1/8*((7*A*a^(3/2) + 12*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (7*A*a^(3/2) +
 12*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(5/2) + 12*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(5/2) - 95*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(
7/2) - 76*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^4*B*a^(7/2) + 53*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*A*a^(9/2) + 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*B*a^(9/2) - 5*A*a^(11/2) - 4*B*a^(11/2))/((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.89 $\int (a + a\cos(c+dx))^{3/2}(A + B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=164

$$\frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(\ldots}{\ldots}$$

```
[Out] (a^(3/2)*(11*A + 14*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^2*(11*A + 14*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]
]) + (a^2*(7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d
*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.400216, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] (a^(3/2)*(11*A + 14*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^2*(11*A + 14*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]
]) + (a^2*(7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d
*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sqrt{a$$

$$= \frac{a^2(7A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(c+$$

$$= \frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+6B)\sec(c+dx)\tan}{12d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+6B)\sec(c+dx)\tan}{12d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^2(11A+14B)\mathrm{t}}{8d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.919483, size = 132, normalized size = 0.8

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(11A+6B)\cos(c+dx)+(33A+42B)\cos(2(c+dx}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*(1
1*A + 14*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (7*(7*A + 6*
B) + 4*(11*A + 6*B)*Cos[c + d*x] + (33*A + 42*B)*Cos[2*(c + d*x)])*Sin[(c +
 d*x)/2]))/(48*d)
```

Maple [B] time = 4.168, size = 1310, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(11*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
```

```
2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+11*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))+14*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+14*B*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(22*A
*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+28*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+33*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-352*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-198*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-198*A*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-384*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)-252*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-25
2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+126
*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+33*A*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+108*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+42*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
^3/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

[Out] Timed out

Fricas [A] time = 1.99327, size = 531, normalized size = 3.24

$$\frac{3\left((11\,A+14\,B)a\cos\left(dx+c\right)^{4}+(11\,A+14\,B)a\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\,\mathrm{s}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{96\left(d\cos\left(dx+c\right)^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((11*A + 14*B)*a*cos(d*x + c)^4 + (11*A + 14*B)*a*cos(d*x + c)^3)*s
qrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) +
 a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*(3*(11*A + 14*B)*a*cos(d*x + c)^2 + 2*(11*A + 6*B)*a*cos(d*x
 + c) + 8*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d
*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.92349, size = 861, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] 1/48*(3*(11*A*a^(3/2) + 14*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(11*A*a^
(3/2) + 14*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(33*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(5/2) + 42*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(
5/2) - 303*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^8*A*a^(7/2) - 822*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^8*B*a^(7/2) + 2394*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(9/2) + 3780*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(9/2) - 1806*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(11/2) - 2508*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(11/2
) + 309*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*A*a^(13/2) + 498*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*B*a^(13/2) - 19*A*a^(15/2) - 30*B*a^(15/2))/((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.90 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(75A}{}$$

```
[Out] (a^(3/2)*(75*A + 88*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a^2*(75*A + 88*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*
x]]) + (a^2*(75*A + 88*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c
 + d*x]]) + (a^2*(9*A + 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d)
```

Rubi [A] time = 0.484684, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(75A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```


```
[Out] (a^(3/2)*(75*A + 88*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a^2*(75*A + 88*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*
x]]) + (a^2*(75*A + 88*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c
 + d*x]]) + (a^2*(9*A + 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
```

```
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \frac{aA\sqrt{a + a\cos(c + dx)}\sec^3(c + dx)\tan(c + dx)}{4d} + \frac{1}{4}\int \sqrt{a$$

$$= \frac{a^2(9A + 8B)\sec^2(c + dx)\tan(c + dx)}{24d\sqrt{a + a\cos(c + dx)}} + \frac{aA\sqrt{a + a\cos(c +$$

$$= \frac{a^2(75A + 88B)\sec(c + dx)\tan(c + dx)}{96d\sqrt{a + a\cos(c + dx)}} + \frac{a^2(9A + 8B)\sec}{24d\sqrt{a +}}$$

$$= \frac{a^2(75A + 88B)\tan(c + dx)}{64d\sqrt{a + a\cos(c + dx)}} + \frac{a^2(75A + 88B)\sec(c + dx)\,t}{96d\sqrt{a + a\cos(c + d}}$$

$$= \frac{a^2(75A + 88B)\tan(c + dx)}{64d\sqrt{a + a\cos(c + dx)}} + \frac{a^2(75A + 88B)\sec(c + dx)\,t}{96d\sqrt{a + a\cos(c + d}}$$

$$= \frac{a^{3/2}(75A + 88B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(75A + 88B)}{64d\sqrt{a + a\,co}}$$

Mathematica [A] time = 1.47133, size = 151, normalized size = 0.72

$$\frac{a\sec\left(\frac{1}{2}(c + dx)\right)\sec^4(c + dx)\sqrt{a(\cos(c + dx) + 1)}\left(\sin\left(\frac{1}{2}(c + dx)\right)((1155A + 1048B)\cos(c + dx) + 4(75A + 88B)\cos(2(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*(7
5*A + 88*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (492*A + 352
*B + (1155*A + 1048*B)*Cos[c + d*x] + 4*(75*A + 88*B)*Cos[2*(c + d*x)] + 22
5*A*Cos[3*(c + d*x)] + 264*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d)
```

Maple [B] time = 4.114, size = 1631, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(75*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))+88*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+88*B*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(75*A
*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+88*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+150*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a+150*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+176*B*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+176*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(825*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+968*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+675*A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+675*A*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a+792*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+792*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^
4-4*(1095*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+1208*B*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+450*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+450*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+528*B*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+528*B*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+1086*A*a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+225*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*a+225*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1008*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+264*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+264*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(
1/2*d*x+1/2*c)-2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/sin(1/2*d*x+1/2*
c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.23668, size = 581, normalized size = 2.78

$$\frac{3\left((75\,A+88\,B)a\cos\left(dx+c\right)^{5}+(75\,A+88\,B)a\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/768*(3*((75*A + 88*B)*a*cos(d*x + c)^5 + (75*A + 88*B)*a*cos(d*x + c)^4)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*(3*(75*A + 88*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d
*x + c)^2 + 8*(15*A + 8*B)*a*cos(d*x + c) + 48*A*a)*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.21408, size = 1083, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] 1/384*(3*(75*A*a^(3/2) + 88*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(75*A*a
^(3/2) + 88*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(225*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(5/2) + 26
4*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*
a^(5/2) - 6261*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^12*A*a^(7/2) - 4008*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^12*B*a^(7/2) + 35925*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(9/2) + 33960*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(9/2) - 127449*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(11/2
) - 131784*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^8*B*a^(11/2) + 101667*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^6*A*a^(13/2) + 108312*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(13/2) - 26079*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(15/2) - 29432*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(15/2)
+ 3303*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*A*a^(17/2) + 3384*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*B*a^(17/2) - 147*A*a^(19/2) - 152*B*a^(19/2))/((sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.91 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{2a^3(209A+194B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(11A+14B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}+\frac{2a^3(803A+}{495d\sqrt{a}}$$

```
[Out] (2*a^3*(803*A + 710*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^3*(209*A + 194*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) - (4*a^2*(803*A + 710*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(346
5*d) + (2*a^2*(11*A + 14*B)*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(1155*d) + (2*a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x]
)/(11*d)
```

Rubi [A] time = 0.648012, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2981, 2759, 2751, 2646}

$$\frac{2a^3(209A+194B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(11A+14B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}+\frac{2a^3(803A+}{495d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*a^3*(803*A + 710*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^3*(209*A + 194*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) - (4*a^2*(803*A + 710*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(346
5*d) + (2*a^2*(11*A + 14*B)*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(1155*d) + (2*a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x]
)/(11*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d} + \frac{2}{11}\int$$

$$= \frac{2a^2(11A+14B)\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx}{99d}$$

$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(11A+1}{}$$

$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(11A+1}{}$$

$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} - \frac{4a^2(803A+}{}$$

$$= \frac{2a^3(803A+710B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(209A+194B)\cos^3(c}{693d\sqrt{a+a\cos}}$$

Mathematica [A] time = 1.08507, size = 127, normalized size = 0.54

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((68552A+69890B)\cos(c+dx)+16(1397A+1625B)\cos(2(c+dx))+5720A\,c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(124366*A + 114640*B + (68552*A + 69890*B)*
Cos[c + d*x] + 16*(1397*A + 1625*B)*Cos[2*(c + d*x)] + 5720*A*Cos[3*(c + d*
x)] + 8675*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 2240*B*Cos[4*(c +
d*x)] + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 1.161, size = 142, normalized size = 0.6

$$\frac{8a^3\sqrt{2}}{3465d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-2520B(\sin(1/2dx+c/2))^{10}+(1540A+10780B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8+(-5940A-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 8/3465*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-2520*B*sin(1/2*d*x+1/2*c
)^10+(1540*A+10780*B)*sin(1/2*d*x+1/2*c)^8+(-5940*A-18810*B)*sin(1/2*d*x+1/
2*c)^6+(9009*A+17325*B)*sin(1/2*d*x+1/2*c)^4+(-6930*A-9240*B)*sin(1/2*d*x+1
/2*c)^2+3465*A+3465*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 3.01999, size = 279, normalized size = 1.18

$$\frac{22\left(35\sqrt{2}a^2\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+225\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+756\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+2100\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/55440*(22*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*
d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(
3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(63
*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c)
+ 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5/2*d*x + 5/
2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2*sin(1/2*d*
x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.65931, size = 366, normalized size = 1.54

$$\frac{2\left(315\,Ba^2\cos\left(dx+c\right)^5+35\left(11\,A+32\,B\right)a^2\cos\left(dx+c\right)^4+5\left(286\,A+355\,B\right)a^2\cos\left(dx+c\right)^3+3\left(803\,A+710\,B\right)a^2\cos\right.}{3465\left(d\cos\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(315*B*a^2*cos(d*x + c)^5 + 35*(11*A + 32*B)*a^2*cos(d*x + c)^4 + 5*
(286*A + 355*B)*a^2*cos(d*x + c)^3 + 3*(803*A + 710*B)*a^2*cos(d*x + c)^2 +
 4*(803*A + 710*B)*a^2*cos(d*x + c) + 8*(803*A + 710*B)*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

3.92 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=175

$$\frac{16a^2(15A+13B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(9A-2B)\sin(c+dx)(a\cos(c+dx)}{63d}$$

```
[Out] (64*a^3*(15*A + 13*B)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(15*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(15
*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(9*A - 2*B
)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*B*(a + a*Cos[c + d*x
])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.279669, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2751, 2647, 2646}

$$\frac{16a^2(15A+13B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(9A-2B)\sin(c+dx)(a\cos(c+dx)}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (64*a^3*(15*A + 13*B)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(15*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(15
*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(9*A - 2*B
)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*B*(a + a*Cos[c + d*x
])^(7/2)*Sin[c + d*x])/(9*a*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^{5/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)$$

$$= \frac{2B(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{2\int (a+a\cos(c+d}{}$$

$$= \frac{2(9A-2B)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2B(a+a\,\mathrm{c}}{}$$

$$= \frac{2a(15A+13B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} + \frac{2(9A-}{}$$

$$= \frac{16a^2(15A+13B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + \frac{2a(15A}{}$$

$$= \frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(15A+13B)\sqrt{a+a\,\mathrm{c}}}{315a}$$

Mathematica [A] time = 0.670721, size = 105, normalized size = 0.6

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((3030A+3116B)\cos(c+dx)+8(90A+127B)\cos(2(c+dx))+90A\cos(3(c+}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(6240*A + 5653*B + (3030*A + 3116*B)*Cos[c
+ d*x] + 8*(90*A + 127*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 260*B*
Cos[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 1.134, size = 123, normalized size = 0.7

$$\frac{8\,a^3\sqrt{2}}{315\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\,B\,(\sin(1/2\,dx+c/2))^8+(-90\,A-540\,B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+(315\,A+819\,B)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*B*sin(1/2*d*x+1/2*c)^8
+(-90*A-540*B)*sin(1/2*d*x+1/2*c)^6+(315*A+819*B)*sin(1/2*d*x+1/2*c)^4+(-42
```

```
0*A-630*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*
a)^(1/2)/d
```

Maxima [A] time = 2.79148, size = 232, normalized size = 1.33

$$\frac{30\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+315\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/2520*(30*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x
 + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c))*A*sqrt(a) + (35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)
*a^2*sin(7/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqr
t(2)*a^2*sin(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sq
rt(a))/d
```

Fricas [A] time = 1.62873, size = 302, normalized size = 1.73

$$\frac{2\left(35\,Ba^2\cos(dx+c)^4+5\,(9\,A+26\,B)a^2\cos(dx+c)^3+3\,(60\,A+73\,B)a^2\cos(dx+c)^2+(345\,A+292\,B)a^2\cos(dx+c\right.}{315\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/315*(35*B*a^2*cos(d*x + c)^4 + 5*(9*A + 26*B)*a^2*cos(d*x + c)^3 + 3*(60*
A + 73*B)*a^2*cos(d*x + c)^2 + (345*A + 292*B)*a^2*cos(d*x + c) + 2*(345*A
+ 292*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.93 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(7A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+5B)\sin(c+dx)(a\cos(c+dx)+a}{35d}$$

```
[Out] (64*a^3*(7*A + 5*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^
2*(7*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(7*A +
5*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.10898, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2647, 2646}

$$\frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(7A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+5B)\sin(c+dx)(a\cos(c+dx)+a}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^
2*(7*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(7*A +
5*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
```

```
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{5/2}(A + B\cos(c+dx))\,dx = \frac{2B(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}(7A+5B)\int (a + a\cos(c+d$$

$$= \frac{2a(7A+5B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+a\cos(c+dx))}{7d}$$

$$= \frac{16a^2(7A+5B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2a(7A+5B)(a+a\cos}{3}$$

$$= \frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(7A+5B)\sqrt{a+a\cos(c+dx)}\sin(c}{105d}$$

Mathematica [A] time = 0.319035, size = 83, normalized size = 0.6

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((392A+505B)\cos(c+dx)+6(7A+20B)\cos(2(c+dx))+1246A+15B\cos(3}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1246*A + 1040*B + (392*A + 505*B)*Cos[c +
d*x] + 6*(7*A + 20*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*
x)/2])/(210*d)
```

Maple [A] time = 1., size = 104, normalized size = 0.8

$$\frac{8a^3\sqrt{2}}{105d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-30B(\sin(1/2dx+c/2))^6+(21A+105B)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+(-70A-140B)\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 8/105*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-30*B*sin(1/2*d*x+1/2*c)^6
+(21*A+105*B)*sin(1/2*d*x+1/2*c)^4+(-70*A-140*B)*sin(1/2*d*x+1/2*c)^2+105*A
+105*B)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 2.64998, size = 188, normalized size = 1.36

$$\frac{14\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+5\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+2\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(14*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
+ 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(3*sqrt(2)*a
^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 77*sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a))
/d
```

Fricas [A] time = 1.61758, size = 248, normalized size = 1.8

$$\frac{2\left(15\,Ba^2\cos(dx+c)^3+3\,(7\,A+20\,B)a^2\cos(dx+c)^2+(98\,A+115\,B)a^2\cos(dx+c)+(301\,A+230\,B)a^2\right)\sqrt{a\cos(dx-}}{105\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a^2*cos(d*x + c)^3 + 3*(7*A + 20*B)*a^2*cos(d*x + c)^2 + (98*A
+ 115*B)*a^2*cos(d*x + c) + (301*A + 230*B)*a^2)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.94 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=142

$$\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(}{}$$

```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(35*A + 32*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(5*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*B*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.414053, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2976, 2981, 2773, 206}

$$\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```



```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(35*A + 32*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(5*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*B*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
```

```
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}\int (a+a\cos(c+$$

$$= \frac{2a^2(5A+8B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2aB(a+a}{}$$

$$= \frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+a\cos(c}}{15d}$$

$$= \frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+a\cos(c}}{15d}$$

$$= \frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^3(35A+32B)\sin(c+}{15d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.380174, size = 104, normalized size = 0.73

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(2(5A+14B)\cos(c+dx)+80A+3B\cos(2(c+dx))+89B)+15\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*A*ArcTanh[Sqrt
[2]*Sin[(c + d*x)/2]] + (80*A + 89*B + 2*(5*A + 14*B)*Cos[c + d*x] + 3*B*Co
s[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d)
```

Maple [B] time = 4.168, size = 311, normalized size = 2.2

$$\frac{1}{15\,d}a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,B\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin(1/2\,dx+c/2))^4-20\,\sqrt{a\,(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*B*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-20*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)*2^(1/2)*(A+4*B)*sin(1/2*d*x+1/2*c)^2+90*A*a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+120
*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(cos(
1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 2.64634, size = 82, normalized size = 0.58

$$\frac{\left(3\,\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\,\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\,\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a)/d
```

Fricas [A] time = 1.7643, size = 462, normalized size = 3.25

$$\frac{15\left(Aa^2\cos\left(dx+c\right)+Aa^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3Ba^2\cos\left(dx+\right.\right.}{30\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/30*(15*(A*a^2*cos(d*x + c) + A*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*c
os(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d
*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*B*a^2*cos(d*x + c)
^2 + (5*A + 14*B)*a^2*cos(d*x + c) + (40*A + 43*B)*a^2)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.8328, size = 306, normalized size = 2.15

$$\frac{\frac{15\,Aa^{\frac{7}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(45\,\sqrt{2}Aa^{5}+60\,\sqrt{2}Ba^{5}+\left(80\,\sqrt{2}Aa^{5}+80\,\sqrt{2}Ba^{5}+\left(35\,\sqrt{2}Aa^{5}+32\,\sqrt{2}Ba^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{5}{2}}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] 1/15*(15*A*a^(7/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a
))/abs(a) + 2*(45*sqrt(2)*A*a^5 + 60*sqrt(2)*B*a^5 + (80*sqrt(2)*A*a^5 + 80
*sqrt(2)*B*a^5 + (35*sqrt(2)*A*a^5 + 32*sqrt(2)*B*a^5)*tan(1/2*d*x + 1/2*c)
^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2
+ a)^(5/2))/d
```

3.95 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(3A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\,\mathrm{t}\ldots}{}$$

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (a^3*(3*A + 14*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (a
^2*(3*A - 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + a*C
os[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.447146, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2976, 2981, 2773, 206}

$$\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(3A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\,\mathrm{t}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (a^3*(3*A + 14*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (a
^2*(3*A - 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + a*C
os[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^2(c + dx)\,dx = \frac{aA(a + a\cos(c + dx))^{3/2}\tan(c + dx)}{d} + \int (a + a\cos(c + $$

$$= -\frac{a^2(3A - 2B)\sqrt{a + a\cos(c + dx)}\sin(c + dx)}{3d} + \frac{aA(a + a$$

$$= \frac{a^3(3A + 14B)\sin(c + dx)}{3d\sqrt{a + a\cos(c + dx)}} - \frac{a^2(3A - 2B)\sqrt{a + a\cos(c + }{3d}$$

$$= \frac{a^3(3A + 14B)\sin(c + dx)}{3d\sqrt{a + a\cos(c + dx)}} - \frac{a^2(3A - 2B)\sqrt{a + a\cos(c + }{3d}$$

$$= \frac{a^{5/2}(5A + 2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^3(3A + 14B)\sin}{3d\sqrt{a + a\cos(}$$

Mathematica [A] time = 0.504775, size = 120, normalized size = 0.83

$$\frac{a^2\sec\left(\frac{1}{2}(c + dx)\right)\sec(c + dx)\sqrt{a(\cos(c + dx) + 1)}\left(2\sin\left(\frac{1}{2}(c + dx)\right)(2(3A + 8B)\cos(c + dx) + 3A + B\cos(2(c + dx)) + }{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*(5
*A + 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(3*A + B + 2*(
3*A + 8*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

Maple [B] time = 3.766, size = 756, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/3*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*B*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+(-24*A*a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-30*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
```

```
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a-30*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-80*B*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-12*B*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a-12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)
*sin(1/2*d*x+1/2*c)^2+18*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+1
5*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+15*A*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+36*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+6*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6*B*ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 4.80759, size = 10954, normalized size = 76.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/252*(1449*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 1260*sqr
t(2)*a^2*sin(1/2*d*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt
(2)*a^2)*sin(5/2*d*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin
(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
- 60*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) - 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (25*sqrt(2)*a^
2*cos(3/2*d*x + 3/2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c))*cos(5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25
*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*
sin(3/2*d*x + 3/2*c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x
+ 3/2*c) + 69*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 198*sqrt(
2)*a^2*sin(1/2*d*x + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 198*sq
rt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(2)*a^2*cos(3/2
*d*x + 3/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(
```

5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*
a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*
x + 3/2*c))*sin(2*d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin
(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin
(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqr
t(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt
(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*
d*x + 2*c))*cos(13/2*d*x + 13/2*c) - 135*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^
2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c
)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) +
2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) +
(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*s
in(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) - 98*(sqrt(2)*a^2*cos(5/2*d*x + 5/2
*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1
/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c
) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c
) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^
2)*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 390*(sqrt(2)*a^2*cos(5/2*d*x +
5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x
+ 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x +
2*c) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/
2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c)^2)*sin(2*d*x + 2*c))*cos(7/2*d*x + 7/2*c) + 21*(50*sqrt(2)*a^2*cos(2*d*x
 + 2*c)^2*cos(1/2*d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) + 50*sqrt(2)*a^2*cos(1/
2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 120*sqrt(2)*a^2*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 10*(5*sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*
x + 1/2*c))*cos(2*d*x + 2*c) + (50*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2
*d*x + 1/2*c) + 189*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 69*sqrt(2)*a^2*sin
(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c) - 21*(60*sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c)^3 - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqr
t(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) + 12*(5*sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*
c) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*co
s(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*c
os(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2
*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^
2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x
 + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2
*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2
*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x
 + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2
*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*
sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(

2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 315*(a^2*cos(1/
2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a
^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^
2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2
*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
+ 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*si
n(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/
2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5
/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d
*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x
+ 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c)
 + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a
^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^
2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x
 + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d
*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/
2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2
)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2
*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d
*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1
/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*c
os(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x
 + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x +
 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2
*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*si
n(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*si
n(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (
a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2
 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c
)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*co
s(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 +
a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*si

n(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*co
s(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x
 + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*s
qrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
35*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2
 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (s
qrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt
(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)
*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x +
2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x +
 5/2*c))*sin(13/2*d*x + 13/2*c) + 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 +
 sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^
2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1
/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a
^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x +
 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*
sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(11/2*d*x + 11/2*c) + 7*(9*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 -
(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sq
rt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 5*(sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2
*d*x + 2*c)^2 - (5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x
 + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*sin(5/2*d*x + 5
/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x +
 1/2*c)^2)*sin(2*d*x + 2*c)^2 - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2
*d*x + 1/2*c) + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 - 4*s
qrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*cos(1/2*d*
x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 4*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 +
sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) - 2*(5*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin
(1/2*d*x + 1/2*c) - 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 9
*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(9/2*d*x + 9/2*
c) - 390*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/
2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^
2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2
*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d
*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + s
qrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*
d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2
*d*x + 5/2*c))*sin(7/2*d*x + 7/2*c) - 21*(69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*
c)^2 + 189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 69*(sqrt(2)*a^2*cos(2*d*x +

2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*cos(2*d*x + 2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*sin(2*d*x + 2*c)^2 + 12*sqrt(2)*a^2 + 138*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 - 50*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 24*sqrt(2)*a^2)*cos(2*d*x + 2*c) - 10*(5*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x + 5/2*c) + 105*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) - 252*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c) - 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 63*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1260*(sqrt(2)*a^2*c

```
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*c
os(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*
x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x +
 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(
2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*
cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2
+ 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/
2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2
*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*
cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) +
 sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*
d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2
*d*x + 5/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*A*sqrt(a)/(((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*cos(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2)*cos(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2*cos(1/2*d*x +
 1/2*c) + cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)*cos(
1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + cos(1/2*d*x + 1
/2*c)^2 + 2*(cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sin(2*d*x + 2*c)^2*s
in(1/2*d*x + 1/2*c) + 2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x
 + 1/2*c))*sin(5/2*d*x + 5/2*c) + sin(1/2*d*x + 1/2*c)^2)*d)
```

Fricas [A] time = 1.94461, size = 516, normalized size = 3.58

$$\frac{3\left((5A+2B)a^2\cos(dx+c)^2+(5A+2B)a^2\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{12\left(d\cos(dx+c)^2+d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/12*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sqr
t(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 4*(2*B*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) + 3*A
```

```
*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)

[Out] Timed out
```

Giac [B] time = 2.82763, size = 460, normalized size = 3.19

$$\frac{3\left(5\,Aa^{\frac{5}{2}}+2\,Ba^{\frac{5}{2}}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)-3\left(5\,Aa^{\frac{5}{2}}+2\,Ba^{\frac{5}{2}}\right)\log\left(\left|\left(\sqrt{\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")

[Out] 1/6*(3*(5*A*a^(5/2) + 2*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*a^(5/2
) + 2*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 12*sqrt(2)*(3*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(7/2) - A*a^(9/2))
/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a +
a^2) + 4*(3*sqrt(2)*A*a^4 + 9*sqrt(2)*B*a^4 + (3*sqrt(2)*A*a^4 + 7*sqrt(2)*
B*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)
^2 + a)^(3/2))/d
```

3.96 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=156

$$-\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(7A+4B)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+$$

```
[Out] (a^(5/2)*(19*A + 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(4*d) - (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
 + (a^2*(7*A + 4*B)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.47386, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2981, 2773, 206}

$$-\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(7A+4B)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```


```
[Out] (a^(5/2)*(19*A + 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(4*d) - (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
 + (a^2*(7*A + 4*B)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a$$

$$= \frac{a^2(7A+4B)\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{4d}+\frac{aA(a+a\,\mathrm{co}}{}$$

$$= -\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+4B)\sqrt{a+a\cos(c+d}}{4d}$$

$$= -\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+4B)\sqrt{a+a\cos(c+d}}{4d}$$

$$= \frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}-\frac{a^3(9A-4B)\sin}{4d\sqrt{a+a\cos(}}$$

Mathematica [A] time = 0.620257, size = 126, normalized size = 0.81

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((11A+4B)\cos(c+dx)+2(A+2B\cos(2(c+dx)\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(1
9*A + 20*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*((11*A + 4
*B)*Cos[c + d*x] + 2*(A + 2*B + 2*B*Cos[2*(c + d*x)]))*Sin[(c + d*x)/2]))/(
8*d)
```

Maple [B] time = 4.404, size = 1016, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((76*A*ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+80*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+80*B
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+64*B*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^4+(-76*A*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a-76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a-44*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-80*B*ln(-4/(-2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-80*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a-80*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/
2*c)^2+19*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+19*A*ln(4/(2*co
```

```
s(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+26*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+20*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+20*B*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+24*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.02725, size = 522, normalized size = 3.35

$$\frac{\left((19\,A+20\,B)a^2\cos\left(dx+c\right)^3+(19\,A+20\,B)a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos\left(dx+c\right)^3+d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((19*A + 20*B)*a^2*cos(d*x + c)^3 + (19*A + 20*B)*a^2*cos(d*x + c)^2)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) + 4*(8*B*a^2*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) +
2*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d
*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.12869, size = 686, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] 1/8*(16*sqrt(2)*B*a^3*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a) + (19*A*a^(5/2) + 20*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (19*A*a^(5/2)
 + 20*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(19*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(7/2) + 12*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(7/2) -
171*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A
*a^(9/2) - 76*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^4*B*a^(9/2) + 89*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2*A*a^(11/2) + 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(11/2) - 9*A*a^(13/2) - 4*B*a^(13/2))/((s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)
^2)/d
```

3.97 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=164

$$\frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(25A+38B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(3A+2B)\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+}}{4d}$$

```
[Out] (a^(5/2)*(25*A + 38*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^3*(49*A + 54*B)*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*(3*A + 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/
(4*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.52581, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2980, 2773, 206}

$$\frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(25A+38B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(3A+2B)\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] (a^(5/2)*(25*A + 38*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^3*(49*A + 54*B)*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*(3*A + 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/
(4*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int$$

$$= \frac{a^2(3A+2B)\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{4d}+$$

$$= \frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(3A+2B)\sqrt{a+a\cos(c}}{}$$

$$= \frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(3A+2B)\sqrt{a+a\cos(c}}{}$$

$$= \frac{a^{5/2}(25A+38B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^3(49A+54B}{24d\sqrt{a+a}}$$

Mathematica [A] time = 1.05021, size = 131, normalized size = 0.8

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(17A+6B)\cos(c+dx)+(75A+66B)\cos(2(c+d}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*
(25*A + 38*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (91*A + 66
*B + 4*(17*A + 6*B)*Cos[c + d*x] + (75*A + 66*B)*Cos[2*(c + d*x)])*Sin[(c +
 d*x)/2]))/(48*d)
```

Maple [B] time = 4.138, size = 1310, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(25*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+25*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))+38*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+38*B*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(50*A
*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+44*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-450*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a-450*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-736*A*a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-684*B*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a-684*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-5
76*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+7
5*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+75*A*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+234*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+114*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+156*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))^3/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 22.5043, size = 10792, normalized size = 65.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] -1/96*((1530*a^2*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*cos(2*d
*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*sin(4*d*x + 4*c)^2*sin(3/2*d*x
+ 3/2*c) + 1530*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 4176*a^2*cos(
7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 2430*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x
 + 2*c) + 678*a^2*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 342*a^2*cos(2*d*x
 + 2*c)*sin(3/2*d*x + 3/2*c) + 10*(a^2*sin(9/2*d*x + 9/2*c) + 17*a^2*sin(3/
2*d*x + 3/2*c))*cos(6*d*x + 6*c)^2 + 10*(a^2*sin(9/2*d*x + 9/2*c) + 17*a^2*
sin(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c)^2 - 56*a^2*sin(3/2*d*x + 3/2*c) + 10
*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*c
os(21/2*d*x + 21/2*c) - 30*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) +
 3*a^2*sin(2*d*x + 2*c))*cos(19/2*d*x + 19/2*c) - 48*(a^2*sin(6*d*x + 6*c)
+ 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(17/2*d*x + 17/2*c) +
 80*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c)
)*cos(15/2*d*x + 15/2*c) + 396*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*
c) + 3*a^2*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 6*(170*a^2*cos(4*d*x
+ 4*c)*sin(3/2*d*x + 3/2*c) + 170*a^2*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
```

```
 - 170*a^2*sin(11/2*d*x + 11/2*c) - 232*a^2*sin(7/2*d*x + 7/2*c) - 135*a^2*
sin(5/2*d*x + 5/2*c) + 19*a^2*sin(3/2*d*x + 3/2*c) + 10*(a^2*cos(4*d*x + 4*
c) + a^2*cos(2*d*x + 2*c) - 25*a^2)*sin(9/2*d*x + 9/2*c))*cos(6*d*x + 6*c)
+ 3060*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c)
 + 4560*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c)
+ 18*(170*a^2*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 232*a^2*sin(7/2*d*x +
 7/2*c) - 135*a^2*sin(5/2*d*x + 5/2*c) + 19*a^2*sin(3/2*d*x + 3/2*c))*cos(4
*d*x + 4*c) - 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x
+ 4*c)^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos
(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) +
sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9
*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2
)*a^2*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4
*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3
*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*
a^2*sin(4*d*x + 4*c) + sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(sqrt(2)*a^2
*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*a^2*cos(
2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*sin(4*d*x +
 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*a^2*
sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(3*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*
cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d
*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqrt(2)*a^2*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 2) + 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x + 4*c)
^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 9*
sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x + 2*c)
```

```
+ sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x + 2*c)
 + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqrt(2
)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) - 10*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x +
 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(21/2*d*x + 21/2*c) + 30*(a^2*cos(
6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(1
9/2*d*x + 19/2*c) + 48*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a
^2*cos(2*d*x + 2*c) + a^2)*sin(17/2*d*x + 17/2*c) - 80*(a^2*cos(6*d*x + 6*c
) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(15/2*d*x + 1
5/2*c) - 396*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d
*x + 2*c) + a^2)*sin(13/2*d*x + 13/2*c) + 2*(510*a^2*sin(4*d*x + 4*c)*sin(3
/2*d*x + 3/2*c) + 510*a^2*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 510*a^2*c
os(11/2*d*x + 11/2*c) + 760*a^2*cos(9/2*d*x + 9/2*c) + 696*a^2*cos(7/2*d*x
+ 7/2*c) + 405*a^2*cos(5/2*d*x + 5/2*c) + 113*a^2*cos(3/2*d*x + 3/2*c) + 30
*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(9/2*d*x + 9/2*c))*sin(6*
d*x + 6*c) - 1020*(3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*s
in(11/2*d*x + 11/2*c) + 10*(9*a^2*cos(4*d*x + 4*c)^2 + 9*a^2*cos(2*d*x + 2*
c)^2 + 9*a^2*sin(4*d*x + 4*c)^2 + 18*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 9*a^2*sin(2*d*x + 2*c)^2 - 450*a^2*cos(2*d*x + 2*c) - 151*a^2 + 18*(a^2*c
os(2*d*x + 2*c) - 25*a^2)*cos(4*d*x + 4*c))*sin(9/2*d*x + 9/2*c) + 6*(510*a
^2*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 696*a^2*cos(7/2*d*x + 7/2*c) + 4
05*a^2*cos(5/2*d*x + 5/2*c) + 113*a^2*cos(3/2*d*x + 3/2*c))*sin(4*d*x + 4*c
) - 1392*(3*a^2*cos(2*d*x + 2*c) + a^2)*sin(7/2*d*x + 7/2*c) - 810*(3*a^2*c
os(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c) - 30*(a^2*cos(6*d*x + 6*c)^2 +
9*a^2*cos(4*d*x + 4*c)^2 + 9*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(6*d*x + 6*c)^
2 + 9*a^2*sin(4*d*x + 4*c)^2 + 18*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9
*a^2*sin(2*d*x + 2*c)^2 + 6*a^2*cos(2*d*x + 2*c) + a^2 + 2*(3*a^2*cos(4*d*x
 + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 6*(3*a^2*cos(2*d
*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 6*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 78*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(4*d*x + 4*c)^2 + 9*a^
2*cos(2*d*x + 2*c)^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(4*d*x + 4*c)^2 +
18*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a^2*sin(2*d*x + 2*c)^2 + 6*a^2
*cos(2*d*x + 2*c) + a^2 + 2*(3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c
) + a^2)*cos(6*d*x + 6*c) + 6*(3*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*
c) + 6*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sin(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 600*(a^2*cos(6*d
*x + 6*c)^2 + 9*a^2*cos(4*d*x + 4*c)^2 + 9*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(6*d*x + 6*c)^2 + 9*a^2*sin(4*d*x + 4*c)^2 + 18*a^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*a^2*sin(2*d*x + 2*c)^2 + 6*a^2*cos(2*d*x + 2*c) + a^2 + 2*(3
*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 6*
```

(3*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 6*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)) + 6*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - (3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s

in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
+ 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2
c)^2 - 3(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(15/2*d*x + 15/2*c) - 5*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin
(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 11*(sqrt(2)*a^2*sin(4*d*x + 4*c) +
2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 45*(sqrt(2)*a^2*si
n(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) - (11
*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 3
8*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*s
in(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
- 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(4*sqrt(2)*a^2
*cos(2*d*x + 2*c) + 27*sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + (20*sqrt(2)*a^2*
cos(2*d*x + 2*c) + 87*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) -
2*(11*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c) + 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(sqrt(2)*a^2*cos(4*d*x + 4*c) +
2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/2*d*x + 15/2*c) + 5*(
sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2

```
)*sin(13/2*d*x + 13/2*c) - 11*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2
*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/2*d*x + 11/2*c) - 45*(sqrt(2)*a^2*c
os(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/2*d*x
 + 9/2*c) - (12*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 20*sqrt
(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 75*sqrt(2)*a^2*cos(7/2*d*x
+ 7/2*c) - 77*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) - 45*sqrt(2)*a^2*cos(3/2*d*x
 + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 4*(17*sqrt(2)*a^2*sin(3/2
*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin
(2*d*x + 2*c))*sin(4*d*x + 4*c) - 6*(2*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 2*s
qrt(2)*a^2*sin(2*d*x + 2*c)^2 + 27*sqrt(2)*a^2*cos(2*d*x + 2*c) + 13*sqrt(2
)*a^2)*sin(7/2*d*x + 7/2*c) - 2*(10*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 10*sqr
t(2)*a^2*sin(2*d*x + 2*c)^2 + 87*sqrt(2)*a^2*cos(2*d*x + 2*c) + 41*sqrt(2)*
a^2)*sin(5/2*d*x + 5/2*c) + 2*(45*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 11*sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*B*sqrt(a)/(2*(2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*
c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 2.04682, size = 544, normalized size = 3.32

$$\frac{3\left((25\,A+38\,B)a^2\cos\left(dx+c\right)^4+(25\,A+38\,B)a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((25*A + 38*B)*a^2*cos(d*x + c)^4 + (25*A + 38*B)*a^2*cos(d*x + c)^
3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*(3*(25*A + 22*B)*a^2*cos(d*x + c)^2 + 2*(17*A + 6*B)*a^2
*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x
+ c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.2778, size = 861, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] 1/48*(3*(25*A*a^(5/2) + 38*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(25*A*a^
(5/2) + 38*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(75*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(7/2) + 114*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^
(7/2) - 1125*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^8*A*a^(9/2) - 1710*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^8*B*a^(9/2) + 6174*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(11/2) + 6804*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(11/2) - 4314*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(13/2) - 4284
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^
(13/2) + 807*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^2*A*a^(15/2) + 858*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*B*a^(15/2) - 49*A*a^(17/2) - 54*B*a^(17/2))/((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.98 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(11A+8B)\tan(c+dx)\sec^2(c+dx)\sqrt{a\,c}}{24d}$$

```
[Out] (a^(5/2)*(163*A + 200*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(64*d) + (a^3*(163*A + 200*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c
+ d*x]]) + (a^3*(95*A + 104*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*
Cos[c + d*x]]) + (a^2*(11*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*
Tan[c + d*x])/(24*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d)
```

Rubi [A] time = 0.608621, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(11A+8B)\tan(c+dx)\sec^2(c+dx)\sqrt{a\,c}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^(5/2)*(163*A + 200*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(64*d) + (a^3*(163*A + 200*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c
+ d*x]]) + (a^3*(95*A + 104*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*
Cos[c + d*x]]) + (a^2*(11*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*
Tan[c + d*x])/(24*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d)
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
```

```
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (a$$

$$= \frac{a^2(11A+8B)\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{24d}$$

$$= \frac{a^3(95A+104B)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(11A+8B)\sqrt{}}{}$$

$$= \frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(95A+104B)\sec(c+d}{96d\sqrt{a+a\cos(c}}$$

$$= \frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(95A+104B)\sec(c+d}{96d\sqrt{a+a\cos(c}}$$

$$= \frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(163A+20}{64d\sqrt{a+}}$$

Mathematica [A] time = 1.65608, size = 152, normalized size = 0.73

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((2203A+2056B)\cos(c+dx)+(652A+544B)\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*
(163*A + 200*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (844*A +
 544*B + (2203*A + 2056*B)*Cos[c + d*x] + (652*A + 544*B)*Cos[2*(c + d*x)]
+ 489*A*Cos[3*(c + d*x)] + 600*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*
d)
```

Maple [B] time = 4.174, size = 1630, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(163*A
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+163*A*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))+200*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+2
00*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(
163*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+200*B*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+326*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+326*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+400*B*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+400*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(1793*A*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2072*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+1467*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1467*A*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1800*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a+1800*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2
*d*x+1/2*c)^4+(-9212*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-3912*
A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-3912*A*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a-9632*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)-4800*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-4800*B*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+2094*A*a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+489*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+489*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+187
2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+600*B*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+600*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/s
in(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.18219, size = 608, normalized size = 2.91

$$\frac{3\left((163\,A+200\,B)a^2\cos\left(dx+c\right)^5+(163\,A+200\,B)a^2\cos\left(dx+c\right)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(\cdots}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/768*(3*((163*A + 200*B)*a^2*cos(d*x + c)^5 + (163*A + 200*B)*a^2*cos(d*x
+ c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 4*(3*(163*A + 200*B)*a^2*cos(d*x + c)^3 + 2*(163*A +
136*B)*a^2*cos(d*x + c)^2 + 8*(23*A + 8*B)*a^2*cos(d*x + c) + 48*A*a^2)*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

[Out] Timed out

Giac [B] time = 3.40891, size = 1083, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] 1/384*(3*(163*A*a^(5/2) + 200*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(163*
A*a^(5/2) + 200*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(489*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(7/2)
+ 600*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^1
4*B*a^(7/2) - 10269*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^12*A*a^(9/2) - 12600*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^12*B*a^(9/2) + 69885*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(11/2) + 103992*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(11/2) - 25
9233*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*
A*a^(13/2) - 339864*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^8*B*a^(13/2) + 209979*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(15/2) + 262920*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(15/2) - 55511*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(17/2) - 736
40*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*
a^(17/2) + 6687*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*A*a^(19/2) + 8808*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*B*a^(19/2) - 299*A*a^(21/2) - 392*B*a^(21/2))/((sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4
)/d
```

3.99 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=254

$$\frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(13A+10B)\tan(c+dx)\sec^3(c+dx)\sqrt{a}}{40d}$$

```
[Out] (a^(5/2)*(283*A + 326*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^3*(283*A + 326*B)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^3*(283*A + 326*B)*Sec[c + d*x]*Tan[c + d*x])/(192*d*Sqrt[a
+ a*Cos[c + d*x]]) + (a^3*(157*A + 170*B)*Sec[c + d*x]^2*Tan[c + d*x])/(240
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(13*A + 10*B)*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c
 + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.713067, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(13A+10B)\tan(c+dx)\sec^3(c+dx)\sqrt{a}}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^(5/2)*(283*A + 326*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^3*(283*A + 326*B)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^3*(283*A + 326*B)*Sec[c + d*x]*Tan[c + d*x])/(192*d*Sqrt[a
+ a*Cos[c + d*x]]) + (a^3*(157*A + 170*B)*Sec[c + d*x]^2*Tan[c + d*x])/(240
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(13*A + 10*B)*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c
 + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{5}\int (a$$

$$= \frac{a^2(13A+10B)\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{40d}$$

$$= \frac{a^3(157A+170B)\sec^2(c+dx)\tan(c+dx)}{240d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(13A+10B}{}$$

$$= \frac{a^3(283A+326B)\sec(c+dx)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(157A+170}{240d}$$

$$= \frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(283A+326B)\sec(c+}{192d\sqrt{a+a\cos(}}$$

$$= \frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(283A+326B)\sec(c+}{192d\sqrt{a+a\cos(}}$$

$$= \frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^3(283A+32}{128d\sqrt{a+}}$$

Mathematica [A] time = 2.02376, size = 176, normalized size = 0.69

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(36(781A+650B)\cos(c+dx)+4(6509A+6730B)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2]
*(283*A + 326*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (24863*
A + 22030*B + 36*(781*A + 650*B)*Cos[c + d*x] + 4*(6509*A + 6730*B)*Cos[2*(
c + d*x)] + 5660*A*Cos[3*(c + d*x)] + 6520*B*Cos[3*(c + d*x)] + 4245*A*Cos[
4*(c + d*x)] + 4890*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d)
```

Maple [B] time = 4.312, size = 1951, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)

[Out] 1/120*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-480*a*(28
3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+283*A*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))+326*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)+326*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^10+
240*(566*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+652*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1415*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a+1415*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1630*B
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1630*B*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^8-80*(3962*A*a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+4564*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4245*A*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a)*sin(1/2*d*x+1/2*c)^6+8*(36224*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+40960*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+21225*A*ln(-4
/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+21225*A*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+24450*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*a+24450*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c
)^4-10*(12556*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+13400*B*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*a+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+4890*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2
```

```
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+22230*A*a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+20940*
B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4890*B*ln(-4/(-2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2
)*cos(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^5/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^5/s
in(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.32016, size = 670, normalized size = 2.64

$$\frac{15\left((283\,A+326\,B)a^2\cos\left(dx+c\right)^6+(283\,A+326\,B)a^2\cos\left(dx+c\right)^5\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="fricas")
```

```
[Out] 1/7680*(15*((283*A + 326*B)*a^2*cos(d*x + c)^6 + (283*A + 326*B)*a^2*cos(d*
x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(15*(283*A + 326*B)*a^2*cos(d*x + c)^4 + 10*(283*
A + 326*B)*a^2*cos(d*x + c)^3 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^2 + 48*(
29*A + 10*B)*a^2*cos(d*x + c) + 384*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x
```

```
 + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)

[Out] Timed out
```

Giac [B] time = 3.56792, size = 1304, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="giac")

[Out] 1/3840*(15*(283*A*a^(5/2) + 326*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(2
83*A*a^(5/2) + 326*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(4245*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*A*a^(7
/2) + 4890*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^18*B*a^(7/2) - 114615*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^16*A*a^(9/2) - 132030*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^16*B*a^(9/2) + 1298820*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(11/2) + 1319880*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*a^(
11/2) - 6176700*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^12*A*a^(13/2) - 6888120*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^12*B*a^(13/2) + 16394598*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(15/2) + 18352620*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(15/2)
 - 14042770*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^8*A*a^(17/2) - 15746180*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
```

```
*d*x + 1/2*c)^2 + a))^8*B*a^(17/2) + 4791060*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(19/2) + 5497320*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(19/2) - 860300
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^
(21/2) - 959320*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*B*a^(21/2) + 75885*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*A*a^(23/2) + 84810*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(23/2) - 2671*A*a^(25/2) - 2990*
B*a^(25/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*a + a^2)^5)/d
```

3.100 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=202

$$\frac{2(7A-B)\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}-\frac{2(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}+\frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A}{\ }$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (4*(49*A - 37*B)*Sin[c + d*x])/(105*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*(7*A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*B*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c
+ d*x]]) - (2*(7*A - 31*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.577639, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(7A-B)\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}-\frac{2(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}+\frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A}{\ }$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (4*(49*A - 37*B)*Sin[c + d*x])/(105*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*(7*A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*B*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c
+ d*x]]) - (2*(7*A - 31*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d)
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(3aB+\frac{1}{2}a(7A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{7a}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{4\int \frac{\cos(c+}{}}{}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{4\int \frac{a^2(7A-}{}}{}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{2(7A-3}{}$$

$$= \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c-}{7d\sqrt{a+}}$$

$$= \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c-}{7d\sqrt{a+}}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-}{35}$$

Mathematica [A] time = 0.64239, size = 111, normalized size = 0.55

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)((169B-28A)\cos(c+dx)+6(7A-B)\cos(2(c+dx))+406A+15B\cos(3(c+dx))\right.}{210d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-420*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*(406*A - 178*
B + (-28*A + 169*B)*Cos[c + d*x] + 6*(7*A - B)*Cos[2*(c + d*x)] + 15*B*Cos[
3*(c + d*x)])*Sin[(c + d*x)/2]))/(210*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 2.797, size = 281, normalized size = 1.4

$$\frac{1}{105d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-240\,B\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{a}\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+168\,\sqrt{2}\sqrt{a\left(\sin\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/105*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+168*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^4-140*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^2-105*2^(1
/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A
+105*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+a))*a*B+210*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(3/2)/sin
(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.66485, size = 494, normalized size = 2.45

$$\frac{4\left(15\,B\cos\left(dx+c\right)^{3}+3\left(7\,A-B\right)\cos\left(dx+c\right)^{2}-\left(7\,A-31\,B\right)\cos\left(dx+c\right)+91\,A-43\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right.}{210\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/210*(4*(15*B*cos(d*x + c)^3 + 3*(7*A - B)*cos(d*x + c)^2 - (7*A - 31*B)*c
os(d*x + c) + 91*A - 43*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 105*sqrt
(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.77195, size = 244, normalized size = 1.21

$$\frac{\frac{105\sqrt{2}(A-B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\left(105\sqrt{2}Aa^3+\left(\left(\sqrt{2}\left(119\,Aa^3-92\,Ba^3\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+7\sqrt{2}\left(37\,Aa^3-16\,Ba^3\right)\right)\tan\left(\frac{1}{2}d\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{7}{2}}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/105*(105*sqrt(2)*(A - B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(105*sqrt(2)*A*a^3 + ((sqrt(2)*(11
9*A*a^3 - 92*B*a^3)*tan(1/2*d*x + 1/2*c)^2 + 7*sqrt(2)*(37*A*a^3 - 16*B*a^3
))*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(2)*(7*A*a^3 - 4*B*a^3))*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

3.101 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}-\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+}{5d\sqrt{a\cos}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c
 + d*x]]) + (2*B*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]
) + (2*(5*A - B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```

Rubi [A] time = 0.384485, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}-\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+}{5d\sqrt{a\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c
 + d*x]]) + (2*B*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]
) + (2*(5*A - B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```


Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos(c+dx)\left(2aB+\frac{1}{2}a(5A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{5a}$$

$$= \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{2aB\cos(c+dx)+\frac{1}{2}a(5A-B)\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{5a}$$

$$= \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15ad} + \frac{4\int \ldots}{\ldots}$$

$$= -\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\cos \ldots}}{15 \ldots}$$

$$= -\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\cos \ldots}}{15 \ldots}$$

$$= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+d \ldots}{5d\sqrt{a+a\ldots}}$$

Mathematica [A] time = 0.319758, size = 94, normalized size = 0.59

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(15(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)(2(5A-B)\cos(c+dx)-10A+3B\cos(2(c+dx \ldots\right)}{15d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(15*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + (-10*A + 29*B +
 2*(5*A - B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d*
Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 2.135, size = 240, normalized size = 1.5

$$\frac{1}{15d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24B\sqrt{2}\sqrt{a(\sin(1/2dx+c/2))^2}\sqrt{a}(\sin(1/2dx+c/2))^4-20\sqrt{2}\sqrt{a(\sin(1/2dx+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/15*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*B*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-20*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+B)*sin(1/2*d*x+1/2*c)^2+15*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-15*2^(1/2)*
ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+30*
B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c
)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.72292, size = 446, normalized size = 2.81

$$\frac{4\left(3\,B\cos\left(dx+c\right)^{2}+\left(5\,A-B\right)\cos\left(dx+c\right)-5\,A+13\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)-\frac{15\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\,\mathrm{l}}{}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/30*(4*(3*B*cos(d*x + c)^2 + (5*A - B)*cos(d*x + c) - 5*A + 13*B)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B
)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(
```

```
a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74188, size = 213, normalized size = 1.34

$$-\frac{\frac{15\left(\sqrt{2}A-\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}-\frac{2\left(15\,\sqrt{2}Ba^{2}-\left(10\,\sqrt{2}Aa^{2}-20\,\sqrt{2}Ba^{2}+\left(10\,\sqrt{2}Aa^{2}-17\,\sqrt{2}Ba^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{5}{2}}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/15*(15*(sqrt(2)*A - sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) - 2*(15*sqrt(2)*B*a^2 - (10*sqr
t(2)*A*a^2 - 20*sqrt(2)*B*a^2 + (10*sqrt(2)*A*a^2 - 17*sqrt(2)*B*a^2)*tan(1
/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*
d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.102 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(3*A - 2*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*B*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.209553, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2751, 2649, 206}

$$\frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(3*A - 2*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*B*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2B\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{aB}{2}+\frac{1}{2}a(3A-2B)\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} + (-A+B)\int \frac{}{\sqrt{a+}}$$

$$= \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{(2(A-B))\,\mathrm{Subst}}{}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\,\mathrm{co}}}{}$$

Mathematica [A] time = 0.155997, size = 78, normalized size = 0.66

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-3(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6A\sin\left(\frac{1}{2}(c+dx)\right)-4B\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 6*A*Sin[(c + d*
x)/2] - 4*B*Sin[(c + d*x)/2]^3))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 2.51, size = 194, normalized size = 1.6

$$\frac{\sqrt{2}}{3\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\,(\sin(1/2\,dx+c/2))^2+6\,A\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*A*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)-3*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a))*a+3*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

[Out] Timed out

Fricas [A] time = 1.66431, size = 400, normalized size = 3.39

$$\frac{4\left(B\cos\left(dx+c\right)+3\,A-B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)-\frac{3\,\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\,\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)}{\sqrt{a}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/6*(4*(B*cos(d*x + c) + 3*A - B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 3
*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt
(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72588, size = 153, normalized size = 1.3

$$\frac{\frac{3\,\sqrt{2}(A-B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\left(\sqrt{2}(3\,Aa-2\,Ba)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,\sqrt{2}Aa\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] 1/3*(3*sqrt(2)*(A - B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(sqrt(2)*(3*A*a - 2*B*a)*tan(1/2*d*x +
 1/2*c)^2 + 3*sqrt(2)*A*a)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 +
 a)^(3/2))/d
```

3.103 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) + (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0712777, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) + (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+(A-B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx\\ &= \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(2(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0632729, size = 60, normalized size = 0.77

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*B*Sin[(c + d*x)/
2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 1.862, size = 160, normalized size = 2.1

$$\frac{\sqrt{2}}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\ln\left(4\,\frac{\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+a}{\cos(1/2\,dx+c/2)}\right)a+2\,B\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}-B\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(4/cos(1/2*d
*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+2*B*a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.64399, size = 363, normalized size = 4.65

$$\frac{4\sqrt{a\cos\left(dx+c\right)+a}B\sin\left(dx+c\right)-\frac{\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c) - sqrt(2)*((A - B)*a*cos(d*x
 + c) + (A - B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [A] time = 1.6717, size = 119, normalized size = 1.53

$$\frac{\frac{2\sqrt{2}B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{\left(\sqrt{2}A-\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*sqrt(2)*B*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - (sqr
t(2)*A - sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a)))/sqrt(a))/d
```

3.104 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.165683, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2985, 2649, 206, 2773}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{a} - (A-B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= -\frac{(2A)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{(2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \right.}{d}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0768419, size = 72, normalized size = 0.79

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*((A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 4.029, size = 268, normalized size = 3.

$$-\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+a}{\cos\left(1/2\,dx+c/2\right)}\right)A-\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}}{\cos\left(1/2\,dx+c/2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2
*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*B-A*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))-A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
))/a^(1/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 2.02556, size = 123, normalized size = 1.35

$$\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.\right.}{2\,\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/(sqrt(a)*d)
```

Fricas [B] time = 1.80238, size = 464, normalized size = 5.1

$$-\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-A\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - A*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
- 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [B] time = 2.7415, size = 227, normalized size = 2.49

$$\frac{\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a}+\frac{2\,A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{a}}-\frac{2\,A\log\left(\left|\left(\sqrt{a}\right.\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] 1/2*(sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + 2*A*log(abs((sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/sqrt(
a) - 2*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + a*(2*sqrt(2) - 3)))/sqrt(a))/d
```

3.105 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -(((A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqr
t[a]*d)) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d
*x]])
```

Rubi [A] time = 0.308313, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2984, 2985, 2649, 206, 2773}

$$-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -(((A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqr
t[a]*d)) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d
*x]])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-2B)+\frac{1}{2}aA\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(A-2B)\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{2a} + (A-$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(A-2B)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{(2(A}{}$$

$$= -\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \cdots$$

Mathematica [A] time = 0.334904, size = 95, normalized size = 0.8

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}(A-2B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(2*(A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*(A - 2*B)*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sec[c + d*x]*Sin[(c + d*x)/2]))/(d*
Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 4.62, size = 812, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(2*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*A-2*2^(1/2)*ln(
4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*B-A*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+2*B*ln(-4/(-2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4/cos(
1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-2*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-A*ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-A*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+2*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*ln
(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x
```

```
+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.15602, size = 699, normalized size = 5.87

$$-\frac{\left((A-2B)\cos(dx+c)^2+(A-2B)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(ad\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/4*(((A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos
(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*s
qrt(a*cos(d*x + c) + a)*A*sin(d*x + c) + 2*sqrt(2)*((A - B)*a*cos(d*x + c)^
2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x
)
```

Giac [B] time = 2.93754, size = 433, normalized size = 3.64

$$-\frac{\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a}+\frac{\left(A\sqrt{a}-2\,B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)}{a}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/2*(sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + (A*sqrt(a) - 2*B*sqrt(a))*log(abs
((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*
(2*sqrt(2) + 3)))/a - (A*sqrt(a) - 2*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a
- 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*A*sqrt(a) - A*a^(3/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.106 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)\text{se}}{2d\sqrt{a\cos(c+}}$$

```
[Out] ((7*A - 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Tan[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*
x]])
```

Rubi [A] time = 0.480372, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2984, 2985, 2649, 206, 2773}

$$-\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)\text{se}}{2d\sqrt{a\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```



```
[Out] ((7*A - 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Tan[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*
x]])
```


Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx = \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-4B)+\frac{3}{2}aA\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{2a}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}a^2(7A-4B)-\frac{1}{4}a^2(A-4B\right.}{\sqrt{a+a\cos}}}{2a}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{(7A-4B)\int \sqrt{a+a\,\mathrm{co}}}{}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(7A-4B)\,\mathrm{Subst}\left(\int \frac{}{a}\right.}{}$$

$$= \frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{(}{4}$$

Mathematica [A] time = 0.771305, size = 114, normalized size = 0.69

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-8(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(7A-4B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\mathrm{s}\right.}{4d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-8*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(7*A - 4*
B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(-A + 4*B + 2*A*Sec[c
 + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 5.027, size = 1240, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a*(-8*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*A+8*2^(1/
2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*B+7*
A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+7*A*ln(-4/(-2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))-4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-4*B*ln
(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(A*a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c
)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-4*B*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a-7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*
(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-2*A*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
-7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-8*B*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d
*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(c
os(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

[Out] Timed out

Fricas [B] time = 2.13617, size = 753, normalized size = 4.56

$$-\frac{\left((7A-4B)\cos(dx+c)^3+(7A-4B)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/16*(((7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) + 4*((A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
+ 8*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*log(-(cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*c
os(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d
*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.09068, size = 722, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/8*(4*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + (7*A*sqrt(a) - 4*B*sqrt(a))*log(
abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
 a*(2*sqrt(2) + 3)))/a - (7*A*sqrt(a) - 4*B*sqrt(a))*log(abs((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)
))/a - 4*sqrt(2)*(17*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^6*A*sqrt(a) - 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(a) - 57*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2) + 76*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(3/2) + 19*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/2) - 36*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(5/2)
 - 3*A*a^(7/2) + 4*B*a^(7/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.107 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}-\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^4(c+}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -((15*A - 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2)) + ((651*A - 799*B)*Sin[c + d*x])/(105*a*d*Sqrt
[a + a*Cos[c + d*x]]) + ((63*A - 67*B)*Cos[c + d*x]^2*Sin[c + d*x])/(70*a*d
*Sqrt[a + a*Cos[c + d*x]]) - ((7*A - 11*B)*Cos[c + d*x]^3*Sin[c + d*x])/(14
*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((273*A - 397*B)*Sqrt[a + a*Cos[c + d*x]]*
Sin[c + d*x])/(210*a^2*d)
```

Rubi [A] time = 0.785725, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}-\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^4(c+}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] -((15*A - 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2)) + ((651*A - 799*B)*Sin[c + d*x])/(105*a*d*Sqrt
[a + a*Cos[c + d*x]]) + ((63*A - 67*B)*Cos[c + d*x]^2*Sin[c + d*x])/(70*a*d
*Sqrt[a + a*Cos[c + d*x]]) - ((7*A - 11*B)*Cos[c + d*x]^3*Sin[c + d*x])/(14
*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((273*A - 397*B)*Sqrt[a + a*Cos[c + d*x]]*
Sin[c + d*x])/(210*a^2*d)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
```

```
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(4a(A-B)-\frac{1}{2}a(7A-11B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(7A-11B)\cos^3(c+dx)\sin(c+dx)}{14ad\sqrt{a+a\cos(c+dx)}} + \int$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - (7$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - (7$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - (7$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} + \frac{(63A-67B)}{70ad}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} + \frac{(63A-67B)}{70ad}$$

$$= -\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

Mathematica [A] time = 1.06607, size = 167, normalized size = 0.64

$$\frac{105(15A-19B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(6(273A-277B)\cos(c}{105d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (105*(15*A - 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - (Cos[(c +
 d*x)/2]^3*(1974*A - 2161*B + 6*(273*A - 277*B)*Cos[c + d*x] + (-84*A + 256
*B)*Cos[2*(c + d*x)] + 42*A*Cos[3*(c + d*x)] - 18*B*Cos[3*(c + d*x)] + 15*B
*Cos[4*(c + d*x)])*Sin[(c + d*x)/2])/2)/(105*d*(a*(1 + Cos[c + d*x]))^(3/2)
*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [A] time = 2.718, size = 448, normalized size = 1.7

$$\frac{1}{420\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(960\,B\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{a}\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-96\,\sqrt{2}\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\,(7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/420*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(960*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^8-96*2^(1/2)*a^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*(7*A+17*B)*sin(1/2*d*x+1/2*c)^6+224*2^(1/2)*a^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*(3*A+8*B)*sin(1/2*d*x+1/2*c)^4-35*2^(1/2)*(48*A*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-45*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-16*B*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+57*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a
)*sin(1/2*d*x+1/2*c)^2-1575*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+1995*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+1785*A*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-1785*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
)/cos(1/2*d*x+1/2*c)/a^(5/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.79064, size = 653, normalized size = 2.5

$$-\frac{105\sqrt{2}\left((15A-19B)\cos(dx+c)^2+2(15A-19B)\cos(dx+c)+15A-19B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{}}{\cos(dx+c)^2+2\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/840*(105*sqrt(2)*((15*A - 19*B)*cos(d*x + c)^2 + 2*(15*A - 19*B)*cos(d*x
 + c) + 15*A - 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(60*B*cos(d*x + c)^4 + 12*(7*A - 3*B)*cos(d*x
+ c)^3 - 28*(3*A - 7*B)*cos(d*x + c)^2 + 12*(63*A - 67*B)*cos(d*x + c) + 10
29*A - 1201*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2
 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.99236, size = 343, normalized size = 1.31

$$\frac{\frac{105\left(15\sqrt{2}A-19\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\left(\frac{105\left(\sqrt{2}Aa^5-\sqrt{2}Ba^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3}+\frac{4\left(693\sqrt{2}Aa^5-877\sqrt{2}Ba^5\right)}{a^3}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots}{}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/420*(105*(15*sqrt(2)*A - 19*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2
*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((((105*(sqrt(2)*A*a^5
 - sqrt(2)*B*a^5)*tan(1/2*d*x + 1/2*c)^2/a^3 + 4*(693*sqrt(2)*A*a^5 - 877*s
qrt(2)*B*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 14*(453*sqrt(2)*A*a^5 - 517*sqr
t(2)*B*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 140*(39*sqrt(2)*A*a^5 - 47*sqrt(2
)*B*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 1785*(sqrt(2)*A*a^5 - sqrt(2)*B*a^5)
/a^3)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

3.108 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=216

$$\frac{(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{30a^2d}+\frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ((11*A - 15*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) - ((65*A - 93*B)*Sin[c + d*x])/(15*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((5*A - 9*B)*Cos[c + d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[
a + a*Cos[c + d*x]]) + ((35*A - 39*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(30*a^2*d)
```

Rubi [A] time = 0.59499, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{30a^2d}+\frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((11*A - 15*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) - ((65*A - 93*B)*Sin[c + d*x])/(15*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((5*A - 9*B)*Cos[c + d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[
a + a*Cos[c + d*x]]) + ((35*A - 39*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(30*a^2*d)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
```

```
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(3a(A-B)-\frac{1}{2}a(5A-9B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\text{co}}{-}}{}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{-a}{-}}{}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} + \frac{(35A}{}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a+a\cos(c+dx)}} - \frac{(5A-9B)\cos^2}{10ad\sqrt{a+}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a+a\cos(c+dx)}} - \frac{(5A-9B)\cos^2}{10ad\sqrt{a+}}$$

$$= \frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(}{1}$$

Mathematica [A] time = 0.870648, size = 142, normalized size = 0.66

$$\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(3(20A-39B)\cos(c+dx)+(6B-10A)\cos(2(c+dx))+85A-3B\cos(3(c+dx))-1}{15d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-15*(11*A - 15*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + Cos[(c +
d*x)/2]^3*(85*A - 141*B + 3*(20*A - 39*B)*Cos[c + d*x] + (-10*A + 6*B)*Cos[
2*(c + d*x)] - 3*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/(15*d*(a*(1 + Cos[c
+ d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [B] time = 2.473, size = 407, normalized size = 1.9

$$\frac{1}{60\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-96\,B\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{a}\left(\sin\left(1/2\,dx+c/2\right)\right)^6+16\,\sqrt{2}\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\,(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/60/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-96*B*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+16*2^(1/2)*a^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+6*B)*sin(1/2*d*x+1/2*c)^4+5*2^(1/2)*(8*A
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-33*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-48*B*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+45*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+a))*a)*sin(1/2*d*x+1/2*c)^2+165*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-225*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)
*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-135*A*a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+255*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.79943, size = 601, normalized size = 2.78

$$-\frac{15\sqrt{2}\left((11A-15B)\cos(dx+c)^2+2(11A-15B)\cos(dx+c)+11A-15B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\cdots}{\cos(dx+c)^2+2\cos\cdots}\cdots\right.}{120\left(a^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/120*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^2 + 2*(11*A - 15*B)*cos(d*x
+ c) + 11*A - 15*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1)) - 4*(12*B*cos(d*x + c)^3 + 4*(5*A - 3*B)*cos(d*x +
c)^2 - 12*(5*A - 9*B)*cos(d*x + c) - 95*A + 147*B)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.92885, size = 273, normalized size = 1.26

$$-\frac{\frac{15\sqrt{2}(11A-15B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\frac{15\sqrt{2}(Aa^3-Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2}+\frac{\sqrt{2}(245Aa^3-381Ba^3)}{a^2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{5\sqrt{2}(73A\cdots}{\cdots}\cdots\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots\right.}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/60*(15*sqrt(2)*(11*A - 15*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + (((15*sqrt(2)*(A*a^3 - B*a^3)*t
an(1/2*d*x + 1/2*c)^2/a^2 + sqrt(2)*(245*A*a^3 - 381*B*a^3)/a^2)*tan(1/2*d*
x + 1/2*c)^2 + 5*sqrt(2)*(73*A*a^3 - 105*B*a^3)/a^2)*tan(1/2*d*x + 1/2*c)^2
 + 15*sqrt(2)*(9*A*a^3 - 17*B*a^3)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x
 + 1/2*c)^2 + a)^(5/2))/d
```

3.109 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}-\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+$$

```
[Out] -((7*A - 11*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 13*B)*Sin[c + d*x])/(3*a*d*Sqrt[a + a
*Cos[c + d*x]]) - ((3*A - 7*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^
2*d)
```

Rubi [A] time = 0.420456, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}-\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((7*A - 11*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 13*B)*Sin[c + d*x])/(3*a*d*Sqrt[a + a
*Cos[c + d*x]]) - ((3*A - 7*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^
2*d)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(A-B)-\frac{1}{2}a(3A-7B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{2a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{2a(A-B)\cos(c+dx)-\frac{1}{2}a(3A-7B)\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{2a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A-7B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A-7B)\sqrt{a+}}{}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A-7B)\sqrt{a+}}{}$$

$$= -\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

Mathematica [A] time = 0.704158, size = 97, normalized size = 0.57

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(12(A-B)\cos(c+dx)+15A+2B\cos(2(c+dx))-17B)-3(7A-11B)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}\right.\right.}{6ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-3*(7*A - 11*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (15*A - 17*B
+ 12*(A - B)*Cos[c + d*x] + 2*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(6*a*d*
Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 2.164, size = 327, normalized size = 1.9

$$-\frac{1}{12d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-16B\sqrt{2}\sqrt{a(\sin(1/2dx+c/2))^2}\sqrt{a}(\cos(1/2dx+c/2))^4+21A\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+}}{\cos(1/2dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/12*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+21*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-33*B*
ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(
1/2)*cos(1/2*d*x+1/2*c)^2*a-24*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*cos(1/2*d*x+1/2*c)^2+40*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)*cos(1/2*d*x+1/2*c)^2-3*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
3*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5
/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.68249, size = 544, normalized size = 3.18

$$-\frac{3\sqrt{2}\left((7A-11B)\cos(dx+c)^2+2(7A-11B)\cos(dx+c)+7A-11B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+}{\cos(dx+c)^2+2\cos(dx+c)+}\right.}{24\left(a^2d\cos(dx+c)^2+2a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^2 + 2*(7*A - 11*B)*cos(d*x + c)
 + 7*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
```

```
cos(d*x + c) + 1)) - 4*(4*B*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 15*A
 - 19*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.91589, size = 227, normalized size = 1.33

$$\frac{\frac{3\left(7\sqrt{2}A-11\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\frac{3\left(\sqrt{2}Aa-\sqrt{2}Ba\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a}+\frac{2\left(15\sqrt{2}Aa-23\sqrt{2}Ba\right)}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+\frac{27\left(\sqrt{2}Aa-\sqrt{\right.}}{a}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/12*(3*(7*sqrt(2)*A - 11*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c)
+ sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((3*(sqrt(2)*A*a - sqrt(2)
*B*a)*tan(1/2*d*x + 1/2*c)^2/a + 2*(15*sqrt(2)*A*a - 23*sqrt(2)*B*a)/a)*tan
(1/2*d*x + 1/2*c)^2 + 27*(sqrt(2)*A*a - sqrt(2)*B*a)/a)*tan(1/2*d*x + 1/2*c
)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2))/d
```

3.110 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2B\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((3*A - 7*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2)) + (2*B*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.22318, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3019, 2751, 2649, 206}

$$\frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2B\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((3*A - 7*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2)) + (2*B*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{3}{2}a(A-B)-2aB\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{(3A-7B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}}{4a}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A-7B)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{2ad}$$

$$= \frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c}{ad\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.392973, size = 104, normalized size = 0.88

$$\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(A-4B\cos(c+dx)-5B)-(3A-7B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-((3*A - 7*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5) + Cos[(c + d*x
)/2]^3*(A - 5*B - 4*B*Cos[c + d*x])*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*
x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [B] time = 2.435, size = 256, normalized size = 2.2

$$\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3A\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/2))^2}+2a}{\cos(1/2dx+c/2)}\right)\sqrt{2}(\cos(1/2dx+c/2))^2a-7B\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(}{\cos(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*ln(2*(2*a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x
+1/2*c)^2*a-7*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2
*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+8*B*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/si
n(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.76719, size = 493, normalized size = 4.18

$$-\frac{\sqrt{2}\left((3A-7B)\cos(dx+c)^2+2(3A-7B)\cos(dx+c)+3A-7B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\cdots\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/8*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*
A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(4*B*cos(d*x + c) - A + 5*B)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.89274, size = 177, normalized size = 1.5

$$-\frac{\frac{\left(\frac{\sqrt{2}(Aa^2-Ba^2)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3}+\frac{\sqrt{2}(Aa^2-9Ba^2)}{a^3}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{\sqrt{2}(3A-7B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/4*((sqrt(2)*(A*a^2 - B*a^2)*tan(1/2*d*x + 1/2*c)^2/a^3 + sqrt(2)*(A*a^2
- 9*B*a^2)/a^3)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + s
qrt(2)*(3*A - 7*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a)))/a^(3/2))/d
```

3.111 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ((A + 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2))
```

Rubi [A] time = 0.0768915, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2750, 2649, 206}

$$\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((A + 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2))
```


Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx &= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A+3B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}\\ &= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A+3B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}\\ &= \frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.184055, size = 63, normalized size = 0.72

$$\frac{\frac{1}{2}(A-B)\sin(c+dx)+(A+3B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + ((A - B)*Sin[c +
d*x])/2)/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 2.506, size = 220, normalized size = 2.5

$$\frac{1}{4\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\left(\sin(1/2\,dx+c/2)\right)^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)\sqrt{2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a+3\,B\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\left(\sin(1/2\,\right.}}{\cos(1/2\,d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(2*(2*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1
/2*c)^2*a+3*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d
*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)-B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/s
in(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.72948, size = 455, normalized size = 5.23

$$\frac{\sqrt{2}\left((A+3B)\cos(dx+c)^2+2(A+3B)\cos(dx+c)+A+3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(d}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A + 3*B
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + 4*sqrt(a*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74244, size = 136, normalized size = 1.56

$$-\frac{\frac{\left(\sqrt{2}A+3\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\sqrt{2}Aa-\sqrt{2}Ba\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{3}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*((sqrt(2)*A + 3*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.112 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2))
```

Rubi [A] time = 0.315465, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2985, 2649, 206, 2773}

$$-\frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2aA-\frac{1}{2}a(A-B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{A\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{a^2} - \frac{(5A-B)\int$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} + \frac{(5A-}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin}{2d(a+a\cos(}$$

Mathematica [A] time = 0.646331, size = 131, normalized size = 1.03

$$\frac{(A-B)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(5A-B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sqrt{2}A\cos^5\left(\frac{1}{2}(c+dx)\right.}{d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((5*A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 4*Sqrt[2]*A*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (A - B)*Cos[(c + d*x)/2]^
3*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^
2))
```

Maple [B] time = 4.757, size = 374, normalized size = 2.9

$$-\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5A\ln\left(2\,\frac{2\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}+2a}{\cos(1/2\,dx+c/2)}\right)\sqrt{2}(\cos(1/2\,dx+c/2))^2a-B\ln\left(2\,\frac{2\sqrt{a}\sqrt{a(\sin}}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*co
s(1/2*d*x+1/2*c)^2*a-B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/
cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(-4*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(
1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(4/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a+A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)-B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2
*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.9338, size = 737, normalized size = 5.8

$$-\frac{\sqrt{2}\left((5\,A-B)\cos\left(dx+c\right)^2+2\,(5\,A-B)\cos\left(dx+c\right)+5\,A-B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2\,a\cos(d}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/8*(sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*A -
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)) - 4*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a
*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos\left(c+dx\right))\sec\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x
)
```

Giac [B] time = 3.08904, size = 289, normalized size = 2.28

$$\frac{\frac{\sqrt{2}\left(5\,A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a^{2}}+\frac{8\,A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)}{a^{\frac{3}{2}}}-\frac{8\,A\log\left(\left|\left(\sqrt{\right.\right.\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/8*(sqrt(2)*(5*A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 8*A*log(abs((sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a
^(3/2) - 8*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) - 2*sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.113 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\tan(c+}{2d(a\cos(c+dx)+}$$

```
[Out] -(((3*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(3/2)*d)) + ((9*A - 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a +
 a*Cos[c + d*x])^(3/2)) + ((3*A - B)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c
+ d*x]])
```

Rubi [A] time = 0.520934, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\tan(c+}{2d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((3*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(3/2)*d)) + ((9*A - 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a +
 a*Cos[c + d*x])^(3/2)) + ((3*A - B)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c
+ d*x]])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(a(3A-B)-\frac{3}{2}a(A-B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{2a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-a^2(3A-2B)+\frac{1}{2}a^2(3A-B)\cos\right.}{\sqrt{a+a\cos(c+}}}{2a^3}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{(9A-5B)\int \frac{1}{\sqrt{a+a\cos(c+dx}}}{4a}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{(9A-5B)\,\text{Subst}\left(\int \frac{1}{2a-x}\right.}{2a}$$

$$= -\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{}{2a}$$

Mathematica [A] time = 1.03759, size = 141, normalized size = 0.83

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(2(9A-5B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-2A\sec(c+dx)-3A+B)+4\sqrt{2}(3A-2B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\right.}{\sin^2\left(\frac{1}{2}(c+dx)\right)-1}\right.}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(2*(9*A - 5*B)*ArcTanh[Sin[(c + d*x)/2]] + (4*Sqrt[2]*(
3*A - 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 + 2*(-3*A +
 B - 2*A*Sec[c + d*x])*Sin[(c + d*x)/2])/(-1 + Sin[(c + d*x)/2]^2)))/(2*d*(
a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 4.868, size = 1051, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-10*B*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1
/2)*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
)*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*c
os(1/2*d*x+1/2*c)^4*a+8*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(
1/2*d*x+1/2*c)^4*a+8*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*
d*x+1/2*c)^4*a-9*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(
1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+5*B*ln(2*(2*a^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^
2*a+6*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2
+6*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+6*
A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a-2*B*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-4*B*ln(-4
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-4*B*ln(4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a-A*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.30093, size = 880, normalized size = 5.18

$$-\frac{\sqrt{2}\left((9\,A-5\,B)\cos\left(dx+c\right)^{3}+2\,(9\,A-5\,B)\cos\left(dx+c\right)^{2}+(9\,A-5\,B)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\,\sqrt{2}\sqrt{a\cos(dx+c)}}{\cos(dx+c)^{2}+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/8*(sqrt(2)*((9*A - 5*B)*cos(d*x + c)^3 + 2*(9*A - 5*B)*cos(d*x + c)^2 +
(9*A - 5*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)) + 2*((3*A - 2*B)*cos(d*x + c)^3 + 2*(3*A - 2*
B)*cos(d*x + c)^2 + (3*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((3*A - B)*co
s(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c
)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + dx))*sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2)
, x)
```

Giac [B] time = 3.22686, size = 504, normalized size = 2.96

$$-\frac{\frac{\sqrt{2}\left(9\,A\sqrt{a}-5\,B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a^{2}}+\frac{4\left(3\,A\sqrt{a}-2\,B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right.}{a^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/8*(sqrt(2)*(9*A*sqrt(a) - 5*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 4*(3*A*sqrt(a) - 2*B*sqrt(a
))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2 - a*(2*sqrt(2) + 3)))/a^2 - 4*(3*A*sqrt(a) - 2*B*sqrt(a))*log(abs((sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*s
qrt(2) - 3)))/a^2 - 16*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a^(3/2))/(((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a) - 2*sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/a^
3)/d
```

3.114 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{(2A-B)\tan(c}{2ad\sqrt{a\cos(}}$$

```
[Out] ((19*A - 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(3/2)*d) - ((13*A - 9*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A - 6*B)*Tan[c + d*x])/(4*
a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a
 + a*Cos[c + d*x])^(3/2)) + ((2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sq
rt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.705256, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$\frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{(2A-B)\tan(c}{2ad\sqrt{a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((19*A - 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(3/2)*d) - ((13*A - 9*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A - 6*B)*Tan[c + d*x])/(4*
a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a
 + a*Cos[c + d*x])^(3/2)) + ((2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sq
rt[a + a*Cos[c + d*x]])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2a(2A-B)-\frac{5}{2}a(A-B)\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{(-a^2(}{}}{}$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)\sec(c+}{2ad\sqrt{a+a}}$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)\sec(c+}{2ad\sqrt{a+a}}$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)\sec(c+}{2ad\sqrt{a+a}}$$

$$= \frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} -$$

Mathematica [A] time = 1.46235, size = 205, normalized size = 0.93

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((6A-8B)\cos(c+dx)+(7A-6B)\cos(2(c+dx))+3(A-2B))+4(13A\right.}{16d\,(\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Sec[c + d*x]^2*(4*(13*A - 9*B)*ArcTanh[Sin[(c + d*x)/2]
]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2 - 2*Sqrt[2]*(19*A - 12*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2
+ 4*(3*(A - 2*B) + (6*A - 8*B)*Cos[c + d*x] + (7*A - 6*B)*Cos[2*(c + d*x)])
*Sin[(c + d*x)/2]))/(16*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/
2]^2))
```

Maple [B] time = 5.48, size = 1540, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(104*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-72*B
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))
*cos(1/2*d*x+1/2*c)^6*a+48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*co
s(1/2*d*x+1/2*c)^6*a+48*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(
1/2*d*x+1/2*c)^6*a-104*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+72*B*ln(2*(2*a^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+
1/2*c)^4*a+28*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+
1/2*c)^4+76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c
)^4*a+76*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^
4*a-24*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^
4-48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-4
8*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+26*
A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2
^(1/2)*cos(1/2*d*x+1/2*c)^2*a-18*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-22*A*a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-19*A*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a-19*A*ln(-4*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)
/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+16*B*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+12*B*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a+12*B*ln(-4*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos
```

```
(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+2*A*a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
)/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*
x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.35302, size = 944, normalized size = 4.27

$$-\frac{2\sqrt{2}\left((13A-9B)\cos(dx+c)^4+2(13A-9B)\cos(dx+c)^3+(13A-9B)\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos}}{\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/16*(2*sqrt(2)*((13*A - 9*B)*cos(d*x + c)^4 + 2*(13*A - 9*B)*cos(d*x + c)
^3 + (13*A - 9*B)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B)*cos(d*x + c)^4 + 2*(
19*A - 12*B)*cos(d*x + c)^3 + (19*A - 12*B)*cos(d*x + c)^2)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(c
os(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*((7*A - 6*B)*cos(d*x + c)^2 + (3*A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 +
 a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.27547, size = 787, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/8*(sqrt(2)*(13*A*sqrt(a) - 9*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + (19*A*sqrt(a) - 12*B*sqrt(a
))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2 - a*(2*sqrt(2) + 3)))/a^2 - (19*A*sqrt(a) - 12*B*sqrt(a))*log(abs((sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*s
qrt(2) - 3)))/a^2 - 2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a - sqr
t(2)*B*a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(29*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(a) - 12*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(a) - 133*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2
) + 76*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
4*B*a^(3/2) + 55*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*A*a^(5/2) - 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*B*a^(5/2) - 7*A*a^(7/2) + 4*B*a^(7/2))/(((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*a))/d
```

3.115 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(85A-157B)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}-\frac{(985A-1729B)\sin(c+dx)}{120a^2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((163*A - 283*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + ((13*A - 21*B)*Cos[c + d*x]^3*Sin[c + d*x]
)/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((985*A - 1729*B)*Sin[c + d*x])/(12
0*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((85*A - 157*B)*Cos[c + d*x]^2*Sin[c +
d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((475*A - 787*B)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(240*a^3*d)
```

Rubi [A] time = 0.798846, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{(85A-157B)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}-\frac{(985A-1729B)\sin(c+dx)}{120a^2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((163*A - 283*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + ((13*A - 21*B)*Cos[c + d*x]^3*Sin[c + d*x]
)/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((985*A - 1729*B)*Sin[c + d*x])/(12
0*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((85*A - 157*B)*Cos[c + d*x]^2*Sin[c +
d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((475*A - 787*B)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(240*a^3*d)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
```

```
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)\left(4a(A-B)-\frac{1}{2}a(5A-13B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \int$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - (85$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - (85$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - (85$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(98}{12}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(98}{12}$$

$$= \frac{(163A-283B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} +$$

Mathematica [A] time = 1.46063, size = 139, normalized size = 0.53

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(-5(479A-887B)\cos(c+dx)+(832B-400A)\cos(2(c+dx))+40A\cos(3(c+dx))-1895A-40B\cos}{240ad(a(\cos(c+dx)-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (30*(163*A - 283*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-1895*A
 + 3491*B - 5*(479*A - 887*B)*Cos[c + d*x] + (-400*A + 832*B)*Cos[2*(c + d*
x)] + 40*A*Cos[3*(c + d*x)] - 40*B*Cos[3*(c + d*x)] + 12*B*Cos[4*(c + d*x)]
)*Tan[(c + d*x)/2])/(240*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 2.654, size = 467, normalized size = 1.8

$$\frac{1}{480\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(768\,B\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{a}\left(\cos\left(1/2\,dx+c/2\right)\right)^{8}+640\,A\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/480*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(768*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^8+640*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-2176*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+2445*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-4245
*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*
2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-2560*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+5248*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-435*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*cos(1/2*d*x+1/2*c)^2+555*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)*cos(1/2*d*x+1/2*c)^2+30*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-30*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*
c)^3/a^(7/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.72255, size = 741, normalized size = 2.84

$$-\frac{15\sqrt{2}\left((163\,A-283\,B)\cos\left(dx+c\right)^{3}+3\,(163\,A-283\,B)\cos\left(dx+c\right)^{2}+3\,(163\,A-283\,B)\cos\left(dx+c\right)+163\,A-283\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/960*(15*sqrt(2)*((163*A - 283*B)*cos(d*x + c)^3 + 3*(163*A - 283*B)*cos(
d*x + c)^2 + 3*(163*A - 283*B)*cos(d*x + c) + 163*A - 283*B)*sqrt(a)*log(-(
a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c)
- 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(96*B*
cos(d*x + c)^4 + 160*(A - B)*cos(d*x + c)^3 - 32*(25*A - 49*B)*cos(d*x + c)
^2 - 5*(503*A - 911*B)*cos(d*x + c) - 1495*A + 2671*B)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.28102, size = 347, normalized size = 1.33

$$-\frac{\frac{15\left(163\sqrt{2}A-283\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{5}{2}}}-\frac{\left(\left(\left(15\left(\frac{2\left(\sqrt{2}Aa^2-\sqrt{2}Ba^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2}-\frac{21\sqrt{2}Aa^2-29\sqrt{2}Ba^2}{a^2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/480*(15*(163*sqrt(2)*A - 283*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1
/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2) - (((15*(2*(sqrt(2)*A*
a^2 - sqrt(2)*B*a^2)*tan(1/2*d*x + 1/2*c)^2/a^2 - (21*sqrt(2)*A*a^2 - 29*sq
rt(2)*B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - (3685*sqrt(2)*A*a^2 - 6733*sqrt(
2)*B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 5*(1133*sqrt(2)*A*a^2 - 1973*sqrt(2
)*B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 15*(155*sqrt(2)*A*a^2 - 291*sqrt(2)*
B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.116 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A}{}$$

```
[Out] -((75*A - 163*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + ((9*A - 17*B)*Cos[c + d*x]^2*Sin[c + d*x])
/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((93*A - 197*B)*Sin[c + d*x])/(24*a^
2*d*Sqrt[a + a*Cos[c + d*x]]) - ((39*A - 95*B)*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.611347, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] -((75*A - 163*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + ((9*A - 17*B)*Cos[c + d*x]^2*Sin[c + d*x])
/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((93*A - 197*B)*Sin[c + d*x])/(24*a^
2*d*Sqrt[a + a*Cos[c + d*x]]) - ((39*A - 95*B)*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(48*a^3*d)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
```

```
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(3a(A-B)-\frac{1}{2}a(3A-11B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos \ldots}{\ldots}}{\ldots}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{a^2(\ldots}{\ldots}}{\ldots}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(39A \ldots}{\ldots}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(93A \ldots}{24a^2 \ldots}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(93A \ldots}{24a^2 \ldots}$$

$$= -\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \ldots$$

Mathematica [A] time = 1.00911, size = 117, normalized size = 0.54

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((255A-479B)\cos(c+dx)+16(3A-5B)\cos(2(c+dx))+195A+8B\cos(3(c+dx))-379B)-6(75A \ldots}{48ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (-6*(75*A - 163*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (195*A -
379*B + (255*A - 479*B)*Cos[c + d*x] + 16*(3*A - 5*B)*Cos[2*(c + d*x)] + 8*
B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(48*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 2.477, size = 397, normalized size = 1.8

$$-\frac{1}{96d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-128B\sqrt{2}\sqrt{a\left(\sin\left(1/2dx+c/2\right)\right)^{2}}\sqrt{a}\left(\cos\left(1/2dx+c/2\right)\right)^{6}+225A\ln\left(2\frac{2\sqrt{a}\sqrt{a\left(\sin\left(1/2dx\right.\right.}}{\cos\left(1/2dx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/96*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-128*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+225*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-489
*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*
2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-192*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*a^(1/2)*cos(1/2*d*x+1/2*c)^4+512*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)*cos(1/2*d*x+1/2*c)^4-63*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)^2+87*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)*cos(1/2*d*x+1/2*c)^2+6*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-6*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)^3/a
^(7/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.7759, size = 687, normalized size = 3.18

$$-\frac{3\sqrt{2}\left((75A-163B)\cos\left(dx+c\right)^{3}+3\left(75A-163B\right)\cos\left(dx+c\right)^{2}+3\left(75A-163B\right)\cos\left(dx+c\right)+75A-163B\right)\sqrt{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^3 + 3*(75*A - 163*B)*cos(d*x
 + c)^2 + 3*(75*A - 163*B)*cos(d*x + c) + 75*A - 163*B)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*cos(d
*x + c)^3 + 32*(3*A - 5*B)*cos(d*x + c)^2 + (255*A - 503*B)*cos(d*x + c) +
147*A - 299*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.20505, size = 275, normalized size = 1.27

$$-\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^6}-\frac{\sqrt{2}\left(15\,Aa^5-23\,Ba^5\right)}{a^6}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{4\,\sqrt{2}\left(75\,Aa^5-167\,Ba^5\right)}{a^6}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{3\,\sqrt{2}\left(83\,Aa^5-155\,Ba^5\right)}{a^6}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}-\frac{3\,\sqrt{2}(75}{}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/96*(((3*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*d*x + 1/2*c)^2/a^6 - sqrt(2)*
(15*A*a^5 - 23*B*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt(2)*(75*A*a^5 - 1
67*B*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(2)*(83*A*a^5 - 155*B*a^5)/a^
```

```
6)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 3*sqrt(2)*(7
5*A - 163*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a)))/a^(5/2))/d
```

3.117 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{(A-9B)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(5A-13B)\sin}{16ad(a\cos(c+}$$

```
[Out] ((19*A - 75*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(4*d
*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*C
os[c + d*x])^(3/2)) - ((A - 9*B)*Sin[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rubi [A] time = 0.420275, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2977, 2968, 3019, 2751, 2649, 206}

$$-\frac{(A-9B)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(5A-13B)\sin}{16ad(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((19*A - 75*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(4*d
*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*C
os[c + d*x])^(3/2)) - ((A - 9*B)*Sin[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(A-B)-\frac{1}{2}a(A-9B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{2a(A-B)\cos(c+dx)-\frac{1}{2}a(A-9B)\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{3}{4}a^2(5A-13B)+}{\sqrt{a+a}}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(A-9B)\sin(}{4a^2d\sqrt{a+a\cos}}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(A-9B)\sin(}{4a^2d\sqrt{a+a\cos}}$$

$$= \frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(}{16}$$

Mathematica [A] time = 0.685333, size = 100, normalized size = 0.59

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((85B-13A)\cos(c+dx)-9A+16B\cos(2(c+dx))+65B)+2(19A-75B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\mathrm{si}\right.}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*(19*A - 75*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-9*A + 65*
B + (-13*A + 85*B)*Cos[c + d*x] + 16*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/
(16*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 2.579, size = 327, normalized size = 1.9

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(19\,A\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)\sqrt{2}\,(\cos(1/2\,dx+c/2))^4\,a-75\,B\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\mathrm{s}}{\mathrm{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)

[Out] 1/32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(19*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-75*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+64*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-13*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+21*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(7/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d

---

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 1.69405, size = 628, normalized size = 3.72

$$-\frac{\sqrt{2}\left((19\,A-75\,B)\cos(dx+c)^3+3\,(19\,A-75\,B)\cos(dx+c)^2+3\,(19\,A-75\,B)\cos(dx+c)+19\,A-75\,B\right)\sqrt{a}\log\Big(-}{64\,\Big(a^3d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -1/64*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*

```
x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*cos(d*x + c)
^2 - (13*A - 85*B)*cos(d*x + c) - 9*A + 49*B)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.18537, size = 244, normalized size = 1.44

$$\frac{\frac{\left(\left(\frac{2\left(\sqrt{2}Aa^6-\sqrt{2}Ba^6\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}-\frac{9\sqrt{2}Aa^6-17\sqrt{2}Ba^6}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{11\sqrt{2}Aa^6-83\sqrt{2}Ba^6}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{\left(19\sqrt{2}A-75\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/32*(((2*(sqrt(2)*A*a^6 - sqrt(2)*B*a^6)*tan(1/2*d*x + 1/2*c)^2/a^8 - (9*s
qrt(2)*A*a^6 - 17*sqrt(2)*B*a^6)/a^8)*tan(1/2*d*x + 1/2*c)^2 - (11*sqrt(2)*
A*a^6 - 83*sqrt(2)*B*a^6)/a^8)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a) - (19*sqrt(2)*A - 75*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x +
1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.118 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A-13B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((5*A + 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```

Rubi [A] time = 0.229703, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3019, 2750, 2649, 206}

$$\frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A-13B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
```

```
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{5}{2}a(A-B)-4aB\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(5A+19B)\int \frac{1}{\sqrt{a+a\cos(}}}{32a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(5A+19B)\,\mathrm{Subst}\left(\int\right.}{1}$$

$$= \frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)}{16ad(a+a\,\mathrm{co}}$$

Mathematica [A] time = 0.461535, size = 87, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((5A-13B)\cos(c+dx)+A-9B)+2(5A+19B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A - 9*B + (
5*A - 13*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^
(3/2))
```

Maple [B] time = 2.665, size = 292, normalized size = 2.3

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5\,A\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)\sqrt{2}\,(\cos(1/2\,dx+c/2))^4\,a+19\,B\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(}}{\mathrm{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*ln(2*(2*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*
d*x+1/2*c)^4*a+19*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos
(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+5*A*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-13*B*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7
/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.67616, size = 586, normalized size = 4.65

$$\frac{\sqrt{2}\Big((5\,A+19\,B)\cos(dx+c)^3+3\,(5\,A+19\,B)\cos(dx+c)^2+3\,(5\,A+19\,B)\cos(dx+c)+5\,A+19\,B\Big)\sqrt{a}\log\Big(-\frac{a\cos(dx+}{}}{64\,\big(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/64*(sqrt(2)*((5*A + 19*B)*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^2
+ 3*(5*A + 19*B)*cos(d*x + c) + 5*A + 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
- 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A - 13*B)*cos(d*x +
 c) + A - 9*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.12077, size = 181, normalized size = 1.44

$$-\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}-\frac{\sqrt{2}\left(3\,Aa^5-11\,Ba^5\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(5\,A+19\,B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/
2*d*x + 1/2*c)^2/a^8 - sqrt(2)*(3*A*a^5 - 11*B*a^5)/a^8)*tan(1/2*d*x + 1/2*
c) + sqrt(2)*(5*A + 19*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.119 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A+5B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((3*A + 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d
*x])^(5/2)) + ((3*A + 5*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)
)
```

Rubi [A] time = 0.10286, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2750, 2650, 2649, 206}

$$\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A+5B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d
*x])^(5/2)) + ((3*A + 5*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)
)
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a} \\
&= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2} \\
&= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(3A+5B)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \cdot\right.}{16a^2d} \\
&= \frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.454785, size = 80, normalized size = 0.63

$$\frac{\sin(c+dx)((3A+5B)\cos(c+dx)+7A+B)+4(3A+5B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*(3*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (7*A + B + (3
*A + 5*B)*Cos[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 2.23, size = 292, normalized size = 2.3

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3\,A\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin\,(1/2\,dx+c/2))^2}+2\,a}{\cos\,(1/2\,dx+c/2)}\right)\sqrt{2}\,(\cos\,(1/2\,dx+c/2))^4\,a+5\,B\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*ln(2*(2*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*
d*x+1/2*c)^4*a+5*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(
1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+3*A*a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+5*B*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2
)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.72635, size = 579, normalized size = 4.6

$$\frac{\sqrt{2}\left((3\,A+5\,B)\cos\,(dx+c)^3+3\,(3\,A+5\,B)\cos\,(dx+c)^2+3\,(3\,A+5\,B)\cos\,(dx+c)+3\,A+5\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2}{}\right.}{64\left(a^3d\cos\,(dx+c)^3+3\,a^3d\cos\,(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 +
3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) -
3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A + 5*B)*cos(d*x + c) +
 7*A + B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.87054, size = 181, normalized size = 1.44

$$\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}+\frac{\sqrt{2}\left(5\,Aa^5+3\,Ba^5\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}(3\,A+5\,B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\right.\right.}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2
*d*x + 1/2*c)^2/a^8 + sqrt(2)*(5*A*a^5 + 3*B*a^5)/a^8)*tan(1/2*d*x + 1/2*c)
 - sqrt(2)*(3*A + 5*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.120 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx}{4d(a\cos(c+dx)+a}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((11*A - 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2))
```

Rubi [A] time = 0.465548, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2985, 2649, 206, 2773}

$$-\frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx}{4d(a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((11*A - 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(4aA-\frac{3}{2}a(A-B)\cos(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(8a^2A-\frac{1}{4}a^2(11A-3B)\cos(\right.}{\sqrt{a+a\cos(c+}}}{8a^4}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a\cos(c+dx}}{a^3}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\,\mathrm{Subst}\left(\int\frac{1}{a-x^2}\,dx\right.}{a^2d}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)}{4d(a+a\,\mathrm{c}}$$

Mathematica [A] time = 1.37921, size = 126, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((3B-11A)\cos(c+dx)-15A+7B)-2(43A-3B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+64\sqrt{2}A}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(43*A - 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 64*Sqrt[2]*
A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-15*A + 7*B + (-1
1*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^(
3/2))
```

Maple [B] time = 4.747, size = 445, normalized size = 2.7

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-43\,A\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2+2\,a}}{\cos(1/2\,dx+c/2)}\right)\sqrt{2}\,(\cos(1/2\,dx+c/2))^4\,a+3\,B\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(}}{\mathrm{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-43*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+3*B*
ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(
1/2)*cos(1/2*d*x+1/2*c)^4*a+32*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)
))*cos(1/2*d*x+1/2*c)^4*a+32*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
cos(1/2*d*x+1/2*c)^4*a-11*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
cos(1/2*d*x+1/2*c)^2+3*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos
(1/2*d*x+1/2*c)^2-2*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/cos(1/2*d*x+1/2*c)^3/
sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.91114, size = 902, normalized size = 5.5

$$-\frac{\sqrt{2}\Big((43\,A-3\,B)\cos\left(dx+c\right)^3+3\,(43\,A-3\,B)\cos\left(dx+c\right)^2+3\,(43\,A-3\,B)\cos\left(dx+c\right)+43\,A-3\,B\Big)\sqrt{a}\log\Big(-\frac{a\cos(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/64*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2
 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
```

```
 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x +
c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 32*(A*cos(d*x + c)^3 + 3
*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((11*A - 3*B)*cos
(d*x + c) + 15*A - 7*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.28883, size = 338, normalized size = 2.06

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}+\frac{\sqrt{2}\left(13Aa^5-5Ba^5\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{\sqrt{2}\left(43A\sqrt{a}-3B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\right.\right.\right.}{a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(
1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(13*A*a^5 - 5*B*a^5)/a^8)*tan(1/2*d*x + 1/
2*c) - sqrt(2)*(43*A*sqrt(a) - 3*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 64*A*log(abs((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) +
3)))/a^(5/2) + 64*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2))/d
```

3.121 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{(35A-11B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(15A-7B)}{16ad(a\cos(c}$$

```
[Out] -(((5*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(5/2)*d)) + ((115*A - 43*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((15*A - 7*B)*Tan[c + d*x])/(16*a*d*(a + a*Co
s[c + d*x])^(3/2)) + ((35*A - 11*B)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rubi [A] time = 0.714781, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$\frac{(35A-11B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(15A-7B)}{16ad(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -(((5*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(5/2)*d)) + ((115*A - 43*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((15*A - 7*B)*Tan[c + d*x])/(16*a*d*(a + a*Co
s[c + d*x])^(3/2)) + ((35*A - 11*B)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(a(5A-B)-\frac{5}{2}a(A-B)\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}a^2(35A-11B)-\frac{3}{4}a^2(\ldots\right.}{\sqrt{a\ldots}}}{\ldots}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A-11B)\tan(c\ldots}{16a^2d\sqrt{a+a\cos(c\ldots}}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A-11B)\tan(c\ldots}{16a^2d\sqrt{a+a\cos(c\ldots}}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A-11B)\tan(c\ldots}{16a^2d\sqrt{a+a\cos(c\ldots}}$$

$$= -\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}\ldots$$

Mathematica [A] time = 3.0522, size = 142, normalized size = 0.69

$$\frac{\tan(c+dx)(10(11A-3B)\cos(c+dx)+(35A-11B)\cos(2(c+dx))+67A-11B)+8(115A-43B)\cos^5\left(\frac{1}{2}(c+dx)\right)\mathrm{ta}\ldots}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (8*(115*A - 43*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 128*Sqrt[2
]*(5*A - 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (67*A
- 11*B + 10*(11*A - 3*B)*Cos[c + d*x] + (35*A - 11*B)*Cos[2*(c + d*x)])*Tan
[c + d*x])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 5.265, size = 1122, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-86*B
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/
2)))*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*cos(1/2*d*x+1/2*c)^6*a+64*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))
*cos(1/2*d*x+1/2*c)^6*a+64*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*co
s(1/2*d*x+1/2*c)^6*a-115*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+43*B*ln(2*(2*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*
x+1/2*c)^4*a+70*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^4+80*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/
2*c)^4*a+80*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c
)^4*a-22*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c
)^4-32*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*
a-32*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-1
5*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+7*B
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2))/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)-2^
(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c
)^2*a)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
```

```
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.46193, size = 1067, normalized size = 5.15

$$-\frac{\sqrt{2}\Big((115\,A-43\,B)\cos(dx+c)^4+3\,(115\,A-43\,B)\cos(dx+c)^3+3\,(115\,A-43\,B)\cos(dx+c)^2+(115\,A-43\,B)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^4 + 3*(115*A - 43*B)*cos(d*x +
c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + (115*A - 43*B)*cos(d*x + c))*sqrt(
a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(
d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
 16*((5*A - 2*B)*cos(d*x + c)^4 + 3*(5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2
*B)*cos(d*x + c)^2 + (5*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((35*A - 11*
B)*cos(d*x + c)^2 + 5*(11*A - 3*B)*cos(d*x + c) + 16*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.76784, size = 552, normalized size = 2.67

$$2\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}+\frac{\sqrt{2}\left(21\,Aa^5-13\,Ba^5\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}\left(115\,A\sqrt{a}-43\,B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\right.\right.\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1
/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(21*A*a^5 - 13*B*a^5)/a^8)*tan(1/2*d*x + 1/
2*c) - sqrt(2)*(115*A*sqrt(a) - 43*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 32*(5*A*sqrt(a) - 2*B*s
qrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^3 + 32*(5*A*sqrt(a) - 2*B*sqrt(a))*log(a
bs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
a*(2*sqrt(2) - 3)))/a^3 + 128*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a^(3/2))/(((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a^2))/d
```

3.122 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=264

$$-\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A-}{1}$$

```
[Out] ((39*A - 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(5/2)*d) - ((219*A - 115*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (7*(9*A - 5*B)*Tan[c + d*x
])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x]
)/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A - 11*B)*Sec[c + d*x]*Tan[c + d*
x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((31*A - 15*B)*Sec[c + d*x]*Tan[c
 + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.922992, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A-}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] ((39*A - 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(5/2)*d) - ((219*A - 115*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (7*(9*A - 5*B)*Tan[c + d*x
])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x]
)/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A - 11*B)*Sec[c + d*x]*Tan[c + d*
x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((31*A - 15*B)*Sec[c + d*x]*Tan[c
 + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(2a(3A-B)-\frac{7}{2}a(A-B)\cos(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\sec(c+dx)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int}{}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\sec(c+dx)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(3}{}$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\,\mathrm{s}}{16ad(a}$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\,\mathrm{s}}{16ad(a}$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\,\mathrm{s}}{16ad(a}$$

$$= \frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx}}\right.}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 5.65572, size = 178, normalized size = 0.67

$$\frac{-8(219A-115B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 32\sqrt{2}(39A-20B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (-8*(219*A - 115*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 32*Sqrt[
2]*(39*A - 20*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-1
58*A + 110*B + (-269*A + 169*B)*Cos[c + d*x] + (-190*A + 110*B)*Cos[2*(c +
```

```
d*x)] - 63*A*Cos[3*(c + d*x)] + 35*B*Cos[3*(c + d*x)])*Sec[c + d*x]^2*Tan[(
c + d*x)/2])/(64*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 5.76, size = 1610, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/8*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-252*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d
*x+1/2*c)^6+140*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^6+624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/
2*c)^8*a+624*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2
*c)^8*a-320*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*
c)^8*a-320*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)
^8*a-876*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+
1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^8*a+460*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^8*a+2*
B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-624*A*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^6*a-624*A*ln(-4*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/
2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a+320*B*ln(4/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^6*a+320*B*ln(-4*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d
*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a+156*A*ln(-4*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*
x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+156*A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-80*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/
2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-80*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-219*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+115*B*
ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(
1/2)*cos(1/2*d*x+1/2*c)^4*a+876*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-460*B*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*co
s(1/2*d*x+1/2*c)^6*a+188*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*c
os(1/2*d*x+1/2*c)^4-100*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*co
s(1/2*d*x+1/2*c)^4-19*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)^2+11*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/
2*d*x+1/2*c)^2)/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^
2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.54037, size = 1131, normalized size = 4.28

$$-\frac{\sqrt{2}\left((219\,A-115\,B)\cos\left(dx+c\right)^{5}+3\,(219\,A-115\,B)\cos\left(dx+c\right)^{4}+3\,(219\,A-115\,B)\cos\left(dx+c\right)^{3}+(219\,A-115\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^5 + 3*(219*A - 115*B)*cos(d*x
+ c)^4 + 3*(219*A - 115*B)*cos(d*x + c)^3 + (219*A - 115*B)*cos(d*x + c)^2)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)) + 4*((39*A - 20*B)*cos(d*x + c)^5 + 3*(39*A - 20*B)*cos(d*x + c)^4 + 3
```

```
*(39*A - 20*B)*cos(d*x + c)^3 + (39*A - 20*B)*cos(d*x + c)^2)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) +
 4*(7*(9*A - 5*B)*cos(d*x + c)^3 + 5*(19*A - 11*B)*cos(d*x + c)^2 + 4*(5*A
- 4*B)*cos(d*x + c) - 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(
d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.76846, size = 837, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(
1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(29*A*a^5 - 21*B*a^5)/a^8)*tan(1/2*d*x + 1
/2*c) - sqrt(2)*(219*A*sqrt(a) - 115*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 8*(39*A*sqrt(a) - 20*
B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^3 + 8*(39*A*sqrt(a) - 20*B*sqrt(a))*l
og(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 + a*(2*sqrt(2) - 3)))/a^3 + 32*sqrt(2)*(41*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(a) - 12*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(a) - 209*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2) + 76
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^
```

```
(3/2) + 91*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*A*a^(5/2) - 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*B*a^(5/2) - 11*A*a^(7/2) + 4*B*a^(7/2))/(((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*a^2))/d
```

3.123 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=159

$$\frac{10a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a(9A+7B)\sin(c+...}{45...}$$

```
[Out] (2*a*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(A + B)*Elliptic
F[(c + d*x)/2, 2])/(21*d) + (10*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (2*a*(9*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(A
 + B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*B*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```

Rubi [A] time = 0.19904, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2639, 2641}

$$\frac{10a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a(9A+7B)\sin(c+...}{45...}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```



```
[Out] (2*a*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(A + B)*Elliptic
F[(c + d*x)/2, 2])/(21*d) + (10*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (2*a*(9*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(A
 + B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*B*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{5}{2}}(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\right.$$

$$= \frac{2aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(9A+\right.$$

$$= \frac{2aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + (a(A+B))\int \cos^{\frac{7}{2}}(c+dx)\,d$$

$$= \frac{2a(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2a(A+B)\cos^{\frac{5}{2}}(c+}{7d}$$

$$= \frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(A+B)\sqrt{\cos(c+dx)}\sin}{21d}$$

$$= \frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Mathematica [C] time = 6.31392, size = 914, normalized size = 5.75

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(9A+7B)\cot(c)}{15d} + \frac{23(A+B)\cos(dx)\sin(c)}{84d} + \frac{(18A+19B)\cos(2dx)\sin(2c)}{180d} + \frac{(A\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((9*A + 7*B
)*Cot[c])/(15*d) + (23*(A + B)*Cos[d*x]*Sin[c])/(84*d) + ((18*A + 19*B)*Cos
[2*d*x]*Sin[2*c])/(180*d) + ((A + B)*Cos[3*d*x]*Sin[3*c])/(28*d) + (B*Cos[4
*d*x]*Sin[4*c])/(72*d) + (23*(A + B)*Cos[c]*Sin[d*x])/(84*d) + ((18*A + 19*
B)*Cos[2*c]*Sin[2*d*x])/(180*d) + ((A + B)*Cos[3*c]*Sin[3*d*x])/(28*d) + (B
*Cos[4*c]*Sin[4*d*x])/(72*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
```

```
ot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*
Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Arc
Tan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (7*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 +
 (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(30*d))
```

Maple [B] time = 3.214, size = 411, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A+2960*B)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-1584*A-3152*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)+(1344*A+1792*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-366*A-408*B)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos(dx+c)^4+(A+B)a\cos(dx+c)^3+Aa\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^4 + (A + B)*a*cos(d*x + c)^3 + A*a*cos(d*x + c)^
2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.124 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{2a(7A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin(c+}{21d}$$

```
[Out] (6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*B)*EllipticF[
(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*B*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.175311, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(7A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin(c+}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*B)*EllipticF[
(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*B*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\right.$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(7A\right.$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (a(A+B))\int \cos^{\frac{5}{2}}(c+dx)$$

$$= \frac{2a(7A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a(A+B)\cos^{\frac{3}{2}}(c}{5a}$$

$$= \frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(7A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

<u>Mathematica</u> [C] time = 6.24721, size = 872, normalized size = 6.61

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{3(A+B)\cot(c)}{5d}+\frac{(28A+23B)\cos(dx)\sin(c)}{84d}+\frac{(A+B)\cos(2dx)\sin(2c)}{10d}+\frac{B\cos(3\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*(A + B)*
Cot[c])/(5*d) + ((28*A + 23*B)*Cos[d*x]*Sin[c])/(84*d) + ((A + B)*Cos[2*d*x
]*Sin[2*c])/(10*d) + (B*Cos[3*d*x]*Sin[3*c])/(28*d) + ((28*A + 23*B)*Cos[c]
*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (B*Cos[3*c]*Sin[
3*d*x])/(28*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*S
qrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2
]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(10*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(10*d))
```

Maple [B] time = 3.418, size = 383, normalized size = 2.9

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-168\,A-528\,B\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-11
2*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ba\cos(dx+c)^{3}+(A+B)a\cos(dx+c)^{2}+Aa\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^3 + (A + B)*a*cos(d*x + c)^2 + A*a*cos(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.125 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (2*a*(5*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF[
(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.158298, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(5*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF[
(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\right.$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}a(5A\right.$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \cos^{\frac{3}{2}}(c+dx)\,d$$

$$= \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\sin(}{3d}$$

$$= \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} +$$

Mathematica [C] time = 6.19786, size = 830, normalized size = 8.22

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(5A+3B)\cot(c)}{5d}+\frac{(A+B)\cos(dx)\sin(c)}{3d}+\frac{B\cos(2dx)\sin(2c)}{10d}+\frac{(A+B)\cos(c)\sin(dx)}{3d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((5*A + 3*B
)*Cot[c])/(5*d) + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (B*Cos[2*d*x]*Sin[2*c])
/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (B*Cos[2*c]*Sin[2*d*x])/(10*d))
 - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^
2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[
c]^2]) - (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (3*
B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 3.14, size = 355, normalized size = 3.5

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(20\,A+44\,B\right)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos(dx+c)^{2}+(A+B)a\cos(dx+c)+Aa\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sqrt(cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.126 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.144455, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a(3A+B)+\frac{3}{2}a(A+B)\cos(c+a}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + (a(A+B))\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}(a(3$$

$$= \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aB\sqrt{\cos(c}}{}$$

Mathematica [C] time = 5.73377, size = 309, normalized size = 4.41

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(-4(3A+B)\sin(c)\sqrt{\csc^2(c)}\sqrt{\sec^2(c)}\cos(c+dx)\sqrt{\cos}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-6*(A + B)*HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c
```

```
]]] + (9*(A + B)*Cos[c - d*x - ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*A*Cos[c +
d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*B*Cos[c + d*x + ArcTan[Tan[c]]]*Csc
[c]*Sec[c] - 12*A*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 12*B*Cos[c + d*x]*Co
t[c]*Sqrt[Sec[c]^2] - 4*(3*A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]
]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sqrt[Sec[c]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 4*B*Cos[c + d
*x]*Sqrt[Sec[c]^2]*Sin[c + d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Maple [B] time = 3.188, size = 321, normalized size = 4.6

$$-\frac{2a}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4+3A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/sqrt(cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.127 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*
x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.147927, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3021, 2748, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```



```
[Out] (-2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*
x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a(A+B)-\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (a(A-B))\int \sqrt{\cos(c+dx)}\,dx + (a(A+B))\int \frac{}{\sqrt{\cos}}$$

$$= -\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aA\sin(c}{d\sqrt{\cos(c+}}$$

Mathematica [C] time = 5.82003, size = 256, normalized size = 3.88

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2(A-B)\sec(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}} - 4(A+B)\sin(c)\sqrt{\csc}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((Csc[c]*(-3*(A - B)*Cos[c - d*x -
 ArcTan[Tan[c]]]*Sec[c] - (A - B)*Cos[c + d*x + ArcTan[Tan[c]]]*Sec[c] + 2*
((2*A - B)*Cos[d*x] - B*Cos[2*c + d*x])*Sqrt[Sec[c]^2]))/Sqrt[Sec[c]^2] - 4
*(A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sin[c] + (2*(A - B)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/(Sqrt[Sec[c]^2]
*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 3.251, size = 240, normalized size = 3.6

$$-2\,\frac{a\left(A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)+A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*a*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/cos(d*x + c)^(
3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.128 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(A
 + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.169493, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(A
 + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}a(A+B)+\frac{1}{2}a(A+3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(a(A+3B))\int \frac{}{\sqrt{\mathrm{co}}}$$

$$= \frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c}{3d\cos^{\frac{3}{2}}(c}$$

Mathematica [C] time = 6.32984, size = 813, normalized size = 8.56

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{A\sec(c)\sin(dx)\sec^2(c+dx)}{3d} + \frac{\sec(c)(A\sin(c)+3A\sin(dx)+3B\sin(dx))\sec(c+dx)}{3d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((A + B)*Csc
[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c +
d*x]*(A*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c + d*x
])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c +
d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTa
n[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sq
```

```
rt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (B*(1 + Cos[c + d*x])*Csc[c
]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 7.918, size = 426, normalized size = 4.5

$$-4\,{\frac{\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}a}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}}\left(1/2\,{\frac{B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{El}}{\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\right.\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(
1/2*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/cos(d*x + c)^(
5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.129 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\,\mathrm{si}}{5d\cos}$$

```
[Out] (-2*a*(3*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*B)*Sin[c
 + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.177031, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\,\mathrm{si}}{5d\cos}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*a*(3*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*B)*Sin[c
 + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}a(A+B)+\frac{1}{2}a(3A+5B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{5}(a(3A+5B))\int \frac{}{\text{co}}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(3A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{3}$$

$$= -\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c}{5d\cos^{\frac{5}{2}}(c}$$

<u>Mathematica</u> [C] time = 6.37362, size = 865, normalized size = 6.55

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{5d} + \frac{\sec(c)(3A\sin(c)+5A\sin(dx)+5B\sin(dx))\sec^2(c+}{15d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 5*B)
*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*A*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (Sec[c]
*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 9*A*Sin[d*x] + 15*B*Sin[d*x]))/(15
*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot
[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
```

```
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
Cot[c]^2]) + (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d
) + (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{
-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Ta
n[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[
c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 10.085, size = 661, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+
2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/2*A
+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*A/(8*sin(1/2*d*x+1/2*
c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2
*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/cos(d*x + c)^(
7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.130 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(6A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{4a^2(9A+8\ldots}{}$$

```
[Out] (4*a^2*(9*A + 8*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(6*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(21*d) + (4*a^2*(9*A + 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
5*d) + (2*a^2*(9*A + 11*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*B*C
os[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.318823, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(6A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{4a^2(9A+8\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```



```
[Out] (4*a^2*(9*A + 8*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(6*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(21*d) + (4*a^2*(9*A + 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
5*d) + (2*a^2*(9*A + 11*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*B*C
os[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2B\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{9d} + \frac{2}{9}\int \text{co}$$

$$= \frac{2B\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{9d} + \frac{2}{9}\int \text{co}$$

$$= \frac{2a^2(9A+11B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2B\cos^{\frac{5}{2}}(c+dx)}{}$$

$$= \frac{2a^2(9A+11B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2B\cos^{\frac{5}{2}}(c+dx)}{}$$

$$= \frac{4a^2(6A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{4a^2(9A+8B)\cos}{}$$

$$= \frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(6A+5B)F\left(\frac{1}{2}(c+dx)\right.}{21d}$$

Mathematica [C] time = 6.26267, size = 944, normalized size = 4.87

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(-\frac{(9A+8B)\cot(c)}{15d} + \frac{(51A+46B)\cos(dx)\sin(c)}{168d} + \frac{(36A+37B)\cos(2dx)\sin(2c)}{360d} + \ldots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((9*A + 8*
B)*Cot[c])/(15*d) + ((51*A + 46*B)*Cos[d*x]*Sin[c])/(168*d) + ((36*A + 37*B
)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((A + 2*B)*Cos[3*d*x]*Sin[3*c])/(56*d) + (
B*Cos[4*d*x]*Sin[4*c])/(144*d) + ((51*A + 46*B)*Cos[c]*Sin[d*x])/(168*d) +
((36*A + 37*B)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((A + 2*B)*Cos[3*c]*Sin[3*d*x
])/(56*d) + (B*Cos[4*c]*Sin[4*d*x])/(144*d)) - (2*A*(a + a*Cos[c + d*x])^2*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c +
 d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
```

```
]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(a +
 a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (4*B*(a + a*Co
s[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

Maple [A] time = 3.137, size = 413, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+1840*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1044*A-2368*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1134*A+1568*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-351*A-387*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+90*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ba^2\cos(dx+c)^4+(A+2B)a^2\cos(dx+c)^3+(2A+B)a^2\cos(dx+c)^2+Aa^2\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^4 + (A + 2*B)*a^2*cos(d*x + c)^3 + (2*A + B)*a
^2*cos(d*x + c)^2 + A*a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

3.131 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=161

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{4a^2(7A+6B}{}$$

```
[Out] (4*a^2*(4*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(7*A + 6*B)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(21*d) + (2*a^2*(7*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35
*d) + (2*B*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.288749, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{4a^2(7A+6B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```


```
[Out] (4*a^2*(4*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(7*A + 6*B)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(21*d) + (2*a^2*(7*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35
*d) + (2*B*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2B\cos^{\frac{3}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d} + \frac{2}{7}\int \surd$$

$$= \frac{2B\cos^{\frac{3}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d} + \frac{2}{7}\int \surd$$

$$= \frac{2a^2(7A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\,(}{}$$

$$= \frac{2a^2(7A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\,(}{}$$

$$= \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)\sqrt{\cos(c+dx}}{21d}$$

$$= \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx\right.}{21d}$$

Mathematica [C] time = 6.22658, size = 898, normalized size = 5.58

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(-\frac{(4A+3B)\cot(c)}{5d} + \frac{(56A+51B)\cos(dx)\sin(c)}{168d} + \frac{(A+2B)\cos(2dx)\sin(2c)}{20d} + \frac{B\cos}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((4*A + 3*
B)*Cot[c])/(5*d) + ((56*A + 51*B)*Cos[d*x]*Sin[c])/(168*d) + ((A + 2*B)*Cos
[2*d*x]*Sin[2*c])/(20*d) + (B*Cos[3*d*x]*Sin[3*c])/(56*d) + ((56*A + 51*B)*
Cos[c]*Sin[d*x])/(168*d) + ((A + 2*B)*Cos[2*c]*Sin[2*d*x])/(20*d) + (B*Cos[
3*c]*Sin[3*d*x])/(56*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
```

```
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (2*A*(a + a*Cos[c + d*x])^2*Csc[
c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) - (3*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec
[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan
[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [A] time = 3.511, size = 385, normalized size = 2.4

$$-\frac{4\,a^2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-84\,A-348\,B\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-9
1*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.132 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2B\sin(c+dx)}{}$$

```
[Out] (4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(15*d) + (2*B*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(5*d)
```

Rubi [A] time = 0.274285, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2B\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(15*d) + (2*B*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(5*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2B\sqrt{\cos(c+dx)}\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(a+a\cos(c+\cdots}{\cdots}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{1}{2}a^2(5A+B)+\cdots}{\cdots}$$

$$= \frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2B\sqrt{\cos(c+dx)}\left(a^2+a^2\cos(\cdots\right.}{5d}$$

$$= \frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2B\sqrt{\cos(c+dx)}\left(a^2+a^2\cos(\cdots\right.}{5d}$$

$$= \frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(5A\cdots}{\cdots}$$

Mathematica [C] time = 6.27118, size = 852, normalized size = 6.76

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(-\frac{(5A+4B)\cot(c)}{5d} + \frac{(A+2B)\cos(dx)\sin(c)}{6d} + \frac{B\cos(2dx)\sin(2c)}{20d} + \frac{(A+2B)\cos(c)\cdots}{6d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((5*A + 4*
B)*Cot[c])/(5*d) + ((A + 2*B)*Cos[d*x]*Sin[c])/(6*d) + (B*Cos[2*d*x]*Sin[2*
c])/(20*d) + ((A + 2*B)*Cos[c]*Sin[d*x])/(6*d) + (B*Cos[2*c]*Sin[2*d*x])/(2
0*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[
1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]
```

```
^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(2*d) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((H
ypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + T
an[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c
]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
)/(5*d)
```

Maple [B] time = 3.53, size = 357, normalized size = 2.8

$$-\frac{4a^2}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(10\,A+32\,B)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2\cos(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

3.133 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}+\frac{4a^2BE\left(\right.}{}$$

```
[Out] (4*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) - (2*a^2*(3*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.26874, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}+\frac{4a^2BE\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```



```
[Out] (4*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) - (2*a^2*(3*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+dx))\left(\frac{1}{2}a(3A\right.}{\sqrt{\cos}}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a^2(3A+B)+\left(-\frac{1}{2}a^2(3A-\right.}{}$$

$$= -\frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a^2(3A-B)\sqrt{c}}{}$$

<u>**Mathematica**</u> **[C]** time = 6.33418, size = 623, normalized size = 5.28

$$-\frac{A\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot\right.}}{d\sqrt{\cot^2(c)+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-A + 2*B
 + A*Cos[2*c] + 2*B*Cos[2*c])*Csc[c]*Sec[c])/(4*d) + (B*Cos[d*x]*Sin[c])/(6
*d) + (B*Cos[c]*Sin[d*x])/(6*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d)) -
 (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^
2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1
+ Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hype
```

```
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
2*d)
```

Maple [B] time = 3.598, size = 388, normalized size = 3.3

$$-\frac{4\,a^2}{3\,d}\left(2\,B\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4-\sqrt{-2\,\left(\sin\left(1/2\,dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^2*(2*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(3*A+B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2\cos(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c) + A)(a\cos(dx+c) + a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

3.134 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-4*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.280193, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-4*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+dx))\left(\frac{1}{2}a(5A\right.}{\cos}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a^2(5A+3B)+\left(-\frac{1}{2}a^2(A-\right.}{}$$

$$= \frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4}{3}\int$$

$$= \frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \left(2a^2\right.$$

$$= -\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(5A+3B}{3d\sqrt{\cos}}$$

Mathematica [C] time = 6.39363, size = 624, normalized size = 5.2

$$\frac{A\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}\right.}}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-4*A - B
 + B*Cos[2*c])*Csc[c]*Sec[c])/(4*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6
*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(6*d))
 - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + C
ot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[
```

```
1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(2*d)
```

Maple [B] time = 3.663, size = 513, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Ba^2\cos\left(dx+c\right)^3+(A+2B)a^2\cos\left(dx+c\right)^2+(2A+B)a^2\cos\left(dx+c\right)+Aa^2}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

3.135 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(4A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+$$

```
[Out] (-4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*Sin[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)) + (4*a^2*(4*A + 5*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.305887, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(4A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (-4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*Sin[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)) + (4*a^2*(4*A + 5*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+dx))\left(\frac{1}{2}a(7A\right.}{\cos}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a^2(7A+5B)+\left(\frac{1}{2}a^2(A+\right.}{}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4}{15}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(2\right.$$

$$= \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(4A+5}{5d\sqrt{\cos}}$$

$$= -\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(7}{1}$$

Mathematica [C] time = 6.46279, size = 883, normalized size = 5.55

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{10d} + \frac{\sec(c)(3A\sin(c)+10A\sin(dx)+5B\sin(dx))\sec^2(c+}{30d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((4*A + 5*B
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c
]*Sec[c + d*x]^2*(3*A*Sin[c] + 10*A*Sin[d*x] + 5*B*Sin[d*x]))/(30*d) + (Sec
[c]*Sec[c + d*x]*(10*A*Sin[c] + 5*B*Sin[c] + 24*A*Sin[d*x] + 30*B*Sin[d*x])
)/(30*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
```

```
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d
*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*
Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]))/(5*d) + (B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d)
```

Maple [B] time = 10.34, size = 741, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+(1/4*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*
x+1/2*c)^2-1)+(1/2*A+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/20*A
/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
```

```
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

3.136 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-4*a^2*(3*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*C
os[c + d*x]^(5/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3
/2)) + (4*a^2*(3*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a^
2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.337491, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^2*(3*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*C
os[c + d*x]^(5/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3
/2)) + (4*a^2*(3*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a^
2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+dx))\left(\frac{1}{2}a(9A\right.}{\cos}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}a^2(9A+7B)+\left(\frac{1}{2}a^2(3A+\right.}{}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4}{35}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{5}\left(2\right.$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(3A+4B)\,\mathrm{s}}{5d\sqrt{\cos(c}}$$

$$= -\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^2(}{}$$

Mathematica [C] time = 6.53333, size = 925, normalized size = 4.77

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(\frac{A\sec(c)\sin(dx)\sec^4(c+dx)}{14d} + \frac{\sec(c)(5A\sin(c)+14A\sin(dx)+7B\sin(dx))\sec^3(c+}{70d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((3*A + 4*B
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c
]*Sec[c + d*x]^3*(5*A*Sin[c] + 14*A*Sin[d*x] + 7*B*Sin[d*x]))/(70*d) + (Sec
[c]*Sec[c + d*x]^2*(42*A*Sin[c] + 21*B*Sin[c] + 60*A*Sin[d*x] + 70*B*Sin[d*
x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(30*A*Sin[c] + 35*B*Sin[c] + 63*A*Sin[d
*x] + 84*B*Sin[d*x]))/(105*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Hyperge
```

```
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/
2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (3*A*(a + a*Cos[c + d*x])^
2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) + (2*B*(a + a*Cos[c + d*x])^2*Csc
[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
 ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [B] time = 12.111, size = 851, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/4
*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*A*(-1/56*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/2*A+1/4*B)/(
8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/si
n(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2), x
)
```

3.137 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{4a^3(121A+105B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{20a^3(22A+21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d}+\frac{4a}{}$$

```
[Out] (4*a^3*(17*A + 15*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(121*A + 10
5*B)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A + 105*B)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(17*A + 15*B)*Cos[c + d*x]^(3/2)*Si
n[c + d*x])/(45*d) + (20*a^3*(22*A + 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(693*d) + (2*a*B*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(
11*d) + (2*(11*A + 15*B)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c
+ d*x])/(99*d)
```

Rubi [A] time = 0.479203, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^3(121A+105B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{20a^3(22A+21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d}+\frac{4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```



```
[Out] (4*a^3*(17*A + 15*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(121*A + 10
5*B)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A + 105*B)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(17*A + 15*B)*Cos[c + d*x]^(3/2)*Si
n[c + d*x])/(45*d) + (20*a^3*(22*A + 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(693*d) + (2*a*B*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(
11*d) + (2*(11*A + 15*B)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c
+ d*x])/(99*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
```

```
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{2}{11}\int \mathrm{co}$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{2(11A+}{}$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{2(11A+}{}$$

$$= \frac{20a^3(22A+21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2aB\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{20a^3(22A+21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2aB\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{4a^3(121A+105B)\sqrt{\cos(c+dx)}\sin(c+dx)}{231d} + \frac{4a^3(17A+15}{}$$

$$= \frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(121A+105B)F\left(\frac{1}{2}(c\right.}{231d}$$

Mathematica [C] time = 6.2976, size = 990, normalized size = 4.18

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(-\frac{(17A+15B)\cot(c)}{30d} + \frac{(2134A+1953B)\cos(dx)\sin(c)}{7392d} + \frac{(73A+75B)\cos(2dx)\sin(2}{720d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((17*A + 1
5*B)*Cot[c])/(30*d) + ((2134*A + 1953*B)*Cos[d*x]*Sin[c])/(7392*d) + ((73*A
 + 75*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + (3*(44*A + 63*B)*Cos[3*d*x]*Sin[3*c
])/(4928*d) + ((A + 3*B)*Cos[4*d*x]*Sin[4*c])/(288*d) + (B*Cos[5*d*x]*Sin[5
*c])/(704*d) + ((2134*A + 1953*B)*Cos[c]*Sin[d*x])/(7392*d) + ((73*A + 75*B
```

```
)*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*(44*A + 63*B)*Cos[3*c]*Sin[3*d*x])/(492
8*d) + ((A + 3*B)*Cos[4*c]*Sin[4*d*x])/(288*d) + (B*Cos[5*c]*Sin[5*d*x])/(7
04*d)) - (11*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sq
rt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(22*d*Sqrt[1 + Cot[c]^2]) - (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(60*d) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/
2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sq
rt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
)/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]))/(4*d)
```

Maple [A] time = 4.067, size = 441, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c)),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*A-43680*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(24200*A+77280*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d
*x+1/2*c)+(-37532*A-72240*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(29722
*A+39270*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8118*A-8820*B)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1815*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3927*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
```

```
/2*d*x+1/2*c),2^(1/2))+1575*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3465*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^5+(A+3B)a^3\cos(dx+c)^4+3(A+B)a^3\cos(dx+c)^3+(3A+B)a^3\cos(dx+c)^2+Aa^3\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^5 + (A + 3*B)*a^3*cos(d*x + c)^4 + 3*(A + B)*a
^3*cos(d*x + c)^3 + (3*A + B)*a^3*cos(d*x + c)^2 + A*a^3*cos(d*x + c))*sqrt
(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

3.138 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=204

$$\frac{4a^3(13A+11B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(24A+23B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(9A}{}$$

```
[Out] (4*a^3*(21*A + 17*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(13*A + 11*
B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(105*d) + (2*a*B*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*
x])/(9*d) + (2*(9*A + 13*B)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin
[c + d*x])/(63*d)
```

Rubi [A] time = 0.447287, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^3(13A+11B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(24A+23B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(9A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```


```
[Out] (4*a^3*(21*A + 17*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(13*A + 11*
B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(105*d) + (2*a*B*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*
x])/(9*d) + (2*(9*A + 13*B)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin
[c + d*x])/(63*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
```

```
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d} + \frac{2}{9}\int \sqrt{c}$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d} + \frac{2(9A+}{}$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d} + \frac{2(9A+}{}$$

$$= \frac{4a^3(24A+23B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB\cos^{\frac{3}{2}}(c+a}{}$$

$$= \frac{4a^3(24A+23B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB\cos^{\frac{3}{2}}(c+a}{}$$

$$= \frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(13A+11B)\sqrt{\cos(c}}{21d}$$

$$= \frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(13A+11B)F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [C] time = 6.25951, size = 944, normalized size = 4.63

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(-\frac{(21A+17B)\cot(c)}{30d} + \frac{(107A+97B)\cos(dx)\sin(c)}{336d} + \frac{(54A+73B)\cos(2dx)\sin(2c)}{720d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((21*A + 1
7*B)*Cot[c])/(30*d) + ((107*A + 97*B)*Cos[d*x]*Sin[c])/(336*d) + ((54*A + 7
3*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + ((A + 3*B)*Cos[3*d*x]*Sin[3*c])/(112*d)
 + (B*Cos[4*d*x]*Sin[4*c])/(288*d) + ((107*A + 97*B)*Cos[c]*Sin[d*x])/(336*
d) + ((54*A + 73*B)*Cos[2*c]*Sin[2*d*x])/(720*d) + ((A + 3*B)*Cos[3*c]*Sin[
3*d*x])/(112*d) + (B*Cos[4*c]*Sin[4*d*x])/(288*d)) - (13*A*(a + a*Cos[c + d
```

```
*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (11*B*(a +
a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Arc
Tan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) -
(7*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ
[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Co
s[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))
/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (17*B
*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

Maple [A] time = 3.167, size = 413, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
```

```
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3\right)\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sqrt(cos(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

3.139 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(7A+$$

```
[Out] (4*a^3*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(21*A + 13*B)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*S
in[c + d*x])/(7*d) + (2*(7*A + 11*B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c +
d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.427102, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(7A+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (4*a^3*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(21*A + 13*B)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*S
in[c + d*x])/(7*d) + (2*(7*A + 11*B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c +
d*x])*Sin[c + d*x])/(35*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
```

```
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2aB\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \frac{(a+a\cos(c+$$

$$= \frac{2aB\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7A+11B)\sqrt{\cos(c}{}$$

$$= \frac{2aB\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7A+11B)\sqrt{\cos(c}{}$$

$$= \frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2aB\sqrt{\cos(c+dx)}(a+a\cos}{7d}$$

$$= \frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2aB\sqrt{\cos(c+dx)}(a+a\cos}{7d}$$

$$= \frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^3}{}$$

Mathematica [C] time = 6.32061, size = 898, normalized size = 5.25

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(-\frac{(9A+7B)\cot(c)}{10d} + \frac{(84A+107B)\cos(dx)\sin(c)}{336d} + \frac{(A+3B)\cos(2dx)\sin(2c)}{40d} + \frac{B\cos}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((9*A + 7*
B)*Cot[c])/(10*d) + ((84*A + 107*B)*Cos[d*x]*Sin[c])/(336*d) + ((A + 3*B)*C
os[2*d*x]*Sin[2*c])/(40*d) + (B*Cos[3*d*x]*Sin[3*c])/(112*d) + ((84*A + 107
*B)*Cos[c]*Sin[d*x])/(336*d) + ((A + 3*B)*Cos[2*c]*Sin[2*d*x])/(40*d) + (B*
Cos[3*c]*Sin[3*d*x])/(112*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*
```

```
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (9*A*(a + a*Cos[c + d*x])
^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (7*B*(a + a*Cos[c + d*x])^3*Cs
c[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
+ ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [A] time = 3.609, size = 385, normalized size = 2.3

$$-\frac{4a^3}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Bigg(120\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-84\,A-432\,B\right)\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/sqrt(cos(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

3.140 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=169

$$\frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-B)\sin}{}$$

```
[Out] (4*a^3*(5*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*B)*El
lipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(15*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]) - (2*(5*A - B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c
 + d*x])/(5*d)
```

Rubi [A] time = 0.434165, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-B)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*(5*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*B)*El
lipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(15*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]) - (2*(5*A - B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c
 + d*x])/(5*d)
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+dx))^2\left(\frac{1}{2}a(5A\right.}{\sqrt{\cos}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-B)\sqrt{\cos(c+dx)}\left(a^3+a^3\right.}{5d}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-B)\sqrt{\cos(c+dx)}\left(a^3+a^3\right.}{5d}$$

$$= -\frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2aA(a+a\cos(c+dx))^2\sin(}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2aA(a+a\cos(c+dx))^2\sin(}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{4a^3(5}{}$$

Mathematica [C] time = 6.41195, size = 888, normalized size = 5.25

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(-\frac{(15\cos(2c)A+5A+18B+18B\cos(2c))\csc(c)\sec(c)}{40d} + \frac{A\sec(c+dx)\sin(dx)\sec(c)}{4d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((5*A + 18
*B + 15*A*Cos[2*c] + 18*B*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + ((A + 3*B)*Cos[
d*x]*Sin[c])/(12*d) + (B*Cos[2*d*x]*Sin[2*c])/(40*d) + ((A + 3*B)*Cos[c]*Si
n[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(4*d) + (B*Cos[2*c]*Sin[2
```

```
*d*x])/(40*d)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
6*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(4*d) - (9*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)
/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(20*d)
```

Maple [B] time = 4.093, size = 519, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^3*(-12*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(5*A+21*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A+9*B)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-27*B*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(3/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

3.141 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(7A+3B)\sin(c\cdots}{3d\sqrt{\cdots}}$$

```
[Out] (-4*a^3*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*(A + B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) - (4*a^3*(4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
+ (2*(7*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d
*x]])
```

Rubi [A] time = 0.430076, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(7A+3B)\sin(c\cdots}{3d\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```



```
[Out] (-4*a^3*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*(A + B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) - (4*a^3*(4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
+ (2*(7*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d
*x]])
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+dx))^2\left(\frac{1}{2}a(7A\right.}{\cos^{\frac{3}{2}}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(7A+3B)\left(a^3+a^3\cos(c+dx)\right)}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(7A+3B)\left(a^3+a^3\cos(c+dx)\right)}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(a+a\cos(c+dx))^2\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(a+a\cos(c+dx))^2\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - 4a^3(4A$$

Mathematica [C] time = 6.48191, size = 879, normalized size = 5.46

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^2(c+dx)}{12d} + \frac{\sec(c)(A\sin(c)+9A\sin(dx)+3B\sin(dx))\sec(c+dx)}{12d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-5*A + B
 + A*Cos[2*c] + 3*B*Cos[2*c])*Csc[c]*Sec[c])/(8*d) + (B*Cos[d*x]*Sin[c])/(1
2*d) + (B*Cos[c]*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*
d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(12*d))
 - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
```

```
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + C
ot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*S
qrt[1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(4*d) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyperg
eometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*
d)
```

Maple [B] time = 4.144, size = 654, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(-4*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
```

```
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2))*a^3/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(5/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

3.142 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(9A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(21\ldots}{15\ldots}$$

```
[Out] (-4*a^3*(9*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(3*d) + (4*a^3*(21*A + 20*B)*Sin[c + d*x])/(15*d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[
c + d*x]^(5/2)) + (2*(9*A + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.461566, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(9A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(21\ldots}{15\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```



```
[Out] (-4*a^3*(9*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(3*d) + (4*a^3*(21*A + 20*B)*Sin[c + d*x])/(15*d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[
c + d*x]^(5/2)) + (2*(9*A + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{(a+a\cos(c+dx))^2\left(\frac{1}{2}a(9A\right.}{\cos}$$

$$=\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(9A+5B)\left(a^3+a^3\cos(c+dx)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(9A+5B)\left(a^3+a^3\cos(c+dx)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}+\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(}{}$$

$$=\frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}+\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(}{}$$

$$=-\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(}{}$$

Mathematica [C] time = 6.52813, size = 890, normalized size = 5.2

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{20d}+\frac{\sec(c)(3A\sin(c)+15A\sin(dx)+5B\sin(dx))\sec^2(c+}{60d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-36*A -
25*B + 5*B*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d
*x])/(20*d) + (Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 15*A*Sin[d*x] + 5*B*Sin[
d*x]))/(60*d) + (Sec[c]*Sec[c + d*x]*(15*A*Sin[c] + 5*B*Sin[c] + 54*A*Sin[d
*x] + 45*B*Sin[d*x]))/(60*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
```

```
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
(d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (9*A*(a + a*Cos[c + d*x])^3
*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (B*(a + a*Cos[c + d*x])^3*Csc[c]
*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [B] time = 10.02, size = 916, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+108*A*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6+100*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+60*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-108*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-100*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-72*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-50*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(7/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

3.143 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(11A+7B)\sin(c+a}{35d\cos}$$

```
[Out] (-4*a^3*(7*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 21*B)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A + 42*B)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(3/2)) + (4*a^3*(7*A + 9*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c
+ d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7
/2)) + (2*(11*A + 7*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c +
 d*x]^(5/2))
```

Rubi [A] time = 0.490751, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(11A+7B)\sin(c+a}{35d\cos}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-4*a^3*(7*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 21*B)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A + 42*B)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(3/2)) + (4*a^3*(7*A + 9*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c
+ d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7
/2)) + (2*(11*A + 7*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c +
 d*x]^(5/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int\frac{(a+a\cos(c+dx))^2\left(\frac{1}{2}a(11A\right.}{\cos}$$

$$=\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(11A+7B)\left(a^3+a^3\cos(c+dx)\right)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(11A+7B)\left(a^3+a^3\cos(c+dx)\right)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(}{}$$

$$=\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(}{}$$

$$=\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(7}{5}$$

$$=-\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a}{}$$

Mathematica [C] time = 6.56303, size = 925, normalized size = 4.53

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^4(c+dx)}{28d}+\frac{\sec(c)(5A\sin(c)+21A\sin(dx)+7B\sin(dx))\sec^3(c+}{140d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((7*A + 9*B
)*Csc[c]*Sec[c])/(10*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[
```

```
c]*Sec[c + d*x]^3*(5*A*Sin[c] + 21*A*Sin[d*x] + 7*B*Sin[d*x]))/(140*d) + (S
ec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 21*B*Sin[c] + 130*A*Sin[d*x] + 105*B*Si
n[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(130*A*Sin[c] + 105*B*Sin[c] + 294*
A*Sin[d*x] + 378*B*Sin[d*x]))/(420*d)) - (13*A*(a + a*Cos[c + d*x])^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])
^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (7*A*(a + a*Cos[
c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (9*B*(a + a*Cos[c + d
*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 13.096, size = 929, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(
```

```
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/8*A+3/8*B)*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(3/8*A+3/8*B)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/
2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(9/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x
)
```

3.144 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(23A+24B)\sin}{105d\cos^{\frac{5}{2}}(c+}$$

```
[Out] (-4*a^3*(17*A + 21*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 13
*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(23*A + 24*B)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(5/2)) + (4*a^3*(11*A + 13*B)*Sin[c + d*x])/(21*d*Cos[c
 + d*x]^(3/2)) + (4*a^3*(17*A + 21*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]
]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) +
 (2*(13*A + 9*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^
(7/2))
```

Rubi [A] time = 0.521141, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(23A+24B)\sin}{105d\cos^{\frac{5}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```


```
[Out] (-4*a^3*(17*A + 21*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 13
*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(23*A + 24*B)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(5/2)) + (4*a^3*(11*A + 13*B)*Sin[c + d*x])/(21*d*Cos[c
 + d*x]^(3/2)) + (4*a^3*(17*A + 21*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]
]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) +
 (2*(13*A + 9*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^
(7/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\cos(c+dx))^2\left(\frac{1}{2}a(13A\right.}{\cos}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+a^3\cos(c+dx)\right.}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+a^3\cos(c+dx)\right.}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^3(17A+}{15d\sqrt{}}$$

$$= -\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} +$$

Mathematica [C] time = 6.63107, size = 967, normalized size = 4.08

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^5(c+dx)}{36d} + \frac{\sec(c)(7A\sin(c)+27A\sin(dx)+9B\sin(dx))\sec^4(c+}{252d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((17*A + 21
*B)*Csc[c]*Sec[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(36*d) + (Se
c[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 27*A*Sin[d*x] + 9*B*Sin[d*x]))/(252*d) +
(Sec[c]*Sec[c + d*x]*(55*A*Sin[c] + 65*B*Sin[c] + 119*A*Sin[d*x] + 147*B*Si
n[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]^3*(135*A*Sin[c] + 45*B*Sin[c] + 238
*A*Sin[d*x] + 189*B*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]^2*(238*A*Sin
[c] + 189*B*Sin[c] + 330*A*Sin[d*x] + 390*B*Sin[d*x]))/(1260*d)) - (11*A*(a
 + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2])
 - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 +
 Cot[c]^2]) + (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(60*d) + (7*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyperge
ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^
2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*
d)
```

Maple [B] time = 14.807, size = 1178, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*A*(-
1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
```

```
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*
x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
)-1/5*(3/8*A+3/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*B*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/8*A+3/8*B)*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*A+1/8*B)*(-1/56*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/
2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
```

```
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3}{\cos(dx+c)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(11/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
x)
```

3.145 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)\text{co}}{5ad}$$

```
[Out] (-3*(5*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(A - B)*EllipticF[(
c + d*x)/2, 2])/(3*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((5*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.198745, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2635, 2641, 2639}

$$\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)\text{co}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```


```
[Out] (-3*(5*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(A - B)*EllipticF[(
c + d*x)/2, 2])/(3*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((5*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(5A-7B)\right.}{a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(5A-7B)\int\cos^{\frac{5}{2}}(c+dx)\,dx}{2a} + \frac{(5(A-B))}{}$$

$$= \frac{5(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(5A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{(A}{}$$

$$= -\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5(A-B)\sqrt{\cos(c+}}{3ad}$$

Mathematica [C] time = 6.5567, size = 1182, normalized size = 7.58

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
(((21*I)/20)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
 (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((2*(5*A - 5*B + 10*A*Cos[c] - 16
*B*Cos[c])*Csc[c])/(5*d) + (4*(A - B)*Cos[d*x]*Sin[c])/(3*d) + (2*B*Cos[2*d
*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Si
n[(d*x)/2]))/d + (4*(A - B)*Cos[c]*Sin[d*x])/(3*d) + (2*B*Cos[2*c]*Sin[2*d*
x])/(5*d)))/(a + a*Cos[c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (5*B*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 3.599, size = 281, normalized size = 1.8

$$-\frac{1}{15\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\,\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*A*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1/2*c)
^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a*cos(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.146 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((3*A - 5*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
+ ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.17957, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2639, 2635, 2641}

$$-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```


```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((3*A - 5*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
+ ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx &= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(3A-5B\right.}{a^2}\\ &= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A-5B)\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a}+\frac{(3(A-B)}{}\\ &= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{(A-B)\,\mathrm{co}}{d(a\,\cdot}\\ &= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(3A-5B)\sqrt{\cos(c+}}{3ad}\end{aligned}$$

Mathematica [C] time = 6.48848, size = 1129, normalized size = 9.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (C
os[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*(1 + 2*Cos[c])*Csc[c])/
d + (4*B*Cos[d*x]*Sin[c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2]))/d + (4*B*Cos[c]*Sin[d*x])/(3*d)))/(a + a*Cos[c + d
*x]) + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*S
qrt[1 + Cot[c]^2]) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
 a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 2.849, size = 262, normalized size = 2.1

$$\frac{1}{3\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(3\,AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(-3*A+
7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos{(dx+c)}+A)\cos{(dx+c)}^{\frac{3}{2}}}{a\cos{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos{(dx+c)}^2+A\cos{(dx+c)}\right)\sqrt{\cos{(dx+c)}}}{a\cos{(dx+c)}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a*cos(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.147 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] -(((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c
 + d*x]))
```

Rubi [A] time = 0.149822, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2977, 2748, 2641, 2639}

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c
 + d*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A-B)-\frac{1}{2}a(A-3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(A-3B)\int\sqrt{\cos(c+dx)}\,dx}{2a} + \frac{(A-B)\int \cdots}{2\cdots}$$

$$= -\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\sqrt{\cos(c+dx)}\cdots}{d(a+a\cos(c+\cdots}$$

Mathematica [C] time = 6.4083, size = 1098, normalized size = 12.92

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((-I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
```

```
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (((3
*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[
c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(-A + B + 2*B*Cos[c])*Csc[c])/d +
(2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d))/(a +
a*Cos[c + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 3.473, size = 244, normalized size = 2.9

$$-\frac{1}{da}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.148 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=83

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A + B)*EllipticF[(c + d*x)/2,
 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*
x]))
```

Rubi [A] time = 0.150444, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A + B)*EllipticF[(c + d*x)/2,
 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*
x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{\frac{1}{2}a(A+B)+\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{(A-B)\int\sqrt{\cos(c+dx)}\,dx}{2a}+\frac{(A+B)\int}{}$$

$$= \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\sin}{d(a+a\cos(c+d}$$

Mathematica [C] time = 6.42271, size = 1094, normalized size = 13.18

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
```

```
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - ((I/4
)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c/2 +
 (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*Csc[c])/d - (2*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d))/(a + a*Cos[c + d*x]) - (
A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 +
Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d
*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 3.329, size = 243, normalized size = 2.9

$$\frac{1}{da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(A\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)/cos(d*x+c)^(1/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^2 + a*cos(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

3.149 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

```
[Out] -(((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((3*A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A -
 B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.173881, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2639, 2641}

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```


```
[Out] -(((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((3*A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A -
 B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A-B)-\frac{1}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} - \frac{(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{(3A-B)\int \frac{1}{\cos^{\frac{3}{2}}}$$

$$= -\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c-$$

$$= -\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} -$$

Mathematica [C] time = 6.60558, size = 1130, normalized size = 9.5

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
((I/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[
c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((2*A + A*Cos[c] - B*Cos[c])*Csc[c/2]*
Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin
[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(a + a*Cos[c + d*x])
 + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c
 + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 6.605, size = 319, normalized size = 2.7

$$-\frac{1}{da}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{-2\,\left(\sin\left(1/2\,dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1/2*c)^2
)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

3.150 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=153

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3(}{}$$

```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- (3*(A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.194849, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2641, 2639}

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```


```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- (3*(A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B)-\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{(5A-3B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a} - \frac{(3(A-B))\ \cdots}{\cdots}$$

$$= \frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+\cdots}$$

$$= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A-3B)\sin(c+dx\cdots}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] time = 6.91151, size = 1167, normalized size = 7.63

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (C
os[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(-(((A - B)*(2 + Cos[c])*Csc[c/2]*Se
c[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[
(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c
 + d*x]*(A*Sin[c] - 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)))/(a + a*Cos[c + d*
x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x]
)*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a +
a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 10.125, size = 493, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((A-B)*(cos(1/
2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*A+2*B)*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^4+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^4 + a*cos(
d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

3.151 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{7(5A-8B)\sin(c+}{15a^2}$$

```
[Out] (-7*(5*A - 8*B)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) + (5*(2*A - 3*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*d) - (7*(5*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*
d) + ((2*A - 3*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.406793, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2635, 2641, 2639}

$$\frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{7(5A-8B)\sin(c+}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (-7*(5*A - 8*B)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) + (5*(2*A - 3*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*d) - (7*(5*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*
d) + ((2*A - 3*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(5A-11B)\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= \frac{(2A-3B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \text{co}\cdots}{\cdots}$$

$$= \frac{(2A-3B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(7(5\cdots}{\cdots}$$

$$= \frac{5(2A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7(5A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \cdots$$

$$= -\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} + \frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{5(2A-3B)\sqrt{\text{co}\cdots}}{\cdots}$$

Mathematica [C] time = 6.76995, size = 1262, normalized size = 6.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] (((-7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2
+ (((28*I)/5)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
2 - (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(
a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[
c + d*x]]*((4*(15*A - 20*B + 20*A*Cos[c] - 36*B*Cos[c])*Csc[c])/(5*d) + (8*
(A - 2*B)*Cos[d*x]*Sin[c])/(3*d) + (4*B*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*Sec
[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 4*B*Sin[(d*x)/2]))/d - (2*Sec[
c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (8*(A
- 2*B)*Cos[c]*Sin[d*x])/(3*d) + (4*B*Cos[2*c]*Sin[2*d*x])/(5*d) - (2*(A - B
)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [A] time = 3.945, size = 465, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*B*cos(1/2
*d*x+1/2*c)^10+80*A*cos(1/2*d*x+1/2*c)^8-352*B*cos(1/2*d*x+1/2*c)^8+60*A*co
s(1/2*d*x+1/2*c)^6+100*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+210
*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+120*B*cos(1/2*d*x+1/2*c)^
6-150*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*B*cos(1/2*d*x+1/
2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-240*A*cos(1/2*d*x+1/2*c)^4+266*B*cos(1/2*d
*x+1/2*c)^4+105*A*cos(1/2*d*x+1/2*c)^2-135*B*cos(1/2*d*x+1/2*c)^2-5*A+5*B)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^4+A\cos(dx+c)^3\right)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/(a^2*cos(
d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x
)
```

3.152 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=166

$$-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5(A-2B)\sin(c+dx)}{3a^2d}$$

```
[Out] ((4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*(A - 2*B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a^2*d) + ((4*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
 + d*x])) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d
*x])^2)
```

Rubi [A] time = 0.389269, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2639, 2635, 2641}

$$-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5(A-2B)\sin(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] ((4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*(A - 2*B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a^2*d) + ((4*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
 + d*x])) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d
*x])^2)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{3}{2}a(A-3B)\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= \frac{(4A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \sqrt{\cdots}}{\cdots}$$

$$= \frac{(4A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(4A\cdots}{\cdots}$$

$$= \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5(A-2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{(4A-7B\cdots}{3a\cdots}$$

$$= \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(A-2B)\sqrt{\cos(c\cdots}}{3a^2\cdots}$$

Mathematica [C] time = 6.682, size = 1218, normalized size = 7.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (((
7*I)/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (1
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((-4*(2*A - 3*B + 2*A*Cos[c] - 4*B*Cos[c])*Csc[c])/d + (8*B*Cos[d*x]*
Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - 3*B*Sin[
(d*x)/2]))/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*
x)/2]))/(3*d) + (8*B*Cos[c]*Sin[d*x])/(3*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 3.277, size = 435, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2*
d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*co
s(1/2*d*x+1/2*c)^6-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-42*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+4
8*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^2*cos(
d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x
)
```

3.153 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\cos}{3d(a\cos(c+dx)+}$$

```
[Out] -(((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A - 5*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.31367, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2977, 2748, 2641, 2639}

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\cos}{3d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A - 5*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(A-7B)\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{2}a \cdots}{\cdots}$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2A \cdots}{\cdots}$$

$$= -\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(2A-5B)\sqrt{\cos(c+ \cdots}}{3a^2d(1+\cos \cdots}$$

Mathematica [C] time = 6.57746, size = 1184, normalized size = 8.71

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((-I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
```

```
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*A*
Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
+ Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]
]*((-4*(-A + 2*B + 2*B*Cos[c])*Csc[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(
A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A
*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Ta
n[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 3.768, size = 421, normalized size = 3.1

$$-\frac{1}{6\,a^2d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
```

```
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B*cos(1/2*d*x+1/2*c)^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2*c)^4+9*A
*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)
^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a^2*cos(d*
x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

3.154 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2} dx$

Optimal. Leaf size=121

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -((B*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((A + 2*B)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d
*x])) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rubi [A] time = 0.27677, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2978, 2748, 2641, 2639}

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -((B*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((A + 2*B)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d
*x])) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A+5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a^2(A+2B)\ldots}{\sqrt{c}\ldots}}{\ldots}$$

$$= \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{B\int\sqrt{\cos(\ldots}}{2a\ldots}$$

$$= -\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx\ldots}{a^2d(1+\cos(c+dx))}$$

Mathematica [C] time = 6.4721, size = 815, normalized size = 6.74

$$-\frac{iB\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\frac{2e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\sqrt{e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)+1}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}-\frac{2\,{}_2F_1\left(-\frac{1}{4}\right.\cdots}{\cdots}\right.\cdots}{2(\cos(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((-I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (2
*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt
[1 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*
x]]*((4*B*Csc[c])/d + (4*B*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (
2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 3.144, size = 350, normalized size = 2.9

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2A\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2+1}\mathrm{EllipticF}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^
2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

3.155 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (A*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A + B)*EllipticF[(c + d*x)/2, 2
])/(3*a^2*d) - (A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.342349, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2978, 2748, 2641, 2639}

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```


```
[Out] (A*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A + B)*EllipticF[(c + d*x)/2, 2
])/(3*a^2*d) - (A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(5A+B)-\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a^2(2A+B)}{\sqrt{c}}}{}$$

$$= -\frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{A\int\sqrt{\cos(}}{2a}$$

$$= \frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} -$$

Mathematica [C] time = 6.48553, size = 815, normalized size = 6.74

$$\frac{iA\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\frac{2e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\sqrt{e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)+1}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)} - \frac{2\,{}_2F_1\left(-\frac{1}{4},\right.}{}\right.}{2(\cos(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*
A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[
1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*C
os[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x
]]*((-4*A*Csc[c])/d - (4*A*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) - (
2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 3.663, size = 350, normalized size = 2.9

$$\frac{1}{6\,a^2 d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6-4\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1
/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
```

```
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3+2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3 + 2*a^
2*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

3.156 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-$$

```
[Out] -(((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - ((5*A - 2*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((4*A - B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*
x]]) - ((5*A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c +
d*x])) - ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^2)
```

Rubi [A] time = 0.359309, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2639, 2641}

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```



```
[Out] -(((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - ((5*A - 2*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((4*A - B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*
x]]) - ((5*A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c +
d*x])) - ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^2)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A-B)-\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \cdots$$

$$= -\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} - \cdots$$

$$= -\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(5A-2B)\sin(c+\cdots}{3a^2d\sqrt{\cos(c+dx)}(1+\cdots}$$

$$= -\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)\sin(c+dx\cdots}{a^2d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.68985, size = 1217, normalized size = 7.24

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
```

```
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (10*A
*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]
]*((2*(2*A + 2*A*Cos[c] - B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c
/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (8*A*Sec[c]
*Sec[c + d*x]*Sin[d*x])/d + (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)
))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 4.243, size = 494, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*(12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*x+1/2*c)^2)
/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4+2\,a^2\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4 + 2*a^
2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

3.157 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-$$

```
[Out] ((7*A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (5*(2*A - B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]
^(3/2)) - ((7*A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((7*A - 4
*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A - B
)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.363733, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2641, 2639}

$$\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```



```
[Out] ((7*A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (5*(2*A - B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]
^(3/2)) - ((7*A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((7*A - 4
*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A - B
)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(3A-B)-\frac{5}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \cdots$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} - \cdots$$

$$= \frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{(7A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+\cdots}$$

$$= \frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{5(2A-B)\sin(c+d\cdots}{3a^2d\cos^{\frac{3}{2}}(c+dx)\cdots}$$

Mathematica [C] time = 7.19355, size = 1258, normalized size = 6.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] (((7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 -
 ((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
```

```
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (2
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((-2*(4*A - 2*B + 3*A*Cos[c] - 2*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/
d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2]))/d
 - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d
) + (8*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*Sec[c + d*x]*(A*
Sin[c] - 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 11.656, size = 750, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*((4*A-2*
B)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-8*A+4*B)*(-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+4*A*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(A-B)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
```

```
-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin
(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)/(-1+sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^5+2\,a^2\cos(dx+c)^4+a^2\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^5 + 2*a^
2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

3.158 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=254

$$\frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{3(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7(17A-33B)\mathrm{s}}{}$$

```
[Out] (-7*(17*A - 33*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A - 21*B)*El
lipticF[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A - 21*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*a^3*d) - (7*(17*A - 33*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30
*a^3*d) + ((A - B)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x
])^3) + ((7*A - 12*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
 + d*x])^2) + (3*(11*A - 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3
+ a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.547762, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2635, 2641, 2639}

$$\frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{3(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7(17A-33B)\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-7*(17*A - 33*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A - 21*B)*El
lipticF[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A - 21*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*a^3*d) - (7*(17*A - 33*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30
*a^3*d) + ((A - B)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x
])^3) + ((7*A - 12*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
 + d*x])^2) + (3*(11*A - 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3
+ a^3*Cos[c + d*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{9}{2}a(A-B)-\frac{5}{2}a(A-3B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}dx}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + 3(1$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + 3(1$$

$$= \frac{(11A-21B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{7(17A-33B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30a^3d}$$

$$= -\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + (11A-21B)$$

Mathematica [C] time = 7.02844, size = 1346, normalized size = 5.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((-119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*H
ypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*
(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^
((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])
^3 + (((231*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
```

```
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I
)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
+ E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d
*x])^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) + (42*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((4*(59*A - 99*B + 60*A*Cos[c] - 132*B*Cos[c])*Csc[c])/(5*d) +
 (16*(A - 3*B)*Cos[d*x]*Sin[c])/(3*d) + (8*B*Cos[2*d*x]*Sin[2*c])/(5*d) + (
4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] - 99*B*Sin[(d*x)/2]))/(5*d
) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(19*A*Sin[(d*x)/2] - 24*B*Sin[(d*x)/2]
))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/
2]))/(5*d) + (16*(A - 3*B)*Cos[c]*Sin[d*x])/(3*d) + (8*B*Cos[2*c]*Sin[2*d*x
])/(5*d) - (4*(19*A - 24*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A -
 B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A] time = 3.471, size = 493, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*B*cos(1/
2*d*x+1/2*c)^12+160*A*cos(1/2*d*x+1/2*c)^10-864*B*cos(1/2*d*x+1/2*c)^10+468
*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-228*B*cos(1/2*d*x+1/
2*c)^8-630*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-1386*B*cos(1/2*
d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1/2*c)^6+1590*B*c
```

```
os(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-744*B*cos(1/2*d*x+1/2*c)^4-4
7*A*cos(1/2*d*x+1/2*c)^2+57*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x
+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{5}+A\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^5 + A*cos(d*x + c)^4)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x
)
```

3.159 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=219

$$-\frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{7(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-33B)\sin}{}$$

```
[Out] (7*(7*A - 17*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 33*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3) + ((A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a
*Cos[c + d*x])^2) + (7*(7*A - 17*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*
(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.518275, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2639, 2635, 2641}

$$-\frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{7(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-33B)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (7*(7*A - 17*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 33*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3) + ((A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a
*Cos[c + d*x])^2) + (7*(7*A - 17*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*
(a^3 + a^3*Cos[c + d*x]))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(3A-13B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} + \int \frac{\cos^{\frac{3}{2}}\ldots}{\ldots}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} + \frac{7(7A\ldots}{\ldots}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} + \frac{7(7A\ldots}{\ldots}$$

$$= \frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} + \frac{(A-B\ldots}{5\ldots}$$

$$= \frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(13A-33B)\sqrt{c\ldots}}{\ldots}$$

Mathematica [C] time = 6.85712, size = 1306, normalized size = 5.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 - (((119*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
```

```
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) - (22*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((-4*(29*A - 59*B + 20*A*Cos[c] - 60*B*Cos[c])*Csc[c])/(5*d) +
 (16*B*Cos[d*x]*Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d
*x)/2] - 59*B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(14*A
*Sin[(d*x)/2] - 19*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]
^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*B*Cos[c]*Sin[d*x])/(3*d)
+ (4*(14*A - 19*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c
/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A] time = 3.536, size = 465, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1/
2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-4
68*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+
1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1
/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B
```

```
)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x
)
```

3.160 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=188

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\cos}{5d(a\cos(c+dx}$$

```
[Out] -((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B)*Ellipt
icF[(c + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.477571, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2977, 2748, 2641, 2639}

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\cos}{5d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```



```
[Out] -((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B)*Ellipt
icF[(c + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(A-11B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A}{}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A}{}$$

$$= -\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-B)\cos^{\frac{5}{2}}(c}{5d(a+a}$$

Mathematica [C] time = 6.79305, size = 1273, normalized size = 6.77

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 + (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^
3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*
(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos
[c + d*x]]*((-4*(-9*A + 29*B + 20*B*Cos[c])*Csc[c])/(5*d) + (4*Sec[c/2]*Sec
[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] - 29*B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]
*Sec[c/2 + (d*x)/2]^3*(9*A*Sin[(d*x)/2] - 14*B*Sin[(d*x)/2]))/(15*d) + (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) - (4*
(9*A - 14*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (
d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 3.592, size = 451, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8-130*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2
*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*
x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x
)
```

3.161 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(\ldots}{5d(a\cos(c+dx)+a\ldots}$$

```
[Out] -((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*
d*(a + a*Cos[c + d*x])^2) + ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.470111, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2978, 2748, 2641, 2639}

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(\ldots}{5d(a\cos(c+dx)+a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*
d*(a + a*Cos[c + d*x])^2) + ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)+\frac{1}{2}a(A+9B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int \frac{1}{2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + (A+$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + (A+$$

$$= -\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-B)\cos^{\frac{3}{2}}(c+d}{5d(a+a\cos(}$$

Mathematica [C] time = 6.69926, size = 1265, normalized size = 7.03

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] ((-I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E
^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)
*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (
((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
```

```
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
 (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*S
qrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a
*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d
*x]]*((4*(A + 9*B)*Csc[c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(4*A*Si
n[(d*x)/2] - 9*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(
A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A
*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) + (4*(4*A - 9*B)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
 + a*Cos[c + d*x])^3
```

Maple [B] time = 3.637, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c
)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a^3*cos(d*
x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

3.162 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B)*EllipticF[(c + d*
x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a
*Cos[c + d*x])^3) + ((A + 4*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
+ a*Cos[c + d*x])^2) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3
 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.464352, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B)*EllipticF[(c + d*
x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a
*Cos[c + d*x])^3) + ((A + 4*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
+ a*Cos[c + d*x])^2) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3
 + a^3*Cos[c + d*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(3A+7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{2}}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(A}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(A}{}$$

$$= \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{5d(a+a\cos(c+}$$

Mathematica [C] time = 6.58703, size = 1264, normalized size = 7.1

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - ((
I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
```

```
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (2*A
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sqrt[1
 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]
]*((-4*(A - B)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/
2] - B*Sin[(d*x)/2]))/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)
/2] - B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x
)/2] + 4*B*Sin[(d*x)/2]))/(15*d) + (4*(A + 4*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/
2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c
 + d*x])^3
```

Maple [B] time = 3.708, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+6*A*
cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*
B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^
3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

3.163 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-B)\sin(c+dx)\sqrt{\cos}}{15ad(a\cos(c+dx)+}$$

```
[Out] ((9*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) - ((6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2) - ((9*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*
d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.480036, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2978, 2748, 2641, 2639}

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-B)\sin(c+dx)\sqrt{\cos}}{15ad(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] ((9*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) - ((6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2) - ((9*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*
d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{\frac{1}{2}a(9A+B)-\frac{3}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{\int \frac{1}{2}\cdots}{\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{(9A\cdots}{\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{(9A\cdots}{\cdots}$$

$$= \frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\sqrt{\cos(c+dx)}\cdots}{5d(a+a\cos(c+\cdots}$$

Mathematica [C] time = 6.65855, size = 1265, normalized size = 6.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
+ ((I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
(2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt
[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*((-4*(9*A + B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[
(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(6*A*S
in[(d*x)/2] - B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A
*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) - (4*(6*A - B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a
*Cos[c + d*x])^3
```

Maple [B] time = 3.121, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+
24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^4+3\,a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+a^3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4 + 3*a^
3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

3.164 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-3B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)\right.}$$

```
[Out] -((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B)*Ellipt
icF[(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - 9*B)*Sin[c + d*x])/(10*a^3*d*Sqrt
[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos
[c + d*x])^3) - ((8*A - 3*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^2) - ((13*A - 3*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a
^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.518832, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2639, 2641}

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-3B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] -((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B)*Ellipt
icF[(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - 9*B)*Sin[c + d*x])/(10*a^3*d*Sqrt
[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos
[c + d*x])^3) - ((8*A - 3*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^2) - ((13*A - 3*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a
^3 + a^3*Cos[c + d*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(11A-B)-\frac{5}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(}{5d\sqrt{\cos(c+dx)}(a+}$$

$$= -\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A-9B)\sin(}{10a^3d\sqrt{\cos(c}}$$

Mathematica [C] time = 6.92708, size = 1305, normalized size = 5.9

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((-49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 + (((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
```

```
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x
])^3*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*
(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos
[c + d*x]]*((2*(20*A + 29*A*Cos[c] - 9*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/
(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2
]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(11*A*Sin[(d*x)/2] - 6*B*Sin[(
d*x)/2]))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin
[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(11*A - 6*B)
*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Ta
n[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 4.433, size = 685, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(
```

```
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x
+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-9*B)*
sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*
d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^5+3\,a^3\cos(dx+c)^4+3\,a^3\cos(dx+c)^3+a^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^5 + 3*a^
3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

3.165 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=254

$$\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-13B)\sin}{6a^3d\cos^{\frac{3}{2}}(c}$$

```
[Out] (7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sin[c + d*x])/(6*a^3*d*Co
s[c + d*x]^(3/2)) - (7*(17*A - 7*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*
x]]) - ((A - B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^
3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]
)^2) - (7*(17*A - 7*B)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Co
s[c + d*x]))
```

Rubi [A] time = 0.602667, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2641, 2639}

$$\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-13B)\sin}{6a^3d\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```


```
[Out] (7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sin[c + d*x])/(6*a^3*d*Co
s[c + d*x]^(3/2)) - (7*(17*A - 7*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*
x]]) - ((A - B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^
3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]
)^2) - (7*(17*A - 7*B)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Co
s[c + d*x]))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A-3B)-\frac{7}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} -$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} -$$

$$= \frac{(33A-13B)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{7(17A-7B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(c+}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= \frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(33A-13B)\sin}{6a^3d\cos^{\frac{3}{2}}(c}$$

<u>Mathematica</u> [C] time = 7.47439, size = 1346, normalized size = 5.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
```

```
3 - (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x]
)^3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((-2*(60*A - 20*B + 59*A*Cos[c] - 29*B*Cos[c])*Csc[c/2]*Sec[c/
2]*Sec[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] - 29*B
*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(16*A*Sin[(d*x)/2]
 - 11*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d
) + (16*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)
 - (4*(16*A - 11*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[
c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 4.655, size = 876, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/60*(4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(165*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/
```

```
2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-168*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(17*A-7*B)*sin(1/2*d*x+1/2*c)^10+8*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1167*A-482*B)*sin(1/2*d*x+1/2*c)
^8-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1111*A-461*B)*s
in(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(404*A-169*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1029*A-439*B)*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(3/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)^5/a^3/sin(1/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^6+3a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+a^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^6 + 3*a^
3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

3.166 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=221

$$\frac{a(8A+7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{5a(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}(8A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{5}{}$$

```
[Out] (5*Sqrt[a]*(8*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (5*a*(8*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (5*a*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*A + 7*B)*Cos[c + d*x]^(5/2)*Sin[c + d
*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.349647, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2770, 2774, 216}

$$\frac{a(8A+7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{5a(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}(8A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (5*Sqrt[a]*(8*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (5*a*(8*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (5*a*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*A + 7*B)*Cos[c + d*x]^(5/2)*Sin[c + d
*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
```

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(8A+7B)\int \cos^{\frac{5}{2}}(c+dx)\sqrt{}$$

$$= \frac{a(8A+7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{7}{2}}(c+dx)\sin(}{4d\sqrt{a+a\cos(c+}}$$

$$= \frac{5a(8A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a(8A+7B)\cos^{\frac{5}{2}}(c}{24d\sqrt{a+a\,c}}$$

$$= \frac{5a(8A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{5a(8A+7B)\cos^{\frac{3}{2}}(}{96d\sqrt{a+a\,c}}$$

$$= \frac{5a(8A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{5a(8A+7B)\cos^{\frac{3}{2}}(}{96d\sqrt{a+a\,c}}$$

$$= \frac{5\sqrt{a}(8A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{5a(8A+7B)\sqrt{\cos(c-}}{64d\sqrt{a+a\,\mathrm{co}}}$$

Mathematica [A] time = 0.985495, size = 135, normalized size = 0.61

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(8A+7B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(2(40A}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(8*A + 7*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(152*A + 133*B + 2*(40*A
+ 53*B)*Cos[c + d*x] + 4*(8*A + 7*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c + d*x
)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.678, size = 428, normalized size = 1.9

$$\frac{(-1+\cos(dx+c))^4}{192\,d\,(\sin(dx+c))^8}\left(64\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^3+144\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^4*(64*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*cos(d*x+c)^3+144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+200*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+56*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+120*A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)+70*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)^2+105*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+120*A*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+1
05*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c))*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^8/(cos(d*x+c)/(1
+cos(d*x+c)))^(7/2)
```

Maxima [B] time = 4.52693, size = 11097, normalized size = 50.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*si
n(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(
arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
```

arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan

```
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*A + (2*(cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(3/4)*((156*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*
x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 39*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 39*sin(4*d*x + 4*c)^
3 + 156*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 39*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c
) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 156*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*si
n(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (32*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + s
in(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 +
 16*sin(4*d*x + 4*c)^2 - 55*cos(4*d*x + 4*c) + 39)*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 55*sin(4*d*x + 4*c
))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 39*cos(4*d*x + 4*
c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 156*(4*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + sin(4*d*x
+ 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (39*cos(4*d*x + 4*c)^3 + 4*(39
*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 86*cos
(4*d*x + 4*c)^2 + 55*cos(4*d*x + 4*c) - 8)*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))^2 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 + 4*(
39*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 + 70*c
os(4*d*x + 4*c)^2 + 23*cos(4*d*x + 4*c) - 8)*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 - 8*cos(4*d*x + 4*c)^2 + (32*(cos(4*d*x + 4*c)^2 +
 sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2
*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^
2 - 55*cos(4*d*x + 4*c) + 39)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 55*sin(4*d*x + 4*c))*sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 39*cos(4*d*x + 4*c))*cos(3/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(39*cos(4*d*x + 4*c)^3 + (39*cos(4*
d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 47*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x +
4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 39*(2*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*
x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*
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d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(39
*cos(4*d*x + 4*c) - 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c))*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((4*(11*s
in(4*d*x + 4*c)^3 + 11*(cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*
d*x + 4*c) - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*
c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 11*cos(4*d*x + 4*c)^2sin(4*d*x
 + 4*c) + 11*sin(4*d*x + 4*c)^3 + 4*(11*sin(4*d*x + 4*c)^3 + 11*(cos(4*d*x
+ 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) - 24*(cos(4*d*x + 4*c)^
2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
c)))^2 + 2(22*sin(4*d*x + 4*c)^3 + 22*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4
*c))*sin(4*d*x + 4*c) + 11*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))*sin(4*d*x + 4*c) - (48*cos(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)^2 - 37
*cos(4*d*x + 4*c) - 11)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 11*cos(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(8*(11*sin(4*d*
x + 4*c)^2 - 24*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 11*(cos(
4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 22
*sin(4*d*x + 4*c)^2 - 37*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 (24*cos(4*d*x + 4*c)^2 + 24*sin(4*d*x + 4*c)^2 + 11*cos(4*d*x + 4*c))*sin(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)) - (11*cos(4*d*x + 4*c)^3 + 4*(11*cos(4*d*x + 4
*c)^3 + (11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)^
2 - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*c
os(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 37*cos(4*d*x + 4*c) +
 24)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (11*cos(4*d*x
 + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 4*(11*cos(4*d*x + 4*c)^3 + (11*cos(4*d*x
 + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 46*cos(4*d*x + 4*c)^2 - 24*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 59*cos(4*d*x + 4*c) + 24)*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 24*cos(4*d*x + 4*c)^2 + 2*(22*cos
(4*d*x + 4*c)^3 + 2*(11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 26*cos(
4*d*x + 4*c)^2 - (48*cos(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)^2 - 37*cos(4*
d*x + 4*c) - 11)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 11*
sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 48*
cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (2

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4*cos(4*d*x + 4*c)^2 + 24*sin(4*d*x + 4*c)^2 + 11*cos(4*d*x + 4*c))*cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(8*((11*cos(4*d*x + 4*c)
+ 24)*sin(4*d*x + 4*c) - 24*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 2*(11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c) - 37*cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 11*(cos(4*d*x + 4*c) +
1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 11*sin(4*d*x + 4*c)*sin(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)))*sqrt(a) + 105*((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2
- 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)
^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(
4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2
(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (4*(cos(4*d*x + 4
*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)
^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2
- cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), c
```

```
os(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)) - 1) - (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c)
+ 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x
+ 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4
*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
 + 1) + (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) +
1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c
)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) -
 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x +
 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(
4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arcta
```

```
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d
*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 2.23, size = 423, normalized size = 1.91

$$\frac{\left(48\,B\cos\left(dx+c\right)^{3}+8\,(8\,A+7\,B)\cos\left(dx+c\right)^{2}+10\,(8\,A+7\,B)\cos\left(dx+c\right)+120\,A+105\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos}}{192\,(d\cos\left(dx+c\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*cos(d*x + c)^3 + 8*(8*A + 7*B)*cos(d*x + c)^2 + 10*(8*A + 7*B)
*cos(d*x + c) + 120*A + 105*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) - 15*((8*A + 7*B)*cos(d*x + c) + 8*A + 7*B)*sqrt(a)*arctan(sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*
x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.167 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{a(6A+5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{aB}{}$$

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*d) + (a*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*(6*A + 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt
[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a +
 a*Cos[c + d*x]])
```

Rubi [A] time = 0.297564, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2770, 2774, 216}

$$\frac{a(6A+5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*d) + (a*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*(6*A + 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt
[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a +
 a*Cos[c + d*x]])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(6A+5B)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{}$$

$$= \frac{a(6A+5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{5}{2}}(c+dx)\sin(}{3d\sqrt{a+a\cos(c+}}$$

$$= \frac{a(6A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(6A+5B)\cos^{\frac{3}{2}}(c+}{12d\sqrt{a+a\,\mathrm{co}}}$$

$$= \frac{a(6A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(6A+5B)\cos^{\frac{3}{2}}(c+}{12d\sqrt{a+a\,\mathrm{co}}}$$

$$= \frac{\sqrt{a}(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a(6A+5B)\sqrt{\cos(c+d}}{8d\sqrt{a+a\cos(}}$$

Mathematica [A] time = 0.530712, size = 118, normalized size = 0.67

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(6A+5B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(2(6A\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*B + 2*(6*A + 5*
B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.721, size = 356, normalized size = 2.

$$-\frac{(-1+\cos(dx+c))^3}{24\,d\,(\sin(dx+c))^6}\left(12\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^2+30\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*cos(d*x+c)^2+30*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)
+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+18*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+10*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)+18*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))*cos(d*x+c)+15*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))*cos(d*x+c))*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
^6/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
```

Maxima [B] time = 3.62505, size = 4024, normalized size = 22.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1)))*A + (4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin
(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
```

```
+ 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
- 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sq
rt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
```

```
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*B)/d
```

Fricas [A] time = 1.99075, size = 373, normalized size = 2.12

$$\frac{\left(8\,B\cos\left(dx+c\right)^{2}+2\,(6\,A+5\,B)\cos\left(dx+c\right)+18\,A+15\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\,((6\,A+5\,B}{24\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 18*A + 15*B)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((6*A + 5*B)*cos(d*
x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.168 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{a}(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) + (a*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rubi [A] time = 0.227166, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2770, 2774, 216}

$$\frac{\sqrt{a}(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) + (a*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(4A+3B)\int \sqrt{\cos(c+dx}$$

$$= \frac{a(4A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{3}{2}}(c+dx)\sin}{2d\sqrt{a+a\cos(c}}$$

$$= \frac{a(4A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{3}{2}}(c+dx)\sin}{2d\sqrt{a+a\cos(c}}$$

$$= \frac{\sqrt{a}(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a(4A+3B)\sqrt{\cos(c+}}{4d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.287942, size = 100, normalized size = 0.76

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(4A+3B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(4A+2}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(4*A + 3*B)*ArcSin[Sq
rt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d*x
])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.681, size = 284, normalized size = 2.2

$$\frac{(-1+\cos(dx+c))^2}{4\,d\,(\sin(dx+c))^4}\left(4\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\cos(dx+c)+4\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}+2\,B\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*c
os(d*x+c)+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3*B*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+3*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] time = 2.72743, size = 2499, normalized size = 19.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
```

```
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2
*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d
*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2
*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*
x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A] time = 1.95919, size = 324, normalized size = 2.47

$$\frac{(2\,B\cos\left(dx+c\right)+4\,A+3\,B)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-((4\,A+3\,B)\cos\left(dx+c\right)+4\,A+3\,B)\sqrt{a}\,\mathrm{ar}}{4\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*cos(d*x + c) + 4*A + 3*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - ((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(
d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos\left(dx+c\right)+A)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

3.169 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a}(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.168901, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2981, 2774, 216}

$$\frac{\sqrt{a}(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}(2A+B)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{aB\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2A+B)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(}{\sqrt{a+a\cos}}\right.}{d}$$

$$= \frac{\sqrt{a}(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{aB\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.152891, size = 83, normalized size = 1.06

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(2A+B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + B)*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 0.692, size = 164, normalized size = 2.1

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2}\left(B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+B\arctan\left(\frac{\sin(}{\cos(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(1/2)*(a*(
1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.66541, size = 1268, normalized size = 16.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
```

```
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A] time = 1.88755, size = 274, normalized size = 3.51

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}B\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(2\,A+B\right)\cos\left(dx+c\right)+2\,A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A + B)*co
s(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(cos(c + d*x))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

3.170 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.164648, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2980, 2774, 216}

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + B\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2B)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c}{\sqrt{a+a\cos}}\right.}{d}$$

$$= \frac{2\sqrt{a}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.162091, size = 86, normalized size = 1.13

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2A\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d
*x]])
```

Maple [A] time = 0.657, size = 109, normalized size = 1.4

$$-2\,\frac{\sqrt{a\,(1+\cos(dx+c))}}{d\sin(dx+c)\sqrt{\cos(dx+c)}}\left(-B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sin(dx+c)+A\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*(a*(1+cos(d*x+c)))^(1/2)*(-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+A*cos(d
*x+c)-A)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [B] time = 2.30896, size = 331, normalized size = 4.36

$$\frac{B\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] (B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos
(d*x + c)) + 2*A*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Fricas [A] time = 1.62316, size = 298, normalized size = 3.92

$$\frac{2\left(\sqrt{a\cos\left(dx+c\right)+a}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(B\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c}}{\sqrt{a}\sin(dx+c)}\right.\right.}{d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 2*(sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - (B*cos(d*x
+ c)^2 + B*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

3.171 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(2*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]])
```

Rubi [A] time = 0.163463, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2980, 2771}

$$\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(2*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
```

```
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{1}{3}(2A+3B)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} \\ &= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.143265, size = 57, normalized size = 0.67

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((2A+3B)\cos(c+dx)+A)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/
2])/(3*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.647, size = 62, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))(2A\cos(dx+c)+3B\cos(dx+c)+A)}{3d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)+3*B*cos(d*x+c)+A)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 1.83452, size = 390, normalized size = 4.59

$$\frac{2\left(\frac{3\,B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}+\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)) + A*(3*sqrt(2)*sqrt(a
)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) +
1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.46101, size = 177, normalized size = 2.08

$$\frac{2\left((2\,A+3\,B)\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{3\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos{(dx+c)}+A)\sqrt{a\cos{(dx+c)}+a}}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

3.172 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*
x]]) + (4*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])
```

Rubi [A] time = 0.218488, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2980, 2772, 2771}

$$\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*
x]]) + (4*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{1}{5}(4A+5B)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} \\ &= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} \\ &= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.239307, size = 78, normalized size = 0.6

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx))+7A+5B)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
+ 5*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.715, size = 86, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(8A(\cos(dx+c))^2+10B(\cos(dx+c))^2+4A\cos(dx+c)+5B\cos(dx+c)+3A\right)}{15d\sin(dx+c)}\sqrt{a(1+\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+4*A*cos(d*x+c)+
5*B*cos(d*x+c)+3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.0141, size = 578, normalized size = 4.45

$$\frac{2\left(\frac{5B\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{A\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{2}\sqrt{a}\sin(dx\cdots}{(\cos(dx+c)+1\cdots}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4\cdots}\right.}\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x
 + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 1)) + A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*
sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.42773, size = 223, normalized size = 1.72

$$\frac{2\left(2\,(4\,A+5\,B)\cos\left(dx+c\right)^{2}+(4\,A+5\,B)\cos\left(dx+c\right)+3\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*co
s(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

3.173 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{8a(6A+7B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(6*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*
x]]) + (8*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + (16*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.28595, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2980, 2772, 2771}

$$\frac{8a(6A+7B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(6*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*
x]]) + (8*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + (16*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])
```


Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(6A+7B)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx}}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx}}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx}}$$

Mathematica [A] time = 0.367869, size = 102, normalized size = 0.58

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos(3(c+dx))+27A}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2
),x]
```

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] + 2
*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d*x
)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))

Maple [A] time = 0.605, size = 108, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(48A(\cos(dx+c))^3+56B(\cos(dx+c))^3+24A(\cos(dx+c))^2+28B(\cos(dx+c))^2+18A\cos\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cos(d*x+c)*a)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)

[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+28*B*cos(d*x+c)^2+18*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+cos(d*x+
c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)

Maxima [B] time = 2.04267, size = 705, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")

[Out] 2/105*(7*B*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 3*A*(35*sqrt(2)*sqrt(a)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)
*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d

```
*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(co
s(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.41965, size = 270, normalized size = 1.54

$$\frac{2\left(8\left(6A+7B\right)\cos\left(dx+c\right)^{3}+4\left(6A+7B\right)\cos\left(dx+c\right)^{2}+3\left(6A+7B\right)\cos\left(dx+c\right)+15A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(d\right.}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*A
 + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

3.174 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}$$

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (a^2*(88*A + 75*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqr
t[a + a*Cos[c + d*x]]) + (a^2*(88*A + 75*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.504117, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (a^2*(88*A + 75*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqr
t[a + a*Cos[c + d*x]]) + (a^2*(88*A + 75*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}\int \cos$$

$$= \frac{a^2(8A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{5}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{a^2(88A+75B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(8A+9B)\cos}{24d\sqrt{a}}$$

$$= \frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(88A+75B)}{96d\sqrt{a}}$$

$$= \frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(88A+75B)}{96d\sqrt{a}}$$

$$= \frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(88A+75B)\sqrt{}}{64d\sqrt{a+}}$$

Mathematica [A] time = 1.09903, size = 136, normalized size = 0.6

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(88A+75B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(2(8\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(88*A + 75*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(296*A + 285*B + 2*(88
*A + 93*B)*Cos[c + d*x] + 4*(8*A + 15*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c +
 d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.517, size = 429, normalized size = 1.9

$$-\frac{a(-1+\cos(dx+c))^3}{192\,d\,(\sin(dx+c))^6}\left(64\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^3+240\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/192/d*a*(-1+cos(d*x+c))^3*(64*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*cos(d*x+c)^3+240*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*
x+c)^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+440*A
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+120*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+264*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)+150*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^2+225*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+26
4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x
+c)+225*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [B] time = 5.03844, size = 12020, normalized size = 52.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
```

```
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(si
n(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1)
 - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arc
tan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
```

```
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(
a))*A + 3*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((9*a*cos(4*d*x + 4*c)^2*sin(4*
d*x + 4*c) + 9*a*sin(4*d*x + 4*c)^3 + 36*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d
*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 36*(a*sin(4*d*x + 4*c)^3 + (a*cos(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*(2*a*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c) - 2*(a*cos
(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*co
s(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(a*sin(4*d*x + 4*c)
^3 + (a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*a*cos(4*d*x + 4*c)^2 + 32
*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c
)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c
) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 9*a*cos(4*d
*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 25*a*cos
(4*d*x + 4*c) + 9*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4
*c) + 25*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 36*(4*a*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a*sin
(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(
3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (9*a*cos(4*d*x + 4*c)^3
 - 8*a*cos(4*d*x + 4*c)^2 + 4*(9*a*cos(4*d*x + 4*c)^3 - 26*a*cos(4*d*x + 4*
c)^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 25*a*cos(4*d*x + 4
*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (9*a*co
s(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 4*(9*a*cos(4*d*x + 4*c)^3 + 10*a
*cos(4*d*x + 4*c)^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 - 7*a
*cos(4*d*x + 4*c) - 8*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + (8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4
*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 - 9*a*cos(4*d*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 +
 16*a*sin(4*d*x + 4*c)^2 - 25*a*cos(4*d*x + 4*c) + 9*a)*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 25*a*sin(4*d*x + 4*c))*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 4*(9*a*cos(4*d*x + 4*c)^3 - 17*a*cos(4*d*x + 4*c)^2
+ (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c))*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2*a*cos(1/2*arctan
```

```
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c
) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(9*a*
cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)*sin(4*d*x + 4*c) + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((7*a
*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 7*a*sin(4*d*x + 4*c)^3 - 48*(a*cos(4
*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4*(7*a*sin(4*d*x + 4*c)^3 +
7*(a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c) - 68*(
a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin
(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + 7*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))*sin(4*d*x + 4*c) + 4*(7*a*sin(4*d*x + 4*c)^3 + 48*a*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (7*a*cos(4*
d*x + 4*c)^2 + 14*a*cos(4*d*x + 4*c) + 19*a)*sin(4*d*x + 4*c) - 68*(a*cos(4
*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 2*(14*a*sin(4*d*x + 4*c)^3 + 7*a*cos(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 14*(a*cos(4*d*x + 4
*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (136*a*cos(4*d*x + 4*c)^2 +
136*a*sin(4*d*x + 4*c)^2 - 129*a*cos(4*d*x + 4*c) - 7*a)*sin(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 2*(6*a*cos(4*d*x + 4*c)^2 + 24*(a*cos(4*d*x + 4*c)^2 + a*si
n(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + 20*a*sin(4*d*x + 4*c)^2 - 129*a*sin(4*d*x + 4*c)
*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*(3*a*cos(4*d*x +
4*c)^2 + 10*a*sin(4*d*x + 4*c)^2 - 68*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 7*(a*cos(4*d*x + 4*c) + a)*cos(1/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) - (68*a*cos(4*d*x + 4*c)^2 + 68*a*sin(4*d*x + 4*c)^2
 + 7*a*cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (7*a*cos(4*d*x
+ 4*c)^3 - 48*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x
+ 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 56*a*c
os(4*d*x + 4*c)^2 + 4*(7*a*cos(4*d*x + 4*c)^3 + 30*a*cos(4*d*x + 4*c)^2 + (
7*a*cos(4*d*x + 4*c) + 44*a)*sin(4*d*x + 4*c)^2 - 93*a*cos(4*d*x + 4*c) - 4
4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*
cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 56*a)*cos(1/2*arctan
```

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2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 7*(a*cos(4*d*x + 4*c) + 8*a)*sin
(4*d*x + 4*c)^2 + 4*(7*a*cos(4*d*x + 4*c)^3 + 70*a*cos(4*d*x + 4*c)^2 + 7*(
a*cos(4*d*x + 4*c) + 8*a)*sin(4*d*x + 4*c)^2 + 119*a*cos(4*d*x + 4*c) - 12*
(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 44*(a*cos(4*d*x + 4*c)
^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 56*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 - 7*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 2*(14*a*cos(4*d*x + 4*c)^3 + 92*a*cos(4*d*x + 4*c)^2
 + 2*(7*a*cos(4*d*x + 4*c) + 53*a)*sin(4*d*x + 4*c)^2 - 7*a*sin(4*d*x + 4*c
)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 112*a*cos(4*d*x +
4*c) - (88*a*cos(4*d*x + 4*c)^2 + 88*a*sin(4*d*x + 4*c)^2 - 81*a*cos(4*d*x
+ 4*c) - 7*a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (44*a*cos(4*d*x + 4*c)^2 +
44*a*sin(4*d*x + 4*c)^2 + 7*a*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 2*(96*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 81*a*cos(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(44*a*cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (7*a*cos(4*d*x + 4*c) + 53*a)*sin
(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 14*(a
*cos(4*d*x + 4*c) + 8*a)*sin(4*d*x + 4*c) + 7*(a*cos(4*d*x + 4*c) + a)*sin(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)))*sqrt(a) + 75*((a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*
sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2
 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4
*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d
*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)
*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4
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*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) -
 (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2
*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^
2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x +
4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a*cos(4*d*x + 4*c)^2
 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) +
a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4
*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*
c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4
*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^
(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1)
 + (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 -
 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c
)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
```

```
 + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x
+ 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*
d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4
*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*
c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x
+ 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 -
4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
+ sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/
d
```

Fricas [A] time = 2.32066, size = 451, normalized size = 1.99

$$\frac{\left(48\,Ba\cos\left(dx+c\right)^{3}+8\,(8\,A+15\,B)a\cos\left(dx+c\right)^{2}+2\,(88\,A+75\,B)a\cos\left(dx+c\right)+3\,(88\,A+75\,B)a\right)\sqrt{a\cos\left(dx+c\right)}}{192\,(d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*a*cos(d*x + c)^3 + 8*(8*A + 15*B)*a*cos(d*x + c)^2 + 2*(88*A +
 75*B)*a*cos(d*x + c) + 3*(88*A + 75*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))*sin(d*x + c) - 3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*B)*
a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(
d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.175 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(14A+11B)\sin(c+dx)\sqrt{\cos(c+d}}{8d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^2*(14*A + 11*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[
a + a*Cos[c + d*x]]) + (a^2*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1
2*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.412213, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(14A+11B)\sin(c+dx)\sqrt{\cos(c+d}}{8d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^2*(14*A + 11*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[
a + a*Cos[c + d*x]]) + (a^2*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1
2*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int \sqrt{}$$

$$= \frac{a^2(6A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(6A+7B)}{12d\sqrt{}}$$

$$= \frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(6A+7B)}{12d\sqrt{}}$$

$$= \frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(14A+11B)\sqrt{}}{8d\sqrt{a+}}$$

Mathematica [A] time = 0.654253, size = 119, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(14A+11B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(2(\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(14*A + 11*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(42*A + 37*B + 2*(6*A
+ 11*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.689, size = 357, normalized size = 2.

$$\frac{a(-1+\cos(dx+c))^2}{24\,d(\sin(dx+c))^4}\left(12\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^2+54\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*a*(-1+cos(d*x+c))^2*(12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*cos(d*x+c)^2+54*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+42*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+22*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*cos(d*x+c)^2+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)+42*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*cos(d*x+c)+33*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))*cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c
)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] time = 3.47846, size = 4081, normalized size = 22.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/
```

```
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*(a*cos(3/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(
3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) -
 (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
```

```
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))) - 1) - a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))*B)/d
```

Fricas [A] time = 1.90417, size = 401, normalized size = 2.23

$$\frac{\left(8\,Ba\cos\left(dx+c\right)^{2}+2\left(6\,A+11\,B\right)a\cos\left(dx+c\right)+3\left(14\,A+11\,B\right)a\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\,((1}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 3*(14*A + 11*
B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((14*A +
```

```
11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.176 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=133

$$\frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}}{2d}$$

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a*B*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rubi [A] time = 0.329873, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2976, 2981, 2774, 216}

$$\frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a*B*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{a+a\cos(c}}{}$$

$$= \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\sqrt{\cos(c+dx)}\sqrt{a+a\cos(}}{2d}$$

$$= \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\sqrt{\cos(c+dx)}\sqrt{a+a\cos(}}{2d}$$

$$= \frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\sin}{4d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.355379, size = 101, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(12A+7B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(4A\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(12*A + 7*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 7*B + 2*B*Cos[c +
d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.677, size = 283, normalized size = 2.1

$$-\frac{a\left(-1+\cos\left(dx+c\right)\right)}{4\,d\left(\sin\left(dx+c\right)\right)^{2}}\left(4\,A\sin\left(dx+c\right)\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{3/2}\cos\left(dx+c\right)+4\,A\sin\left(dx+c\right)\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{3/2}+2\,B\,\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*a*(-1+cos(d*x+c))*(4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
cos(d*x+c)+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+7*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)+12*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+7*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [B] time = 2.55572, size = 2543, normalized size = 19.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
```

```
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
```

```
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*B)/d
```

Fricas [A] time = 1.8741, size = 343, normalized size = 2.58

$$\frac{(2\,Ba\cos\left(dx+c\right)+(4\,A+7\,B)a)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-((12\,A+7\,B)a\cos\left(dx+c\right)+(12\,A+7}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*a*cos(d*x + c) + (4*A + 7*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))*sin(d*x + c) - ((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*sq
rt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

3.177 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^2*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]])
```

Rubi [A] time = 0.33107, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2981, 2774, 216}

$$\frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^2*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{1}{2}a(2A\right.}{\sqrt{c}}$$

$$= -\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}\sin(}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}\sin(}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\sin(c}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.310588, size = 107, normalized size = 0.85

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(2A+3B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(2A+B\cos(c+dx))\right)}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + 3*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + B*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(2*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.596, size = 300, normalized size = 2.4

$$\frac{a}{d\left(1+\cos\left(dx+c\right)\right)}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(2A\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+3*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))*cos(d*x+c)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*a/(1+cos(d*x+c))/cos(d*x+
c)^(1/2)
```

Maxima [B] time = 2.45104, size = 2431, normalized size = 19.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
```

```
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A] time = 2.05926, size = 362, normalized size = 2.87

$$\frac{(Ba\cos(dx+c)+2\,Aa)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-\left((2\,A+3\,B)a\cos(dx+c)^2+(2\,A+3\,B)a\cos(dx\right.}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] ((B*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

3.178 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(4*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2))
```

Rubi [A] time = 0.315417, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2774, 216}

$$\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(4*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{1}{2}a(4\ldots\right.}{\cos^{\frac{3}{2}}(\ldots}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+\ldots}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+\ldots}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \ldots$$

Mathematica [A] time = 0.36892, size = 106, normalized size = 0.85

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((5A+3B)\cos(c+dx)+A)+3\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*B*ArcSin[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3*B)*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(3*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.596, size = 211, normalized size = 1.7

$$-\frac{2a}{3d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-3B\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*a*(a*(1+cos(d*x+c)))^(1/2)*(-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c)-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+5*A*cos(d*x+c)^2+3*B*cos(d*
x+c)^2-4*A*cos(d*x+c)-3*B*cos(d*x+c)-A)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.10996, size = 1517, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(3*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4) + 8*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*
sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [A] time = 1.93496, size = 359, normalized size = 2.87

$$\frac{2\left(\left(\left(5\,A+3\,B\right)a\cos\left(dx+c\right)+Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(Ba\cos\left(dx+c\right)^{3}+Ba\cos\left(dx+c\right)\right.\right.}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
```

```
ithm="fricas")
```

```
[Out] 2/3*(((5*A + 3*B)*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c) - 3*(B*a*cos(d*x + c)^3 + B*a*cos(d*x + c)^2)*sqrt(a)
*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

3.179 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*a^2*(6*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(18*A + 25*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*
Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.339731, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2975, 2980, 2771}

$$\frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*(6*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(18*A + 25*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*
Cos[c + d*x]^(5/2))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{1}{2}a(6A\right.}{\mathrm{co}}\\ &= \frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)}\\ &= \frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+}}\end{aligned}$$

Mathematica [A] time = 0.29614, size = 80, normalized size = 0.6

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(9A+5B)\cos(c+dx)+(18A+25B)\cos(2(c+dx))+24A+25B)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] + (
18*A + 25*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.567, size = 87, normalized size = 0.7

$$-\frac{2a\left(-1+\cos\left(dx+c\right)\right)\left(18A\left(\cos\left(dx+c\right)\right)^{2}+25B\left(\cos\left(dx+c\right)\right)^{2}+9A\cos\left(dx+c\right)+5B\cos\left(dx+c\right)+3A\right)}{15d\sin\left(dx+c\right)}\sqrt{a\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(18*A*cos(d*x+c)^2+25*B*cos(d*x+c)^2+9*A*cos(d*x+
c)+5*B*cos(d*x+c)+3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.67331, size = 464, normalized size = 3.46

$$\frac{4\left(\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)B}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}+\frac{3\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 4/15*(5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)) + 3*(5*sqrt(2)*a^(3/2)*sin(d*x + c)
/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/
2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.65871, size = 231, normalized size = 1.72

$$\frac{2\left((18\,A+25\,B)a\cos\left(dx+c\right)^{2}+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4
+ d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2
), x)
```

3.180 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(52A+63B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a^2*(8*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Cos[c + d*x]]) + (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[
c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.431021, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{2a^2(52A+63B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (2*a^2*(8*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Cos[c + d*x]]) + (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[
c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{1}{2}a(8A\right.}{\cos}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(52A+63B)\sin(c+}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(}}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(52A+63B)\sin(c+}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(}}$$

Mathematica [A] time = 0.503632, size = 102, normalized size = 0.56

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(3(78A+77B)\cos(c+dx)+(52A+63B)\cos(2(c+dx))+52A\cos(3(c+dx))+}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x] +
 (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c + d
*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.576, size = 109, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(104A(\cos(dx+c))^3+126B(\cos(dx+c))^3+52A(\cos(dx+c))^2+63B(\cos(dx+c))^2+39\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+52*A*cos(
d*x+c)^2+63*B*cos(d*x+c)^2+39*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+cos(
```

```
d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 1.67757, size = 649, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 4/105*(21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
+ (105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^(3/2
)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.78047, size = 285, normalized size = 1.57

$$\frac{2\left(2\,(52\,A+63\,B)a\cos\left(dx+c\right)^{3}+(52\,A+63\,B)a\cos\left(dx+c\right)^{2}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)+15\,Aa\right)\sqrt{a\cos\left(dx+c\right)+}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2
), x)
```

3.181 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{8a^2(34A+39B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sin[c + d*x])
/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.505171, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{8a^2(34A+39B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```



```
[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sin[c + d*x])
/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{1}{2}a(10A\right.}{\mathrm{co}}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+39B)\sin(c+dx}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+39B)\sin(c+dx}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+39B)\sin(c+dx}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}$$

Mathematica [A] time = 0.652843, size = 124, normalized size = 0.54

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((374A+324B)\cos(c+dx)+11(34A+39B)\cos(2(c+dx))+68A\cos(3(c+dx))}{315d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x]
 + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3*(
c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2]
)/(315*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.639, size = 131, normalized size = 0.6

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(272\,A\,(\cos(dx+c))^4+312\,B\,(\cos(dx+c))^4+136\,A\,(\cos(dx+c))^3+156\,B\,(\cos(dx+c))^3+10\right.}{315\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+136*A*cos
(d*x+c)^3+156*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+85*A*cos
(d*x+c)+45*B*cos(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c
)^(9/2)
```

Maxima [B] time = 1.75113, size = 774, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 4/315*(3*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)
*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)) + (315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x +
c) + 1) - 840*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sq
rt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 - 94*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1
)))/d
```

Fricas [A] time = 1.86035, size = 335, normalized size = 1.47

$$\frac{2\left(8\,(34\,A+39\,B)a\cos\left(dx+c\right)^{4}+4\,(34\,A+39\,B)a\cos\left(dx+c\right)^{3}+3\,(34\,A+39\,B)a\cos\left(dx+c\right)^{2}+5\,(17\,A+9\,B)a\cos}{315\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
+ 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A*a
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^
6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/
2), x)
```

3.182 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=274

$$\frac{a^3(170A+157B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\,c}{4}$$

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^3*(326*A + 283*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(326*A + 283*B)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(170*A + 157*B)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(10*A + 13*
B)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B*
Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.708842, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^3(170A+157B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\,c}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```



```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^3*(326*A + 283*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(326*A + 283*B)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(170*A + 157*B)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(10*A + 13*
B)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B*
Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}\int$$

$$= \frac{a^2(10A+13B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+d}{40d}$$

$$= \frac{a^3(170A+157B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(10A+1}{}$$

$$= \frac{a^3(326A+283B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(170A+}{24}$$

$$= \frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(326A+}{19}$$

$$= \frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(326A+}{19}$$

$$= \frac{a^{5/2}(326A+283B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d}+\frac{a^3(326A+28}{128}$$

Mathematica [A] time = 1.87209, size = 159, normalized size = 0.58

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(326A+283B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(326*A + 283*B
)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(5810*A + 5521*B
+ (3620*A + 3874*B)*Cos[c + d*x] + 4*(230*A + 331*B)*Cos[2*(c + d*x)] + 120
*A*Cos[3*(c + d*x)] + 348*B*Cos[3*(c + d*x)] + 48*B*Cos[4*(c + d*x)])*Sin[(
c + d*x)/2]))/(3840*d)
```

Maple [B] time = 0.561, size = 503, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/1920/d*a^2*(-1+cos(d*x+c))^3*(480*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)+2320*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*c
os(d*x+c)^3+384*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+5100*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+1392*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+8150*A*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+2264*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+4890*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)+2830*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2
+4245*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+4890*A*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+4245
*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+
c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/sin(d*x+c)^6/(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.48549, size = 540, normalized size = 1.97

$$\frac{\left(384\,Ba^2\cos\left(dx+c\right)^4+48\,(10\,A+29\,B)a^2\cos\left(dx+c\right)^3+8\,(230\,A+283\,B)a^2\cos\left(dx+c\right)^2+10\,(326\,A+283\,B)a^2\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/1920*((384*B*a^2*cos(d*x + c)^4 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^3 + 8
*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 10*(326*A + 283*B)*a^2*cos(d*x + c) +
 15*(326*A + 283*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*
x + c) - 15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.183 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^3(104A+95B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{a^{5/2}(200A+163$$

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^3*(200*A + 163*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(104*A + 95*B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 11*B)*Cos[c + d*x]^(3/
2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d) + (a*B*Cos[c + d*x]^(3/2)*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.71324, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^3(104A+95B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{a^{5/2}(200A+163$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^3*(200*A + 163*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(104*A + 95*B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 11*B)*Cos[c + d*x]^(3/
2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d) + (a*B*Cos[c + d*x]^(3/2)*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
```

```
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{1}{4}\int$$

$$= \frac{a^2(8A+11B)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{24d}$$

$$= \frac{a^3(104A+95B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(8A+11B)}{}$$

$$= \frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(104A+9}{96}$$

$$= \frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(104A+9}{96}$$

$$= \frac{a^{5/2}(200A+163B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(200A+16}{64d}$$

Mathematica [A] time = 1.2012, size = 137, normalized size = 0.6

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(200A+163B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(200*A + 163*B)
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(632*A + 581*B + (
272*A + 362*B)*Cos[c + d*x] + 4*(8*A + 23*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*
(c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.642, size = 431, normalized size = 1.9

$$\frac{a^2(-1+\cos(dx+c))^2}{192\,d\,(\sin(dx+c))^4}\left(64\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^3+336\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/192/d*a^2*(-1+cos(d*x+c))^2*(64*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*cos(d*x+c)^3+336*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d
*x+c)^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+872*
A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+184*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+600*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)+326*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^2+489*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+6
00*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c)+489*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*
cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^4/(cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] time = 5.00659, size = 12710, normalized size = 55.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
```

```
3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(
1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
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*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) - 1))*sqrt(a))*A + (10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(
4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*
d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(
4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(
a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) +
 a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 3*(2*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d
*x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)
^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)
^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 +
2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a
^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin
(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
12*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x +
4*c)^2 + a^2*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*
cos(4*d*x + 4*c)^3 - 8*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3
 - 14*a^2*cos(4*d*x + 4*c)^2 + 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x +
 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))^2 + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^
2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d
*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4
*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d
*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)
^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16
*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 11*a^2*cos(4*d*

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x + 4*c)^2 + 8*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(
4*d*x + 4*c)^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*(2
*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
+ a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) - 4*(4*(3*a^2*cos(4*d*x + 4*c) - 8*a^2)*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) -
 8*a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1)^(1/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3
*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^3 + 4*(3*a^2*sin(4*d*x + 4*c)^3 + 3*(a^2*cos(4*d*x + 4*c)^
2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4
*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 4*(3*a^2*sin(4*d*x + 4*c)^3 + 160*a^2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x
 + 4*c)^2 + 6*a^2*cos(4*d*x + 4*c) + 43*a^2)*sin(4*d*x + 4*c) - 160*(a^2*co
s(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*s
in(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(6*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 6*(a^2*
cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (320*a^2*cos(
4*d*x + 4*c)^2 + 320*a^2*sin(4*d*x + 4*c)^2 - 317*a^2*cos(4*d*x + 4*c) - 3*
a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(20*a^2*cos(4*d*x + 4*c)^2 + 26*a^
2*sin(4*d*x + 4*c)^2 - 317*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*
c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 8*(10*a^2*cos(4*d*x + 4*c)^2 + 13*a^2*sin(4*d*x + 4*c)^2
 - 160*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) - 10*a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 3*(a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - (160*a^2*cos(4*d*x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d
*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 + 1
20*a^2*cos(4*d*x + 4*c)^2 - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
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(4*d*x + 4*c)))^3 - 3*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 + 74*a^2*cos(4*d*x + 4*c
)^2 - 197*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) + 80*a^2)*sin(4*d*
x + 4*c)^2 + 120*a^2 - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
- 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(a^2*
cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3
 + 126*a^2*cos(4*d*x + 4*c)^2 + 243*a^2*cos(4*d*x + 4*c) + 3*(a^2*cos(4*d*x
 + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2*cos(4*d*x + 4*c)^2
 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(
4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*(6*a^2*cos(4*d*x + 4*c)^3 + 214*a^2*cos(4*d*x + 4*c)^2 - 3*a^2*si
n(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 240*a
^2*cos(4*d*x + 4*c) + 2*(3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c)
^2 - (160*a^2*cos(4*d*x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 - 157*a^2*cos
(4*d*x + 4*c) - 3*a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (80*a^2*cos(4*d*x
+ 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*cos(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(320*a^2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 157*a^2*cos(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(80*a^2*cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a^2*cos
(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - 6*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c) +
 3*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x
+ 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d
*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^
2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*co
s(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4
*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
+ a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*

x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/
4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*co
s(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
 + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
- a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x
 + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)
*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) -
 (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)
^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*s
in(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*s
in(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2

```
 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/
2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c
)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2
*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d
*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*ar
ctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*
d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d
*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos
(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x +
4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*
c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x
+ 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))))/d
```

Fricas [A] time = 2.00076, size = 478, normalized size = 2.11

$$\frac{\left(48\,Ba^{2}\cos\left(dx+c\right)^{3}+8\,(8\,A+23\,B)a^{2}\cos\left(dx+c\right)^{2}+2\,(136\,A+163\,B)a^{2}\cos\left(dx+c\right)+3\,(200\,A+163\,B)a^{2}\right)\sqrt{a\cos}}{192\,(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*a^2*cos(d*x + c)^3 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^2 + 2*(13
6*A + 163*B)*a^2*cos(d*x + c) + 3*(200*A + 163*B)*a^2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((200*A + 163*B)*a^2*cos(d*x + c)
```

```
+ (200*A + 163*B)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.184 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{\cos(c+a}}{4d}$$

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^3*(54*A + 49*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) + (a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.547004, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2976, 2981, 2774, 216}

$$\frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{\cos(c+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```



```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^3*(54*A + 49*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) + (a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{(a+a\cos(c$$

$$= \frac{a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{aB\sqrt{\cos(}}{}$$

$$= \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(2A+3B)\sqrt{\cos(c+dx)}}{}$$

$$= \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(2A+3B)\sqrt{\cos(c+dx)}}{}$$

$$= \frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}}{24d\sqrt{a+a\cos(c+}}$$

Mathematica [A] time = 0.731708, size = 121, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(38A+25B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(2\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(38*A + 25*B)*A
rcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(66*A + 79*B + 2*(6*
A + 17*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.708, size = 357, normalized size = 2.

$$-\frac{a^2(-1+\cos(dx+c))}{24\,d\,(\sin(dx+c))^2}\left(12\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^2+78\,A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*cos(d*x+c)^2+78*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+
c)+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+66*A*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+34*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2+75*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)+114*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos
(d*x+c))*cos(d*x+c)+75*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 4.6714, size = 4146, normalized size = 23.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
```

+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*
(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c)
- (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*
a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a

```
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a
^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1
))*sqrt(a))*B)/d
```

Fricas [A] time = 1.67226, size = 414, normalized size = 2.3

$$\frac{\left(8\,Ba^2\cos\left(dx+c\right)^2+2\left(6\,A+17\,B\right)a^2\cos\left(dx+c\right)+3\left(22\,A+25\,B\right)a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 3*(22*A +
```

```
 25*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((
38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arctan(sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)
 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

3.185 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a}}{2d}$$

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) - (a^3*(4*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Cos[c + d*x]]) - (a^2*(4*A - B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.552139, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) - (a^3*(4*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Cos[c + d*x]]) - (a^2*(4*A - B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{1}{2}a\right.}{\sqrt{}}$$

$$= -\frac{a^2(4A-B)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{2aA(a+a}{}$$

$$= -\frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(4A-B)\sqrt{\cos(c+dx)}\sqrt{a}}{2d}$$

$$= -\frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(4A-B)\sqrt{\cos(c+dx)}\sqrt{a}}{2d}$$

$$= \frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\sin(}{4d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.66313, size = 126, normalized size = 0.71

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(20A+19B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)((4A\right.}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(20*A + 19*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*A + B + (4*A + 11*B
)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Cos[c + d
*x]])
```

Maple [B] time = 0.689, size = 336, normalized size = 1.9

$$\frac{a^2}{4\,d\,(1+\cos(dx+c))}\sqrt{a\,(1+\cos(dx+c))}\left(20\,A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(20*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+2*B*
sin(d*x+c)*cos(d*x+c)^2+19*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+4*A*cos(d*x+
c)*sin(d*x+c)+20*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+11*B*sin(d*x+c)*cos(d*x+c)+19*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))+8*A*sin(d*x+c))*a^2/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [B] time = 3.37, size = 2808, normalized size = 15.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*
x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
```

```
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 4*(2*
(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 8*(a
^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
(a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4))/d
```

Fricas [A] time = 1.66248, size = 432, normalized size = 2.43

$$\frac{\left(2\,Ba^2\cos\left(dx+c\right)^2+\left(4\,A+11\,B\right)a^2\cos\left(dx+c\right)+8\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(20\,A+19\,B\right.\right.}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - ((20*A + 19*B)*a^2*
cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

3.186 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=173

$$\frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} - \frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2(2A+B)\sin(c+dx)\sqrt{a\cos(c+dx)+}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^3*(14*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.532847, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2981, 2774, 216}

$$\frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} - \frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2(2A+B)\sin(c+dx)\sqrt{a\cos(c+dx)+}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^3*(14*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3}{2}a\right.}{}$$

$$= \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos(c+dx))^{3/}}{3d\cos^{\frac{3}{2}}(c+d}$$

$$= -\frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+a\cos(c+}}{d\sqrt{\cos(c+d}}$$

$$= -\frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+a\cos(c+}}{d\sqrt{\cos(c+d}}$$

$$= \frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\sin(c}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.708525, size = 130, normalized size = 0.75

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(2A+5B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+\sin\left(\frac{1}{2}(c+dx)\right)(4(8A+\right.}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(2*A + 5*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A + 3*B + 4*(8*A + 3*
B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d*Cos[c + d*x
]^(3/2))
```

Maple [B] time = 0.514, size = 484, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*sin(d*x+c)^2*(a*(1+cos(d*x+c)))^(1/2)*(6*A*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))+15*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+12*A*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))+30*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*sin(d*x+c)*cos(d*x+c)^2+16*A*cos(d*x
+c)*sin(d*x+c)+6*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*a^2/(-1+cos(d*x+c)
)/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.88245, size = 3200, normalized size = 18.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
```

```
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqr
t(a) + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4) + 2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(2*d*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x
 + 2*c) + 4*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x +
2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x
 + 2*c) - a^2 + 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*sqrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2
+ 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x +
2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2
*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
```

```
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.5839, size = 436, normalized size = 2.52

$$\frac{\left(3\,Ba^2\cos\left(dx+c\right)^2+2\,(8\,A+3\,B)a^2\cos\left(dx+c\right)+2\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left((2\,A+5\,B\right.}{3\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^2)*sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((2*A + 5*B)*a^2
*cos(d*x + c)^3 + (2*A + 5*B)*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^
3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

3.187 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\cdots$$

```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(32*A + 35*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d
*Cos[c + d*x]^(3/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d
*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.506073, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2774, 216}

$$\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(32*A + 35*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d
*Cos[c + d*x]^(3/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d
*Cos[c + d*x]^(5/2))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{1}{2}\right.}{\cos}$$

$$= \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx}{5d\cos^{\frac{5}{2}}(c}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+}}{15d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+}}{15d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} +$$

Mathematica [A] time = 0.725331, size = 130, normalized size = 0.76

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(14A+5B)\cos(c+dx)+(43A+40B)\cos(2(c+dx))+49A\right.}{30d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sqrt[2]*B*ArcSin[Sqrt[
2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40*B + 2*(14*A + 5*B)*C
os[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(30*d*Cos[
c + d*x]^(5/2))
```

Maple [B] time = 0.674, size = 306, normalized size = 1.8

$$-\frac{2a^2}{15d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-15B\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{}{1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*a^2*(a*(1+cos(d*x+c)))^(1/2)*(-15*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))-30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-15*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))+43*A*cos(d*x+c)^3+40*B*cos(d*x+c)^3-29*A*cos(
d*x+c)^2-35*B*cos(d*x+c)^2-11*A*cos(d*x+c)-5*B*cos(d*x+c)-3*A)/sin(d*x+c)/c
os(d*x+c)^(5/2)
```

Maxima [B] time = 3.19795, size = 2090, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/30*(5*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2
*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3
*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
```

```
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqr
t(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 16*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)))/d
```

Fricas [A] time = 1.34738, size = 424, normalized size = 2.47

$$\frac{2\left(\left((43\,A+40\,B)a^2\cos\left(dx+c\right)^2+(14\,A+5\,B)a^2\cos\left(dx+c\right)+3\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\right.}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(((43*A + 40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + 3
*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*(B*a^
2*cos(d*x + c)^4 + B*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^4 + d*cos
(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2
), x)
```

3.188 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3(230A+301B)\sin(c+d}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}$$

```
[Out] (2*a^3*(10*A + 11*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^3*(230*A + 301*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.55189, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2975, 2980, 2771}

$$\frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3(230A+301B)\sin(c+d}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(10*A + 11*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^3*(230*A + 301*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{1}{2}a\right.}{}$$

$$= \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx)}{7d\cos^{\frac{7}{2}}(c+}$$

$$= \frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+a}}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(230A+301B)\sin(c+d}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.600312, size = 104, normalized size = 0.57

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((930A+987B)\cos(c+dx)+2(115A+98B)\cos(2(c+dx))+230A\cos(3(c+dx}{210d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d*
x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*Cos
[3*(c + d*x)])*Tan[(c + d*x)/2])/(210*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.601, size = 111, normalized size = 0.6

$$-\frac{2\,a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(230\,A\left(\cos\left(dx+c\right)\right)^{3}+301\,B\left(\cos\left(dx+c\right)\right)^{3}+115\,A\left(\cos\left(dx+c\right)\right)^{2}+98\,B\left(\cos\left(dx+c\right)\right)^{2}+\cdots\right.}{105\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(230*A*cos(d*x+c)^3+301*B*cos(d*x+c)^3+115*A*c
os(d*x+c)^2+98*B*cos(d*x+c)^2+60*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+c
os(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 2.00511, size = 535, normalized size = 2.96

$$\frac{8\left(\frac{7\left(\frac{15\,\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{28\,\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{8\,\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)B}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}+\frac{5\left(\frac{21\,\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\,\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\,\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{36\,\sqrt{2}a^{\frac{5}{2}}\cdots}{\cdots}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(\cdots}{(\cos(dx\cdots}\right.}\cdots\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 8/105*(7*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2)) + 5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c)
 + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)
```

```
*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1
)))/d
```

Fricas [A] time = 1.26924, size = 297, normalized size = 1.64

$$\frac{2\left((230\,A+301\,B)a^2\cos\left(dx+c\right)^3+(115\,A+98\,B)a^2\cos\left(dx+c\right)^2+3\,(20\,A+7\,B)a^2\cos\left(dx+c\right)+15\,Aa^2\right)\sqrt{a\cos\left(dx\right.}}{105\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c)
^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2
), x)
```

3.189 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^3(292A+345B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)}}{21d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.7035, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{2a^3(292A+345B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)}}{21d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3}{2}a\right.}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))}{9d\cos^{\frac{9}{2}}(c+}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+}}{21d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(292A+345B)\sin(c+}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(}}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(292A+345B)\sin(c+}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(}}$$

Mathematica [A] time = 0.829134, size = 126, normalized size = 0.55

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((1396A+1215B)\cos(c+dx)+2(803A+870B)\cos(2(c+dx))+292A\cos(3(c+}{630d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
+ d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 345*
B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Tan[(
c + d*x)/2])/(630*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.621, size = 133, normalized size = 0.6

$$-\frac{2a^2(-1+\cos(dx+c))\left(584A(\cos(dx+c))^4+690B(\cos(dx+c))^4+292A(\cos(dx+c))^3+345B(\cos(dx+c))^3+\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+292*A*c
os(d*x+c)^3+345*B*cos(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+130*A*
cos(d*x+c)+45*B*cos(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*
x+c)^(9/2)
```

Maxima [B] time = 2.45213, size = 720, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 8/315*(15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + (315*sqrt(2)*a
^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^(5/2)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 572*sqrt
(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*sqrt(2)*a^(5/2)*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.19209, size = 351, normalized size = 1.54

$$\frac{2\left(2\,(292\,A+345\,B)a^2\cos\left(dx+c\right)^4+(292\,A+345\,B)a^2\cos\left(dx+c\right)^3+3\,(73\,A+60\,B)a^2\cos\left(dx+c\right)^2+5\,(26\,A+9\,B\right.}{315\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x + c
) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*c
os(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/
2), x)
```

3.190 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=275

$$\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+}$$

```
[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(
5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^3*(710*A + 803*B)*S
in[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/
2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(1
1/2))
```

Rubi [A] time = 0.713863, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x
]
```


```
[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(
5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^3*(710*A + 803*B)*S
in[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/
2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(1
1/2))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+a\cos(c+dx))^{3/2}}{}$$

$$= \frac{2a^2(14A+11B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+}{11d\cos^{\frac{11}{2}}}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(14A+11B)\sqrt{a+a\cos(}{99d\cos^{\frac{9}{2}}(c+}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A+803B)\sin(c}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A+803B)\sin(c}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A+803B)\sin(c}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.959139, size = 147, normalized size = 0.53

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((25070A+24827B)\cos(c+dx)+(9230A+9284B)\cos(2(c+dx))+9230A\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos[
c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] + 1
0439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*x)]
 + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(69
30*d*Cos[c + d*x]^(11/2))
```

Maple [A] time = 0.635, size = 155, normalized size = 0.6

$$-\frac{2a^2(-1+\cos(dx+c))\left(5680A(\cos(dx+c))^5+6424B(\cos(dx+c))^5+2840A(\cos(dx+c))^4+3212B(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+2840
*A*cos(d*x+c)^4+3212*B*cos(d*x+c)^4+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3
+1775*A*cos(d*x+c)^2+1430*B*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*x+c)
+315*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 2.35852, size = 845, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 8/3465*(11*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(
2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 1)) + 5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1
) - 2310*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)
*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x
 + c) + 1)^9 - 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 +
 200*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.12629, size = 414, normalized size = 1.51

$$\frac{2\left(8\,(710\,A + 803\,B)a^{2}\cos\left(dx + c\right)^{5} + 4\,(710\,A + 803\,B)a^{2}\cos\left(dx + c\right)^{4} + 3\,(710\,A + 803\,B)a^{2}\cos\left(dx + c\right)^{3} + 5\,(355\,A\right.}{3465\left(d\cos\left(dx + c\right)^{7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d*
x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*cos
(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*x +
c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx + c\right) + A)(a\cos\left(dx + c\right) + a)^{\frac{5}{2}}}{\cos\left(dx + c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/
2), x)
```

3.191 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=190

$$-\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+$$

```
[Out] -((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((4*A - B)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.594787, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((4*A - B)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aB}{2}+\frac{1}{2}a(4A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\frac{1}{4}a^2(4A-B)-}{\sqrt{\cos(c+}}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(4A-7B)\int}{}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{(4A-7B)\,\mathrm{S}}{}$$

$$= -\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right.}{\sqrt{a}d}$$

Mathematica [C] time = 1.82602, size = 348, normalized size = 1.83

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(4A+2B\cos(c+dx)-B)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-8i\sqrt{2}(A-B)\log\left(1+e^{i(c+dx)}\right)+i(4A-7B)\sinh^{-1}(\right.}{}\right.}{8\sqrt{a(\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(-4*A*d*x + 7*B*d*x + I*(4*A - 7*B)*ArcSinh[E^(I*(c +
d*x))] - (8*I)*Sqrt[2]*(A - B)*Log[1 + E^(I*(c + d*x))] - (4*I)*A*Log[1 + S
qrt[1 + E^((2*I)*(c + d*x))]] + (7*I)*B*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x)
)]] + (8*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))]] - (8*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^
((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (4*Sqrt[Cos[c + d
*x]]*(4*A - B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2])/d))/(8*Sqrt[a*(1 + Cos[
c + d*x])])
```

Maple [B] time = 0.73, size = 346, normalized size = 1.8

$$\frac{(-1+\cos(dx+c))^3}{4d(\sin(dx+c))^6 a}(\cos(dx+c))^{\frac{3}{2}}\sqrt{a(1+\cos(dx+c))}\left(-4A\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\cos(dx+c)-4A\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(-4*A*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)-4*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^2+4*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)-4*B
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)+B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)-7*B*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))/sin(d*x+c)^6/(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

Fricas [A] time = 20.8539, size = 516, normalized size = 2.72

$$\frac{(2B\cos(dx+c)+4A-B)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)+((4A-7B)\cos(dx+c)+4A-7B)\sqrt{a}\,\mathrm{arc}}{4(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*cos(d*x + c) + 4*A - B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c) + ((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2
)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) +
 a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

3.192 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a
]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.39654, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a
]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\frac{aB}{2}+\frac{1}{2}a(2A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(2A-B)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a} + (-A+B)\int \frac{}{\sqrt{co}}$$

$$= \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(2a(A-B))\operatorname{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin}{\sqrt{\cos(c+dx)}}\right.}{d}$$

$$= \frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] time = 1.21748, size = 222, normalized size = 1.57

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4B\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}}{d} - \frac{i\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left((2A-B)\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}(A-B)\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+(B\right.}{d\sqrt{1+e^{2i(c+dx)}}}\right.}{2\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*(((-I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c
+ d*x)))/E^(I*(c + d*x))]*((2*A - B)*ArcSinh[E^(I*(c + d*x))] + 2*Sqrt[2]*(
A - B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]
)] + (-2*A + B)*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*
I)*(c + d*x))]) + (4*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2])/d))/(2*Sqrt[a*(
1 + Cos[c + d*x])])
```

Maple [A] time = 0.62, size = 216, normalized size = 1.5

$$\frac{(-1+\cos(dx+c))^2}{d(\sin(dx+c))^4 a}(\cos(dx+c))^{\frac{3}{2}}\sqrt{a(1+\cos(dx+c))}\left(A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2} - B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(A*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-B*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.9809, size = 466, normalized size = 3.3

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}B\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(2\,A-B\right)\cos\left(dx+c\right)+2\,A-B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*co
s(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(a*cos(d*x + c) + a),
 x)
```

3.193 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.242146, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
```

```
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = (A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx + \frac{B\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a}$$

$$= -\frac{(2a(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{(2B)\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.138071, size = 82, normalized size = 0.82

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.749, size = 149, normalized size = 1.5

$$\frac{-1+\cos(dx+c)}{da(\sin(dx+c))^2}\sqrt{a(1+\cos(dx+c))}\sqrt{\cos(dx+c)}\left(A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}-B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(A*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*2^(1/2)-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/
2)-2*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/a/sin(d*x+c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.53816, size = 278, normalized size = 2.78

$$-\frac{\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2B\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*B*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}\sqrt{\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x)
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx + c) + A}{\sqrt{a\cos(dx + c) + a}\sqrt{\cos(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

3.194 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.192438, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2984, 12, 2782, 205}

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2\int -\frac{a(A-B)}{2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + (-A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{(2a(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{}{\sqrt{\cos}}\right.}{d}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+}$$

Mathematica [C] time = 1.52052, size = 203, normalized size = 2.05

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(10B\cos(c+dx)-(A-B)\left(\frac{1}{2}\sin(c+dx)\tan(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\sec(c+dx)\sin^2\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*(10*B*Cos[c + d*x] - (A - B)*((-5*(1 +
 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] +
ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*S
ec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c +
d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Cos[c + d*x]^(3/2)*Sqrt[a*
(1 + Cos[c + d*x])])
```

Maple [B] time = 0.593, size = 230, normalized size = 2.3

$$\frac{1}{da\left(1+\cos\left(dx+c\right)\right)}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(A\arcsin\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)\sqrt{2}\cos\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}-B\arcsin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*(A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*sin
(d*x+c))/a/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28712, size = 396, normalized size = 4.

$$\frac{2\sqrt{a\cos(dx+c)+a}A\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}}\right)}{\sqrt{a}}}{ad\cos(dx+c)^2+ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (2*sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(2)*((A
 - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*
x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a(\cos(c+dx)+1)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/
2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

3.195 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.335527, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2984, 12, 2782, 205}

$$-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```



```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-3B)+aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + (A -$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2a(}{}$$

$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+a}$$

Mathematica [C] time = 6.75526, size = 627, normalized size = 4.42

$$2(A-B)\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-12\left(3\sin^4\left(\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (8*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d
*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) +
 (2*(A - B)*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(-12*Cos[(c + d*x)/2]^4
*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin
[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/
2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]
^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2
 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3
*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 +
(d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*
ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*
Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 +
(d*x)/2]^2)^(7/2))
```

Maple [B] time = 0.609, size = 383, normalized size = 2.7

$$\frac{(\sin(dx+c))^2}{3\,da\,(-1+\cos(dx+c))\,(1+\cos(dx+c))^2}\sqrt{a\,(1+\cos(dx+c))}\left(3\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/3/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(3*A*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)^2-3*B*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d
*x+c)^2+6*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^
```

```
(3/2)*2^(1/2)*cos(d*x+c)-6*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)-3*B*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+2*A*cos(d*x+c)*sin(d*
x+c)-6*B*sin(d*x+c)*cos(d*x+c)-2*A*sin(d*x+c))/a/(-1+cos(d*x+c))/(1+cos(d*x
+c))^2/cos(d*x+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37235, size = 447, normalized size = 3.15

$$-\frac{2\left((A-3B)\cos(dx+c)-A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx+c)^2\right)\arctan\left(\right.}{\sqrt{a}}}{3\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 3*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*
x + c)^2)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x
 + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

3.196 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=187

$$-\frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15*d*C
os[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A - 5*B)*Sin[c + d*x])
/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.608732, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2984, 12, 2782, 205}

$$-\frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15*d*C
os[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A - 5*B)*Sin[c + d*x])
/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
```

```
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-5B)+2aA\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{15}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{15}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{15}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

Mathematica [C] time = 7.91586, size = 1728, normalized size = 9.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(5*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (16*B*Cos[c/2 + (d*x)/2]*(Sin[c/2 +
(d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1
 - 2*Sin[c/2 + (d*x)/2]^2]))/(15*d*Sqrt[a*(1 + Cos[c + d*x])]) - (2*(A - B)
*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48825
*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x
)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2,
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10
- 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 51
8760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 +
```

```
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*S
in[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2
+ (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*
Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d
*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2
]^2)))/(675*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)
*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

Maple [B] time = 0.68, size = 519, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/15/d*sin(d*x+c)^4*(a*(1+cos(d*x+c)))^(1/2)*(15*A*2^(1/2)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-15*B*2^(1
/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+45*A*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcs
in((-1+cos(d*x+c))/sin(d*x+c))-45*B*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+45*A*2^(1/2)*cos(d*x+c)*
```

```
(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-45*B*2
^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+15*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))-15*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))+26*A*sin(d*x+c)*cos(d*x+c)^2-10*B*sin(d*x+c)*cos(
d*x+c)^2-2*A*cos(d*x+c)*sin(d*x+c)+10*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c
))/a/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30428, size = 491, normalized size = 2.63

$$\frac{2\left((13\,A-5\,B)\cos\left(dx+c\right)^{2}-(A-5\,B)\cos\left(dx+c\right)+3\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{15\,\sqrt{2}\left((A-B)a\,\mathrm{co}\right.}{}}{15\left(ad\cos\left(dx+c\right)^{4}+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*
cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c)
)*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(7/2)
), x)
```

3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-(A-$$

```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3
/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((A - 3*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.636338, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-(A-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3
/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((A - 3*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)-a(A-3B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{-\frac{1}{2}a^{\cdots}}{\cdots}}{\cdots}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{(5A-\cdots}{\cdots}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{(5A-\cdots}{\cdots}$$

$$= \frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-9B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 2.10118, size = 362, normalized size = 1.84

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(-A+2B\cos(c+dx)+3B)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(i\sqrt{2}(5A-9B)\log\left(1+e^{i(c+dx)}\right)-\cdots}{\cdots}\right.}{2(a(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(4*A*d*x - 6*B*d*x - (2*I)*(2*A - 3*B)*ArcSinh[E^(I*
(c + d*x))] + I*Sqrt[2]*(5*A - 9*B)*Log[1 + E^(I*(c + d*x))] + (4*I)*A*Log[
1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (6*I)*B*Log[1 + Sqrt[1 + E^((2*I)*(c +
 d*x))]] - (5*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))]] + (9*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1
```

```
 + E^((2*I)*(c + d*x))]]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (2*Sqrt[Cos[
c + d*x]]*(-A + 3*B + 2*B*Cos[c + d*x])*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/
d))/(2*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.602, size = 379, normalized size = 1.9

$$-\frac{(-1+\cos(dx+c))^3}{4\,d\,(\sin(dx+c))^7\,a^2}(\cos(dx+c))^{\frac{3}{2}}\sqrt{a\,(1+\cos(dx+c))}\left(2\,A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^2+5\,A\arcsin\left(\frac{-1\,\cdots}{\mathrm{s}\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+5*A*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^3-9*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos
(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*A*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)-2*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-12*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+6*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/
a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A] time = 40.7271, size = 644, normalized size = 3.27

$$\frac{\sqrt{2}\Big((5\,A-9\,B)\cos\left(dx+c\right)^2+2\,(5\,A-9\,B)\cos\left(dx+c\right)+5\,A-9\,B\Big)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\,(2\,B\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A
 - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) + 2*(2*B*cos(d*x + c) - A + 3*B)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*
(2*A - 3*B)*cos(d*x + c) + 2*A - 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)\cos\left(dx+c\right)^{\frac{3}{2}}}{(a\cos\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

3.198 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.403173, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2977, 2982, 2782, 205, 2774, 216}

$$\frac{(A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+2aB\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A-5B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{4a} + \frac{B \ldots}{\ldots}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-5B)\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\,\mathrm{s}\ldots}{\sqrt{\cos(c+d\ldots}}\right.}{2d}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-\ldots}{2\ldots}$$

Mathematica [C] time = 1.8249, size = 226, normalized size = 1.56

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{(A-B)\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{d} - \frac{ie^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-\sqrt{2}(A-5B)\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+4B\sinh^{-1}\left(e^{\ldots}\right.\right.}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x))]*(4*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(A - 5*B)*ArcTa
nh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 4*B*Arc
Tanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x
))]) + ((A - B)*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/d))/(
a*(1 + Cos[c + d*x]))^(3/2)
```

Maple [B] time = 0.579, size = 298, normalized size = 2.1

$$-\frac{(-1+\cos(dx+c))^2}{4d(\sin(dx+c))^5a^2}\sqrt{\cos(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(2A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^2 + A\arcsin\left(\frac{-1+\cos\ldots}{\sin(d\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+A*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
sin(d*x+c)*2^(1/2)*cos(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-2*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-8*B*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+2*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A] time = 34.5713, size = 571, normalized size = 3.94

$$-\frac{\sqrt{2}\left((A-5B)\cos(dx+c)^2+2(A-5B)\cos(dx+c)+A-5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos(dx}{4\left(a^2d\cos(dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*arctan(s
```

```
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sqrt{\cos(c + dx)}}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3
/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx + c) + A)\sqrt{\cos(dx + c)}}{(a\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

3.199 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.216302, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2978, 12, 2782, 205}

$$\frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{a(3A+B)}{2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$
$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(3A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$
$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{(3A+B)\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx,x,-\frac{\cdots}{\sqrt{\cos(\cdots}}\right)}{2d}$$
$$= \frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+d\cdots}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [C] time = 1.11506, size = 212, normalized size = 1.98

$$\frac{\frac{1}{2}i(A-B)e^{-\frac{1}{2}i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\sqrt{\cos(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)+i(3A+B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\cdots}{d\sqrt{1+e^{2i(c+dx)}}(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (I*(3*A + B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d
*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])
]*Cos[(c + d*x)/2]^3 + ((I/2)*(A - B)*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]])/E^((I/2)*(c + d*x)))/(
d*Sqrt[1 + E^((2*I)*(c + d*x))]*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.55, size = 247, normalized size = 2.3

$$-\frac{-1+\cos(dx+c)}{4\,a^2d\,(\sin(dx+c))^3}\sqrt{a\,(1+\cos(dx+c))}\left(2\,A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^2-3\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*cos(d*x+c)^2-3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*
2^(1/2)*cos(d*x+c)-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*
cos(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-2*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*cos(d*x+c)^2+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/a^2/cos(d*x+c)^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [A] time = 1.64034, size = 447, normalized size = 4.18

$$\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\sqrt{a\cdots}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A + B
)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x
+ c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x
+ c) + a)*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d
*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

3.200 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a}$$

```
[Out] -((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*
d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A - B)*Sin[c + d*x])
/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.384014, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$-\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```



```
[Out] -((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*
d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A - B)*Sin[c + d*x])
/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A-B)-a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+$$

Mathematica [C] time = 3.68397, size = 423, normalized size = 2.71

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{(A+3B)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx)\right.\right.\right.\right.}{2\cos^{\frac{3}{2}}(c+dx)}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(30*(A - B)*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c
+ d*x]]] - 30*(A - B)*ArcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] -
 (20*(A - B)*Sqrt[Cos[c + d*x]])/(-1 + Sin[(c + d*x)/2]) - (20*(A - B)*Sqrt
[Cos[c + d*x]])/(1 + Sin[(c + d*x)/2]) + (5*(A - B)*(-1 + 2*Sin[(c + d*x)/2
]))/(Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2) - (5*(A -
B)*(1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(-1 + Sin[(c + d*x)/2])) +
 ((A + 3*B)*Csc[(c + d*x)/2]^3*(5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(
1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c
+ d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c
 + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))
/(2*Cos[c + d*x]^(3/2))))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.577, size = 299, normalized size = 1.9

$$\frac{1}{4\,a^2 d \sin\left(dx+c\right)\left(1+\cos\left(dx+c\right)\right)}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(7\,A\sin\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\cos\left(dx+c\right)\arcsin\left(\frac{-1+\cdots}{\sin\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(7*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-3*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*2^(1/2)+7*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*2^(1/2)-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-10*A*cos(d*x+c)^2+2*B*cos(d*
x+c)^2+2*A*cos(d*x+c)-2*B*cos(d*x+c)+8*A)/a^2/sin(d*x+c)/(1+cos(d*x+c))/cos
(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.74568, size = 531, normalized size = 3.4

$$-\frac{\sqrt{2}\left(\left(7\,A-3\,B\right)\cos\left(dx+c\right)^3+2\left(7\,A-3\,B\right)\cos\left(dx+c\right)^2+\left(7\,A-3\,B\right)\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos\cdots}}{2\left(a\cos(dx+c)^2+a\cos\cdots\right.}\right.}{4\left(a^2 d\cos\left(dx+c\right)^3+2\,a^2 d\cos\left(dx+c\right)^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^3 + 2*(7*A - 3*B)*cos(d*x + c)^2 +
(7*A - 3*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) +
a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x +
c))) - 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

3.201 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+}$$

```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*
d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B)*Sin[c + d*x
])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((19*A - 15*B)*Sin
[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.555461, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*
d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B)*Sin[c + d*x
])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((19*A - 15*B)*Sin
[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A-3B)-2a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}}$$

$$= \frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c}$$

Mathematica [C] time = 6.80029, size = 1054, normalized size = 5.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] -((A - B)*Cos[c/2 + (d*x)/2]^3*(1 - 2*Sin[c/2 + (d*x)/2]))/(6*d*(a*(1 + Cos
[c + d*x]))^(3/2)*(1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/
2)) + ((A - B)*Cos[c/2 + (d*x)/2]^3*(1 + 2*Sin[c/2 + (d*x)/2]))/(6*d*(a*(1
+ Cos[c + d*x]))^(3/2)*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(3/2)) - ((A - B)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2
])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 + Sin[c/2 + (d*x)/2])/((1 - Sin[c
/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 +
(d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))^(3/2)) +
((A - B)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 -
 2*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/
```

```
2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]
)/(1 + Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((A + 3*B)*
Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^2*(-12*Cos[(c + d*x)/2]^4*Hypergeom
etricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*
x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[c
/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*
Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2
]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*
Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2
)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sq
rt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 +
(d*x)/2]^2))))/(63*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]
^2)^(7/2))
```

Maple [B] time = 0.623, size = 443, normalized size = 2.2

$$\frac{\sin(dx+c)}{12\,a^2d\,(-1+\cos(dx+c))\,(1+\cos(dx+c))^2}\sqrt{a\,(1+\cos(dx+c))}\left(33\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2\sqrt{2}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/12/d*sin(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(33*A*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-
21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)+66*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(
d*x+c)*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-42*B*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-38*A*cos(d*x+c)^3+30*B*co
s(d*x+c)^3+14*A*cos(d*x+c)^2-6*B*cos(d*x+c)^2+32*A*cos(d*x+c)-24*B*cos(d*x+
c)-8*A)/a^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.72612, size = 586, normalized size = 2.89

$$\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^4+2(11A-7B)\cos(dx+c)^3+(11A-7B)\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\cdots}{2\left(a\cos(dx+c)\cdots\right.}\right.}{12\left(a^2d\cos(dx+c)^4+2a^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^4 + 2*(11*A - 7*B)*cos(d*x + c)^
3 + (11*A - 7*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos
(d*x + c))) - 2*((19*A - 15*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) - 4
*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*
x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

3.202 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(43A-115B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5
/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((7*A - 15*B
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((
11*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rubi [A] time = 0.836839, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(43A-115B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5
/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((7*A - 15*B
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((
11*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-a(A-5B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \int$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - (1$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - (1$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - (1$$

$$= \frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-115B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+}\right.}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 3.30646, size = 376, normalized size = 1.53

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((55B-15A)\cos(c+dx)-11A+8B\cos(2(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(32*A*d*x - 80*B*d*x - (16*I)*(2*A - 5*B)*ArcSinh[E^
(I*(c + d*x))] + I*Sqrt[2]*(43*A - 115*B)*Log[1 + E^(I*(c + d*x))] + (32*I)
*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (80*I)*B*Log[1 + Sqrt[1 + E^((2
*I)*(c + d*x))]] - (43*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[
1 + E^((2*I)*(c + d*x))]] + (115*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] + Sqrt[
Cos[c + d*x]]*(-11*A + 43*B + (-15*A + 55*B)*Cos[c + d*x] + 8*B*Cos[2*(c +
d*x)])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5
/2))
```

Maple [B] time = 0.632, size = 647, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/32/d*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*(30*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+43*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+22*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)*cos(d*x+c)^2-32*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-115
*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+64*A*
arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^
2*sin(d*x+c)+43*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos
(d*x+c)-30*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-160*B*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c
)-78*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-115*B*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+64*A*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)-22*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)-160*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+40*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)^2+70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/
sin(d*x+c)^11/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [A] time = 98.4708, size = 830, normalized size = 3.37

$$\frac{\sqrt{2}\left((43\,A-115\,B)\cos\left(dx+c\right)^{3}+3\,(43\,A-115\,B)\cos\left(dx+c\right)^{2}+3\,(43\,A-115\,B)\cos\left(dx+c\right)+43\,A-115\,B\right)\sqrt{a}\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c
)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(16
*B*cos(d*x + c)^2 - 5*(3*A - 11*B)*cos(d*x + c) - 11*A + 35*B)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 32*((2*A - 5*B)*cos(d*x + c)
^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B)
*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

3.203 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{(3A-43B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(3A-11}{16a}$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*A - 11*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.582239, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2977, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-43B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(3A-11}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*A - 11*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)+4aB\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int}{}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(3}{}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(3}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A}{}$$

Mathematica [C] time = 2.03008, size = 246, normalized size = 1.27

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((7A-15B)\cos(c+dx)+3A-11B) - \frac{i\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c-}}}{}\right.}{8d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(
c + d*x)))/E^(I*(c + d*x))]*(32*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(3*A -
 43*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]
)] - 32*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d
*x))] + Sqrt[Cos[c + d*x]]*(3*A - 11*B + (7*A - 15*B)*Cos[c + d*x])*Sec[(c
+ d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.593, size = 515, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/32/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(14*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+3*A*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*cos(d*x+c)^2-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c
)^2*sin(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*co
s(d*x+c)-14*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-43*B*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-64*B*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-30*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-6*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)-64*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))*sin(d*x+c)*cos(d*x+c)+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)^2+22*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^9/(cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [A] time = 69.3504, size = 736, normalized size = 3.79

$$-\frac{\sqrt{2}\left((3A-43B)\cos(dx+c)^3+3(3A-43B)\cos(dx+c)^2+3(3A-43B)\cos(dx+c)+3A-43B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2
 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((7*A - 15
*B)*cos(d*x + c) + 3*A - 11*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) + 64*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c)
 + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

3.204 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((A + 7*B)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.376915, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2978, 12, 2782, 205}

$$\frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((A + 7*B)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+a(A+3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \overline{4\sqrt{\cdots}}}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(5A\cdots}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(5A\cdots}{}$$

$$= \frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx\cdots}{4d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [C] time = 1.40628, size = 198, normalized size = 1.29

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((A+7B)\cos(c+dx)+5A+3B)+\frac{i(5A+3B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i\cdots}}}{\cdots}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(5*A + 3*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
+ E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c + d*x
]]*(5*A + 3*B + (A + 7*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]
)/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.563, size = 413, normalized size = 2.7

$$\frac{(-1+\cos(dx+c))^3}{32\,d\,(\sin(dx+c))^7\,a^3}\sqrt{\cos(dx+c)}\sqrt{a\,(1+\cos(dx+c))}\left(2\,A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^3+10\,A\left(\frac{\cos(dx+c)}{1+\cos(dx+\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+10*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*cos(d*x+c)^2+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x
+c)^2*sin(d*x+c)+3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^
2*sin(d*x+c)-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+5*A*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+14*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c)*2^(1/2)*cos(d*x+c)-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [A] time = 1.3748, size = 571, normalized size = 3.71

$$\frac{\sqrt{2}\Big((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\Big)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos}}{2(}\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
+ a*cos(d*x + c))) + 2*((A + 7*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

3.205 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.439014, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2978, 12, 2782, 205}

$$\frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```


```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{\int \frac{\frac{1}{2}a(7A+B)-a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(9A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{\int}{}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(9A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{(19}{}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(9A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}-\frac{(19}{}$$

$$= \frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+}{4d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [C] time = 1.36854, size = 200, normalized size = 1.28

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{2}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((9A-B)\cos(c+dx)+13A-5B)+\frac{i(19A+5B)e^{\frac{1}{2}i(c+dx)}}{}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(19*A + 5*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[c + d*
x]]*(13*A - 5*B + (9*A - B)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/
2])/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.561, size = 413, normalized size = 2.7

$$\frac{(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\sqrt{a(1+\cos(dx+c))}\left(18\,A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^3-19\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(18*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)*cos(d*x+c)^3-19*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)
*cos(d*x+c)^2*sin(d*x+c)+26*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^
2-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-19
*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-18*A*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*cos(d*x+c)^3-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2
)*cos(d*x+c)-26*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2+10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/a^3/cos(d*x+c)^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [A] time = 1.38651, size = 578, normalized size = 3.71

$$\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}\cdots}{\cdots}\right)\cdots}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
+ 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c
)^2 + a*cos(d*x + c))) - 2*((9*A - B)*cos(d*x + c) + 13*A - 5*B)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

3.206 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a\cos(c+dx)$$

```
[Out] -((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/
(4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 5*B)*Sin[c +
 d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((49*A - 9*
B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.56656, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a\cos(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```



```
[Out] -((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/
(4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 5*B)*Sin[c +
 d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((49*A - 9*
B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A-B)-2a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx}{4d\sqrt{\cos(c+dx)}(a+a\cos(}$$

Mathematica [C] time = 2.5741, size = 217, normalized size = 1.07

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(2(85A-13B)\cos(c+dx)+(49A-9B)\cos(2(c+dx))+113A-9B)}{4\sqrt{\cos(c+dx)}} - \frac{i(75A-19B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx}\right.}}{\sqrt{1+e^{2i(c+dx)}}}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*(75*A - 19*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((
2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((113*A - 9*
B + 2*(85*A - 13*B)*Cos[c + d*x] + (49*A - 9*B)*Cos[2*(c + d*x)])*Sec[(c +
d*x)/2]^3*Tan[(c + d*x)/2])/(4*Sqrt[Cos[c + d*x]])))/(4*d*(a*(1 + Cos[c + d
*x]))^(5/2))
```

Maple [B] time = 0.632, size = 443, normalized size = 2.2

$$\frac{-1+\cos(dx+c)}{32\,da^3(\sin(dx+c))^3(1+\cos(dx+c))}\sqrt{a(1+\cos(dx+c))}\left(-75\,A(\cos(dx+c))^2\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(-75*A*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))+19*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2
)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-150*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+38*B*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*2^(1/2)-75*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+19*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+98*A*cos(d*x+c)^3-
18*B*cos(d*x+c)^3+72*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2-106*A*cos(d*x+c)+26*B*
cos(d*x+c)-64*A)/a^3/sin(d*x+c)^3/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.40351, size = 666, normalized size = 3.28

$$-\frac{\sqrt{2}\left((75\,A-19\,B)\cos(dx+c)^4+3\,(75\,A-19\,B)\cos(dx+c)^3+3\,(75\,A-19\,B)\cos(dx+c)^2+(75\,A-19\,B)\cos(dx\right.}{32\left(a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^4 + 3*(75*A - 19*B)*cos(d*x + c)
^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + (75*A - 19*B)*cos(d*x + c))*sqrt(a)*a
rctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((49*A - 9*B)*cos(d*x + c)
^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

3.207 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{(95A-39B)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sin[c +
 d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39
*B)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) -
((299*A - 147*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])
```

Rubi [A] time = 0.751786, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(95A-39B)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sin[c +
 d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39
*B)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) -
((299*A - 147*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(11A-3B)-3a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+d}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+d}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+d}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+d}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+d}$$

$$= \frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+a}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

Mathematica [C] time = 3.33274, size = 239, normalized size = 0.96

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((1537A-825B)\cos(c+dx)+2(503A-255B)\cos(2(c+dx))+299A\cos(3(c+dx))+878A-147B\cos(3(c+dx)}{8\cos^{\frac{3}{2}}(c+dx)}\right.}{12d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((3*I)*(163*A - 75*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^
((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((878*A -
```

```
510*B + (1537*A - 825*B)*Cos[c + d*x] + 2*(503*A - 255*B)*Cos[2*(c + d*x)]
+ 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(
c + d*x)/2])/(8*Cos[c + d*x]^(3/2))))/(12*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.527, size = 571, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/96/d*(a*(1+cos(d*x+c)))^(1/2)*(489*A*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-225*B*sin
(d*x+c)*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))+1467*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)
^2*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-675*B*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)+1467*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-675*B*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+4
89*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)-225*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-598*A*cos(d*x+c)^4+294*B*cos(d*x+c)^4-
408*A*cos(d*x+c)^3+216*B*cos(d*x+c)^3+686*A*cos(d*x+c)^2-318*B*cos(d*x+c)^2
+384*A*cos(d*x+c)-192*B*cos(d*x+c)-64*A)/a^3/sin(d*x+c)/(1+cos(d*x+c))^2/co
s(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.44291, size = 728, normalized size = 2.91

$$3\sqrt{2}\Big((163\,A-75\,B)\cos(dx+c)^5+3\,(163\,A-75\,B)\cos(dx+c)^4+3\,(163\,A-75\,B)\cos(dx+c)^3+(163\,A-75\,B)\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^5 + 3*(163*A - 75*B)*cos(d*x +
 c)^4 + 3*(163*A - 75*B)*cos(d*x + c)^3 + (163*A - 75*B)*cos(d*x + c)^2)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((299*A - 147*B)*co
s(d*x + c)^3 + (503*A - 255*B)*cos(d*x + c)^2 + 32*(5*A - 3*B)*cos(d*x + c)
 - 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos
(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

3.208 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=293

$$\frac{(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{7(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}-\text{(1}$$

```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7
/2)*d) - ((177*A - 637*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[
c + d*x]^(7/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*A - 7*B
)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((
79*A - 259*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*
x])^(3/2)) - (7*(7*A - 27*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.04402, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{7(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}-\text{(1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7
/2)*d) - ((177*A - 637*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[
c + d*x]^(7/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*A - 7*B
)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((
79*A - 259*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*
x])^(3/2)) - (7*(7*A - 27*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-a(A-7B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \int$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + (79$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + (79$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + (79$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + (79$$

$$= \frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} - \frac{(177A-637B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(}}\right.}{64\sqrt{2}a^{7/2}d}$$

Mathematica [C] time = 5.51663, size = 396, normalized size = 1.35

$$\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)((3172B-724A)\cos(c+dx)+(1099B-247A)\cos(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((3*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]*(128*A*d*x - 448*B*d*x - (64*I)*(2*A - 7*B)*ArcSin
h[E^(I*(c + d*x))] + I*Sqrt[2]*(177*A - 637*B)*Log[1 + E^(I*(c + d*x))] + (
128*I)*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (448*I)*B*Log[1 + Sqrt[1
+ E^((2*I)*(c + d*x))]] - (177*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))]] + (637*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x
)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))]
 + (Sqrt[Cos[c + d*x]]*(-541*A + 2233*B + (-724*A + 3172*B)*Cos[c + d*x] +
(-247*A + 1099*B)*Cos[2*(c + d*x)] + 96*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2
]^5*Tan[(c + d*x)/2])/4))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.665, size = 887, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*cos(d*x+c)^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*(494*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+531*A*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+724*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*cos(d*x+c)^3-384*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
5-1911*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)
+768*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(
d*x+c)^3*sin(d*x+c)+1062*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d
*x+c)^2*sin(d*x+c)-200*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2-181
4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-2688*B*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-3822
*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+1536*
A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c
)^2*sin(d*x+c)+531*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*
cos(d*x+c)-724*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-686*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-5376*B*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-1911*B*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+768*A*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)-2
94*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1750*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)^2-2688*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
```

```
)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+1134*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c))/sin(d*x+c)^15/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/a^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 141.259, size = 1018, normalized size = 3.47

$$\frac{3\sqrt{2}\Big((177\,A-637\,B)\cos(dx+c)^4+4\,(177\,A-637\,B)\cos(dx+c)^3+6\,(177\,A-637\,B)\cos(dx+c)^2+4\,(177\,A-637}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(d*
x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x + c
) + 177*A - 637*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(192*B*cos(d*x + c)^3 - (247*A - 109
9*B)*cos(d*x + c)^2 - 2*(181*A - 721*B)*cos(d*x + c) - 147*A + 567*B)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 384*((2*A - 7*B)*cos(
d*x + c)^4 + 4*(2*A - 7*B)*cos(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 +
4*(2*A - 7*B)*cos(d*x + c) + 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

3.209 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}} dx$

Optimal. Leaf size=241

$$\frac{(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(5A-177B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{(A$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) +
 ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((5*A - 17*B)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A - 49*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.765032, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(5A-177B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```


```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) +
 ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((5*A - 17*B)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A - 49*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
```

```
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)+6aB\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}dx}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \int$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(5}{}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(5}{}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(5}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} + \frac{(5A-177B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{(}{}$$

Mathematica [C] time = 3.03985, size = 266, normalized size = 1.1

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(4(25A-181B)\cos(c+dx)+(67A-247B)\cos(2(c\right.}{48d(a(\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-3*I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]*(128*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(5*
A - 177*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*
x))])] - 128*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(
c + d*x))] + (Sqrt[Cos[c + d*x]]*(97*A - 541*B + 4*(25*A - 181*B)*Cos[c + d
*x] + (67*A - 247*B)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])
/4))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.635, size = 703, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)^(5/2)*(134*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+15*A*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+100*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*cos(d*x+c)^3-531*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos
(d*x+c)^3*sin(d*x+c)+30*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*
x+c)^2*sin(d*x+c)-104*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2-494*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-768*B*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-1062*B*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*A*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-100*A*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-230*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^3-1536*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-531*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)*2^(1/2)*cos(d*x+c)-30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+430*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-768*B*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+294*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^13/(cos(d*x+c)/(1+cos
(d*x+c)))^(7/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A] time = 98.4538, size = 905, normalized size = 3.76

$$-\frac{3\sqrt{2}\left((5A-177B)\cos(dx+c)^4+4(5A-177B)\cos(dx+c)^3+6(5A-177B)\cos(dx+c)^2+4(5A-177B)\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x +
 c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5*A
 - 177*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) - 2*((67*A - 247*B)*cos(d*x + c)^2 + 2*(25*A - 18
1*B)*cos(d*x + c) + 15*A - 147*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c) + 768*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x
+ c)^2 + 4*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

3.210 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx}{6d(a\cos(c+dx)+a)^{7/2}}$$

```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A - 13*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((17*A + 67*B)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.587176, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2978, 12, 2782, 205}

$$\frac{(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx}{6d(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A - 13*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((17*A + 67*B)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)+a(A+5B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}dx}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{\int}{}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(17}{}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(17}{}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(17}{}$$

$$= \frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{6d(a+a\cos(c+dx))^{7/2}}$$

Mathematica [C] time = 2.24854, size = 217, normalized size = 1.08

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{8}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(20(7A+5B)\cos(c+dx)+(17A+67B)\cos(2(c+d\right.}{24d(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(7*A + 5*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2
*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c +
 d*x]]*(59*A + 97*B + 20*(7*A + 5*B)*Cos[c + d*x] + (17*A + 67*B)*Cos[2*(c
+ d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d*x
]))^(7/2))
```

Maple [B] time = 0.649, size = 549, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*(34*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+140*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*cos(d*x+c)^3+21*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos
(d*x+c)^3*sin(d*x+c)+15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*
x+c)^3*sin(d*x+c)+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+42*A*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+134*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+30*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-140*A*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)+21*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)
*cos(d*x+c)-34*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+15*B*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-42*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)-70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-
30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^11/(cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A] time = 1.7205, size = 702, normalized size = 3.49

$$\frac{3\sqrt{2}\Big((7A+5B)\cos(dx+c)^4+4(7A+5B)\cos(dx+c)^3+6(7A+5B)\cos(dx+c)^2+4(7A+5B)\cos(dx+c)+7}{384\Big(a^4d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)^3
 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B)*s
qrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*((17*A + 67*B)*cos
(d*x + c)^2 + 10*(7*A + 5*B)*cos(d*x + c) + 21*A + 15*B)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

3.211 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(A+3B)\sin(c+dx)\sqrt{\cos(c+d}}{16ad(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A + 3*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 17*B)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.579293, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2978, 12, 2782, 205}

$$-\frac{(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(A+3B)\sin(c+dx)\sqrt{\cos(c+d}}{16ad(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A + 3*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 17*B)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+2a(A+2B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{4}a^2}{}}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{(5A}{1}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{(5A}{1}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{(5A}{1}$$

$$= \frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+d}{6d(a+a\cos(c+dx))^{7/2}}$$

Mathematica [C] time = 1.99237, size = 215, normalized size = 1.07

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{8}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(4(A+35B)\cos(c+dx)+(17B-5A)\cos(2(c+dx))+\right.}{24d(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(13*A + 7*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((
2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c
+ d*x]]*(73*A + 59*B + 4*(A + 35*B)*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c +
 d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d*x]
))^(7/2))
```

Maple [B] time = 0.62, size = 549, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(10*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4-39*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*cos(d*x+c)^3-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+
c)^3*sin(d*x+c)-78*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^
2*sin(d*x+c)-88*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2-34*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-42*B*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-39*A*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+4*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)-106*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-21*B*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+78*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)+98*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+42*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^9/(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A] time = 1.65976, size = 705, normalized size = 3.51

$$\frac{3\sqrt{2}\Big((13A+7B)\cos(dx+c)^4+4(13A+7B)\cos(dx+c)^3+6(13A+7B)\cos(dx+c)^2+4(13A+7B)\cos(dx+c}{384\left(a^4d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x + c)
^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A + 7
*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((5*A - 17*B)
*cos(d*x + c)^2 - 2*(A + 35*B)*cos(d*x + c) - 39*A - 21*B)*sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*c
os(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

3.212 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - ((5*A - B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((103*A + 5*B)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.591436, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2978, 12, 2782, 205}

$$-\frac{(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```


```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - ((5*A - B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((103*A + 5*B)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(11A+B)-2a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \int$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - (1$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - (1$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - (1$$

$$= \frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+}{6d(a+a\cos(c+dx))^{7/2}}$$

Mathematica [C] time = 2.01223, size = 216, normalized size = 1.06

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{8}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)((532A-4B)\cos(c+dx)+(103A+5B)\cos(2(c+\right.}{24d(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(63*A + 13*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^(
(2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*S
qrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[c
 + d*x]]*(493*A - 73*B + (532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(
c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d
*x]))^(7/2))
```

Maple [B] time = 0.606, size = 549, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(206*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*cos(d*x+c)^4-189*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*cos(d*x+c)^3*sin(d*x+c)+532*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*
x+c)^3-39*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x
+c)-378*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c
)+184*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+10*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)^4-78*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*cos(d*x+c)^2*sin(d*x+c)-189*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)*2^(1/2)*cos(d*x+c)-532*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
-14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-39*B*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-390*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)-74*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+78*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
a^4/cos(d*x+c)^(1/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.686, size = 716, normalized size = 3.53

$$\frac{3\sqrt{2}\Big((63A+13B)\cos(dx+c)^4+4(63A+13B)\cos(dx+c)^3+6(63A+13B)\cos(dx+c)^2+4(63A+13B)\cos(dx}{384\left(a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x +
c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 63*A
 + 13*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((103*A
+ 5*B)*cos(d*x + c)^2 + 2*(133*A - B)*cos(d*x + c) + 195*A - 39*B)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c
))), x)
```

3.213 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=250

$$\frac{(691A-103B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{si}}{\sqrt{2}\sqrt{\cos(c+dx}}\right.}{64\sqrt{2}a^{7/2}d}$$

```
[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x
])/(6*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)) - ((19*A - 7*B)*Sin[
c + d*x])/(48*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((199*A
- 43*B)*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3
/2)) + ((691*A - 103*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])
```

Rubi [A] time = 0.803849, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(691A-103B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{si}}{\sqrt{2}\sqrt{\cos(c+dx}}\right.}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```


```
[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x
])/(6*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)) - ((19*A - 7*B)*Sin[
c + d*x])/(48*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((199*A
- 43*B)*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3
/2)) + ((691*A - 103*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(13A-B)-3a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{(A-B)\sin(c+}{6d\sqrt{\cos(c+dx)}(a+a\cos}$$

Mathematica [C] time = 2.73043, size = 240, normalized size = 0.96

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(9(941A-121B)\cos(c+dx)+4(937A-133B)\cos(2(c+dx))+691A\cos(3(c+dx))+5284A-103B\cos(3(c+dx))-}{16\sqrt{\cos(c+dx)}}\right.}{24d(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-9*I)*(121*A - 21*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E
^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((5284*A
- 532*B + 9*(941*A - 121*B)*Cos[c + d*x] + 4*(937*A - 133*B)*Cos[2*(c + d*x
)] + 691*A*Cos[3*(c + d*x)] - 103*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^5*Ta
```

```
n[(c + d*x)/2])/(16*Sqrt[Cos[c + d*x]])))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2
))
```

Maple [B] time = 0.625, size = 581, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(1089*A*sin(d*x+c)*2^(1/
2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))-189*B*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+3267*A*cos(d*x+c)^2*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-
567*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+3267*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-567*B*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*2^(1/2)+1089*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-1382*A*cos(d*x+c)^4-189*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2
)+206*B*cos(d*x+c)^4-2366*A*cos(d*x+c)^3+326*B*cos(d*x+c)^3+550*A*cos(d*x+c
)^2-142*B*cos(d*x+c)^2+2430*A*cos(d*x+c)-390*B*cos(d*x+c)+768*A)/a^4/sin(d*
x+c)^5/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.73288, size = 810, normalized size = 3.24

$$-\frac{9\sqrt{2}\left((121\,A-21\,B)\cos\left(dx+c\right)^5+4\,(121\,A-21\,B)\cos\left(dx+c\right)^4+6\,(121\,A-21\,B)\cos\left(dx+c\right)^3+4\,(121\,A-21\,B)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^5 + 4*(121*A - 21*B)*cos(d*x
 + c)^4 + 6*(121*A - 21*B)*cos(d*x + c)^3 + 4*(121*A - 21*B)*cos(d*x + c)^2
 + (121*A - 21*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x +
 c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(
d*x + c))) - 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(937*A - 133*B)*cos(d*x
+ c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^
4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/
2)), x)
```

3.214 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=297

$$\frac{(579A-199B)\sin(c+dx)}{192a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{(1887A-691B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}}$$

```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x]
)/(6*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sin[
c + d*x])/(48*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((109*A
- 41*B)*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)) + ((579*A - 199*B)*Sin[c + d*x])/(192*a^3*d*Cos[c + d*x]^(3/2)*Sqrt[a +
 a*Cos[c + d*x]]) - ((1887*A - 691*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.03211, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(579A-199B)\sin(c+dx)}{192a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{(1887A-691B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```


```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x]
)/(6*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sin[
c + d*x])/(48*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((109*A
- 41*B)*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)) + ((579*A - 199*B)*Sin[c + d*x])/(192*a^3*d*Cos[c + d*x]^(3/2)*Sqrt[a +
 a*Cos[c + d*x]]) - ((1887*A - 691*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{3}{2}a(5A-B)-4a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= \frac{(1015A-363B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{(A-B)\sin(c+}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

Mathematica [C] time = 5.10489, size = 262, normalized size = 0.88

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(4(9415A-3579B)\cos(c+dx)+8(3069A-1145B)\cos(2(c+dx))+10164A\cos(3(c+dx))+1887A\cos(4(c+dx)}{32\cos^{\frac{3}{2}}(c+dx)}\right.}{24d(a(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(1015*A - 363*B)*E^((I/2)*(c + d*x))*Sqrt[(1 +
E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((21641*
A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos[2*(
c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*A*Cos
[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2]
)/(32*Cos[c + d*x]^(3/2))))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.484, size = 715, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(3045*A*2^(1/2)*sin(d*x+c)
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))-1089*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+12180*A*sin(d*x+c)*2^(1/2)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-
4356*B*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))+18270*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-6534*B*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)+12180*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x
+c)*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-4356*B*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)+3045*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-1089*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-3774*A*cos(d*x+c)^5+13
82*B*cos(d*x+c)^5-6390*A*cos(d*x+c)^4+2366*B*cos(d*x+c)^4+1662*A*cos(d*x+c)
^3-550*B*cos(d*x+c)^3+6710*A*cos(d*x+c)^2-2430*B*cos(d*x+c)^2+2048*A*cos(d*
x+c)-768*B*cos(d*x+c)-256*A)/a^4/sin(d*x+c)^3/(1+cos(d*x+c))^2/cos(d*x+c)^(
3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.88757, size = 878, normalized size = 2.96

$$\frac{3\sqrt{2}\big((1015\,A-363\,B)\cos(dx+c)^6+4\,(1015\,A-363\,B)\cos(dx+c)^5+6\,(1015\,A-363\,B)\cos(dx+c)^4+4\,(1015\,A-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^6 + 4*(1015*A - 363*B)*cos(
d*x + c)^5 + 6*(1015*A - 363*B)*cos(d*x + c)^4 + 4*(1015*A - 363*B)*cos(d*x
 + c)^3 + (1015*A - 363*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 + a*cos(d*x + c))) - 2*((1887*A - 691*B)*cos(d*x + c)^4 + 2*(2541*A - 93
7*B)*cos(d*x + c)^3 + 39*(109*A - 41*B)*cos(d*x + c)^2 + 128*(7*A - 3*B)*co
s(d*x + c) - 128*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4
+ 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(5/
2)), x)
```

3.215 $\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=105

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(4aA+3bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4aA+3bB)+\frac{bB\sin(c+$$

```
[Out] ((4*a*A + 3*b*B)*x)/8 + ((A*b + a*B)*Sin[c + d*x])/d + ((4*a*A + 3*b*B)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.169751, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(4aA+3bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4aA+3bB)+\frac{bB\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```



```
[Out] ((4*a*A + 3*b*B)*x)/8 + ((A*b + a*B)*Sin[c + d*x])/d + ((4*a*A + 3*b*B)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^2(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+\right.$$

$$= \frac{bB\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\int \cos^2(c+dx)(4aA+3bB$$

$$= \frac{bB\cos^3(c+dx)\sin(c+dx)}{4d} + (Ab+aB)\int \cos^3(c+dx)\,dx$$

$$= \frac{(4aA+3bB)\cos(c+dx)\sin(c+dx)}{8d} + \frac{bB\cos^3(c+dx)\sin(}{4d}$$

$$= \frac{1}{8}(4aA+3bB)x + \frac{(Ab+aB)\sin(c+dx)}{d} + \frac{(4aA+3bB)\cos}{}$$

Mathematica [A] time = 0.219568, size = 91, normalized size = 0.87

$$\frac{-32(aB+Ab)\sin^3(c+dx)+96(aB+Ab)\sin(c+dx)+24(aA+bB)\sin(2(c+dx))+48aAc+48aAdx+3bB\sin(4(c}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (48*a*A*c + 36*b*B*c + 48*a*A*d*x + 36*b*B*d*x + 96*(A*b + a*B)*Sin[c + d*x
] - 32*(A*b + a*B)*Sin[c + d*x]^3 + 24*(a*A + b*B)*Sin[2*(c + d*x)] + 3*b*B
*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.042, size = 107, normalized size = 1.

$$\frac{1}{d}\left(Bb\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{aB\left(2+(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.998019, size = 136, normalized size = 1.3

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ab+3\,(12}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 32*(sin(d*x + c)^3 - 3*sin(
d*x + c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b + 3*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b)/d
```

Fricas [A] time = 1.39391, size = 205, normalized size = 1.95

$$\frac{3\,(4\,Aa+3\,Bb)dx+\left(6\,Bb\cos\left(dx+c\right)^{3}+8\,(Ba+Ab)\cos\left(dx+c\right)^{2}+16\,Ba+16\,Ab+3\,(4\,Aa+3\,Bb)\cos\left(dx+c\right)\right)\sin}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A*a + 3*B*b)*d*x + (6*B*b*cos(d*x + c)^3 + 8*(B*a + A*b)*cos(d*x
 + c)^2 + 16*B*a + 16*A*b + 3*(4*A*a + 3*B*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.30528, size = 252, normalized size = 2.4

$$\begin{cases}\frac{Aax\sin^{2}(c+dx)}{2}+\frac{Aax\cos^{2}(c+dx)}{2}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Ab\sin^{3}(c+dx)}{3d}+\frac{Ab\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{2Ba\sin^{3}(c+dx)}{3d}+\frac{Ba\sin(c+dx)\mathrm{cc}}{d}\\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)\cos^{2}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + 2*A*b*sin(c + d*x)**3/(3*d) + A*b*sin(c + d*x)*co
s(c + d*x)**2/d + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(c + d*
x)**2/d + 3*B*b*x*sin(c + d*x)**4/8 + 3*B*b*x*sin(c + d*x)**2*cos(c + d*x)*
*2/4 + 3*B*b*x*cos(c + d*x)**4/8 + 3*B*b*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 5*B*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(
a + b*cos(c))*cos(c)**2, True))
```

Giac [A] time = 1.43524, size = 120, normalized size = 1.14

$$\frac{1}{8}\,(4\,Aa+3\,Bb)x+\frac{Bb\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(Ba+Ab)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{(Aa+Bb)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{3\,(Ba+Ab)\sin\left(dx\,+\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + 3*B*b)*x + 1/32*B*b*sin(4*d*x + 4*c)/d + 1/12*(B*a + A*b)*sin(
3*d*x + 3*c)/d + 1/4*(A*a + B*b)*sin(2*d*x + 2*c)/d + 3/4*(B*a + A*b)*sin(d
*x + c)/d
```

3.216 $\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{(3aA+2bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] ((A*b + a*B)*x)/2 + ((3*a*A + 2*b*B)*Sin[c + d*x])/(3*d) + ((A*b + a*B)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (b*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0896968, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2968, 3023, 2734}

$$\frac{(3aA+2bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((A*b + a*B)*x)/2 + ((3*a*A + 2*b*B)*Sin[c + d*x])/(3*d) + ((A*b + a*B)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (b*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right.$$
$$= \frac{bB\cos^2(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\int \cos(c+dx)(3aA+2bB+3$$
$$= \frac{1}{2}(Ab+aB)x + \frac{(3aA+2bB)\sin(c+dx)}{3d} + \frac{(Ab+aB)\cos(c+}{2d}$$

Mathematica [A] time = 0.151935, size = 75, normalized size = 0.89

$$\frac{3(4aA+3bB)\sin(c+dx)+3(aB+Ab)\sin(2(c+dx))+6aBc+6aBdx+6Abc+6Abdx+bB\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 3*(4*a*A + 3*b*B)*Sin[c + d*x]
 + 3*(A*b + a*B)*Sin[2*(c + d*x)] + b*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.037, size = 85, normalized size = 1.

$$\frac{1}{d}\left(\frac{Bb\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) + aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*B*b*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c))
```

Maxima [A] time = 1.11735, size = 107, normalized size = 1.27

$$\frac{3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba+3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ab-4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Bb+12\,Aa\sin}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*A*b - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b + 12*A*a*sin(d*x + c))
/d
```

Fricas [A] time = 1.36506, size = 149, normalized size = 1.77

$$\frac{3\left(Ba+Ab\right)dx+\left(2\,Bb\cos\left(dx+c\right)^{2}+6\,Aa+4\,Bb+3\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/6*(3*(B*a + A*b)*d*x + (2*B*b*cos(d*x + c)^2 + 6*A*a + 4*B*b + 3*(B*a + A
*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 0.668947, size = 168, normalized size = 2.

$$\begin{cases}\frac{Aa\sin\left(c+dx\right)}{d}+\frac{Abx\sin^{2}\left(c+dx\right)}{2}+\frac{Abx\cos^{2}\left(c+dx\right)}{2}+\frac{Ab\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Bax\sin^{2}\left(c+dx\right)}{2}+\frac{Bax\cos^{2}\left(c+dx\right)}{2}+\frac{Ba\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+ \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)\cos\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d + A*b*x*sin(c + d*x)**2/2 + A*b*x*cos(c + d*x
)**2/2 + A*b*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*b*sin(c +
 d*x)**3/(3*d) + B*b*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*c
os(c))*(a + b*cos(c))*cos(c), True))
```

Giac [A] time = 1.32706, size = 92, normalized size = 1.1

$$\frac{1}{2}(Ba + Ab)x + \frac{Bb\sin(3\,dx + 3\,c)}{12\,d} + \frac{(Ba + Ab)\sin(2\,dx + 2\,c)}{4\,d} + \frac{(4\,Aa + 3\,Bb)\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(B*a + A*b)*x + 1/12*B*b*sin(3*d*x + 3*c)/d + 1/4*(B*a + A*b)*sin(2*d*x
 + 2*c)/d + 1/4*(4*A*a + 3*B*b)*sin(d*x + c)/d
```

3.217 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(2aA+bB)+\frac{bB\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] ((2*a*A + b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (b*B*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rubi [A] time = 0.0228202, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(2aA+bB)+\frac{bB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((2*a*A + b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (b*B*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx=\frac{1}{2}(2aA+bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{bB\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0839046, size = 51, normalized size = 0.98

$$\frac{4(aB+Ab)\sin(c+dx)+4aAdx+bB\sin(2(c+dx))+2bBc+2bBdx}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*b*B*c + 4*a*A*d*x + 2*b*B*d*x + 4*(A*b + a*B)*Sin[c + d*x] + b*B*Sin[2*(
c + d*x)])/(4*d)
```

Maple [A] time = 0.038, size = 57, normalized size = 1.1

$$\frac{1}{d}\left(Bb\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab\sin(dx+c)+aB\sin(dx+c)+aA(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b*sin(d*x+c)+a*B*sin(d
*x+c)+a*A*(d*x+c))
```

Maxima [A] time = 1.01024, size = 74, normalized size = 1.42

$$\frac{4(dx+c)Aa+(2dx+2c+\sin(2dx+2c))Bb+4Ba\sin(dx+c)+4Ab\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*b + 4*B*a*sin(d*x
 + c) + 4*A*b*sin(d*x + c))/d
```

Fricas [A] time = 1.35626, size = 104, normalized size = 2.

$$\frac{(2Aa+Bb)dx+(Bb\cos(dx+c)+2Ba+2Ab)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*A*a + B*b)*d*x + (B*b*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/d
```

Sympy [A] time = 0.316311, size = 94, normalized size = 1.81

$$\begin{cases} Aax + \frac{Ab\sin(c+dx)}{d} + \frac{Ba\sin(c+dx)}{d} + \frac{Bbx\sin^2(c+dx)}{2} + \frac{Bbx\cos^2(c+dx)}{2} + \frac{Bb\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(A + B\cos(c)\right)\left(a + b\cos(c)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x + A*b*sin(c + d*x)/d + B*a*sin(c + d*x)/d + B*b*x*sin(c +
d*x)**2/2 + B*b*x*cos(c + d*x)**2/2 + B*b*sin(c + d*x)*cos(c + d*x)/(2*d),
Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c)), True))
```

Giac [A] time = 1.36641, size = 61, normalized size = 1.17

$$\frac{1}{2}\,(2\,Aa + Bb)x + \frac{Bb\sin(2\,dx + 2\,c)}{4\,d} + \frac{(Ba + Ab)\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*b)*x + 1/4*B*b*sin(2*d*x + 2*c)/d + (B*a + A*b)*sin(d*x + c)
/d
```

3.218 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=35

$$x(aB+Ab)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}$$

```
[Out] (A*b + a*B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Sin[c + d*x])/d
```

Rubi [A] time = 0.105084, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2968, 3023, 2735, 3770}

$$x(aB+Ab)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (A*b + a*B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Sin[c + d*x])/d
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx &= \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sec(c+d\\ &= \frac{bB\sin(c+dx)}{d}+\int (aA+(Ab+aB)\cos(c+dx))\sec(c+dx\\ &= (Ab+aB)x+\frac{bB\sin(c+dx)}{d}+(aA)\int \sec(c+dx)\,dx\\ &= (Ab+aB)x+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0265727, size = 46, normalized size = 1.31

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+aBx+Abx+\frac{bB\sin(c)\cos(dx)}{d}+\frac{bB\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] A*b*x + a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Cos[d*x]*Sin[c])/d + (
b*B*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.06, size = 56, normalized size = 1.6

$$Abx+aBx+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Abc}{d}+\frac{Bb\sin(dx+c)}{d}+\frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] A*b*x+a*B*x+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b*c+b*B*sin(d*x+c)/d+1/
d*B*a*c
```

Maxima [A] time = 1.09902, size = 63, normalized size = 1.8

$$\frac{(dx+c)Ba + (dx+c)Ab + Aa\log(\sec(dx+c) + \tan(dx+c)) + Bb\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
```

```
[Out] ((d*x + c)*B*a + (d*x + c)*A*b + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*b
*sin(d*x + c))/d
```

Fricas [A] time = 1.4341, size = 142, normalized size = 4.06

$$\frac{2(Ba + Ab)dx + Aa\log(\sin(dx+c) + 1) - Aa\log(-\sin(dx+c) + 1) + 2Bb\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(B*a + A*b)*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c)
+ 1) + 2*B*b*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\cos(c+dx))(a + b\cos(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 1.59196, size = 107, normalized size = 3.06

$$\frac{Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Ba+Ab)(dx+c)+\frac{2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (B*a + A*b)*(d*x + c) + 2*B*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.219 $\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^2(c + dx)\,dx$

Optimal. Leaf size=35

$$\frac{(aB + Ab)\tanh^{-1}(\sin(c + dx))}{d} + \frac{aA\tan(c + dx)}{d} + bBx$$

```
[Out] b*B*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rubi [A] time = 0.113709, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3021, 2735, 3770}

$$\frac{(aB + Ab)\tanh^{-1}(\sin(c + dx))}{d} + \frac{aA\tan(c + dx)}{d} + bBx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*B*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sec^2(c+\\ &= \frac{aA\tan(c+dx)}{d}+\int (Ab+aB+bB\cos(c+dx))\sec(c+dx\\ &= bBx+\frac{aA\tan(c+dx)}{d}-(-Ab-aB)\int \sec(c+dx)\,dx\\ &= bBx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0139163, size = 43, normalized size = 1.23

$$\frac{aA\tan(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*B*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
A*Tan[c + d*x])/d
```

Maple [A] time = 0.07, size = 65, normalized size = 1.9

$$bBx+\frac{A\tan(dx+c)\,a}{d}+\frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bbc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] b*B*x+a*A*tan(d*x+c)/d+1/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B*ln(sec(d*x
+c)+tan(d*x+c))+1/d*B*b*c
```

Maxima [B] time = 1.10044, size = 99, normalized size = 2.83

$$\frac{2 (dx + c)Bb + Ba(\log (\sin (dx + c) + 1) - \log (\sin (dx + c) - 1)) + Ab(\log (\sin (dx + c) + 1) - \log (\sin (dx + c) - 1)) + 2 A}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*b + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ A*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c))
/d
```

Fricas [B] time = 1.50891, size = 225, normalized size = 6.43

$$\frac{2 Bbdx \cos (dx + c) + (Ba + Ab) \cos (dx + c) \log (\sin (dx + c) + 1) - (Ba + Ab) \cos (dx + c) \log (- \sin (dx + c) + 1) + 2 A}{2 d \cos (dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] 1/2*(2*B*b*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x + c) + 1
) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B \cos (c + dx)) (a + b \cos (c + dx)) \sec^2 (c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.28301, size = 113, normalized size = 3.23

$$\frac{(dx+c)Bb+(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] ((d*x + c)*B*b + (B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + A*
b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.220 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=61

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] ((a*A + 2*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*Tan[c + d*x])/d
+ (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.147897, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 8, 3770}

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((a*A + 2*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*Tan[c + d*x])/d
+ (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sec^3(c+$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (2(Ab+aB)+(aA+2bB$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+(Ab+aB)\int \sec^2(c+dx)\,dx$$
$$= \frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\sec(c+dx)\tan(c+d}{2d}$$
$$= \frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(Ab+aB)\tan(c+dx)}{d}+$$

Mathematica [A] time = 0.0221397, size = 75, normalized size = 1.23

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)}{d}+\frac{Ab\tan(c+dx)}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (b*B*ArcTanh[Sin[c + d*x]])/d + (A*b*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.1, size = 86, normalized size = 1.4

$$\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aB\tan(dx+c)}{d}+\frac{Ab\tan(dx+c)}{d}+\frac{Bb\ln(\sec(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B
*tan(d*x+c)+1/d*A*b*tan(d*x+c)+1/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.979855, size = 128, normalized size = 2.1

$$-\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2Bb(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*B*a*tan(d*x + c) - 4*A*b*tan(d*x + c))/d
```

Fricas [A] time = 1.50592, size = 247, normalized size = 4.05

$$\frac{(Aa+2Bb)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa+2Bb)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(Aa+2(Ba+Ab)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] 1/4*((A*a + 2*B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a + 2*B*b)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a + 2*(B*a + A*b)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44335, size = 204, normalized size = 3.34

$$\frac{(Aa+2Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(Aa+2Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\cdots\right.}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1
/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c)
 + 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

3.221 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=93

$$\frac{(2aA+3bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*A + 3*b*B)*Tan[c + d*x])/
(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.163321, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2aA+3bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*A + 3*b*B)*Tan[c + d*x])/
(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
```

```
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sec^4(c+
$$
$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int (3(Ab+aB)+(2aA+3b$$
$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d} + (Ab+aB)\int \sec^3(c+dx)\,dx +$$
$$= \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{aA\sec^2(c+dx)\tan(c+$$
$$= \frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(2aA+3bB)\tan(c+dx)}{3d} +$$

Mathematica [A] time = 0.263559, size = 67, normalized size = 0.72

$$\frac{3(aB+Ab)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(aB+Ab)\sec(c+dx)+2aA\tan^2(c+dx)+6aA+6bB\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (3*(A*b + a*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*A + 6*b*B + 3*(A*b
 + a*B)*Sec[c + d*x] + 2*a*A*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.073, size = 128, normalized size = 1.4

$$\frac{2A\tan(dx+c)a}{3d} + \frac{aA(\sec(dx+c))^2\tan(dx+c)}{3d} + \frac{aB\sec(dx+c)\tan(dx+c)}{2d} + \frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*a*B*sec(d*x+c)
*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b*sec(d*x+c)*tan(d*
x+c)+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b*tan(d*x+c)
```

Maxima [A] time = 1.03363, size = 171, normalized size = 1.84

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa-3Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3Ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*B*b*tan(d*x + c))/d
```

Fricas [A] time = 1.40996, size = 298, normalized size = 3.2

$$\frac{3\left(Ba+Ab\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba+Ab\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(2Aa+3Bb\right)c\right.}{12d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(B*a + A*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a + A*b)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A*a + 3*B*b)*cos(d*x + c)^2 +
 2*A*a + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35696, size = 284, normalized size = 3.05

$$\frac{3\,(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/
2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A
*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x +
1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.222 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aA+4bB)\tan(c+dx)\sec(c$$

```
[Out] ((3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((A*b + a*B)*Tan[c + d*x])/
d + ((3*a*A + 4*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.178587, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 3768, 3770}

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aA+4bB)\tan(c+dx)\sec(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```



```
[Out] ((3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((A*b + a*B)*Tan[c + d*x])/
d + ((3*a*A + 4*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
```

```
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sec^5(c+a$$

$$= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (4(Ab+aB)+(3aA+4b$$

$$= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+(Ab+aB)\int \sec^4(c+dx)\,dx+$$

$$= \frac{(3aA+4bB)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec^3(c+dx)\tan(c}{4d}$$

$$= \frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(Ab+aB)\tan(c+dx)}{d}+$$

Mathematica [A] time = 0.583825, size = 85, normalized size = 0.75

$$\frac{3(3aA+4bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(aB+Ab)(\cos(2(c+dx))+2)\sec(c+dx)+6aA\sec^2(c\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*a*A + 12*b*B + 8
*(A*b + a*B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*a*A*Sec[c + d*x]^2)*Ta
n[c + d*x])/(24*d)
```

Maple [A] time = 0.076, size = 171, normalized size = 1.5

$$\frac{aA(\sec(dx+c))^3\tan(dx+c)}{4d}+\frac{3aA\sec(dx+c)\tan(dx+c)}{8d}+\frac{3aA\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2aB\tan(dx}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A
*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a*B*tan(d*x+c)+1/3/d*a*B*tan(d*x+c)*sec(d*
x+c)^2+2/3/d*A*b*tan(d*x+c)+1/3/d*A*b*tan(d*x+c)*sec(d*x+c)^2+1/2/d*B*b*tan
(d*x+c)*sec(d*x+c)+1/2/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.08998, size = 220, normalized size = 1.93

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ab-3Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*b - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
```

```
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.41719, size = 352, normalized size = 3.09

$$\frac{3\,(3\,Aa + 4\,Bb)\cos\left(dx + c\right)^4\log\left(\sin\left(dx + c\right) + 1\right) - 3\,(3\,Aa + 4\,Bb)\cos\left(dx + c\right)^4\log\left(-\sin\left(dx + c\right) + 1\right) + 2\,\big(16\,(Ba + A}{48\,d\cos\left(dx + c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(3*A*a + 4*B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a + 4
*B*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B*a + A*b)*cos(d*x + c
)^3 + 3*(3*A*a + 4*B*b)*cos(d*x + c)^2 + 6*A*a + 8*(B*a + A*b)*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36246, size = 410, normalized size = 3.6

$$\frac{3\,(3\,Aa + 4\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(3\,Aa + 4\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^7 - 24\,Ba\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*B
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 -
 24*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan
(1/2*d*x + 1/2*c)^7 + 9*A*a*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*tan(1/2*d*x + 1
/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*b*tan(1/2*d*x + 1/2*c)^5 + 9
*A*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/
2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 15*A*a*tan(1/2*d*x + 1/2
*c) + 24*B*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c) + 12*B*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.223 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{(4a^2A+6abB+3Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2A+6abB+3Ab^2\right)-\frac{(5a(aB+2Ab)+4b^2B)\sin^3(c+dx)}{15d}+\frac{(5$$

```
[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*x)/8 + ((4*b^2*B + 5*a*(2*A*b + a*B))*Sin[c
+ d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (b*(5*A*b + 6*a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(20*d) + (b*B*Cos[c
+ d*x]^3*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) - ((4*b^2*B + 5*a*(2*A*b
+ a*B))*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.311075, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2990, 3023, 2748, 2635, 8, 2633}

$$\frac{(4a^2A+6abB+3Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2A+6abB+3Ab^2\right)-\frac{(5a(aB+2Ab)+4b^2B)\sin^3(c+dx)}{15d}+\frac{(5$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```


```
[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*x)/8 + ((4*b^2*B + 5*a*(2*A*b + a*B))*Sin[c
+ d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (b*(5*A*b + 6*a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(20*d) + (b*B*Cos[c
+ d*x]^3*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) - ((4*b^2*B + 5*a*(2*A*b
+ a*B))*Sin[c + d*x]^3)/(15*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}\int \cos^2(c$$

$$= \frac{b(5Ab+6aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bB\cos^3(c+dx)(a}{}$$

$$= \frac{b(5Ab+6aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bB\cos^3(c+dx)(a}{}$$

$$= \frac{\left(4a^2A+3Ab^2+6abB\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{b(5Ab+6}{}$$

$$= \frac{1}{8}\left(4a^2A+3Ab^2+6abB\right)x + \frac{\left(4b^2B+5a(2Ab+aB)\right)\sin(c}{5d}$$

Mathematica [A] time = 0.456626, size = 146, normalized size = 0.77

$$\frac{60(c+dx)\left(4a^2A+6abB+3Ab^2\right)+60\left(6a^2B+12aAb+5b^2B\right)\sin(c+dx)+120\left(a^2A+2abB+Ab^2\right)\sin(2(c+dx))+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (60*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*(c + d*x) + 60*(12*a*A*b + 6*a^2*B + 5*b^
2*B)*Sin[c + d*x] + 120*(a^2*A + A*b^2 + 2*a*b*B)*Sin[2*(c + d*x)] + 10*(8*
a*A*b + 4*a^2*B + 5*b^2*B)*Sin[3*(c + d*x)] + 15*b*(A*b + 2*a*B)*Sin[4*(c +
 d*x)] + 6*b^2*B*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.043, size = 184, normalized size = 1.

$$\frac{1}{d}\left(a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{Ba^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2\,Aab\left(2+(\cos(dx+c))^2\right)\sin(d}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*B*a^2*(2+cos(d*x+c
)^2)*sin(d*x+c)+2/3*A*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+2*B*a*b*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*b^2*(1/4*(cos(d*x+c)^3+3/
```

```
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*b^2*B*(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.09796, size = 238, normalized size = 1.26

$$\frac{120\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^2 - 160\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Ba^2 - 320\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 - 160*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b + 30*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b + 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^2 + 32*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^2)/d
```

Fricas [A] time = 1.40327, size = 350, normalized size = 1.85

$$\frac{15\left(4\,Aa^2 + 6\,Bab + 3\,Ab^2\right)dx + \left(24\,Bb^2\cos(dx + c)^4 + 30\left(2\,Bab + Ab^2\right)\cos(dx + c)^3 + 80\,Ba^2 + 160\,Aab + 64\,Bb^2 + \right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*d*x + (24*B*b^2*cos(d*x + c)^4 + 30
*(2*B*a*b + A*b^2)*cos(d*x + c)^3 + 80*B*a^2 + 160*A*a*b + 64*B*b^2 + 8*(5*
B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^2 + 15*(4*A*a^2 + 6*B*a*b + 3*A*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.44535, size = 459, normalized size = 2.43

$$\begin{cases} \frac{Aa^2x\sin^2(c+dx)}{2} + \frac{Aa^2x\cos^2(c+dx)}{2} + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{4Aab\sin^3(c+dx)}{3d} + \frac{2Aab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Ab^2x\sin^4(c+dx)}{8} + \frac{3Ab^2x}{} \\ x\,(A + B\cos(c))\,(a + b\cos(c))^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*A*a*b*sin(c + d*x)**3/(3*d) + 2*A*a*b*
sin(c + d*x)*cos(c + d*x)**2/d + 3*A*b**2*x*sin(c + d*x)**4/8 + 3*A*b**2*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*b**2*x*cos(c + d*x)**4/8 + 3*A*b**2
*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*b**2*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*B*a*b*x*sin(c + d*x)**4/4 + 3*B*a*b*x*sin(c + d*x)**2*cos(c + d*x)
**2/2 + 3*B*a*b*x*cos(c + d*x)**4/4 + 3*B*a*b*sin(c + d*x)**3*cos(c + d*x)/
(4*d) + 5*B*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*B*b**2*sin(c + d*x)*
*5/(15*d) + 4*B*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**2*sin(c +
 d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**2*cos
(c)**2, True))
```

Giac [A] time = 1.43544, size = 211, normalized size = 1.12

$$\frac{Bb^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(4\,Aa^2+6\,Bab+3\,Ab^2\right)x+\frac{\left(2\,Bab+Ab^2\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(4\,Ba^2+8\,Aab+5\,Bb^2\right)\sin(3\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*b^2*sin(5*d*x + 5*c)/d + 1/8*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*x + 1/32*
(2*B*a*b + A*b^2)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^2 + 8*A*a*b + 5*B*b^2)*s
in(3*d*x + 3*c)/d + 1/4*(A*a^2 + 2*B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1/8*
(6*B*a^2 + 12*A*a*b + 5*B*b^2)*sin(d*x + c)/d
```

3.224 $\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=170

$$\frac{\left(4a^2Ab+a^3(-B)+8ab^2B+4Ab^3\right)\sin(c+dx)}{6bd}+\frac{\left(-2a^2B+8aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2B+8aAb\right.$$

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*x)/8 + ((4*a^2*A*b + 4*A*b^3 - a^3*B + 8*a*b
^2*B)*Sin[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B + 9*b^2*B)*Cos[c + d*x]*S
in[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
12*b*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.233517, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2753, 2734}

$$\frac{\left(4a^2Ab+a^3(-B)+8ab^2B+4Ab^3\right)\sin(c+dx)}{6bd}+\frac{\left(-2a^2B+8aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2B+8aAb\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```



```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*x)/8 + ((4*a^2*A*b + 4*A*b^3 - a^3*B + 8*a*b
^2*B)*Sin[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B + 9*b^2*B)*Cos[c + d*x]*S
in[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
12*b*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx &= \int (a+b\cos(c+dx))^2\left(A\cos(c+dx)+B\cos^2(c+dx)\right)\,dx\\ &= \frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4bd}+\frac{\int (a+b\cos(c+dx))^2(3b\\ &= \frac{(4Ab-aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12bd}+\frac{B(a+b\cos(c+\\ &= \frac{1}{8}\left(8aAb+4a^2B+3b^2B\right)x+\frac{\left(4a^2Ab+4Ab^3-a^3B+8ab^2B\right)}{6bd}\end{aligned}$$

Mathematica [A] time = 0.430768, size = 118, normalized size = 0.69

$$\frac{12(c+dx)\left(4a^2B+8aAb+3b^2B\right)+24\left(4a^2A+6abB+3Ab^2\right)\sin(c+dx)+24\left(a^2B+2aAb+b^2B\right)\sin(2(c+dx))+8b(}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a*A*b + 4*a^2*B + 3*b^2*B)*(c + d*x) + 24*(4*a^2*A + 3*A*b^2 + 6*a*b
*B)*Sin[c + d*x] + 24*(2*a*A*b + a^2*B + b^2*B)*Sin[2*(c + d*x)] + 8*b*(A*b
```

```
+ 2*a*B)*Sin[3*(c + d*x)] + 3*b^2*B*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.047, size = 152, normalized size = 0.9

$$\frac{1}{d}\left(a^2A\sin(dx+c)+Ba^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,Aab\,(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*sin(d*x+c)+B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a
*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2/3*B*a*b*(2+cos(d*x+c)^2)*sin
(d*x+c)+1/3*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*B*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.09274, size = 192, normalized size = 1.13

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+48\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab-64\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Bab-32}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 48*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a*b - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b - 32*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*A*b^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*B*b^2 + 96*A*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.42785, size = 274, normalized size = 1.61

$$\frac{3\left(4\,Ba^2+8\,Aab+3\,Bb^2\right)dx+\left(6\,Bb^2\cos(dx+c)^3+24\,Aa^2+32\,Bab+16\,Ab^2+8\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+3\,(4}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*d*x + (6*B*b^2*cos(d*x + c)^3 + 24*A*
a^2 + 32*B*a*b + 16*A*b^2 + 8*(2*B*a*b + A*b^2)*cos(d*x + c)^2 + 3*(4*B*a^2
 + 8*A*a*b + 3*B*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.7511, size = 338, normalized size = 1.99

$$\begin{cases} \frac{Aa^2\sin(c+dx)}{d} + Aabx\sin^2(c+dx) + Aabx\cos^2(c+dx) + \frac{Aab\sin(c+dx)\cos(c+dx)}{d} + \frac{2Ab^2\sin^3(c+dx)}{3d} + \frac{Ab^2\sin(c+dx)\cos^2(c+dx)}{d} + \\ x(A+B\cos(c))(a+b\cos(c))^2\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d + A*a*b*x*sin(c + d*x)**2 + A*a*b*x*cos(c
+ d*x)**2 + A*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*A*b**2*sin(c + d*x)**3/(3
*d) + A*b**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*x*sin(c + d*x)**2/2 +
B*a**2*x*cos(c + d*x)**2/2 + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*B*a
*b*sin(c + d*x)**3/(3*d) + 2*B*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b**
2*x*sin(c + d*x)**4/8 + 3*B*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*
b**2*x*cos(c + d*x)**4/8 + 3*B*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
B*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
+ b*cos(c))**2*cos(c), True))
```

Giac [A] time = 1.56091, size = 167, normalized size = 0.98

$$\frac{Bb^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(4\,Ba^2 + 8\,Aab + 3\,Bb^2\right)x + \frac{\left(2\,Bab + Ab^2\right)\sin(3\,dx+3\,c)}{12\,d} + \frac{\left(Ba^2 + 2\,Aab + Bb^2\right)\sin(2\,dx+2\,c}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*B*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*x + 1/12*
(2*B*a*b + A*b^2)*sin(3*d*x + 3*c)/d + 1/4*(B*a^2 + 2*A*a*b + B*b^2)*sin(2*
d*x + 2*c)/d + 1/4*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*sin(d*x + c)/d
```

3.225 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=107

$$\frac{2\left(a^2B+3aAb+b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^2A+2abB+Ab^2\right)+\frac{b(2aB+3Ab)\sin(c+dx)\cos(c+dx)}{6d}+\frac{B\sin(c+dx)}{}$$

```
[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*x)/2 + (2*(3*a*A*b + a^2*B + b^2*B)*Sin[c + d*
x])/(3*d) + (b*(3*A*b + 2*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + b
*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0935754, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$\frac{2\left(a^2B+3aAb+b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^2A+2abB+Ab^2\right)+\frac{b(2aB+3Ab)\sin(c+dx)\cos(c+dx)}{6d}+\frac{B\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```


```
[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*x)/2 + (2*(3*a*A*b + a^2*B + b^2*B)*Sin[c + d*
x])/(3*d) + (b*(3*A*b + 2*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + b
*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int (a+b\cos(c+dx))(3aA+2bB+$$
$$= \frac{1}{2}\left(2a^2A+Ab^2+2abB\right)x + \frac{2\left(3aAb+a^2B+b^2B\right)\sin(c+dx)}{3d} + \frac{b(3Ab}{}$$

Mathematica [A] time = 0.213408, size = 90, normalized size = 0.84

$$\frac{6(c+dx)\left(2a^2A+2abB+Ab^2\right)+3\left(4a^2B+8aAb+3b^2B\right)\sin(c+dx)+3b(2aB+Ab)\sin(2(c+dx))+b^2B\sin(3(c+dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*(2*a^2*A + A*b^2 + 2*a*b*B)*(c + d*x) + 3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*
Sin[c + d*x] + 3*b*(A*b + 2*a*B)*Sin[2*(c + d*x)] + b^2*B*Sin[3*(c + d*x)])
/(12*d)
```

Maple [A] time = 0.039, size = 114, normalized size = 1.1

$$\frac{1}{d}\left(\frac{b^2B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + Ab^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,Bab\,(1/2\,\cos(dx+c)\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*B*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a*b*s
in(d*x+c)+B*a^2*sin(d*x+c)+a^2*A*(d*x+c))
```

Maxima [A] time = 1.0408, size = 146, normalized size = 1.36

$$\frac{12\,(dx+c)Aa^2+6\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab^2-4\left(\sin(dx+c)^3-3\,\sin(dx+\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b + 3*(2*
d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B
*b^2 + 12*B*a^2*sin(d*x + c) + 24*A*a*b*sin(d*x + c))/d
```

Fricas [A] time = 1.33801, size = 201, normalized size = 1.88

$$\frac{3\left(2\,Aa^2+2\,Bab+Ab^2\right)dx+\left(2\,Bb^2\cos\left(dx+c\right)^2+6\,Ba^2+12\,Aab+4\,Bb^2+3\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*d*x + (2*B*b^2*cos(d*x + c)^2 + 6*B*a^2
+ 12*A*a*b + 4*B*b^2 + 3*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 0.839749, size = 199, normalized size = 1.86

$$\begin{cases} Aa^2x+\frac{2Aab\sin\left(c+dx\right)}{d}+\frac{Ab^2x\sin^2\left(c+dx\right)}{2}+\frac{Ab^2x\cos^2\left(c+dx\right)}{2}+\frac{Ab^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Ba^2\sin\left(c+dx\right)}{d}+Babx\sin^2\left(c+dx\right)+Bab \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x + 2*A*a*b*sin(c + d*x)/d + A*b**2*x*sin(c + d*x)**2/2 +
 A*b**2*x*cos(c + d*x)**2/2 + A*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*
*2*sin(c + d*x)/d + B*a*b*x*sin(c + d*x)**2 + B*a*b*x*cos(c + d*x)**2 + B*a
*b*sin(c + d*x)*cos(c + d*x)/d + 2*B*b**2*sin(c + d*x)**3/(3*d) + B*b**2*si
n(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**
2, True))
```

Giac [A] time = 1.3853, size = 126, normalized size = 1.18

$$\frac{Bb^2\sin(3\,dx+3\,c)}{12\,d}+\frac{1}{2}\left(2\,Aa^2+2\,Bab+Ab^2\right)x+\frac{\left(2\,Bab+Ab^2\right)\sin(2\,dx+2\,c)}{4\,d}+\frac{\left(4\,Ba^2+8\,Aab+3\,Bb^2\right)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*sin(3*d*x + 3*c)/d + 1/2*(2*A*a^2 + 2*B*a*b + A*b^2)*x + 1/4*(2*
B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*sin(d
*x + c)/d
```

3.226 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}x\left(2a^2B+4aAb+b^2B\right)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(3aB+2Ab)\sin(c+dx)}{2d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

```
[Out] ((4*a*A*b + 2*a^2*B + b^2*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (b*(2
*A*b + 3*a*B)*Sin[c + d*x])/(2*d) + (b*B*(a + b*Cos[c + d*x])*Sin[c + d*x])
/(2*d)
```

Rubi [A] time = 0.176197, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2990, 3023, 2735, 3770}

$$\frac{1}{2}x\left(2a^2B+4aAb+b^2B\right)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(3aB+2Ab)\sin(c+dx)}{2d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*a*A*b + 2*a^2*B + b^2*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (b*(2
*A*b + 3*a*B)*Sin[c + d*x])/(2*d) + (b*B*(a + b*Cos[c + d*x])*Sin[c + d*x])
/(2*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{bB(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int \Big(2a^2A+\Big(4aAb+2a\\ &= \frac{b(2Ab+3aB)\sin(c+dx)}{2d}+\frac{bB(a+b\cos(c+dx))\sin(c+dx}{2d}\\ &= \frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{b(2Ab+3aB)\sin(c+dx)}{2d}+\frac{bB(a}{}\\ &= \frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(2A}{}\end{aligned}$$

Mathematica [A] time = 0.221827, size = 120, normalized size = 1.4

$$\frac{2(c+dx)\left(2a^2B+4aAb+b^2B\right)-4a^2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^2A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(4*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) - 4*a^2*A*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 4*a^2*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*b*(
```

```
A*b + 2*a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.07, size = 120, normalized size = 1.4

$$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d}+a^2Bx+\frac{Ba^2c}{d}+2\,Aabx+2\,\frac{Aabc}{d}+2\,\frac{Bab\sin(dx+c)}{d}+\frac{Ab^2\sin(dx+c)}{d}+\frac{b^2B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*x+1/d*B*a^2*c+2*A*a*b*x+2/d*A*a*b
*c+2/d*B*a*b*sin(d*x+c)+1/d*A*b^2*sin(d*x+c)+1/2/d*b^2*B*cos(d*x+c)*sin(d*x
+c)+1/2*b^2*B*x+1/2/d*b^2*B*c
```

Maxima [A] time = 1.04364, size = 124, normalized size = 1.44

$$\frac{4\,(dx+c)Ba^2+8\,(dx+c)Aab+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^2+4\,Aa^2\log(\sec(dx+c)+\tan(dx+c))+8\,Bab\sin(}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 + 8*(d*x + c)*A*a*b + (2*d*x + 2*c + sin(2*d*x + 2*c
))*B*b^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*B*a*b*sin(d*x + c)
+ 4*A*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.54185, size = 213, normalized size = 2.48

$$\frac{Aa^2\log(\sin(dx+c)+1)-Aa^2\log(-\sin(dx+c)+1)+\left(2\,Ba^2+4\,Aab+Bb^2\right)dx+\left(Bb^2\cos(dx+c)+4\,Bab+2\,Ab^2\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*(A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*x + c) + 1) + (2*B*a^2
+ 4*A*a*b + B*b^2)*d*x + (B*b^2*cos(d*x + c) + 4*B*a*b + 2*A*b^2)*sin(d*x +
 c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))^2 \sec(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 1.297, size = 240, normalized size = 2.79

$$\frac{2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \left(2\,Ba^2 + 4\,Aab + Bb^2\right)(dx + c) + \frac{2\left(4\,Bab\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (2*B*a^2 + 4*A*a*b + B*b^2)*(d*x + c) + 2*(4*B*a*b*tan(
1/2*d*x + 1/2*c)^3 + 2*A*b^2*tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1
/2*c)^3 + 4*B*a*b*tan(1/2*d*x + 1/2*c) + 2*A*b^2*tan(1/2*d*x + 1/2*c) + B*b
^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.227 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=60

$$\frac{a^2A\tan(c+dx)}{d}+\frac{a(aB+2Ab)\tanh^{-1}(\sin(c+dx))}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

```
[Out] b*(A*b + 2*a*B)*x + (a*(2*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b^2*B*Sin[
c + d*x])/d + (a^2*A*Tan[c + d*x])/d
```

Rubi [A] time = 0.168526, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2988, 3023, 2735, 3770}

$$\frac{a^2A\tan(c+dx)}{d}+\frac{a(aB+2Ab)\tanh^{-1}(\sin(c+dx))}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*(A*b + 2*a*B)*x + (a*(2*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b^2*B*Sin[
c + d*x])/d + (a^2*A*Tan[c + d*x])/d
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a^2A\tan(c+dx)}{d} - \int \left(-a(2Ab+aB)-b(Ab+2aB)\cos(c+\right.$$
$$= \frac{b^2B\sin(c+dx)}{d} + \frac{a^2A\tan(c+dx)}{d} - \int (-a(2Ab+aB)-b($$
$$= b(Ab+2aB)x + \frac{b^2B\sin(c+dx)}{d} + \frac{a^2A\tan(c+dx)}{d} + (a(2A$$
$$= b(Ab+2aB)x + \frac{a(2Ab+aB)\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^2B\sin}{}$$

Mathematica [A] time = 0.473277, size = 109, normalized size = 1.82

$$\frac{a^2A\tan(c+dx)+b(c+dx)(2aB+Ab)-a(aB+2Ab)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(aB+2Ab)\log\left(\sin\left(\frac{1}{2}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(A*b + 2*a*B)*(c + d*x) - a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2
*B*Sin[c + d*x] + a^2*A*Tan[c + d*x])/d
```

Maple [A] time = 0.071, size = 104, normalized size = 1.7

$$Ab^2x + 2Babx + \frac{a^2A\tan(dx+c)}{d} + 2\frac{Aab\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Ab^2c}{d} + \frac{b^2B\sin(dx+c)}{d} + \frac{Ba^2\ln(\sec(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] A*b^2*x+2*B*a*b*x+a^2*A*tan(d*x+c)/d+2/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/
d*A*b^2*c+b^2*B*sin(d*x+c)/d+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a*b*
c
```

Maxima [A] time = 0.969159, size = 139, normalized size = 2.32

$$\frac{4(dx+c)Bab + 2(dx+c)Ab^2 + Ba^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Aab(\log(\sin(dx+c)+1) - \log}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(4*(d*x + c)*B*a*b + 2*(d*x + c)*A*b^2 + B*a^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*A*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*B*b^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.40506, size = 294, normalized size = 4.9

$$\frac{2\left(2Bab + Ab^2\right)dx\cos(dx+c) + \left(Ba^2 + 2Aab\right)\cos(dx+c)\log(\sin(dx+c)+1) - \left(Ba^2 + 2Aab\right)\cos(dx+c)\log(-\mathrm{s}}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*(2*B*a*b + A*b^2)*d*x*cos(d*x + c) + (B*a^2 + 2*A*a*b)*cos(d*x + c)*
log(sin(d*x + c) + 1) - (B*a^2 + 2*A*a*b)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(B*b^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51963, size = 205, normalized size = 3.42

$$\frac{\left(2\,Bab + Ab^2\right)(dx + c) + \left(Ba^2 + 2\,Aab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Ba^2 + 2\,Aab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(Aa^2 \ldots\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((2*B*a*b + A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x +
 1/2*c) + B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.228 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{\left(a^2A+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}+\frac{a(aB+2Ab)\tan(c+dx)}{d}+b^2Bx$$

```
[Out] b^2*B*x + ((a^2*A + 2*A*b^2 + 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2
*A*b + a*B)*Tan[c + d*x])/d + (a^2*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.19951, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2988, 3021, 2735, 3770}

$$\frac{\left(a^2A+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}+\frac{a(aB+2Ab)\tan(c+dx)}{d}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```


```
[Out] b^2*B*x + ((a^2*A + 2*A*b^2 + 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2
*A*b + a*B)*Tan[c + d*x])/d + (a^2*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a^2A\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}\int\Big(-2a(2Ab+aB)-\Big(a^2A$$
$$= \frac{a(2Ab+aB)\tan(c+dx)}{d}+\frac{a^2A\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}$$
$$= b^2Bx+\frac{a(2Ab+aB)\tan(c+dx)}{d}+\frac{a^2A\sec(c+dx)\tan(c+$$
$$= b^2Bx+\frac{\left(a^2A+2Ab^2+4abB\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2Ab$$

Mathematica [A] time = 0.265642, size = 67, normalized size = 0.84

$$\frac{\left(a^2A+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))+a\tan(c+dx)(aA\sec(c+dx)+2aB+4Ab)+2b^2Bdx}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*B*d*x + (a^2*A + 2*A*b^2 + 4*a*b*B)*ArcTanh[Sin[c + d*x]] + a*(4*A*b
 + 2*a*B + a*A*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.088, size = 133, normalized size = 1.7

$$\frac{a^2A\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Ba^2\tan(dx+c)}{d}+2\frac{Aab\tan(dx+c)}{d}+2\frac{Bab\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^2*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/d
*B*a^2*tan(d*x+c)+2/d*A*a*b*tan(d*x+c)+2/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+
1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^2*B*x+1/d*b^2*B*c
```

Maxima [A] time = 1.03715, size = 189, normalized size = 2.36

$$\frac{4(dx+c)Bb^2-Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Bab(\log(\sin(dx+c)+1)-\log(\mathrm{si}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*B*b^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a*b*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 4*B*a^2*tan(d*x + c) + 8*A*a*b*tan(d*x + c))/d
```

Fricas [A] time = 1.41339, size = 335, normalized size = 4.19

$$\frac{4Bb^2dx\cos(dx+c)^2+\left(Aa^2+4Bab+2Ab^2\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-\left(Aa^2+4Bab+2Ab^2\right)\cos(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*B*b^2*d*x*cos(d*x + c)^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) + 2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5992, size = 257, normalized size = 3.21

$$\frac{2\,(dx+c)Bb^2 + \left(Aa^2 + 4\,Bab + 2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Aa^2 + 4\,Bab + 2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*B*b^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) + 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*A
*a*b*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x + 1/2*c) + 2*B*a^2*tan(1/2*
d*x + 1/2*c) + 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
)/d
```

3.229 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{(2a^2A+6abB+3Ab^2)\tan(c+dx)}{3d}+\frac{(a^2B+2aAb+2b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a(\ldots$$

```
[Out] ((2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a^2*A + 3*A
*b^2 + 6*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b + a*B)*Sec[c + d*x]*Tan[c +
 d*x])/(2*d) + (a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.270161, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2988, 3021, 2748, 3767, 8, 3770}

$$\frac{(2a^2A+6abB+3Ab^2)\tan(c+dx)}{3d}+\frac{(a^2B+2aAb+2b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] ((2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a^2*A + 3*A
*b^2 + 6*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b + a*B)*Sec[c + d*x]*Tan[c +
 d*x])/(2*d) + (a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a^2A\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{1}{3}\int \Big(-3a(2Ab+aB)-\Big(2a^2$$

$$= \frac{a(2Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2A\sec^2(c+dx)\tan}{3d}$$

$$= \frac{a(2Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2A\sec^2(c+dx)\tan}{3d}$$

$$= \frac{\left(2aAb+a^2B+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(2Ab+aB)\sec}{}$$

$$= \frac{\left(2aAb+a^2B+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\left(2a^2A+3Ab^2\right.}{}$$

Mathematica [A] time = 0.450794, size = 92, normalized size = 0.79

$$\frac{3\left(a^2B+2aAb+2b^2B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(2\left(a^2A\tan^2(c+dx)+3a^2A+6abB+3Ab^2\right)+3a(aB+2A\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (3*(2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(2
*A*b + a*B)*Sec[c + d*x] + 2*(3*a^2*A + 3*A*b^2 + 6*a*b*B + a^2*A*Tan[c + d
*x]^2)))/(6*d)
```

Maple [A] time = 0.083, size = 174, normalized size = 1.5

$$\frac{2a^2A\tan(dx+c)}{3d}+\frac{a^2A(\sec(dx+c))^2\tan(dx+c)}{3d}+\frac{Ba^2\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba^2\ln(\sec(dx+c)+\tan(dx-}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^2*A*tan(d*x+c)/d+1/3*a^2*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*B*a^2*sec(
d*x+c)*tan(d*x+c)+1/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*b*sec(d*x+c
)*tan(d*x+c)+1/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a*b*tan(d*x+c)+1/d*A
*b^2*tan(d*x+c)+1/d*b^2*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.05812, size = 232, normalized size = 2.

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2-3Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-6Aab\left(\frac{2\sin}{\sin(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 3*B*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*A*
```

```
a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
24*B*a*b*tan(d*x + c) + 12*A*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.45872, size = 371, normalized size = 3.2

$$\frac{3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*
A*a^2 + 2*(2*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^2 + 3*(B*a^2 + 2*A*a*b
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49652, size = 397, normalized size = 3.42

$$\frac{3\left(Ba^2+2\,Aab+2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ba^2+2\,Aab+2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(
B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*
tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d
*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2
*c)^5 - 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 -
12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*ta
n(1/2*d*x + 1/2*c) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x +
1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.230 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{\left(2a^2B+4aAb+3b^2B\right)\tan(c+dx)}{3d}+\frac{\left(3a^2A+8abB+4Ab^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(3a^2A+8abB+4Ab^2\right)\tan(c+dx)}{8d}$$

```
[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*A*b + 2
*a^2*B + 3*b^2*B)*Tan[c + d*x])/(3*d) + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*(2*A*b + a*B)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.293472, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2988, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(2a^2B+4aAb+3b^2B\right)\tan(c+dx)}{3d}+\frac{\left(3a^2A+8abB+4Ab^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(3a^2A+8abB+4Ab^2\right)\tan(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*A*b + 2
*a^2*B + 3*b^2*B)*Tan[c + d*x])/(3*d) + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*(2*A*b + a*B)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
```

```
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{a^2A\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}\int \Big(-4a(2Ab+aB) - \Big(3a^2$$
$$= \frac{a(2Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2A\sec^3(c+dx)\tan}{4d}$$
$$= \frac{a(2Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2A\sec^3(c+dx)\tan}{4d}$$
$$= \frac{\left(3a^2A+4Ab^2+8abB\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a(2Ab+}{}$$
$$= \frac{\left(3a^2A+4Ab^2+8abB\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\left(4aAb+2a^2\right.}{}$$

Mathematica [A] time = 0.693543, size = 120, normalized size = 0.77

$$\frac{3\left(3a^2A+8abB+4Ab^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(3a^2A+8abB+4Ab^2\right)\sec(c+dx)+24\left(a^2B+2aAb+b\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(
2*a*A*b + a^2*B + b^2*B) + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[c + d*x] + 6
*a^2*A*Sec[c + d*x]^3 + 8*a*(2*A*b + a*B)*Tan[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.084, size = 241, normalized size = 1.5

$$\frac{a^2A\left(\sec(dx+c)\right)^3\tan(dx+c)}{4d} + \frac{3a^2A\sec(dx+c)\tan(dx+c)}{8d} + \frac{3a^2A\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{2Ba^2\tan(d}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a^2*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^2*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d
*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*a^2*tan(d*x+c)+1/3/d*B*a^2*tan(d*x
+c)*sec(d*x+c)^2+4/3/d*A*a*b*tan(d*x+c)+2/3/d*A*a*b*tan(d*x+c)*sec(d*x+c)^2
+1/d*B*a*b*tan(d*x+c)*sec(d*x+c)+1/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*
A*b^2*tan(d*x+c)*sec(d*x+c)+1/2/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*B
```

```
*tan(d*x+c)
```

Maxima [A] time = 1.10652, size = 308, normalized size = 1.97

$$16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba^2+32\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aab-3\,Aa^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a*b - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 24*B*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*b^2*tan(d*x +
 c))/d
```

Fricas [A] time = 1.47, size = 443, normalized size = 2.84

$$3\left(3\,Aa^2+8\,Bab+4\,Ab^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Aa^2+8\,Bab+4\,Ab^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(8*(2*B*a^2 + 4*A*a*b + 3*B*b^2)*cos(d*x + c)^3 + 6*A*a^2 + 3*(3*A*a^2 + 8
*B*a*b + 4*A*b^2)*cos(d*x + c)^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49832, size = 645, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*
A*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*t
an(1/2*d*x + 1/2*c)^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*
d*x + 1/2*c)^7 - 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 40*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 80*A*a*b*tan(1/2*d*x + 1/2*c)^5
- 24*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*
b^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^2*tan(
1/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*d*x + 1/2*
c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 24*B*a^2*tan(1/2*d*x + 1/2*c) + 48*A
*a*b*tan(1/2*d*x + 1/2*c) + 24*B*a*b*tan(1/2*d*x + 1/2*c) + 12*A*b^2*tan(1/
2*d*x + 1/2*c) + 24*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

3.231 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=269

$$-\frac{(15a^2Ab+5a^3B+12ab^2B+4Ab^3)\sin^3(c+dx)}{15d}+\frac{(15a^2Ab+5a^3B+12ab^2B+4Ab^3)\sin(c+dx)}{5d}+\frac{b(14a^2B+18aA}{}$$

```
[Out] ((8*a^3*A + 18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*x)/16 + ((15*a^2*A*b + 4*A*b
^3 + 5*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(5*d) + ((8*a^3*A + 18*a*A*b^2 + 1
8*a^2*b*B + 5*b^3*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a*A*b + 14*
a^2*B + 5*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*(3*A*b + 4*a*B)
*Cos[c + d*x]^4*Sin[c + d*x])/(15*d) + (b*B*Cos[c + d*x]^3*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(6*d) - ((15*a^2*A*b + 4*A*b^3 + 5*a^3*B + 12*a*b^2*B)
*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.507479, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2990, 3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{(15a^2Ab+5a^3B+12ab^2B+4Ab^3)\sin^3(c+dx)}{15d}+\frac{(15a^2Ab+5a^3B+12ab^2B+4Ab^3)\sin(c+dx)}{5d}+\frac{b(14a^2B+18aA}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```


```
[Out] ((8*a^3*A + 18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*x)/16 + ((15*a^2*A*b + 4*A*b
^3 + 5*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(5*d) + ((8*a^3*A + 18*a*A*b^2 + 1
8*a^2*b*B + 5*b^3*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a*A*b + 14*
a^2*B + 5*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*(3*A*b + 4*a*B)
*Cos[c + d*x]^4*Sin[c + d*x])/(15*d) + (b*B*Cos[c + d*x]^3*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(6*d) - ((15*a^2*A*b + 4*A*b^3 + 5*a^3*B + 12*a*b^2*B)
*Sin[c + d*x]^3)/(15*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
```

```
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{1}{6}\int \cos$$

$$= \frac{b^2(3Ab+4aB)\cos^4(c+dx)\sin(c+dx)}{15d} + \frac{bB\cos^3(c+dx)}{}$$

$$= \frac{b\left(18aAb+14a^2B+5b^2B\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{b^2(3}{}$$

$$= \frac{b\left(18aAb+14a^2B+5b^2B\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{b^2(3}{}$$

$$= \frac{\left(8a^3A+18aAb^2+18a^2bB+5b^3B\right)\cos(c+dx)\sin(c+dx}{16d}$$

$$= \frac{1}{16}\left(8a^3A+18aAb^2+18a^2bB+5b^3B\right)x + \frac{\left(15a^2Ab+4Ab\right.}{}$$

Mathematica [A] time = 0.653479, size = 289, normalized size = 1.07

$$\frac{120\left(18a^2Ab+6a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)+15\left(16a^3A+48a^2bB+48aAb^2+15b^3B\right)\sin(2(c+dx))+240a^2A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (480*a^3*A*c + 1080*a*A*b^2*c + 1080*a^2*b*B*c + 300*b^3*B*c + 480*a^3*A*d*
x + 1080*a*A*b^2*d*x + 1080*a^2*b*B*d*x + 300*b^3*B*d*x + 120*(18*a^2*A*b +
 5*A*b^3 + 6*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 15*(16*a^3*A + 48*a*A*b^2 +
 48*a^2*b*B + 15*b^3*B)*Sin[2*(c + d*x)] + 240*a^2*A*b*Sin[3*(c + d*x)] + 1
00*A*b^3*Sin[3*(c + d*x)] + 80*a^3*B*Sin[3*(c + d*x)] + 300*a*b^2*B*Sin[3*(
c + d*x)] + 90*a*A*b^2*Sin[4*(c + d*x)] + 90*a^2*b*B*Sin[4*(c + d*x)] + 45*
b^3*B*Sin[4*(c + d*x)] + 12*A*b^3*Sin[5*(c + d*x)] + 36*a*b^2*B*Sin[5*(c +
d*x)] + 5*b^3*B*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.052, size = 270, normalized size = 1.

$$\frac{1}{d}\left(Aa^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Aa^2b\left(2+(\cos(dx+c))^2\right)\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+A*a^2*b*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*B*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a*b^2*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*B*a*b^2*(8/3+cos(d*x+c)^4+4
/3*cos(d*x+c)^2)*sin(d*x+c)+1/5*A*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*s
in(d*x+c)+B*b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*
x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.00011, size = 359, normalized size = 1.33

$$\frac{240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3-320\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^3-960\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b + 90*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b + 90*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^2 + 192*(3*sin(d*x +
 c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a*b^2 + 64*(3*sin(d*x + c)^5
 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*b^3 - 5*(4*sin(2*d*x + 2*c)^3 - 6
0*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^3)/d
```

Fricas [A] time = 1.58892, size = 517, normalized size = 1.92

$$\frac{15\left(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3\right)dx+\left(40\,Bb^3\cos(dx+c)^5+48\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^4+160\,Ba^3+480\,Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*d*x + (40*B*b^3*cos
(d*x + c)^5 + 48*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^4 + 160*B*a^3 + 480*A*a^2
*b + 384*B*a*b^2 + 128*A*b^3 + 10*(18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*cos(d
*x + c)^3 + 16*(5*B*a^3 + 15*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c)^2
 + 15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [A] time = 7.0304, size = 721, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*x*sin(c + d*x)**2/2 + A*a**3*x*cos(c + d*x)**2/2 + A*a**3
*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*b*sin(c + d*x)**3/d + 3*A*a**2*
b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a*b**2*x*sin(c + d*x)**4/8 + 9*A*a*b
**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*A*a*b**2*x*cos(c + d*x)**4/8 +
9*A*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*A*a*b**2*sin(c + d*x)*co
s(c + d*x)**3/(8*d) + 8*A*b**3*sin(c + d*x)**5/(15*d) + 4*A*b**3*sin(c + d*
x)**3*cos(c + d*x)**2/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 2*B*a
**3*sin(c + d*x)**3/(3*d) + B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**
2*b*x*sin(c + d*x)**4/8 + 9*B*a**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
9*B*a**2*b*x*cos(c + d*x)**4/8 + 9*B*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8
*d) + 15*B*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*a*b**2*sin(c + d
*x)**5/(5*d) + 4*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a*b**2*si
n(c + d*x)*cos(c + d*x)**4/d + 5*B*b**3*x*sin(c + d*x)**6/16 + 15*B*b**3*x*
sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**3*x*sin(c + d*x)**2*cos(c + d*
x)**4/16 + 5*B*b**3*x*cos(c + d*x)**6/16 + 5*B*b**3*sin(c + d*x)**5*cos(c +
 d*x)/(16*d) + 5*B*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*B*b**3*s
in(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos
(c))**3*cos(c)**2, True))
```

Giac [A] time = 1.30493, size = 311, normalized size = 1.16

$$\frac{Bb^3\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3\right)x+\frac{\left(3\,Bab^2+Ab^3\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{3\left(2\,Ba^2b+2\,Aab^2\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*b^3*sin(6*d*x + 6*c)/d + 1/16*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 +
5*B*b^3)*x + 1/80*(3*B*a*b^2 + A*b^3)*sin(5*d*x + 5*c)/d + 3/64*(2*B*a^2*b
+ 2*A*a*b^2 + B*b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^3 + 12*A*a^2*b + 15*B
*a*b^2 + 5*A*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*A*a^3 + 48*B*a^2*b + 48*A*a
*b^2 + 15*B*b^3)*sin(2*d*x + 2*c)/d + 1/8*(6*B*a^3 + 18*A*a^2*b + 15*B*a*b^
2 + 5*A*b^3)*sin(d*x + c)/d
```

3.232 $\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=243

$$\frac{(15a^3Ab+52a^2b^2B-3a^4B+60aAb^3+16b^4B)\sin(c+dx)}{30bd}+\frac{(-3a^2B+15aAb+16b^2B)\sin(c+dx)(a+b\cos(c+dx))}{60bd}$$

```
[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*x)/8 + ((15*a^3*A*b + 60*a*A*
b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(30*b*d) + ((30*a^2*
A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(120*d) +
 ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*
b*d) + ((5*A*b - a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (B*(a
 + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.333154, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2753, 2734}

$$\frac{(15a^3Ab+52a^2b^2B-3a^4B+60aAb^3+16b^4B)\sin(c+dx)}{30bd}+\frac{(-3a^2B+15aAb+16b^2B)\sin(c+dx)(a+b\cos(c+dx))}{60bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```


```
[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*x)/8 + ((15*a^3*A*b + 60*a*A*
b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(30*b*d) + ((30*a^2*
A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(120*d) +
 ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*
b*d) + ((5*A*b - a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (B*(a
 + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^3\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$
$$= \frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5bd} + \frac{\int (a+b\cos(c+dx))^3(4b\ldots}{\ldots}$$
$$= \frac{(5Ab-aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20bd} + \frac{B(a+b\cos(c+\ldots}{5\ldots}$$
$$= \frac{\left(15aAb-3a^2B+16b^2B\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60bd} + \frac{(\ldots}{\ldots}$$
$$= \frac{1}{8}\left(12a^2Ab+3Ab^3+4a^3B+9ab^2B\right)x + \frac{\left(15a^3Ab+60aAb^3\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.671014, size = 176, normalized size = 0.72

$$\frac{60(c+dx)\left(12a^2Ab+4a^3B+9ab^2B+3Ab^3\right)+10b\left(12a^2B+12aAb+5b^2B\right)\sin(3(c+dx))+60\left(8a^3A+18a^2bB+18aA\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (60*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*(c + d*x) + 60*(8*a^3*A +
18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*Sin[c + d*x] + 120*(3*a^2*A*b + A*b^3 +
a^3*B + 3*a*b^2*B)*Sin[2*(c + d*x)] + 10*b*(12*a*A*b + 12*a^2*B + 5*b^2*B)*
Sin[3*(c + d*x)] + 15*b^2*(A*b + 3*a*B)*Sin[4*(c + d*x)] + 6*b^3*B*Sin[5*(c
 + d*x)])/(480*d)
```

Maple [A] time = 0.125, size = 227, normalized size = 0.9

$$\frac{1}{d}\left(Aa^3\sin(dx+c)+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3\,Aa^2b\,(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*A*a
^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*b*B*(2+cos(d*x+c)^2)*sin
(d*x+c)+A*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*a*b^2*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*
x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*B*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))
```

Maxima [A] time = 1.11803, size = 293, normalized size = 1.21

$$\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3+360\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b-480\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b - 480
*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^2 + 45*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c)
 + 8*sin(2*d*x + 2*c))*A*b^3 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 1
```

```
5*sin(d*x + c))*B*b^3 + 480*A*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.5275, size = 423, normalized size = 1.74

$$\frac{15\left(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3\right)dx+\left(24\,Bb^3\cos\left(dx+c\right)^4+120\,Aa^3+240\,Ba^2b+240\,Aab^2+64\,Bb^3+30\left(3\,Bab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*d*x + (24*B*b^3*cos(
d*x + c)^4 + 120*A*a^3 + 240*B*a^2*b + 240*A*a*b^2 + 64*B*b^3 + 30*(3*B*a*b
^2 + A*b^3)*cos(d*x + c)^3 + 8*(15*B*a^2*b + 15*A*a*b^2 + 4*B*b^3)*cos(d*x
+ c)^2 + 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [A] time = 3.85204, size = 551, normalized size = 2.27

$$\begin{cases} \frac{Aa^3\sin\left(c+dx\right)}{d}+\frac{3Aa^2bx\sin^2\left(c+dx\right)}{2}+\frac{3Aa^2bx\cos^2\left(c+dx\right)}{2}+\frac{3Aa^2b\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Aab^2\sin^3\left(c+dx\right)}{d}+\frac{3Aab^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{3A}{} \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)^3\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)/d + 3*A*a**2*b*x*sin(c + d*x)**2/2 + 3*A*a**
2*b*x*cos(c + d*x)**2/2 + 3*A*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*
a*b**2*sin(c + d*x)**3/d + 3*A*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*
b**3*x*sin(c + d*x)**4/8 + 3*A*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3
*A*b**3*x*cos(c + d*x)**4/8 + 3*A*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) +
 5*A*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*x*sin(c + d*x)**2/2 +
 B*a**3*x*cos(c + d*x)**2/2 + B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*
a**2*b*sin(c + d*x)**3/d + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*
a*b**2*x*sin(c + d*x)**4/8 + 9*B*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4
 + 9*B*a*b**2*x*cos(c + d*x)**4/8 + 9*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)
/(8*d) + 15*B*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*b**3*sin(c +
d*x)**5/(15*d) + 4*B*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**3*si
```

```
n(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**
3*cos(c), True))
```

Giac [A] time = 1.34094, size = 254, normalized size = 1.05

$$\frac{Bb^3\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3\right)x+\frac{\left(3\,Bab^2+Ab^3\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(12\,Ba^2b+12\,Aab^2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/80*B*b^3*sin(5*d*x + 5*c)/d + 1/8*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A
*b^3)*x + 1/32*(3*B*a*b^2 + A*b^3)*sin(4*d*x + 4*c)/d + 1/48*(12*B*a^2*b +
12*A*a*b^2 + 5*B*b^3)*sin(3*d*x + 3*c)/d + 1/4*(B*a^3 + 3*A*a^2*b + 3*B*a*b
^2 + A*b^3)*sin(2*d*x + 2*c)/d + 1/8*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5
*B*b^3)*sin(d*x + c)/d
```

3.233 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$\frac{\left(16a^2Ab+3a^3B+12ab^2B+4Ab^3\right)\sin(c+dx)}{6d}+\frac{b\left(6a^2B+20aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^3A+12a^2b\right.$$

```
[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*x)/8 + ((16*a^2*A*b + 4*A*b^
3 + 3*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(6*d) + (b*(20*a*A*b + 6*a^2*B + 9*
b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A*b + 3*a*B)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(12*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d
)
```

Rubi [A] time = 0.196937, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$\frac{\left(16a^2Ab+3a^3B+12ab^2B+4Ab^3\right)\sin(c+dx)}{6d}+\frac{b\left(6a^2B+20aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^3A+12a^2b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*x)/8 + ((16*a^2*A*b + 4*A*b^
3 + 3*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(6*d) + (b*(20*a*A*b + 6*a^2*B + 9*
b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A*b + 3*a*B)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(12*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d
)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int (a+b\cos(c+dx))^2(4aA+3bB$$

$$= \frac{(4Ab+3aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d}+\frac{B(a+b\cos(c+dx))^3\sin}{4d}$$

$$= \frac{1}{8}\left(8a^3A+12aAb^2+12a^2bB+3b^3B\right)x+\frac{\left(16a^2Ab+4Ab^3+3a^3B+12\right.}{6d}$$

Mathematica [A] time = 0.381848, size = 140, normalized size = 0.82

$$\frac{12(c+dx)\left(8a^3A+12a^2bB+12aAb^2+3b^3B\right)+24b\left(3a^2B+3aAb+b^2B\right)\sin(2(c+dx))+24\left(12a^2Ab+4a^3B+9ab^2\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*(c + d*x) + 24*(12*a^2*A*
b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x] + 24*b*(3*a*A*b + 3*a^2*B +
 b^2*B)*Sin[2*(c + d*x)] + 8*b^2*(A*b + 3*a*B)*Sin[3*(c + d*x)] + 3*b^3*B*S
in[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.045, size = 180, normalized size = 1.1

$$\frac{1}{d}\left(Bb^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3\,dx}{8}+\frac{3\,c}{8}\right)+\frac{Ab^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Bab^2\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(B*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*A*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*
```

```
a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*B*(1/2*cos(d*x+c)*s
in(d*x+c)+1/2*d*x+1/2*c)+3*A*a^2*b*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c
))
```

Maxima [A] time = 1.16311, size = 231, normalized size = 1.35

$$\frac{96\,(dx+c)Aa^3+72\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2b+72\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab^2-96\left(\sin(dx+c)^3-3\,\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*A*a^3 + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b + 72
*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^2 - 96*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*B*a*b^2 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^3 + 3*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b^3 + 96*B*a^3*sin(d*x + c) +
 288*A*a^2*b*sin(d*x + c))/d
```

Fricas [A] time = 1.34661, size = 321, normalized size = 1.88

$$\frac{3\left(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3\right)dx+\left(6\,Bb^3\cos(dx+c)^3+24\,Ba^3+72\,Aa^2b+48\,Bab^2+16\,Ab^3+8\left(3\,Bab^2+A\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*d*x + (6*B*b^3*cos(d*
x + c)^3 + 24*B*a^3 + 72*A*a^2*b + 48*B*a*b^2 + 16*A*b^3 + 8*(3*B*a*b^2 + A
*b^3)*cos(d*x + c)^2 + 9*(4*B*a^2*b + 4*A*a*b^2 + B*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [A] time = 1.76105, size = 386, normalized size = 2.26

$$\begin{cases} Aa^3x+\frac{3Aa^2b\sin(c+dx)}{d}+\frac{3Aab^2x\sin^2(c+dx)}{2}+\frac{3Aab^2x\cos^2(c+dx)}{2}+\frac{3Aab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Ab^3\sin^3(c+dx)}{3d}+\frac{Ab^3\sin(c+dx)\cos^2(c+\right.}{d} \\ x\,(A+B\cos(c))\,(a+b\cos(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*x + 3*A*a**2*b*sin(c + d*x)/d + 3*A*a*b**2*x*sin(c + d*x)
**2/2 + 3*A*a*b**2*x*cos(c + d*x)**2/2 + 3*A*a*b**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 2*A*b**3*sin(c + d*x)**3/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x
)**2/d + B*a**3*sin(c + d*x)/d + 3*B*a**2*b*x*sin(c + d*x)**2/2 + 3*B*a**2*
b*x*cos(c + d*x)**2/2 + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*a*
b**2*sin(c + d*x)**3/d + 3*B*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b*
*3*x*sin(c + d*x)**4/8 + 3*B*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B
*b**3*x*cos(c + d*x)**4/8 + 3*B*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5
*B*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
 + b*cos(c))**3, True))
```

Giac [A] time = 1.36552, size = 200, normalized size = 1.17

$$\frac{Bb^3\sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\left(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3\right)x+\frac{\left(3\,Bab^2+Ab^3\right)\sin(3\,dx+3\,c)}{12\,d}+\frac{\left(3\,Ba^2b+3\,Aab^2+\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*B*b^3*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*
B*b^3)*x + 1/12*(3*B*a*b^2 + A*b^3)*sin(3*d*x + 3*c)/d + 1/4*(3*B*a^2*b + 3
*A*a*b^2 + B*b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^
2 + 3*A*b^3)*sin(d*x + c)/d
```

3.234 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=137

$$\frac{b\left(8a^2B+9aAb+2b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(6a^2Ab+2a^3B+3ab^2B+Ab^3\right)+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2(5aB+3Ab)}{}$$

```
[Out] ((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*
x]])/d + (b*(9*a*A*b + 8*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*d) + (b^2*(3*A*b
 + 5*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*B*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(3*d)
```

Rubi [A] time = 0.323917, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2990, 3033, 3023, 2735, 3770}

$$\frac{b\left(8a^2B+9aAb+2b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(6a^2Ab+2a^3B+3ab^2B+Ab^3\right)+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2(5aB+3Ab)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] ((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*
x]])/d + (b*(9*a*A*b + 8*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*d) + (b^2*(3*A*b
 + 5*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*B*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(3*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int (a+b\cos(c+dx))($$
$$= \frac{b^2(3Ab+5aB)\cos(c+dx)\sin(c+dx)}{6d} + \frac{bB(a+b\cos(c+dx}{3d}$$
$$= \frac{b\left(9aAb+8a^2B+2b^2B\right)\sin(c+dx)}{3d} + \frac{b^2(3Ab+5aB)\cos(c-}{6d}$$
$$= \frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x + \frac{b\left(9aAb+8a^2B+2b^2\right.}{3d}$$
$$= \frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x + \frac{a^3A\tanh^{-1}(\sin(c+dx}{d}$$

Mathematica [A] time = 0.370471, size = 159, normalized size = 1.16

$$\frac{6(c+dx)\left(6a^2Ab+2a^3B+3ab^2B+Ab^3\right)+9b\left(4a^2B+4aAb+b^2B\right)\sin(c+dx)-12a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (6*(6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*(c + d*x) - 12*a^3*A*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*A*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*b*(4*a*A*b + 4*a^2*B + b^2*B)*Sin[c + d*x] + 3*b^2*(A*b + 3*a*
B)*Sin[2*(c + d*x)] + b^3*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.077, size = 207, normalized size = 1.5

$$\frac{Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{d} + a^3Bx + \frac{Ba^3c}{d} + 3\,Aa^2bx + 3\,\frac{Aa^2bc}{d} + 3\,\frac{a^2bB\sin(dx+c)}{d} + 3\,\frac{Aab^2\sin(dx+c)}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*x+1/d*B*a^3*c+3*A*a^2*b*x+3/d*A*a
^2*b*c+3/d*a^2*b*B*sin(d*x+c)+3/d*A*a*b^2*sin(d*x+c)+3/2/d*B*a*b^2*cos(d*x+
c)*sin(d*x+c)+3/2*B*a*b^2*x+3/2/d*B*a*b^2*c+1/2/d*A*b^3*cos(d*x+c)*sin(d*x+
```

```
c)+1/2*A*b^3*x+1/2/d*A*b^3*c+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*b^3+2/3/d*B*b^
3*sin(d*x+c)
```

Maxima [A] time = 1.13425, size = 196, normalized size = 1.43

$$\frac{12\,(dx+c)Ba^3+36\,(dx+c)Aa^2b+9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab^2+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab^3-4\,\big(\sin(}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^3 + 36*(d*x + c)*A*a^2*b + 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*B*a*b^2 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^3 - 4*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*B*b^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)) +
 36*B*a^2*b*sin(d*x + c) + 36*A*a*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.46861, size = 317, normalized size = 2.31

$$\frac{3\,Aa^3\log(\sin(dx+c)+1)-3\,Aa^3\log(-\sin(dx+c)+1)+3\left(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3\right)dx+\left(2\,Bb^3\cos(dx+c}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-sin(d*x + c) + 1) + 3*(2*
B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*d*x + (2*B*b^3*cos(d*x + c)^2 + 18*B
*a^2*b + 18*A*a*b^2 + 4*B*b^3 + 3*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60186, size = 424, normalized size = 3.09

$$6\,Aa^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(2\,Ba^{3}+6\,Aa^{2}b+3\,Bab^{2}+Ab^{3}\right)(dx+c)+\frac{2(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*(d*x + c) +
 2*(18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 -
 9*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^
3*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*t
an(1/2*d*x + 1/2*c)^3 + 4*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*B*a^2*b*tan(1/2
*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c) + 9*B*a*b^2*tan(1/2*d*x + 1
/2*c) + 3*A*b^3*tan(1/2*d*x + 1/2*c) + 6*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.235 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=131

$$-\frac{b\left(2a^2A-3abB-Ab^2\right)\sin(c+dx)}{d}+\frac{1}{2}bx\left(6a^2B+6aAb+b^2B\right)+\frac{a^2(aB+3Ab)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(2aA-bB)}{}$$

```
[Out] (b*(6*a*A*b + 6*a^2*B + b^2*B)*x)/2 + (a^2*(3*A*b + a*B)*ArcTanh[Sin[c + d*
x]])/d - (b*(2*a^2*A - A*b^2 - 3*a*b*B)*Sin[c + d*x])/d - (b^2*(2*a*A - b*B
)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*(a + b*Cos[c + d*x])^2*Tan[c + d*
x])/d
```

Rubi [A] time = 0.331931, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2989, 3033, 3023, 2735, 3770}

$$-\frac{b\left(2a^2A-3abB-Ab^2\right)\sin(c+dx)}{d}+\frac{1}{2}bx\left(6a^2B+6aAb+b^2B\right)+\frac{a^2(aB+3Ab)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(2aA-bB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(6*a*A*b + 6*a^2*B + b^2*B)*x)/2 + (a^2*(3*A*b + a*B)*ArcTanh[Sin[c + d*
x]])/d - (b*(2*a^2*A - A*b^2 - 3*a*b*B)*Sin[c + d*x])/d - (b^2*(2*a*A - b*B
)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*(a + b*Cos[c + d*x])^2*Tan[c + d*
x])/d
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+b\cos(c+dx)$$

$$= -\frac{b^2(2aA-bB)\cos(c+dx)\sin(c+dx)}{2d} + \frac{aA(a+b\cos(c+$$

$$= -\frac{b\left(2a^2A-Ab^2-3abB\right)\sin(c+dx)}{d} - \frac{b^2(2aA-bB)\cos(c}{2d}$$

$$= \frac{1}{2}b\left(6aAb+6a^2B+b^2B\right)x - \frac{b\left(2a^2A-Ab^2-3abB\right)\sin(c}{d}$$

$$= \frac{1}{2}b\left(6aAb+6a^2B+b^2B\right)x + \frac{a^2(3Ab+aB)\tanh^{-1}(\sin(c+}{d}$$

Mathematica [A] time = 0.650147, size = 217, normalized size = 1.66

$$\frac{2b(c+dx)\left(6a^2B+6aAb+b^2B\right) - 4a^2(aB+3Ab)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2(aB+3Ab)\log\left(\sin\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(6*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - 4*a^2*(3*A*b + a*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^2*(3*A*b + a*B)*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 4*b^2*(A*b + 3*a*B)*Sin[c + d*x] + b^3*B*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.075, size = 168, normalized size = 1.3

$$\frac{Aa^3\tan(dx+c)}{d} + \frac{a^3B\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3\frac{Aa^2b\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3a^2bBx + 3\frac{Ba^2bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*a^3*tan(d*x+c)+1/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a^2*b*ln(sec
(d*x+c)+tan(d*x+c))+3*a^2*b*B*x+3/d*B*a^2*b*c+3*A*a*b^2*x+3/d*A*a*b^2*c+3/d
```

```
*B*a*b^2*sin(d*x+c)+1/d*A*b^3*sin(d*x+c)+1/2/d*B*b^3*cos(d*x+c)*sin(d*x+c)+
1/2*b^3*B*x+1/2/d*B*b^3*c
```

Maxima [A] time = 1.16678, size = 194, normalized size = 1.48

$$\frac{12\,(dx+c)Ba^2b+12\,(dx+c)Aab^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^3+2\,Ba^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a^2*b + 12*(d*x + c)*A*a*b^2 + (2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*b^3 + 2*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
6*A*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*a*b^2*sin(
d*x + c) + 4*A*b^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.56437, size = 369, normalized size = 2.82

$$\frac{\left(6\,Ba^2b+6\,Aab^2+Bb^3\right)dx\cos(dx+c)+\left(Ba^3+3\,Aa^2b\right)\cos(dx+c)\log(\sin(dx+c)+1)-\left(Ba^3+3\,Aa^2b\right)\cos(dx+c}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*((6*B*a^2*b + 6*A*a*b^2 + B*b^3)*d*x*cos(d*x + c) + (B*a^3 + 3*A*a^2*b)
*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^3 + 3*A*a^2*b)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (B*b^3*cos(d*x + c)^2 + 2*A*a^3 + 2*(3*B*a*b^2 + A*b^3
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.54625, size = 316, normalized size = 2.41

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \left(6\,Ba^2b + 6\,Aab^2 + Bb^3\right)(dx+c) - 2\left(Ba^3 + 3\,Aa^2b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 2\left(Ba^3 + 3\,Aa^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*B*a^2*
b + 6*A*a*b^2 + B*b^3)*(d*x + c) - 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) + 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(6*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 - B
*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(
1/2*d*x + 1/2*c) + B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

3.236 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{a\left(a^2A+6abB+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{2d}+b^2x(3aB+Ab)$$

```
[Out] b^2*(A*b + 3*a*B)*x + (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])
/(2*d) - (b^2*(a*A - 2*b*B)*Sin[c + d*x])/(2*d) + (a^2*(2*A*b + a*B)*Tan[c
+ d*x])/d + (a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.339024, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2989, 3031, 3023, 2735, 3770}

$$\frac{a\left(a^2A+6abB+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{2d}+b^2x(3aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```



```
[Out] b^2*(A*b + 3*a*B)*x + (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])
/(2*d) - (b^2*(a*A - 2*b*B)*Sin[c + d*x])/(2*d) + (a^2*(2*A*b + a*B)*Tan[c
+ d*x])/d + (a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (a+b$$
$$= \frac{a^2(2Ab+aB)\tan(c+dx)}{d} + \frac{aA(a+b\cos(c+dx))^2\sec(c+$$
$$= -\frac{b^2(aA-2bB)\sin(c+dx)}{2d} + \frac{a^2(2Ab+aB)\tan(c+dx)}{d} + \frac{a}{}$$
$$= b^2(Ab+3aB)x - \frac{b^2(aA-2bB)\sin(c+dx)}{2d} + \frac{a^2(2Ab+aB)}{d}$$
$$= b^2(Ab+3aB)x + \frac{a\left(a^2A+6Ab^2+6abB\right)\tanh^{-1}(\sin(c+dx}{2d}$$

Mathematica [B] time = 2.03464, size = 277, normalized size = 2.23

$$\frac{-2a\left(a^2A+6abB+6Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2a\left(a^2A+6abB+6Ab^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (4*b^2*(A*b + 3*a*B)*(c + d*x) - 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^2 + (4*a^2*(3*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]) - (a^3*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a^2*(3*A*b
 + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*b^3*B*S
in[c + d*x])/(4*d)
```

Maple [A] time = 0.085, size = 172, normalized size = 1.4

$$\frac{Aa^3\sec(dx+c)\tan(dx+c)}{2d} + \frac{Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{a^3B\tan(dx+c)}{d} + 3\,\frac{Aa^2b\tan(dx+c)}{d} + 3\,\frac{a^2bB\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d
*a^3*B*tan(d*x+c)+3/d*A*a^2*b*tan(d*x+c)+3/d*a^2*b*B*ln(sec(d*x+c)+tan(d*x+
c))+3/d*A*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+3*B*a*b^2*x+3/d*B*a*b^2*c+A*b^3*x
+1/d*A*b^3*c+1/d*B*b^3*sin(d*x+c)
```

Maxima [A] time = 1.1441, size = 228, normalized size = 1.84

$$\frac{12\,(dx+c)Bab^2+4\,(dx+c)Ab^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Ba^2b(\log\left(\sin\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a*b^2 + 4*(d*x + c)*A*b^3 - A*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^2*
b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*A*a*b^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1)) + 4*B*b^3*sin(d*x + c) + 4*B*a^3*tan(d*x
 + c) + 12*A*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 1.46356, size = 401, normalized size = 3.23

$$\frac{4\left(3\,Bab^2+Ab^3\right)dx\cos\left(dx+c\right)^2+\left(Aa^3+6\,Ba^2b+6\,Aab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(Aa^3+6\,Ba^2b+6\,A\right.}{4\,d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*(3*B*a*b^2 + A*b^3)*d*x*cos(d*x + c)^2 + (A*a^3 + 6*B*a^2*b + 6*A*a*
b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*B*b^3*cos(d*x + c)^2 + A*a^3
+ 2*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70623, size = 323, normalized size = 2.6

$$\frac{\frac{4 Bb^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+1} + 2 \left(3 Bab^2 + Ab^3\right)(dx + c) + \left(Aa^3 + 6 Ba^2 b + 6 Aab^2\right) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - \left(Aa^3 + 6 Ba^2 b + 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*B*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*B*a*b
^2 + A*b^3)*(d*x + c) + (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c)^
3 - 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a^3*tan(1/2*d*x + 1/2*c) + 2*B*a^3
*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2)/d
```

3.237 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{a\left(2a^2A+9abB+8Ab^2\right)\tan(c+dx)}{3d}+\frac{\left(3a^2Ab+a^3B+6ab^2B+2Ab^3\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3aB+5Ab)\tan(c+}{6d}$$

```
[Out] b^3*B*x + ((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a^2*(5*A*b
 + 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2*Se
c[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.345814, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2989, 3031, 3021, 2735, 3770}

$$\frac{a\left(2a^2A+9abB+8Ab^2\right)\tan(c+dx)}{3d}+\frac{\left(3a^2Ab+a^3B+6ab^2B+2Ab^3\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3aB+5Ab)\tan(c+}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] b^3*B*x + ((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a^2*(5*A*b
 + 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2*Se
c[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int(a$$
$$= \frac{a^2(5Ab+3aB)\sec(c+dx)\tan(c+dx)}{6d}+\frac{aA(a+b\cos(c+$$
$$= \frac{a\left(2a^2A+8Ab^2+9abB\right)\tan(c+dx)}{3d}+\frac{a^2(5Ab+3aB)\sec$$
$$= b^3Bx+\frac{a\left(2a^2A+8Ab^2+9abB\right)\tan(c+dx)}{3d}+\frac{a^2(5Ab+3$$
$$= b^3Bx+\frac{\left(3a^2Ab+2Ab^3+a^3B+6ab^2B\right)\tanh^{-1}(\sin(c+dx$$

Mathematica [A] time = 0.565952, size = 108, normalized size = 0.74

$$\frac{3\left(3a^2Ab+a^3B+6ab^2B+2Ab^3\right)\tanh^{-1}(\sin(c+dx))+3a\tan(c+dx)\left(2a^2A+a(aB+3Ab)\sec(c+dx)+6abB+6A\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (6*b^3*B*d*x + 3*(3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c +
d*x]] + 3*a*(2*a^2*A + 6*A*b^2 + 6*a*b*B + a*(3*A*b + a*B)*Sec[c + d*x])*Ta
n[c + d*x] + 2*a^3*A*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.086, size = 223, normalized size = 1.5

$$\frac{2\,Aa^3\tan(dx+c)}{3\,d}+\frac{Aa^3\tan(dx+c)(\sec(dx+c))^2}{3\,d}+\frac{a^3B\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{a^3B\ln(\sec(dx+c)+\tan(dx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3/d*A*a^3*tan(d*x+c)+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^3*B*sec(
d*x+c)*tan(d*x+c)+1/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a^2*b*sec(d
*x+c)*tan(d*x+c)+3/2/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*B*tan(d*
x+c)+3/d*A*a*b^2*tan(d*x+c)+3/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b^3
```

```
*ln(sec(d*x+c)+tan(d*x+c))+b^3*B*x+1/d*B*b^3*c
```

Maxima [A] time = 1.15364, size = 292, normalized size = 2.01

$$4\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Aa^3 + 12(dx+c)Bb^3 - 3Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*b^3 - 3*B*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 9*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 18*B*a*b^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 6*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 36*B*a^2*b*tan(d*x + c) + 36*A*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.50364, size = 458, normalized size = 3.16

$$12Bb^3dx\cos(dx+c)^3 + 3\left(Ba^3 + 3Aa^2b + 6Bab^2 + 2Ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1) - 3\left(Ba^3 + 3Aa^2b + 6Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*B*b^3*d*x*cos(d*x + c)^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*
b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^
2 + 2*A*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*A*a^3 + 2*(2*A*a^
3 + 9*B*a^2*b + 9*A*a*b^2)*cos(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48595, size = 454, normalized size = 3.13

$$6(dx+c)Bb^3+3\left(Ba^3+3Aa^2b+6Bab^2+2Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(Ba^3+3Aa^2b+6Bab^2+2Ab^3\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*B*b^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*
B*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^2*
b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^3*tan(
1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2
*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 3*B*a^3*tan(1/2*d*x + 1/2*
c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*
A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.238 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=188

$$\frac{\left(6a^2Ab+2a^3B+9ab^2B+3Ab^3\right)\tan(c+dx)}{3d}+\frac{\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(3a^2A+12\right.}{}$$

```
[Out] ((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((6*a^2*A*b + 3*A*b^3 + 2*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(3*d) + (a*(3
*a^2*A + 10*A*b^2 + 12*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(3*A*
b + 2*a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.457663, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2989, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(6a^2Ab+2a^3B+9ab^2B+3Ab^3\right)\tan(c+dx)}{3d}+\frac{\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(3a^2A+12\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((6*a^2*A*b + 3*A*b^3 + 2*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(3*d) + (a*(3
*a^2*A + 10*A*b^2 + 12*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(3*A*
b + 2*a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
```

```
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (a+$$
$$= \frac{a^2(3Ab+2aB)\sec^2(c+dx)\tan(c+dx)}{6d}+\frac{aA(a+b\cos(c+$$
$$= \frac{a\left(3a^2A+10Ab^2+12abB\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(3A$$
$$= \frac{a\left(3a^2A+10Ab^2+12abB\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(3A$$
$$= \frac{\left(3a^3A+12aAb^2+12a^2bB+8b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$
$$= \frac{\left(3a^3A+12aAb^2+12a^2bB+8b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$

Mathematica [A] time = 0.803062, size = 140, normalized size = 0.74

$$\frac{3\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(9a\left(a^2A+4abB+4Ab^2\right)\sec(c+dx)+24\left(3a^2\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(24*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B) + 9*a*(a^2*A + 4*A*b
^2 + 4*a*b*B)*Sec[c + d*x] + 6*a^3*A*Sec[c + d*x]^3 + 8*a^2*(3*A*b + a*B)*T
an[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.098, size = 290, normalized size = 1.5

$$\frac{Aa^3\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{3Aa^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{3Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2a^3B\tan(d}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+3/8/d
*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^3*B*tan(d*x+c)+1/3/d*a^3*B*tan(d*x
+c)*sec(d*x+c)^2+2/d*A*a^2*b*tan(d*x+c)+1/d*A*a^2*b*tan(d*x+c)*sec(d*x+c)^2
+3/2/d*a^2*b*B*tan(d*x+c)*sec(d*x+c)+3/2/d*a^2*b*B*ln(sec(d*x+c)+tan(d*x+c)
)+3/2/d*A*a*b^2*tan(d*x+c)*sec(d*x+c)+3/2/d*A*a*b^2*ln(sec(d*x+c)+tan(d*x+c
))+3/d*B*a*b^2*tan(d*x+c)+1/d*A*b^3*tan(d*x+c)+1/d*B*b^3*ln(sec(d*x+c)+tan(
d*x+c))
```

Maxima [A] time = 1.14916, size = 369, normalized size = 1.96

$$\frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^3+48\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^2b-3\,Aa^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^2*b - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 36*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*A*a*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*b^3*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 144*B*a*b^2*tan(d*x + c) + 4
8*A*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.57739, size = 510, normalized size = 2.71

$$\frac{3\left(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(6*A*a^3 + 8*(2*B*a^3 + 6*A*a^2*b + 9*B*a*
b^2 + 3*A*b^3)*cos(d*x + c)^3 + 9*(A*a^3 + 4*B*a^2*b + 4*A*a*b^2)*cos(d*x +
 c)^2 + 8*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36687, size = 791, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan
(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d
*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^3*tan(1/2*d*x + 1/2
*c)^5 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^
5 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A
*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^3*tan(1/
2*d*x + 1/2*c)^3 + 15*A*a^3*tan(1/2*d*x + 1/2*c) + 24*B*a^3*tan(1/2*d*x + 1
```

```
/2*c) + 72*A*a^2*b*tan(1/2*d*x + 1/2*c) + 36*B*a^2*b*tan(1/2*d*x + 1/2*c) +
 36*A*a*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*A*b
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.239 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{(8a^3A+30a^2bB+30aAb^2+15b^3B)\tan(c+dx)}{15d}+\frac{(9a^2Ab+3a^3B+12ab^2B+4Ab^3)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4a^2A+$$

```
[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((8*a^3*A + 30*a*A*b^2 + 30*a^2*b*B + 15*b^3*B)*Tan[c + d*x])/(15*d) + ((
9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a^2*(7*A*b + 5*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.488813, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2989, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(8a^3A+30a^2bB+30aAb^2+15b^3B)\tan(c+dx)}{15d}+\frac{(9a^2Ab+3a^3B+12ab^2B+4Ab^3)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4a^2A+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```


```
[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((8*a^3*A + 30*a*A*b^2 + 30*a^2*b*B + 15*b^3*B)*Tan[c + d*x])/(15*d) + ((
9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a^2*(7*A*b + 5*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
```

```
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (a+$$
$$= \frac{a^2(7Ab+5aB)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{aA(a+b\cos(c+$$
$$= \frac{a\left(4a^2A+12Ab^2+15abB\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{a^2(7A$$
$$= \frac{a\left(4a^2A+12Ab^2+15abB\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{a^2(7A$$
$$= \frac{\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)\sec(c+dx)\tan(c+dx)}{8d}+$$
$$= \frac{\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(8$$

Mathematica [A] time = 3.2125, size = 181, normalized size = 0.77

$$15\left(9a^2Ab+3a^3B+12ab^2B+4Ab^3\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5a\left(2a^2A+3abB+3Ab^2\right)\tan^2(c+dx)+15\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x] +
30*a^2*(3*A*b + a*B)*Sec[c + d*x]^3 + 8*(15*(a^3*A + 3*a*A*b^2 + 3*a^2*b*B
+ b^3*B) + 5*a*(2*a^2*A + 3*A*b^2 + 3*a*b*B)*Tan[c + d*x]^2 + 3*a^3*A*Tan[c
```

```
+ d*x]^4)))/(120*d)
```

Maple [A] time = 0.092, size = 382, normalized size = 1.6

$$\frac{8\,Aa^3\tan(dx+c)}{15\,d}+\frac{Aa^3\tan(dx+c)\,(\sec(dx+c))^4}{5\,d}+\frac{4\,Aa^3\tan(dx+c)\,(\sec(dx+c))^2}{15\,d}+\frac{a^3B\tan(dx+c)\,(\sec(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 8/15/d*A*a^3*tan(d*x+c)+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*a^3*ta
n(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*B*sec(d
*x+c)*tan(d*x+c)+3/8/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*A*a^2*b*tan(d*
x+c)*sec(d*x+c)^3+9/8/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+9/8/d*A*a^2*b*ln(sec(
d*x+c)+tan(d*x+c))+2/d*a^2*b*B*tan(d*x+c)+1/d*a^2*b*B*tan(d*x+c)*sec(d*x+c)
^2+2/d*A*a*b^2*tan(d*x+c)+1/d*A*a*b^2*tan(d*x+c)*sec(d*x+c)^2+3/2/d*B*a*b^2
*tan(d*x+c)*sec(d*x+c)+3/2/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b^3*
tan(d*x+c)*sec(d*x+c)+1/2/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b^3*tan(d
*x+c)
```

Maxima [A] time = 1.11634, size = 460, normalized size = 1.95

$$\frac{16\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^3+240\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^2b+240\left(\tan(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b + 240*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*a*b^2 - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 45*A*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 180*B*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) - 60*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2
```

```
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*b^3*tan(d*x +
 c))/d
```

Fricas [A] time = 1.52949, size = 612, normalized size = 2.59

$$\frac{15\left(3 Ba^3 + 9 Aa^2b + 12 Bab^2 + 4 Ab^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right) - 15\left(3 Ba^3 + 9 Aa^2b + 12 Bab^2 + 4 Ab^3\right)\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c)^5*log(s
in(d*x + c) + 1) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x
+ c)^5*log(-sin(d*x + c) + 1) + 2*(8*(8*A*a^3 + 30*B*a^2*b + 30*A*a*b^2 + 1
5*B*b^3)*cos(d*x + c)^4 + 24*A*a^3 + 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 +
 4*A*b^3)*cos(d*x + c)^3 + 8*(4*A*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d*x +
c)^2 + 30*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52396, size = 975, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*t
an(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan
(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d
*x + 1/2*c)^9 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^3*tan(1/2*d*x + 1
/2*c)^7 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*B*a^2*b*tan(1/2*d*x + 1/2
*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 360*B*a*b^2*tan(1/2*d*x + 1/2*
c)^7 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^7
+ 464*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 16
0*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*A*a^2
*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*A*a*b^
2*tan(1/2*d*x + 1/2*c)^3 - 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*b^3*t
an(1/2*d*x + 1/2*c)^3 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/
2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x +
1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c
) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c) + 60*A*b^3*tan(1/2*d*x + 1/2*c) + 120*
B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.240 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=366

$$-\frac{\left(140a^3Ab+168a^2b^2B+35a^4B+112aAb^3+24b^4B\right)\sin^3(c+dx)}{105d}+\frac{\left(140a^3Ab+168a^2b^2B+35a^4B+112aAb^3+24b^4B\right.}{35d}$$

```
[Out] ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*x)/16 + ((140
*a^3*A*b + 112*a*A*b^3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x])
/(35*d) + ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (b*(224*a^2*A*b + 35*A*b^3 + 104*a^3*B + 1
40*a*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(168*d) + (b^2*(49*a*A*b + 31*a^2*
B + 18*b^2*B)*Cos[c + d*x]^4*Sin[c + d*x])/(105*d) + (b*(7*A*b + 10*a*B)*Co
s[c + d*x]^3*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(42*d) + (b*B*Cos[c + d*x
]^3*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) - ((140*a^3*A*b + 112*a*A*b^
3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.837696, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2990, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{\left(140a^3Ab+168a^2b^2B+35a^4B+112aAb^3+24b^4B\right)\sin^3(c+dx)}{105d}+\frac{\left(140a^3Ab+168a^2b^2B+35a^4B+112aAb^3+24b^4B\right.}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*x)/16 + ((140
*a^3*A*b + 112*a*A*b^3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x])
/(35*d) + ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (b*(224*a^2*A*b + 35*A*b^3 + 104*a^3*B + 1
40*a*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(168*d) + (b^2*(49*a*A*b + 31*a^2*
B + 18*b^2*B)*Cos[c + d*x]^4*Sin[c + d*x])/(105*d) + (b*(7*A*b + 10*a*B)*Co
s[c + d*x]^3*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(42*d) + (b*B*Cos[c + d*x
]^3*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) - ((140*a^3*A*b + 112*a*A*b^
3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x]^3)/(105*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{7d}+\frac{1}{7}\int \cos^2($$

$$= \frac{b(7Ab+10aB)\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{42d}$$

$$= \frac{b^2\left(49aAb+31a^2B+18b^2B\right)\cos^4(c+dx)\sin(c+dx)}{105d}+\frac{b(7$$

$$= \frac{b\left(224a^2Ab+35Ab^3+104a^3B+140ab^2B\right)\cos^3(c+dx)\sin}{168d}$$

$$= \frac{b\left(224a^2Ab+35Ab^3+104a^3B+140ab^2B\right)\cos^3(c+dx)\sin}{168d}$$

$$= \frac{\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B\right)\cos(c+dx)\,s}{16d}$$

$$= \frac{1}{16}\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B\right)x+\frac{(140a}{}$$

Mathematica [A] time = 0.864315, size = 408, normalized size = 1.11

$$105\left(192a^3Ab+240a^2b^2B+48a^4B+160aAb^3+35b^4B\right)\sin(c+dx)+105\left(96a^2Ab^2+16a^4A+64a^3bB+60ab^3B+15\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3360*a^4*A*c + 15120*a^2*A*b^2*c + 2100*A*b^4*c + 10080*a^3*b*B*c + 8400*a
*b^3*B*c + 3360*a^4*A*d*x + 15120*a^2*A*b^2*d*x + 2100*A*b^4*d*x + 10080*a^
3*b*B*d*x + 8400*a*b^3*B*d*x + 105*(192*a^3*A*b + 160*a*A*b^3 + 48*a^4*B +
240*a^2*b^2*B + 35*b^4*B)*Sin[c + d*x] + 105*(16*a^4*A + 96*a^2*A*b^2 + 15*
A*b^4 + 64*a^3*b*B + 60*a*b^3*B)*Sin[2*(c + d*x)] + 2240*a^3*A*b*Sin[3*(c +
 d*x)] + 2800*a*A*b^3*Sin[3*(c + d*x)] + 560*a^4*B*Sin[3*(c + d*x)] + 4200*
a^2*b^2*B*Sin[3*(c + d*x)] + 735*b^4*B*Sin[3*(c + d*x)] + 1260*a^2*A*b^2*Si
n[4*(c + d*x)] + 315*A*b^4*Sin[4*(c + d*x)] + 840*a^3*b*B*Sin[4*(c + d*x)]
+ 1260*a*b^3*B*Sin[4*(c + d*x)] + 336*a*A*b^3*Sin[5*(c + d*x)] + 504*a^2*b^
2*B*Sin[5*(c + d*x)] + 147*b^4*B*Sin[5*(c + d*x)] + 35*A*b^4*Sin[6*(c + d*x
)] + 140*a*b^3*B*Sin[6*(c + d*x)] + 15*b^4*B*Sin[7*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.047, size = 368, normalized size = 1.

$$\frac{1}{d}\left(Aa^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{4\,Aa^3b\left(2+(\cos(dx+c))^2\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^4*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+4/3*A*a^3*b*(2+cos(d*x+c)^2)*sin(d*x+c)+4*B*a^3*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*a^2*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*B*a^2*b^2*(8/3+cos(d*
x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/5*A*a*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c)+4*B*a*b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(
d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+A*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)
^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/7*B*b^4*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.17136, size = 494, normalized size = 1.35

$$\frac{1680\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^4 - 2240\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Ba^4 - 8960\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6720*(1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 - 2240*(sin(d*x + c)^3
- 3*sin(d*x + c))*B*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3*b +
840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3*b + 1260*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b^2 + 2688*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^2*b^2 + 1792*(3*
sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a*b^3 - 140*(4*sin(
2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*
B*a*b^3 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*A*b^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*
sin(d*x + c)^3 - 35*sin(d*x + c))*B*b^4)/d
```

Fricas [A] time = 1.69899, size = 717, normalized size = 1.96

$$\frac{105\left(8\,Aa^4 + 24\,Ba^3b + 36\,Aa^2b^2 + 20\,Bab^3 + 5\,Ab^4\right)dx + \left(240\,Bb^4\cos(dx + c)^6 + 280\left(4\,Bab^3 + Ab^4\right)\cos(dx + c)^5 + 11\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*d*
x + (240*B*b^4*cos(d*x + c)^6 + 280*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^5 + 11
20*B*a^4 + 4480*A*a^3*b + 5376*B*a^2*b^2 + 3584*A*a*b^3 + 768*B*b^4 + 96*(2
1*B*a^2*b^2 + 14*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4 + 70*(24*B*a^3*b + 36*A*
a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c)^3 + 16*(35*B*a^4 + 140*A*a^3*b
 + 168*B*a^2*b^2 + 112*A*a*b^3 + 24*B*b^4)*cos(d*x + c)^2 + 105*(8*A*a^4 +
24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c))*sin(d*x + c
))/d
```

Sympy [A] time = 11.8799, size = 1017, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*x*sin(c + d*x)**2/2 + A*a**4*x*cos(c + d*x)**2/2 + A*a**4
*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*A*a**3*b*sin(c + d*x)**3/(3*d) + 4*A*a
**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a**2*b**2*x*sin(c + d*x)**4/4 +
9*A*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*A*a**2*b**2*x*cos(c +
 d*x)**4/4 + 9*A*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*A*a**2*b
**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*A*a*b**3*sin(c + d*x)**5/(15*d)
 + 16*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*A*a*b**3*sin(c + d
*x)*cos(c + d*x)**4/d + 5*A*b**4*x*sin(c + d*x)**6/16 + 15*A*b**4*x*sin(c +
 d*x)**4*cos(c + d*x)**2/16 + 15*A*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/1
6 + 5*A*b**4*x*cos(c + d*x)**6/16 + 5*A*b**4*sin(c + d*x)**5*cos(c + d*x)/(
16*d) + 5*A*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*A*b**4*sin(c +
d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*sin(c
 + d*x)*cos(c + d*x)**2/d + 3*B*a**3*b*x*sin(c + d*x)**4/2 + 3*B*a**3*b*x*s
in(c + d*x)**2*cos(c + d*x)**2 + 3*B*a**3*b*x*cos(c + d*x)**4/2 + 3*B*a**3*
b*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a**3*b*sin(c + d*x)*cos(c + d*x)
**3/(2*d) + 16*B*a**2*b**2*sin(c + d*x)**5/(5*d) + 8*B*a**2*b**2*sin(c + d*
x)**3*cos(c + d*x)**2/d + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*
B*a*b**3*x*sin(c + d*x)**6/4 + 15*B*a*b**3*x*sin(c + d*x)**4*cos(c + d*x)**
2/4 + 15*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/4 + 5*B*a*b**3*x*cos(c
+ d*x)**6/4 + 5*B*a*b**3*sin(c + d*x)**5*cos(c + d*x)/(4*d) + 10*B*a*b**3*s
in(c + d*x)**3*cos(c + d*x)**3/(3*d) + 11*B*a*b**3*sin(c + d*x)*cos(c + d*x
)**5/(4*d) + 16*B*b**4*sin(c + d*x)**7/(35*d) + 8*B*b**4*sin(c + d*x)**5*co
s(c + d*x)**2/(5*d) + 2*B*b**4*sin(c + d*x)**3*cos(c + d*x)**4/d + B*b**4*s
in(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))*
*4*cos(c)**2, True))
```

Giac [A] time = 1.46736, size = 423, normalized size = 1.16

$$\frac{Bb^4 \sin(7\,dx + 7\,c)}{448\,d} + \frac{1}{16}\left(8\,Aa^4 + 24\,Ba^3b + 36\,Aa^2b^2 + 20\,Bab^3 + 5\,Ab^4\right)x + \frac{\left(4\,Bab^3 + Ab^4\right)\sin(6\,dx + 6\,c)}{192\,d} + \frac{(24\,Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gi
```

ac")

```
[Out] 1/448*B*b^4*sin(7*d*x + 7*c)/d + 1/16*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
+ 20*B*a*b^3 + 5*A*b^4)*x + 1/192*(4*B*a*b^3 + A*b^4)*sin(6*d*x + 6*c)/d +
1/320*(24*B*a^2*b^2 + 16*A*a*b^3 + 7*B*b^4)*sin(5*d*x + 5*c)/d + 1/64*(8*B*
a^3*b + 12*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*sin(4*d*x + 4*c)/d + 1/192*(16
*B*a^4 + 64*A*a^3*b + 120*B*a^2*b^2 + 80*A*a*b^3 + 21*B*b^4)*sin(3*d*x + 3*
c)/d + 1/64*(16*A*a^4 + 64*B*a^3*b + 96*A*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4)*
sin(2*d*x + 2*c)/d + 1/64*(48*B*a^4 + 192*A*a^3*b + 240*B*a^2*b^2 + 160*A*a
*b^3 + 35*B*b^4)*sin(d*x + c)/d
```

3.241 $\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=325

$$\frac{\left(224a^2Ab^3+24a^4Ab+121a^3b^2B-4a^5B+128ab^4B+32Ab^5\right)\sin(c+dx)}{60bd}+\frac{\left(-4a^2B+24aAb+25b^2B\right)\sin(c+dx)(a}{120bd}$$

```
[Out] ((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*x)/16 + ((24*
a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*a^3*b^2*B + 128*a*b^4*B)
*Sin[c + d*x])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^3 - 8*a^4*B + 178*a^2*b^
2*B + 75*b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(240*d) + ((24*a^2*A*b + 32*A*b^
3 - 4*a^3*B + 53*a*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(120*b*d) +
((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*
b*d) + ((6*A*b - a*B)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + (B*(a
 + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.509253, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2753, 2734}

$$\frac{\left(224a^2Ab^3+24a^4Ab+121a^3b^2B-4a^5B+128ab^4B+32Ab^5\right)\sin(c+dx)}{60bd}+\frac{\left(-4a^2B+24aAb+25b^2B\right)\sin(c+dx)(a}{120bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```


```
[Out] ((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*x)/16 + ((24*
a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*a^3*b^2*B + 128*a*b^4*B)
*Sin[c + d*x])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^3 - 8*a^4*B + 178*a^2*b^
2*B + 75*b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(240*d) + ((24*a^2*A*b + 32*A*b^
3 - 4*a^3*B + 53*a*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(120*b*d) +
((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*
b*d) + ((6*A*b - a*B)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + (B*(a
 + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^4\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$= \frac{B(a+b\cos(c+dx))^5\sin(c+dx)}{6bd} + \frac{\int (a+b\cos(c+dx))^4(5$$

$$= \frac{(6Ab-aB)(a+b\cos(c+dx))^4\sin(c+dx)}{30bd} + \frac{B(a+b\cos(c$$

$$= \frac{\left(24aAb-4a^2B+25b^2B\right)(a+b\cos(c+dx))^3\sin(c+dx)}{120bd} +$$

$$= \frac{\left(24a^2Ab+32Ab^3-4a^3B+53ab^2B\right)(a+b\cos(c+dx))^2\sin}{120bd}$$

$$= \frac{1}{16}\left(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B\right)x + \frac{(24a}{}$$

Mathematica [A] time = 1.05533, size = 333, normalized size = 1.02

$$\frac{120\left(36a^2Ab^2+8a^4A+24a^3bB+20ab^3B+5Ab^4\right)\sin(c+dx)+15\left(64a^3Ab+96a^2b^2B+16a^4B+64aAb^3+15b^4B\right)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (1920*a^3*A*b*c + 1440*a*A*b^3*c + 480*a^4*B*c + 2160*a^2*b^2*B*c + 300*b^4
*B*c + 1920*a^3*A*b*d*x + 1440*a*A*b^3*d*x + 480*a^4*B*d*x + 2160*a^2*b^2*B
*d*x + 300*b^4*B*d*x + 120*(8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B +
 20*a*b^3*B)*Sin[c + d*x] + 15*(64*a^3*A*b + 64*a*A*b^3 + 16*a^4*B + 96*a^2
*b^2*B + 15*b^4*B)*Sin[2*(c + d*x)] + 480*a^2*A*b^2*Sin[3*(c + d*x)] + 100*
A*b^4*Sin[3*(c + d*x)] + 320*a^3*b*B*Sin[3*(c + d*x)] + 400*a*b^3*B*Sin[3*(
c + d*x)] + 120*a*A*b^3*Sin[4*(c + d*x)] + 180*a^2*b^2*B*Sin[4*(c + d*x)] +
 45*b^4*B*Sin[4*(c + d*x)] + 12*A*b^4*Sin[5*(c + d*x)] + 48*a*b^3*B*Sin[5*(
c + d*x)] + 5*b^4*B*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.084, size = 316, normalized size = 1.

$$\frac{1}{d}\left(Aa^4\sin(dx+c)+a^4B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4\,Aa^3b\,(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*A*a
^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4/3*B*a^3*b*(2+cos(d*x+c)^2)
*sin(d*x+c)+2*A*a^2*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+6*B*a^2*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*A*a*b^3*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*B*a*b^3*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/5*A*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c)+B*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(
d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.04622, size = 414, normalized size = 1.27

$$\frac{240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^4+960\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3b-1280\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^3*b - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3*b - 19
20*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b^2 + 180*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b^2 + 120*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^3 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*B*a*b^3 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)
^3 + 15*sin(d*x + c))*A*b^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*s
in(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^4 + 960*A*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.57297, size = 587, normalized size = 1.81

$$\frac{15\left(8\,Ba^4+32\,Aa^3b+36\,Ba^2b^2+24\,Aab^3+5\,Bb^4\right)dx+\left(40\,Bb^4\cos(dx+c)^5+240\,Aa^4+640\,Ba^3b+960\,Aa^2b^2+512\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*d*x
+ (40*B*b^4*cos(d*x + c)^5 + 240*A*a^4 + 640*B*a^3*b + 960*A*a^2*b^2 + 512*
B*a*b^3 + 128*A*b^4 + 48*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 10*(36*B*a^2*
b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^3 + 32*(10*B*a^3*b + 15*A*a^2*b^2
+ 8*B*a*b^3 + 2*A*b^4)*cos(d*x + c)^2 + 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2
*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 6.86899, size = 811, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*sin(c + d*x)/d + 2*A*a**3*b*x*sin(c + d*x)**2 + 2*A*a**3*
b*x*cos(c + d*x)**2 + 2*A*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*A*a**2*b**
2*sin(c + d*x)**3/d + 6*A*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a*
b**3*x*sin(c + d*x)**4/2 + 3*A*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3
*A*a*b**3*x*cos(c + d*x)**4/2 + 3*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 5*A*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*A*b**4*sin(c + d*x)*
*5/(15*d) + 4*A*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*b**4*sin(c +
 d*x)*cos(c + d*x)**4/d + B*a**4*x*sin(c + d*x)**2/2 + B*a**4*x*cos(c + d*x
)**2/2 + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*B*a**3*b*sin(c + d*x)**
3/(3*d) + 4*B*a**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**2*b**2*x*sin(c
 + d*x)**4/4 + 9*B*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*B*a**2
*b**2*x*cos(c + d*x)**4/4 + 9*B*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d
) + 15*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*B*a*b**3*sin(c +
 d*x)**5/(15*d) + 16*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*B*a
*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*b**4*x*sin(c + d*x)**6/16 + 15*B
*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**4*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 5*B*b**4*x*cos(c + d*x)**6/16 + 5*B*b**4*sin(c + d*x)**5
*cos(c + d*x)/(16*d) + 5*B*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*
B*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
 + b*cos(c))**4*cos(c), True))
```

Giac [A] time = 1.71884, size = 355, normalized size = 1.09

$$\frac{Bb^4 \sin(6\,dx + 6\,c)}{192\,d} + \frac{1}{16}\left(8\,Ba^4 + 32\,Aa^3b + 36\,Ba^2b^2 + 24\,Aab^3 + 5\,Bb^4\right)x + \frac{\left(4\,Bab^3 + Ab^4\right)\sin(5\,dx + 5\,c)}{80\,d} + \frac{(12\,Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*B*b^4*sin(6*d*x + 6*c)/d + 1/16*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2
+ 24*A*a*b^3 + 5*B*b^4)*x + 1/80*(4*B*a*b^3 + A*b^4)*sin(5*d*x + 5*c)/d + 1
/64*(12*B*a^2*b^2 + 8*A*a*b^3 + 3*B*b^4)*sin(4*d*x + 4*c)/d + 1/48*(16*B*a^
3*b + 24*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*sin(3*d*x + 3*c)/d + 1/64*(16*B*
a^4 + 64*A*a^3*b + 96*B*a^2*b^2 + 64*A*a*b^3 + 15*B*b^4)*sin(2*d*x + 2*c)/d
 + 1/8*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*sin(d*x
 + c)/d
```

3.242 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=241

$$\frac{\left(95a^3Ab+112a^2b^2B+12a^4B+80aAb^3+16b^4B\right)\sin(c+dx)}{30d}+\frac{\left(12a^2B+35aAb+16b^2B\right)\sin(c+dx)(a+b\cos(c+d}{60d}$$

```
[Out] ((8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*x)/8 + ((95*a
^3*A*b + 80*a*A*b^3 + 12*a^4*B + 112*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(3
0*d) + (b*(130*a^2*A*b + 45*A*b^3 + 24*a^3*B + 116*a*b^2*B)*Cos[c + d*x]*Si
n[c + d*x])/(120*d) + ((35*a*A*b + 12*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(60*d) + ((5*A*b + 4*a*B)*(a + b*Cos[c + d*x])^3*Sin[c +
d*x])/(20*d) + (B*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.337711, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$\frac{\left(95a^3Ab+112a^2b^2B+12a^4B+80aAb^3+16b^4B\right)\sin(c+dx)}{30d}+\frac{\left(12a^2B+35aAb+16b^2B\right)\sin(c+dx)(a+b\cos(c+d}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*x)/8 + ((95*a
^3*A*b + 80*a*A*b^3 + 12*a^4*B + 112*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(3
0*d) + (b*(130*a^2*A*b + 45*A*b^3 + 24*a^3*B + 116*a*b^2*B)*Cos[c + d*x]*Si
n[c + d*x])/(120*d) + ((35*a*A*b + 12*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(60*d) + ((5*A*b + 4*a*B)*(a + b*Cos[c + d*x])^3*Sin[c +
d*x])/(20*d) + (B*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}\int (a+b\cos(c+dx))^3(5aA+4bB$$
$$=\frac{(5Ab+4aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20d}+\frac{B(a+b\cos(c+dx))^4\sin($$
$$=\frac{\left(35aAb+12a^2B+16b^2B\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60d}+\frac{(5Ab+4aB$$
$$=\frac{1}{8}\left(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B\right)x+\frac{\left(95a^3Ab+80aAb^3\right.}{}$$

Mathematica [A] time = 0.619624, size = 263, normalized size = 1.09

$$\frac{60\left(32a^3Ab+36a^2b^2B+8a^4B+24aAb^3+5b^4B\right)\sin(c+dx)+120b\left(6a^2Ab+4a^3B+4ab^2B+Ab^3\right)\sin(2(c+dx))+14}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (480*a^4*A*c + 1440*a^2*A*b^2*c + 180*A*b^4*c + 960*a^3*b*B*c + 720*a*b^3*B
*c + 480*a^4*A*d*x + 1440*a^2*A*b^2*d*x + 180*A*b^4*d*x + 960*a^3*b*B*d*x +
 720*a*b^3*B*d*x + 60*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5
*b^4*B)*Sin[c + d*x] + 120*b*(6*a^2*A*b + A*b^3 + 4*a^3*B + 4*a*b^2*B)*Sin[
2*(c + d*x)] + 160*a*A*b^3*Sin[3*(c + d*x)] + 240*a^2*b^2*B*Sin[3*(c + d*x)
] + 50*b^4*B*Sin[3*(c + d*x)] + 15*A*b^4*Sin[4*(c + d*x)] + 60*a*b^3*B*Sin[
4*(c + d*x)] + 6*b^4*B*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.04, size = 258, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Bb^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ab^4\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*B*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*b^4*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*B*a*b^3*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a*b^3*(2+cos(d*x+c)
^2)*sin(d*x+c)+2*B*a^2*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+6*A*a^2*b^2*(1/2*cos
(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*B*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*
d*x+1/2*c)+4*A*a^3*b*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [A] time = 1.15258, size = 332, normalized size = 1.38

$$\frac{480\,(dx+c)Aa^4 + 480\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^3b + 720\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^2b^2 - 960\,\left(\sin\left(dx+c\right)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(480*(d*x + c)*A*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3*b +
 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b^2 - 960*(sin(d*x + c)^3 - 3*s
in(d*x + c))*B*a^2*b^2 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^3 + 60
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^3 + 15*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^4 + 32*(3*sin(d*x +
c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^4 + 480*B*a^4*sin(d*x + c)
+ 1920*A*a^3*b*sin(d*x + c))/d
```

Fricas [A] time = 1.5214, size = 478, normalized size = 1.98

$$\frac{15\left(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4\right)dx+\left(24\,Bb^4\cos\left(dx+c\right)^4+120\,Ba^4+480\,Aa^3b+480\,Ba^2b^2+320\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*d*x
+ (24*B*b^4*cos(d*x + c)^4 + 120*B*a^4 + 480*A*a^3*b + 480*B*a^2*b^2 + 320*
```

```
A*a*b^3 + 64*B*b^4 + 30*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^3 + 16*(15*B*a^2*b
^2 + 10*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 15*(16*B*a^3*b + 24*A*a^2*b^2 +
 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.67264, size = 580, normalized size = 2.41

$$\begin{cases} Aa^4x + \frac{4Aa^3b\sin(c+dx)}{d} + 3Aa^2b^2x\sin^2(c+dx) + 3Aa^2b^2x\cos^2(c+dx) + \frac{3Aa^2b^2\sin(c+dx)\cos(c+dx)}{d} + \frac{8Aab^3\sin^3(c+dx)}{3d} + \frac{4Aa \\ x(A+B\cos(c))(a+b\cos(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*x + 4*A*a**3*b*sin(c + d*x)/d + 3*A*a**2*b**2*x*sin(c + d
*x)**2 + 3*A*a**2*b**2*x*cos(c + d*x)**2 + 3*A*a**2*b**2*sin(c + d*x)*cos(c
 + d*x)/d + 8*A*a*b**3*sin(c + d*x)**3/(3*d) + 4*A*a*b**3*sin(c + d*x)*cos(
c + d*x)**2/d + 3*A*b**4*x*sin(c + d*x)**4/8 + 3*A*b**4*x*sin(c + d*x)**2*c
os(c + d*x)**2/4 + 3*A*b**4*x*cos(c + d*x)**4/8 + 3*A*b**4*sin(c + d*x)**3*
cos(c + d*x)/(8*d) + 5*A*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**4*s
in(c + d*x)/d + 2*B*a**3*b*x*sin(c + d*x)**2 + 2*B*a**3*b*x*cos(c + d*x)**2
 + 2*B*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*B*a**2*b**2*sin(c + d*x)**3/d
 + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*b**3*x*sin(c + d*x)
**4/2 + 3*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*B*a*b**3*x*cos(c +
 d*x)**4/2 + 3*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a*b**3*sin
(c + d*x)*cos(c + d*x)**3/(2*d) + 8*B*b**4*sin(c + d*x)**5/(15*d) + 4*B*b**
4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**4*sin(c + d*x)*cos(c + d*x)*
*4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**4, True))
```

Giac [A] time = 1.42656, size = 286, normalized size = 1.19

$$\frac{Bb^4\sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\left(8\,Aa^4 + 16\,Ba^3b + 24\,Aa^2b^2 + 12\,Bab^3 + 3\,Ab^4\right)x + \frac{\left(4\,Bab^3 + Ab^4\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{\left(24\,Ba^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*b^4*sin(5*d*x + 5*c)/d + 1/8*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 +
12*B*a*b^3 + 3*A*b^4)*x + 1/32*(4*B*a*b^3 + A*b^4)*sin(4*d*x + 4*c)/d + 1/4
```

```
8*(24*B*a^2*b^2 + 16*A*a*b^3 + 5*B*b^4)*sin(3*d*x + 3*c)/d + 1/4*(4*B*a^3*b
 + 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sin(2*d*x + 2*c)/d + 1/8*(8*B*a^4 + 32*
A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*sin(d*x + c)/d
```

3.243 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=200

$$\frac{b\left(34a^2Ab+19a^3B+16ab^2B+4Ab^3\right)\sin(c+dx)}{6d}+\frac{b^2\left(26a^2B+32aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(32a^3Ab\right.$$

```
[Out] ((32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*x)/8 + (a^4*A
*ArcTanh[Sin[c + d*x]])/d + (b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*
B)*Sin[c + d*x])/(6*d) + (b^2*(32*a*A*b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]*
Sin[c + d*x])/(24*d) + (b*(4*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*
x])/(12*d) + (b*B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.547011, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2990, 3049, 3033, 3023, 2735, 3770}

$$\frac{b\left(34a^2Ab+19a^3B+16ab^2B+4Ab^3\right)\sin(c+dx)}{6d}+\frac{b^2\left(26a^2B+32aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(32a^3Ab\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] ((32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*x)/8 + (a^4*A
*ArcTanh[Sin[c + d*x]])/d + (b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*
B)*Sin[c + d*x])/(6*d) + (b^2*(32*a*A*b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]*
Sin[c + d*x])/(24*d) + (b*(4*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*
x])/(12*d) + (b*B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int (a+b\cos(c+dx))^2$$
$$= \frac{b(4Ab+7aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d}+\frac{bB(a+b\cos}{}$$
$$= \frac{b^2\left(32aAb+26a^2B+9b^2B\right)\cos(c+dx)\sin(c+dx)}{24d}+\frac{b(4Ab}{}$$
$$= \frac{b\left(34a^2Ab+4Ab^3+19a^3B+16ab^2B\right)\sin(c+dx)}{6d}+\frac{b^2\left(32aA\right.}{}$$
$$= \frac{1}{8}\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)x+\frac{b\left(34a^2\right.}{}$$
$$= \frac{1}{8}\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)x+\frac{a^4A\tan}{}$$

Mathematica [A] time = 0.580132, size = 210, normalized size = 1.05

$$\frac{12(c+dx)\left(32a^3Ab+24a^2b^2B+8a^4B+16aAb^3+3b^4B\right)+24b\left(24a^2Ab+16a^3B+12ab^2B+3Ab^3\right)\sin(c+dx)+24b^2(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (12*(32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*(c + d*x)
- 96*a^4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*a^4*A*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 24*b*(24*a^2*A*b + 3*A*b^3 + 16*a^3*B + 12*a
*b^2*B)*Sin[c + d*x] + 24*b^2*(4*a*A*b + 6*a^2*B + b^2*B)*Sin[2*(c + d*x)]
+ 8*b^3*(A*b + 4*a*B)*Sin[3*(c + d*x)] + 3*b^4*B*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.076, size = 319, normalized size = 1.6

$$\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+a^4Bx+\frac{Ba^4c}{d}+4\,Aa^3bx+4\,\frac{Aa^3bc}{d}+4\,\frac{Ba^3b\sin(dx+c)}{d}+6\,\frac{Aa^2b^2\sin(dx+c)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*x+1/d*B*a^4*c+4*A*a^3*b*x+4/d*A*a
^3*b*c+4/d*B*a^3*b*sin(d*x+c)+6/d*A*a^2*b^2*sin(d*x+c)+3/d*B*a^2*b^2*cos(d*
x+c)*sin(d*x+c)+3*B*a^2*b^2*x+3/d*B*a^2*b^2*c+2/d*A*a*b^3*cos(d*x+c)*sin(d*
x+c)+2*A*a*b^3*x+2/d*A*a*b^3*c+4/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a*b^3+8/3/d*
B*a*b^3*sin(d*x+c)+1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*b^4+2/3/d*A*b^4*sin(d*x+
c)+1/4/d*B*b^4*sin(d*x+c)*cos(d*x+c)^3+3/8/d*B*b^4*cos(d*x+c)*sin(d*x+c)+3/
8*b^4*B*x+3/8/d*B*b^4*c
```

Maxima [A] time = 1.12496, size = 281, normalized size = 1.4

$$\frac{96\,(dx+c)Ba^4 + 384\,(dx+c)Aa^3b + 144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2b^2 + 96\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab^3 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(96*(d*x + c)*B*a^4 + 384*(d*x + c)*A*a^3*b + 144*(2*d*x + 2*c + sin(2
*d*x + 2*c))*B*a^2*b^2 + 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^3 - 128*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b^3 - 32*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*A*b^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b
^4 + 96*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 384*B*a^3*b*sin(d*x + c) +
 576*A*a^2*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.54827, size = 447, normalized size = 2.23

$$\frac{12\,Aa^4\log(\sin(dx+c)+1) - 12\,Aa^4\log(-\sin(dx+c)+1) + 3\left(8\,Ba^4 + 32\,Aa^3b + 24\,Ba^2b^2 + 16\,Aab^3 + 3\,Bb^4\right)dx + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/24*(12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*log(-sin(d*x + c) + 1) + 3*
(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*d*x + (6*B*b^4
```

```
*cos(d*x + c)^3 + 96*B*a^3*b + 144*A*a^2*b^2 + 64*B*a*b^3 + 16*A*b^4 + 8*(4
*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 3*(24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*
cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53986, size = 814, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3
+ 3*B*b^4)*(d*x + c) + 2*(96*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 144*A*a^2*b^2
*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b^3*
tan(1/2*d*x + 1/2*c)^7 + 96*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^4*tan(1
/2*d*x + 1/2*c)^7 - 15*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 288*B*a^3*b*tan(1/2*d
*x + 1/2*c)^5 + 432*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*a^2*b^2*tan(1/2
*d*x + 1/2*c)^5 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*B*a*b^3*tan(1/2*d
*x + 1/2*c)^5 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 9*B*b^4*tan(1/2*d*x + 1/2
*c)^5 + 288*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 432*A*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2
*c)^3 + 160*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^
3 - 9*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 96*B*a^3*b*tan(1/2*d*x + 1/2*c) + 144*
A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 48*A*a
*b^3*tan(1/2*d*x + 1/2*c) + 96*B*a*b^3*tan(1/2*d*x + 1/2*c) + 24*A*b^4*tan(
1/2*d*x + 1/2*c) + 15*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

3.244 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=195

$$-\frac{b\left(6a^3A-17a^2bB-12aAb^2-2b^3B\right)\sin(c+dx)}{3d}-\frac{b^2\left(6a^2A-8abB-3Ab^2\right)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}bx\left(12a^2Ab\right.$$

```
[Out] (b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*x)/2 + (a^3*(4*A*b + a*B)*Arc
Tanh[Sin[c + d*x]])/d - (b*(6*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Si
n[c + d*x])/(3*d) - (b^2*(6*a^2*A - 3*A*b^2 - 8*a*b*B)*Cos[c + d*x]*Sin[c +
 d*x])/(6*d) - (b*(3*a*A - b*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
+ (a*A*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.569693, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b\left(6a^3A-17a^2bB-12aAb^2-2b^3B\right)\sin(c+dx)}{3d}-\frac{b^2\left(6a^2A-8abB-3Ab^2\right)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}bx\left(12a^2Ab\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```


```
[Out] (b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*x)/2 + (a^3*(4*A*b + a*B)*Arc
Tanh[Sin[c + d*x]])/d - (b*(6*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Si
n[c + d*x])/(3*d) - (b^2*(6*a^2*A - 3*A*b^2 - 8*a*b*B)*Cos[c + d*x]*Sin[c +
 d*x])/(6*d) - (b*(3*a*A - b*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
+ (a*A*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
```

```
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\tan(c+dx)}{d} + \int (a+b\cos(c+dx)$$

$$= -\frac{b(3aA-bB)(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{aA(a+b\,c}{}$$

$$= -\frac{b^2\left(6a^2A-3Ab^2-8abB\right)\cos(c+dx)\sin(c+dx)}{6d} - \frac{b(3a}{}$$

$$= -\frac{b\left(6a^3A-12aAb^2-17a^2bB-2b^3B\right)\sin(c+dx)}{3d} - \frac{b^2\left(6a\right.}{}$$

$$= \frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x - \frac{b\left(6a^3A-12aAb^2\right.}{}$$

$$= \frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x + \frac{a^3(4Ab+aB)\tan}{d}$$

Mathematica [A] time = 1.01931, size = 257, normalized size = 1.32

$$\frac{6b(c+dx)\left(12a^2Ab+8a^3B+4ab^2B+Ab^3\right)+3b^2\left(24a^2B+16aAb+3b^2B\right)\sin(c+dx)-12a^3(aB+4Ab)\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (6*b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*(c + d*x) - 12*a^3*(4*A*b +
 a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*(4*A*b + a*B)*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + 3*b^2*(16*a*A*b + 24*a^2*B + 3*b^2*B)*Sin[c + d*x]
+ 3*b^3*(A*b + 4*a*B)*Sin[2*(c + d*x)] + b^4*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.084, size = 255, normalized size = 1.3

$$\frac{Aa^4\tan(dx+c)}{d} + \frac{a^4B\ln(\sec(dx+c)+\tan(dx+c))}{d} + 4\frac{Aa^3b\ln(\sec(dx+c)+\tan(dx+c))}{d} + 4Ba^3bx + 4\frac{Ba^3bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*a^4*tan(d*x+c)+1/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^3*b*ln(sec
(d*x+c)+tan(d*x+c))+4*B*a^3*b*x+4/d*B*a^3*b*c+6*A*a^2*b^2*x+6/d*A*a^2*b^2*c
+6/d*B*a^2*b^2*sin(d*x+c)+4/d*A*a*b^3*sin(d*x+c)+2/d*B*a*b^3*cos(d*x+c)*sin
(d*x+c)+2*B*a*b^3*x+2/d*B*a*b^3*c+1/2/d*A*b^4*cos(d*x+c)*sin(d*x+c)+1/2*A*b
^4*x+1/2/d*A*b^4*c+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*b^4+2/3/d*B*b^4*sin(d*x+
c)
```

Maxima [A] time = 1.12913, size = 266, normalized size = 1.36

$$48\,(dx+c)Ba^3b + 72\,(dx+c)Aa^2b^2 + 12\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab^3 + 3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab^4 - 4\,(\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(48*(d*x + c)*B*a^3*b + 72*(d*x + c)*A*a^2*b^2 + 12*(2*d*x + 2*c + sin
(2*d*x + 2*c))*B*a*b^3 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 4*(sin(
d*x + c)^3 - 3*sin(d*x + c))*B*b^4 + 6*B*a^4*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 24*A*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 72*B*a^2*b^2*sin(d*x + c) + 48*A*a*b^3*sin(d*x + c) + 12*A*a^4*tan(d*
x + c))/d
```

Fricas [A] time = 1.60192, size = 471, normalized size = 2.42

$$3\left(8\,Ba^3b + 12\,Aa^2b^2 + 4\,Bab^3 + Ab^4\right)dx\cos(dx+c) + 3\left(Ba^4 + 4\,Aa^3b\right)\cos(dx+c)\log(\sin(dx+c)+1) - 3\left(Ba^4 + 4\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c) + 3*
(B*a^4 + 4*A*a^3*b)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(B*a^4 + 4*A*a^3
```

```
*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*B*b^4*cos(d*x + c)^3 + 6*A*a^4
 + 3*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 4*(9*B*a^2*b^2 + 6*A*a*b^3 + B*b^
4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.52787, size = 501, normalized size = 2.57

$$-\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\left(8\,Ba^3b + 12\,Aa^2b^2 + 4\,Bab^3 + Ab^4\right)(dx+c) - 6\left(Ba^4 + 4\,Aa^3b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(8*B*a
^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - 6*(B*a^4 + 4*A*a^3*b)*
log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(B*a^4 + 4*A*a^3*b)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(36*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*a*b^3*ta
n(1/2*d*x + 1/2*c)^5 - 12*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^4*tan(1/2*
d*x + 1/2*c)^5 + 6*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*B*b^4*tan(1/2*d*x + 1/2*
c)^3 + 36*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*A*a*b^3*tan(1/2*d*x + 1/2*c)
+ 12*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3*A*b^4*tan(1/2*d*x + 1/2*c) + 6*B*b^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.245 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=209

$$-\frac{b\left(13a^2Ab+4a^3B-8ab^2B-2Ab^3\right)\sin(c+dx)}{2d}+\frac{a^2\left(a^2A+8abB+12Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(2a^2B+6aAb-\right.}{}$$

```
[Out] (b^2*(8*a*A*b + 12*a^2*B + b^2*B)*x)/2 + (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*
ArcTanh[Sin[c + d*x]])/(2*d) - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2
*B)*Sin[c + d*x])/(2*d) - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Cos[c + d*x]*Sin
[c + d*x])/(2*d) + (a*(5*A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/
(2*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.615265, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3047, 3033, 3023, 2735, 3770}

$$-\frac{b\left(13a^2Ab+4a^3B-8ab^2B-2Ab^3\right)\sin(c+dx)}{2d}+\frac{a^2\left(a^2A+8abB+12Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(2a^2B+6aAb-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```


```
[Out] (b^2*(8*a*A*b + 12*a^2*B + b^2*B)*x)/2 + (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*
ArcTanh[Sin[c + d*x]])/(2*d) - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2
*B)*Sin[c + d*x])/(2*d) - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Cos[c + d*x]*Sin
[c + d*x])/(2*d) + (a*(5*A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/
(2*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
```

```
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a+b$$
$$=\frac{a(5Ab+2aB)(a+b\cos(c+dx))^2\tan(c+dx)}{2d}+\frac{aA(a+b\,\mathrm{co}}{}$$
$$=-\frac{b^2\left(6aAb+2a^2B-b^2B\right)\cos(c+dx)\sin(c+dx)}{2d}+\frac{a(5Ab+}{}$$
$$=-\frac{b\left(13a^2Ab-2Ab^3+4a^3B-8ab^2B\right)\sin(c+dx)}{2d}-\frac{b^2\left(6aA\right.}{}$$
$$=\frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x-\frac{b\left(13a^2Ab-2Ab^3+4a^3B-\right.}{2d}$$
$$=\frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x+\frac{a^2\left(a^2A+12Ab^2+8abB\right)\mathrm{ta}}{2d}$$

Mathematica [A] time = 2.38423, size = 310, normalized size = 1.48

$$2b^2(c+dx)\left(12a^2B+8aAb+b^2B\right)-2a^2\left(a^2A+8abB+12Ab^2\right)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+2a^2\left(a^2A+8a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*(8*a*A*b + 12*a^2*B + b^2*B)*(c + d*x) - 2*a^2*(a^2*A + 12*A*b^2 + 8
*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*(a^2*A + 12*A*b^2
+ 8*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^4*A)/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]) - (a^4*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]) + 4*b^3*(A*b + 4*a*B)*Sin[c + d*x] + b^4*B*Sin[2*(c + d*x)])/(4*d
)
```

Maple [A] time = 0.092, size = 236, normalized size = 1.1

$$\frac{Aa^4\sec(dx+c)\tan(dx+c)}{2d}+\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{a^4B\tan(dx+c)}{d}+4\frac{Aa^3b\tan(dx+c)}{d}+4\frac{Ba^3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d
*a^4*B*tan(d*x+c)+4/d*A*a^3*b*tan(d*x+c)+4/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+
c))+6/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+6*B*a^2*b^2*x+6/d*B*a^2*b^2*c+4
*A*a*b^3*x+4/d*A*a*b^3*c+4/d*B*a*b^3*sin(d*x+c)+1/d*A*b^4*sin(d*x+c)+1/2/d*
B*b^4*cos(d*x+c)*sin(d*x+c)+1/2*b^4*B*x+1/2/d*B*b^4*c
```

Maxima [A] time = 1.09024, size = 282, normalized size = 1.35

$$\frac{24(dx+c)Ba^2b^2+16(dx+c)Aab^3+(2dx+2c+\sin(2dx+2c))Bb^4-Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(24*(d*x + c)*B*a^2*b^2 + 16*(d*x + c)*A*a*b^3 + (2*d*x + 2*c + sin(2*d
*x + 2*c))*B*b^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 8*B*a^3*b*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 12*A*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 16*B*a*b^3*sin(d*x + c) + 4*A*b^4*sin(d*x + c) + 4*B*a^4*tan(d*x
 + c) + 16*A*a^3*b*tan(d*x + c))/d
```

Fricas [A] time = 1.5289, size = 479, normalized size = 2.29

$$\frac{2\left(12Ba^2b^2+8Aab^3+Bb^4\right)dx\cos(dx+c)^2+\left(Aa^4+8Ba^3b+12Aa^2b^2\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-\left(Aa^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*d*x*cos(d*x + c)^2 + (A*a^4 + 8*B
*a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^4 + 8*B*
a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*b^4*cos(
d*x + c)^3 + A*a^4 + 2*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 2*(B*a^4 + 4*A*
a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.55699, size = 710, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*(d*x + c) + (A*a^4 + 8*B*a^3*b + 12
*A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a^4 + 8*B*a^3*b + 12*A*
a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^4*tan(1/2*d*x + 1/2*c)
^7 - 2*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 8*
B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^7 - B*b^4*tan
(1/2*d*x + 1/2*c)^7 + 3*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 2*B*a^4*tan(1/2*d*x
+ 1/2*c)^5 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 8*B*a*b^3*tan(1/2*d*x + 1/2
*c)^5 - 2*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 3
*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 8*A*a^3*b*
tan(1/2*d*x + 1/2*c)^3 - 8*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^4*tan(1/2
*d*x + 1/2*c)^3 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^3 + A*a^4*tan(1/2*d*x + 1/2*
```

```
c) + 2*B*a^4*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b*tan(1/2*d*x + 1/2*c) + 8*B*a*
b^3*tan(1/2*d*x + 1/2*c) + 2*A*b^4*tan(1/2*d*x + 1/2*c) + B*b^4*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.246 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=198

$$-\frac{b^2\left(3a^2B+8aAb-6b^2B\right)\sin(c+dx)}{6d}+\frac{a^2\left(2a^2A+9abB+9Ab^2\right)\tan(c+dx)}{3d}+\frac{a\left(4a^2Ab+a^3B+12ab^2B+8Ab^3\right)\tan}{2d}$$

```
[Out] b^3*(A*b + 4*a*B)*x + (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*ArcTanh
[Sin[c + d*x]])/(2*d) - (b^2*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Sin[c + d*x])/(6
*d) + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b +
a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*(a + b*
Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.580128, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3047, 3031, 3023, 2735, 3770}

$$-\frac{b^2\left(3a^2B+8aAb-6b^2B\right)\sin(c+dx)}{6d}+\frac{a^2\left(2a^2A+9abB+9Ab^2\right)\tan(c+dx)}{3d}+\frac{a\left(4a^2Ab+a^3B+12ab^2B+8Ab^3\right)\tan}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] b^3*(A*b + 4*a*B)*x + (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*ArcTanh
[Sin[c + d*x]])/(2*d) - (b^2*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Sin[c + d*x])/(6
*d) + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b +
a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*(a + b*
Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
```

```
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (a+$$

$$= \frac{a(2Ab+aB)(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}+$$

$$= \frac{a^2\left(2a^2A+9Ab^2+9abB\right)\tan(c+dx)}{3d}+\frac{a(2Ab+aB)(a+b}{}$$

$$= -\frac{b^2\left(8aAb+3a^2B-6b^2B\right)\sin(c+dx)}{6d}+\frac{a^2\left(2a^2A+9Ab^2\right.}{3}$$

$$= b^3(Ab+4aB)x-\frac{b^2\left(8aAb+3a^2B-6b^2B\right)\sin(c+dx)}{6d}+\frac{a^2}{}$$

$$= b^3(Ab+4aB)x+\frac{a\left(4a^2Ab+8Ab^3+a^3B+12ab^2B\right)\tanh^{-1}}{2d}$$

Mathematica [B] time = 5.9631, size = 415, normalized size = 2.1

$$\frac{\frac{8a^2\left(a^2A+6abB+9Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{8a^2\left(a^2A+6abB+9Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-6a\left(4a^2Ab+a^3B+12ab^2B+8Ab^3\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (12*b^3*(A*b + 4*a*B)*(c + d*x) - 6*a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b
^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*a*(4*a^2*A*b + 8*A*b^3 +
 a^3*B + 12*a*b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*(12*A*
b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*a^4*A*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (8*a^2*(a^2*A + 9*A*b
^2 + 6*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*
a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (a^3*(12*
A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (8*a^2*(a^2*A
```

```
 + 9*A*b^2 + 6*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 12*b^4*B*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.102, size = 262, normalized size = 1.3

$$\frac{2\,Aa^4\tan(dx+c)}{3\,d}+\frac{Aa^4\tan(dx+c)\,(\sec(dx+c))^2}{3\,d}+\frac{a^4B\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{a^4B\ln(\sec(dx+c)+\tan(dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3/d*A*a^4*tan(d*x+c)+1/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*B*sec(
d*x+c)*tan(d*x+c)+1/2/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a^3*b*sec(d*x
+c)*tan(d*x+c)+2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*B*a^3*b*tan(d*x+c)
+6/d*A*a^2*b^2*tan(d*x+c)+6/d*B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a*b
^3*ln(sec(d*x+c)+tan(d*x+c))+4*B*a*b^3*x+4/d*B*a*b^3*c+A*b^4*x+1/d*A*b^4*c+
1/d*B*b^4*sin(d*x+c)
```

Maxima [A] time = 1.16211, size = 331, normalized size = 1.67

$$\frac{4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+48\,(dx+c)Bab^3+12\,(dx+c)Ab^4-3\,Ba^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 48*(d*x + c)*B*a*b^3 + 12
*(d*x + c)*A*b^4 - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*a^3*b*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*B*a^2*b^
2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*A*a*b^3*(log(sin(d*x
 + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*b^4*sin(d*x + c) + 48*B*a^3*b*ta
n(d*x + c) + 72*A*a^2*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.54464, size = 524, normalized size = 2.65

$$\frac{12\left(4\,Bab^3+Ab^4\right)dx\cos\left(dx+c\right)^3+3\left(Ba^4+4\,Aa^3b+12\,Ba^2b^2+8\,Aab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba^4+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*(4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c)^3 + 3*(B*a^4 + 4*A*a^3*b + 12
*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^4 + 4
*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
+ 2*(6*B*b^4*cos(d*x + c)^3 + 2*A*a^4 + 4*(A*a^4 + 6*B*a^3*b + 9*A*a^2*b^2)
*cos(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61798, size = 522, normalized size = 2.64

$$\frac{\frac{12\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(4\,Bab^3+Ab^4\right)\left(dx+c\right)+3\left(Ba^4+4\,Aa^3b+12\,Ba^2b^2+8\,Aab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(B\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*B*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(4*B*a*
b^3 + A*b^4)*(d*x + c) + 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*l
og(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8
*A*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^4*tan(1/2*d*x + 1/2
*c)^5 - 3*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*b*tan(1/2*d*x + 1/2*c)^5
+ 24*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 -
 4*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan
(1/2*d*x + 1/2*c) + 12*A*a^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b*tan(1/2*d*
x + 1/2*c) + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^3)/d
```

3.247 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=216

$$\frac{a\left(16a^2Ab+4a^3B+34ab^2B+19Ab^3\right)\tan(c+dx)}{6d}+\frac{\left(24a^2Ab^2+3a^4A+16a^3bB+32ab^3B+8Ab^4\right)\tanh^{-1}(\sin(c+dx)}{8d}$$

```
[Out] b^4*B*x + ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*Arc
Tanh[Sin[c + d*x]])/(8*d) + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*
B)*Tan[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Sec[c + d*x]*
Tan[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*
x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d)
```

Rubi [A] time = 0.597489, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3047, 3031, 3021, 2735, 3770}

$$\frac{a\left(16a^2Ab+4a^3B+34ab^2B+19Ab^3\right)\tan(c+dx)}{6d}+\frac{\left(24a^2Ab^2+3a^4A+16a^3bB+32ab^3B+8Ab^4\right)\tanh^{-1}(\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```


```
[Out] b^4*B*x + ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*Arc
Tanh[Sin[c + d*x]])/(8*d) + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*
B)*Tan[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Sec[c + d*x]*
Tan[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*
x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
```

```
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (a+$$
$$= \frac{a(7Ab+4aB)(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{12d}+$$
$$= \frac{a^2\left(9a^2A+26Ab^2+32abB\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{a(7A$$
$$= \frac{a\left(16a^2Ab+19Ab^3+4a^3B+34ab^2B\right)\tan(c+dx)}{6d}+\frac{a^2\left(9a^2\right.}{}$$
$$= b^4Bx+\frac{a\left(16a^2Ab+19Ab^3+4a^3B+34ab^2B\right)\tan(c+dx)}{6d}+$$
$$= b^4Bx+\frac{\left(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B\right)\tanh}{8d}$$

Mathematica [A] time = 1.01386, size = 160, normalized size = 0.74

$$\frac{3\left(24a^2Ab^2+3a^4A+16a^3bB+32ab^3B+8Ab^4\right)\tanh^{-1}(\sin(c+dx))+3a\tan(c+dx)\left(a\left(3a^2A+16abB+24Ab^2\right)\sec(c\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (24*b^4*B*d*x + 3*(3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3
*B)*ArcTanh[Sin[c + d*x]] + 3*a*(8*(4*a^2*A*b + 4*A*b^3 + a^3*B + 6*a*b^2*B
) + a*(3*a^2*A + 24*A*b^2 + 16*a*b*B)*Sec[c + d*x] + 2*a^3*A*Sec[c + d*x]^3
)*Tan[c + d*x] + 8*a^3*(4*A*b + a*B)*Tan[c + d*x]^3)/(24*d)
```

Maple [A] time = 0.098, size = 338, normalized size = 1.6

$$\frac{Aa^4 \tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{3Aa^4 \sec(dx+c)\tan(dx+c)}{8d} + \frac{3Aa^4 \ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{2a^4B\tan}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^4*sec(d*x+c)*tan(d*x+c)+3/8/d
*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^4*B*tan(d*x+c)+1/3/d*a^4*B*tan(d*x
+c)*sec(d*x+c)^2+8/3/d*A*a^3*b*tan(d*x+c)+4/3/d*A*a^3*b*tan(d*x+c)*sec(d*x+
c)^2+2/d*B*a^3*b*tan(d*x+c)*sec(d*x+c)+2/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+c)
)+3/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)+3/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c
))+6/d*B*a^2*b^2*tan(d*x+c)+4/d*A*a*b^3*tan(d*x+c)+4/d*B*a*b^3*ln(sec(d*x+c
)+tan(d*x+c))+1/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+b^4*B*x+1/d*B*b^4*c
```

Maxima [A] time = 1.13339, size = 428, normalized size = 1.98

$$16\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ba^4 + 64\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Aa^3b + 48(dx+c)Bb^4 - 3Aa^4\left(\frac{2\left(3\sin(dx+c\right.}{\sin(dx+c)^4-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 64*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^3*b + 48*(d*x + c)*B*b^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 48*B*a^3*b*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 72*A*a^2*b^2*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 96*B*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*
A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 288*B*a^2*b^2*tan(d
*x + c) + 192*A*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.54221, size = 603, normalized size = 2.79

$$48Bb^4dx\cos(dx+c)^4 + 3\left(3Aa^4 + 16Ba^3b + 24Aa^2b^2 + 32Bab^3 + 8Ab^4\right)\cos(dx+c)^4\log(\sin(dx+c)+1) - 3\left(3A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(48*B*b^4*d*x*cos(d*x + c)^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
+ 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a^4 +
 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(-sin(
d*x + c) + 1) + 2*(6*A*a^4 + 16*(B*a^4 + 4*A*a^3*b + 9*B*a^2*b^2 + 6*A*a*b^
3)*cos(d*x + c)^3 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*cos(d*x + c)^2
+ 8*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.86199, size = 857, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*(d*x + c)*B*b^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*
b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^4 + 16*B*a^3*b
 + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
+ 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 96
*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*
A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^4*
tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 48*B*a^3*b*ta
```

```
n(1/2*d*x + 1/2*c)^5 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 432*B*a^2*b^2*
tan(1/2*d*x + 1/2*c)^5 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^4*tan(1
/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*b*tan(1/2*d
*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*
x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*A*a*b^3*tan(1/2*d
*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) + 24*B*a^4*tan(1/2*d*x + 1/2*
c) + 96*A*a^3*b*tan(1/2*d*x + 1/2*c) + 48*B*a^3*b*tan(1/2*d*x + 1/2*c) + 72
*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*A
*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.248 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=267

$$\frac{\left(60a^2Ab^2+8a^4A+40a^3bB+60ab^3B+15Ab^4\right)\tan(c+dx)}{15d}+\frac{\left(12a^3Ab+24a^2b^2B+3a^4B+16aAb^3+8b^4B\right)\tanh^{-1}(\text{si}}{8d}$$

```
[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*ArcTanh[Sin[c
 + d*x]])/(8*d) + ((8*a^4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b
^3*B)*Tan[c + d*x])/(15*d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b
^2*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b
*B)*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Cos[c + d*x])^3*
Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.722923, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2989, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(60a^2Ab^2+8a^4A+40a^3bB+60ab^3B+15Ab^4\right)\tan(c+dx)}{15d}+\frac{\left(12a^3Ab+24a^2b^2B+3a^4B+16aAb^3+8b^4B\right)\tanh^{-1}(\text{si}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```


```
[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*ArcTanh[Sin[c
 + d*x]])/(8*d) + ((8*a^4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b
^3*B)*Tan[c + d*x])/(15*d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b
^2*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b
*B)*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Cos[c + d*x])^3*
Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
```

```
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (a+$$

$$= \frac{a(8Ab+5aB)(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{20d}$$

$$= \frac{a^2\left(8a^2A+18Ab^2+25abB\right)\sec^2(c+dx)\tan(c+dx)}{30d}+\frac{a(8}{}$$

$$= \frac{a\left(60a^2Ab+56Ab^3+15a^3B+110ab^2B\right)\sec(c+dx)\tan(c+}{40d}$$

$$= \frac{a\left(60a^2Ab+56Ab^3+15a^3B+110ab^2B\right)\sec(c+dx)\tan(c+}{40d}$$

$$= \frac{\left(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B\right)\tanh^{-1}(\sin(c}{8d}$$

$$= \frac{\left(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B\right)\tanh^{-1}(\sin(c}{8d}$$

Mathematica [A] time = 4.15113, size = 198, normalized size = 0.74

$$\frac{15\left(12a^3Ab+24a^2b^2B+3a^4B+16aAb^3+8b^4B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(80a^2\left(a^2A+2abB+3Ab^2\right)\tan^2(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (15*(12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(120*(a^4*A + 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B +
4*a*b^3*B) + 15*a*(12*a^2*A*b + 16*A*b^3 + 3*a^3*B + 24*a*b^2*B)*Sec[c + d*
x] + 30*a^3*(4*A*b + a*B)*Sec[c + d*x]^3 + 80*a^2*(a^2*A + 3*A*b^2 + 2*a*b*
B)*Tan[c + d*x]^2 + 24*a^4*A*Tan[c + d*x]^4))/(120*d)
```

Maple [A] time = 0.116, size = 431, normalized size = 1.6

$$\frac{8\,Aa^4\tan(dx+c)}{15\,d}+\frac{Aa^4\tan(dx+c)(\sec(dx+c))^4}{5\,d}+\frac{4\,Aa^4\tan(dx+c)(\sec(dx+c))^2}{15\,d}+\frac{a^4B\tan(dx+c)(\sec(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 8/15/d*A*a^4*tan(d*x+c)+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*a^4*ta
n(d*x+c)*sec(d*x+c)^2+1/4/d*a^4*B*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^4*B*sec(d
*x+c)*tan(d*x+c)+3/8/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a^3*b*tan(d*x+
c)*sec(d*x+c)^3+3/2/d*A*a^3*b*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^3*b*ln(sec(d*
x+c)+tan(d*x+c))+8/3/d*B*a^3*b*tan(d*x+c)+4/3/d*B*a^3*b*tan(d*x+c)*sec(d*x+
c)^2+4/d*A*a^2*b^2*tan(d*x+c)+2/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+3/d*B*a
^2*b^2*tan(d*x+c)*sec(d*x+c)+3/d*B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*
a*b^3*tan(d*x+c)*sec(d*x+c)+2/d*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+4/d*B*a*b
^3*tan(d*x+c)+1/d*A*b^4*tan(d*x+c)+1/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.05269, size = 521, normalized size = 1.95

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+320\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3b+480\left(\tan(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
320*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3*b + 480*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*a^2*b^2 - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 60*A*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
c) - 1)) - 360*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 240*A*a*b^3*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*b^4*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 960*B*a*b^3*tan(d*x + c) +
240*A*b^4*tan(d*x + c))/d
```

Fricas [A] time = 1.57572, size = 687, normalized size = 2.57

$$\frac{15\left(3\,Ba^4+12\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+8\,Bb^4\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,Ba^4+12\,Aa^3b+24\,Ba^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*cos(
d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2
+ 16*A*a*b^3 + 8*B*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*A*a^4
 + 8*(8*A*a^4 + 40*B*a^3*b + 60*A*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4)*cos(d*x
+ c)^4 + 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3)*cos(d*x + c)
^3 + 16*(2*A*a^4 + 10*B*a^3*b + 15*A*a^2*b^2)*cos(d*x + c)^2 + 30*(B*a^4 +
4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70156, size = 1148, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 +
16*A*a*b^3 + 8*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^4*tan
(1/2*d*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 300*A*a^3*b*tan(1/2
*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*tan(1/
2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*a*b^3*tan(1
/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^4*tan(1/2*
d*x + 1/2*c)^9 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^4*tan(1/2*d*x +
1/2*c)^7 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 1280*B*a^3*b*tan(1/2*d*x +
1/2*c)^7 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^2*b^2*tan(1/2*d*
x + 1/2*c)^7 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 1920*B*a*b^3*tan(1/2*d*
x + 1/2*c)^7 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 2400*A*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 720*A*b^4*tan(1/2*d*x +
1/2*c)^5 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^4*tan(1/2*d*x + 1/2*c)
^3 - 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1280*B*a^3*b*tan(1/2*d*x + 1/2*c)
^3 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*B*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1920*B*a*b^3*tan(1/2*d*x + 1/
2*c)^3 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^4*tan(1/2*d*x + 1/2*c)
+ 75*B*a^4*tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1/2*d*x + 1/2*c) + 480*B*
a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 360*B*a^2
*b^2*tan(1/2*d*x + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*
tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

3.249 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal. Leaf size=324

$$\frac{\left(32a^3Ab+60a^2b^2B+8a^4B+40aAb^3+15b^4B\right)\tan(c+dx)}{15d}+\frac{\left(36a^2Ab^2+5a^4A+24a^3bB+32ab^3B+8Ab^4\right)\tanh^{-1}(\mathrm{si}$$

```
[Out] ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[Sin[c
 + d*x]])/(16*d) + ((32*a^3*A*b + 40*a*A*b^3 + 8*a^4*B + 60*a^2*b^2*B + 15*
b^4*B)*Tan[c + d*x])/(15*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b
*B + 32*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a*(16*a^2*A*b + 13*A*
b^3 + 4*a^3*B + 27*a*b^2*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (a^2*(25*
a^2*A + 48*A*b^2 + 72*a*b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (a*(3*A
*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a
*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.802034, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2989, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(32a^3Ab+60a^2b^2B+8a^4B+40aAb^3+15b^4B\right)\tan(c+dx)}{15d}+\frac{\left(36a^2Ab^2+5a^4A+24a^3bB+32ab^3B+8Ab^4\right)\tanh^{-1}(\mathrm{si}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```


```
[Out] ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[Sin[c
 + d*x]])/(16*d) + ((32*a^3*A*b + 40*a*A*b^3 + 8*a^4*B + 60*a^2*b^2*B + 15*
b^4*B)*Tan[c + d*x])/(15*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b
*B + 32*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a*(16*a^2*A*b + 13*A*
b^3 + 4*a^3*B + 27*a*b^2*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (a^2*(25*
a^2*A + 48*A*b^2 + 72*a*b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (a*(3*A
*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a
*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{6}\int (a$$

$$= \frac{a(3Ab+2aB)(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{10d}$$

$$= \frac{a^2\left(25a^2A+48Ab^2+72abB\right)\sec^3(c+dx)\tan(c+dx)}{120d}+$$

$$= \frac{a\left(16a^2Ab+13Ab^3+4a^3B+27ab^2B\right)\sec^2(c+dx)\tan(c+$$

$$= \frac{a\left(16a^2Ab+13Ab^3+4a^3B+27ab^2B\right)\sec^2(c+dx)\tan(c+$$

$$= \frac{\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)\sec(c+dx}{16d}$$

$$= \frac{\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)\tanh^{-1}(\sin}{16d}$$

Mathematica [A] time = 2.64182, size = 244, normalized size = 0.75

$$15\left(36a^2Ab^2+5a^4A+24a^3bB+32ab^3B+8Ab^4\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(160a\left(4a^2Ab+a^3B+3ab^2B+2A\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] (15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(240*(4*a^3*A*b + 4*a*A*b^3 + a^4*B + 6*a^2*b^2*
B + b^4*B) + 15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B
)*Sec[c + d*x] + 10*a^2*(5*a^2*A + 36*A*b^2 + 24*a*b*B)*Sec[c + d*x]^3 + 40
*a^4*A*Sec[c + d*x]^5 + 160*a*(4*a^2*A*b + 2*A*b^3 + a^3*B + 3*a*b^2*B)*Tan
[c + d*x]^2 + 48*a^3*(4*A*b + a*B)*Tan[c + d*x]^4))/(240*d)
```

Maple [A] time = 0.098, size = 550, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x)
```

```
[Out] 32/15/d*A*a^3*b*tan(d*x+c)+3/2/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+9/4/d*A*
a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b^4*tan(d*x+c)+3/2/d*A*a^2*b^2*tan(
d*x+c)*sec(d*x+c)^3+2/d*B*a*b^3*tan(d*x+c)*sec(d*x+c)+4/3/d*A*a*b^3*tan(d*x
+c)*sec(d*x+c)^2+4/5/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^4+2/d*B*a^2*b^2*tan(d*
x+c)*sec(d*x+c)^2+1/d*B*a^3*b*tan(d*x+c)*sec(d*x+c)^3+8/15/d*a^4*B*tan(d*x+
c)+4/15/d*a^4*B*tan(d*x+c)*sec(d*x+c)^2+4/d*B*a^2*b^2*tan(d*x+c)+8/3/d*A*a*
b^3*tan(d*x+c)+2/d*B*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+5/16/d*A*a^4*ln(sec(d*
x+c)+tan(d*x+c))+1/2/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+5/24/d*A*a^4*tan(d*x
+c)*sec(d*x+c)^3+16/15/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^2+3/2/d*B*a^3*b*tan(
d*x+c)*sec(d*x+c)+9/4/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)+1/6/d*A*a^4*tan(d*x
+c)*sec(d*x+c)^5+1/2/d*A*b^4*tan(d*x+c)*sec(d*x+c)+1/5/d*a^4*B*tan(d*x+c)*s
ec(d*x+c)^4+5/16/d*A*a^4*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.02618, size = 640, normalized size = 1.98

$$\frac{32\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)Ba^{4}+128\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 +
128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3*b + 960*
(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b^2 + 640*(tan(d*x + c)^3 + 3*tan(d
*x + c))*A*a*b^3 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 120*B*a^3*b*(2*(3*sin
(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*l
og(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*A*a^2*b^2*(2*(3*sin(d
*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 480*B*a*b^3*(2*sin(d*x + c)
/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12
0*A*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) + 480*B*b^4*tan(d*x + c))/d
```

Fricas [A] time = 1.67112, size = 797, normalized size = 2.46

$$\frac{15\left(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+32\,Bab^3+8\,Ab^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(5\,Aa^4+24\,Ba^3b+36\,Aa^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(
d*x + c)^6*log(sin(d*x + c) + 1) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
+ 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(8*B*
a^4 + 32*A*a^3*b + 60*B*a^2*b^2 + 40*A*a*b^3 + 15*B*b^4)*cos(d*x + c)^5 + 4
0*A*a^4 + 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*c
os(d*x + c)^4 + 32*(2*B*a^4 + 8*A*a^3*b + 15*B*a^2*b^2 + 10*A*a*b^3)*cos(d*
x + c)^3 + 10*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2)*cos(d*x + c)^2 + 48*(B*
a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70022, size = 1601, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 +
32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*A*a^4*tan
(1/2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*A*a^3*b*tan(
1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*A*a^2*b^2*t
an(1/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*A*a*b
^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*A*b^
4*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x + 1/2*c)^11 + 25*A*a^4*ta
n(1/2*d*x + 1/2*c)^9 + 560*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*A*a^3*b*tan(
1/2*d*x + 1/2*c)^9 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1260*A*a^2*b^2*ta
n(1/2*d*x + 1/2*c)^9 + 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*A*a*b^3
*tan(1/2*d*x + 1/2*c)^9 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 360*A*b^4*t
an(1/2*d*x + 1/2*c)^9 + 1200*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^4*tan(1
/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*A*a^3*b*tan(1/
2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*A*a^2*b^2*tan(1
/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*A*b^4*tan(1
/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 450*A*a^4*tan(1/2*d
*x + 1/2*c)^5 + 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*A*a^3*b*tan(1/2*d*
x + 1/2*c)^5 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*A*a^2*b^2*tan(1/2*d
*x + 1/2*c)^5 + 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*A*a*b^3*tan(1/
2*d*x + 1/2*c)^5 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*A*b^4*tan(1/2*d
*x + 1/2*c)^5 + 2400*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^4*tan(1/2*d*x +
1/2*c)^3 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*A*a^3*b*tan(1/2*d*x + 1/
2*c)^3 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*A*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*A*a*b^3*tan(1/2*d*x
 + 1/2*c)^3 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*b^4*tan(1/2*d*x +
 1/2*c)^3 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*A*a^4*tan(1/2*d*x + 1/2
*c) + 240*B*a^4*tan(1/2*d*x + 1/2*c) + 960*A*a^3*b*tan(1/2*d*x + 1/2*c) + 6
00*B*a^3*b*tan(1/2*d*x + 1/2*c) + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 1440
*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 960*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*
a*b^3*tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c) + 240*B*b^4*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.250 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{\left(-3a^2B+3aAb-2b^2B\right)\sin(c+dx)}{3b^3d}-\frac{2a^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)(Ab-aB)}{2b^4}+\frac{(Ab-aB)\,\mathrm{s}}{}$$

```
[Out] ((2*a^2 + b^2)*(A*b - a*B)*x)/(2*b^4) - (2*a^3*(A*b - a*B)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a
*A*b - 3*a^2*B - 2*b^2*B)*Sin[c + d*x])/(3*b^3*d) + ((A*b - a*B)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^2*d) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.493183, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2990, 3049, 3023, 2735, 2659, 205}

$$-\frac{\left(-3a^2B+3aAb-2b^2B\right)\sin(c+dx)}{3b^3d}-\frac{2a^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)(Ab-aB)}{2b^4}+\frac{(Ab-aB)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*(A*b - a*B)*x)/(2*b^4) - (2*a^3*(A*b - a*B)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a
*A*b - 3*a^2*B - 2*b^2*B)*Sin[c + d*x])/(3*b^3*d) + ((A*b - a*B)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^2*d) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(2aB+2bB\cos(c+dx)+3(Ab-aB)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{3b}$$

$$= \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{B\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{3a(Ab-aB)+\cdots}{\cdots}}{\cdots}$$

$$= -\frac{\left(3aAb-3a^2B-2b^2B\right)\sin(c+dx)}{3b^3d} + \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{B\cdots}{\cdots}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)x}{2b^4} - \frac{\left(3aAb-3a^2B-2b^2B\right)\sin(c+dx)}{3b^3d} + \frac{(Ab-aB)\cos(\cdots}{\cdots}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)x}{2b^4} - \frac{\left(3aAb-3a^2B-2b^2B\right)\sin(c+dx)}{3b^3d} + \frac{(Ab-aB)\cos(\cdots}{\cdots}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)x}{2b^4} - \frac{2a^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d} - \frac{\left(3aAb-3a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.434912, size = 152, normalized size = 0.85

$$\frac{6\left(2a^2+b^2\right)(c+dx)(Ab-aB)+3b\left(4a^2B-4aAb+3b^2B\right)\sin(c+dx)-\frac{24a^3(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+3b^2(Ab-a\cdots}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (6*(2*a^2 + b^2)*(A*b - a*B)*(c + d*x) - (24*a^3*(-(A*b) + a*B)*ArcTanh[((a
 - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(-4*a*A*b
 + 4*a^2*B + 3*b^2*B)*Sin[c + d*x] + 3*b^2*(A*b - a*B)*Sin[2*(c + d*x)] + b
^3*B*Sin[3*(c + d*x)])/(12*b^4*d)
```

Maple [B] time = 0.115, size = 641, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A*a-1/d/b/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^
3*tan(1/2*d*x+1/2*c)^5*a^2*B+1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x
+1/2*c)^5*B*a+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B-4/d/b
^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A*a+4/d/b^3/(1+tan(1/2*d
*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*a^2*B+4/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^
3*tan(1/2*d*x+1/2*c)^3*B-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)*A*a+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*a^2*B+2/d/b/(
1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*B+1/d/b/(1+tan(1/2*d*x+1/2*c)^
2)^3*tan(1/2*d*x+1/2*c)*A-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/
2*c)*B*a+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*A*a^2+1/d/b*arctan(tan(1/2*d*x+
1/2*c))*A-2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3*B-1/d/b^2*arctan(tan(1/2*d
*x+1/2*c))*B*a-2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a
-b)/((a-b)*(a+b))^(1/2))*A+2/d*a^4/b^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82331, size = 1164, normalized size = 6.54

$$\left[-\frac{3\left(2\,Ba^5-2\,Aa^4b-Ba^3b^2+Aa^2b^3-Bab^4+Ab^5\right)dx-3\left(Ba^4-Aa^3b\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{\cdots}}{b^2\cos(dx+c)^2+2\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d*
x - 3*(B*a^4 - A*a^3*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*B*a^4
*b - 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3 - B*b^5
)*cos(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^4 - b^6)*d), -1/6*(3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b
^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d*x - 6*(B*a^4 - A*a^3*b)*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^4*b
- 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3 - B*b^5)*c
os(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.54545, size = 486, normalized size = 2.73

$$-\frac{\frac{3\left(2Ba^3-2Aa^2b+Bab^2-Ab^3\right)(dx+c)}{b^4}+\frac{12\left(Ba^4-Aa^3b\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}-\frac{2\left(6Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 \cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*(d*x + c)/b^4 + 12*(B*a^4 -
 A*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 -
b^2)*b^4) - 2*(6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c
```

```
)^5 + 3*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B
*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*A*a*b*ta
n(1/2*d*x + 1/2*c)^3 + 4*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x
 + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 3*B*a*b*tan(1/2*d*x + 1/2*c) + 3
*A*b^2*tan(1/2*d*x + 1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 + 1)^3*b^3))/d
```

3.251 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{2b^3}+\frac{(Ab-aB)\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] -((2*a*A*b - 2*a^2*B - b^2*B)*x)/(2*b^3) + (2*a^2*(A*b - a*B)*ArcTan[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((
A*b - a*B)*Sin[c + d*x])/(b^2*d) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.287214, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2990, 3023, 2735, 2659, 205}

$$\frac{2a^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{2b^3}+\frac{(Ab-aB)\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```


```
[Out] -((2*a*A*b - 2*a^2*B - b^2*B)*x)/(2*b^3) + (2*a^2*(A*b - a*B)*ArcTan[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((
A*b - a*B)*Sin[c + d*x])/(b^2*d) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{aB+bB\cos(c+dx)+2(Ab-aB)\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{abB-\left(2aAb-2a^2B-b^2B\right)\cos(}{a+b\cos(c+dx)}}{2b^2}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)x}{2b^3} + \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)x}{2b^3} + \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)x}{2b^3} + \frac{2a^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} + \frac{(Ab-a}{}$$

Mathematica [A] time = 0.289127, size = 121, normalized size = 0.9

$$\frac{2(c+dx)\left(2a^2B-2aAb+b^2B\right)+\frac{8a^2(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+4b(Ab-aB)\sin(c+dx)+b^2B\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(-2*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) + (8*a^2*(-(A*b) + a*B)*ArcTanh[(
(a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*b*(A*b -
a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*b^3*d)
```

Maple [B] time = 0.109, size = 367, normalized size = 2.7

$$2\,\frac{(\tan(1/2\,dx+c/2))^3\,A}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - 2\,\frac{(\tan(1/2\,dx+c/2))^3\,Ba}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - \frac{B}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-2/d/b^2/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*a-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^3*B+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A
-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B*a+1/d/b/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B-2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*A*
a+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*B*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c))
*B+2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(
a+b))^(1/2))*A-2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a
-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89433, size = 918, normalized size = 6.85

$$\left[\frac{\left(2\,Ba^4-2\,Aa^3b-Ba^2b^2+2\,Aab^3-Bb^4\right)dx+\left(Ba^3-Aa^2b\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(d}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+}\right.}{2\left(a^2b^3-b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*d*x + (B*a^3 -
A*a^2*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^3*b - 2*A*a^2*b^
2 - 2*B*a*b^3 + 2*A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^2*b^3 - b^5)*d), 1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b
^4)*d*x - 2*(B*a^3 - A*a^2*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
```

```
(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*
A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33727, size = 306, normalized size = 2.28

$$\frac{\frac{(2\,Ba^2-2\,Aab+Bb^2)(dx+c)}{b^3}+\frac{4\,(Ba^3-Aa^2b)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{2\left(2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab\tan\left(\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*B*a^2 - 2*A*a*b + B*b^2)*(d*x + c)/b^3 + 4*(B*a^3 - A*a^2*b)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^3) - 2*(2
*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*
x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*
b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.252 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{2a(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{x(Ab-aB)}{b^2}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] ((A*b - a*B)*x)/b^2 - (2*a*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.174094, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 12, 2735, 2659, 205}

$$-\frac{2a(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{x(Ab-aB)}{b^2}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 - (2*a*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{B\sin(c+dx)}{bd} + \frac{\int \frac{(Ab-aB)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{B\sin(c+dx)}{bd} + \frac{(Ab-aB)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{(Ab-aB)x}{b^2} + \frac{B\sin(c+dx)}{bd} - \frac{(a(Ab-aB))\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}$$
$$= \frac{(Ab-aB)x}{b^2} + \frac{B\sin(c+dx)}{bd} - \frac{(2a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c\right.\right.}{b^2d}$$
$$= \frac{(Ab-aB)x}{b^2} - \frac{2a(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d} + \frac{B\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.195344, size = 85, normalized size = 0.96

$$\frac{-\frac{2a(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + (c+dx)(Ab-aB) + bB\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*(c + d*x) - (2*a*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)
/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*B*Sin[c + d*x])/(b^2*d)
```

Maple [B] time = 0.116, size = 172, normalized size = 1.9

$$2\,\frac{B\tan(1/2\,dx+c/2)}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)} + 2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,A}{bd} - 2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,Ba}{b^2d} - 2\,\frac{aA}{bd\sqrt{(a-b)(a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b*arctan(tan(1/2*d*
x+1/2*c))*A-2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*B*a-2/d*a/b/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d*a^2/b^2/((a-
b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86228, size = 689, normalized size = 7.74

$$\left[-\frac{2\left(Ba^3 - Aa^2b - Bab^2 + Ab^3\right)dx - \left(Ba^2 - Aab\right)\sqrt{-a^2 + b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right.}{2\left(a^2b^2 - b^4\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*a*b)*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*
b^2 - b^4)*d), -((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*a*b)*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29543, size = 192, normalized size = 2.16

$$-\frac{\frac{(Ba-Ab)(dx+c)}{b^2}-\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}+\frac{2\left(Ba^2-Aab\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((B*a - A*b)*(d*x + c)/b^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*b) + 2*(B*a^2 - A*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/(sqrt(a^2 - b^2)*b^2))/d
```

3.253 $\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{Bx}{b}$$

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0777636, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2735, 2659, 205}

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{Bx}{b}-\frac{(-Ab+aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{Bx}{b}+\frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\ &= \frac{Bx}{b}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.11225, size = 68, normalized size = 1.01

$$\frac{\frac{2(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+B(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(c + d*x) + (2*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2])/(b*d)
```

Maple [A] time = 0.098, size = 113, normalized size = 1.7

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)B}{bd}+2\,\frac{A}{d\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan\left(1/2\,dx+c/2\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)-2\,\frac{aB}{bd\sqrt{(a-b)(a+b)}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))*B+2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-2/d/b/((a-b)*(a+b))^(1/2)*arctan(tan
(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*a*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8237, size = 524, normalized size = 7.82

$$\left[\frac{2\left(Ba^2-Bb^2\right)dx+(Ba-Ab)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2b-b^3\right)d},\ \frac{\left(Ba^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(B*a^2 - B*b^2)*d*x + (B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)))/((a^2*b - b^3)*d), ((B*a^2 - B*b^2)*d*x - (B*a - A*b)*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2*b -
 b^3)*d)]
```

Sympy [A] time = 127.075, size = 524, normalized size = 7.82

$$\begin{cases} \frac{\tilde{\infty}x(A+B\cos(c))}{\cos(c)} \\ \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd}+\frac{Bx}{b}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} \\ \frac{A}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{Bx}{b}+\frac{B}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} \\ \frac{Ax+\frac{B\sin(c+dx)}{d}}{a} \\ \frac{x(A+B\cos(c))}{a+b\cos(c)} \\ \frac{Ab\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{Ab\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{Badx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{Ba\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cos(c))/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (A
*tan(c/2 + d*x/2)/(b*d) + B*x/b - B*tan(c/2 + d*x/2)/(b*d), Eq(a, b)), (A/(
b*d*tan(c/2 + d*x/2)) + B*x/b + B/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((A*x
 + B*sin(c + d*x)/d)/a, Eq(b, 0)), (x*(A + B*cos(c))/(a + b*cos(c)), Eq(d,
0)), (A*b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt
(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - A*b*log(s
qrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a - b) - b/
(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + B*a*d*x*sqrt(-a/(a - b) -
 b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) -
b/(a - b))) - B*a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*
b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + B
*a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a -
 b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - B*b*d*x*sqrt(-a/(
a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a
 - b) - b/(a - b))), True))
```

Giac [A] time = 1.58013, size = 136, normalized size = 2.03

$$\frac{\frac{(dx+c)B}{b}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ba-Ab)}{\sqrt{a^2-b^2}\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arct
an(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(B*
a - A*b)/(sqrt(a^2 - b^2)*b))/d
```

3.254 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (-2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rubi [A] time = 0.115054, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3001, 3770, 2659, 205}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{A\int\sec(c+dx)\,dx}{a}+\frac{(-Ab+aB)\int\frac{1}{a+b\cos(c+dx)}\,dx}{a}\\ &= \frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(2(Ab-aB))\,\mathrm{Subst}\left(\int\frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+d\right.\right.}{ad}\\ &= -\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.154502, size = 112, normalized size = 1.47

$$\frac{\frac{2(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+A\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[
-a^2 + b^2] + A*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]))/(a*d)
```

Maple [A] time = 0.151, size = 135, normalized size = 1.8

$$-\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\,\frac{Ab}{da\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan(1/2\,dx+c/2)\,(a-b)}{\sqrt{(a-b)(a+b)}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/d*A/a*ln(tan(1/2*d*x+1/2*c)-1)+1/d*A/a*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a/((
a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b+
2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.65766, size = 689, normalized size = 9.07

$$\left[\frac{(Ba-Ab)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(Aa^2-Ab^2\right)\log\left(\sin\left(dx\right.\right.}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [1/2*((B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*
cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^2 - A*b^2
)*log(sin(d*x + c) + 1) - (A*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a
*b^2)*d), 1/2*(2*(B*a - A*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(
sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^2 - A*b^2)*log(sin(d*x + c) + 1) - (A
*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.48512, size = 171, normalized size = 2.25

$$\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ba-Ab)}{\sqrt{a^2-b^2}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(B*a - A*b)/(s
qrt(a^2 - b^2)*a))/d
```

3.255 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{2b(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

```
[Out] (2*b*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) - ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (
A*Tan[c + d*x])/(a*d)
```

Rubi [A] time = 0.183228, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 12, 2747, 3770, 2659, 205}

$$\frac{2b(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) - ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (
A*Tan[c + d*x])/(a*d)
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
```

```
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\tan(c+dx)}{ad} + \frac{\int \frac{(-Ab+aB)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{A\tan(c+dx)}{ad} + \frac{(-Ab+aB)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{A\tan(c+dx)}{ad} - \frac{(Ab-aB)\int \sec(c+dx)\,dx}{a^2} + \frac{(b(Ab-aB))\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2}$$
$$= -\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{A\tan(c+dx)}{ad} + \frac{(2b(Ab-aB))\,\mathrm{Subst}\left(\int \frac{}{a\cdot}\right.}{}$$
$$= \frac{2b(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} - \frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{A\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.517713, size = 129, normalized size = 1.3

$$\frac{-\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + (Ab-aB)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-2*b*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2] + (A*b - a*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a*A*Tan[c + d*x])/(a^2*d)
```

Maple [B] time = 0.181, size = 228, normalized size = 2.3

$$-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{Ab}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) - \frac{B}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) - \frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{Ab}{a^2d}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/d*A/a/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A*b-1/d/a*
ln(tan(1/2*d*x+1/2*c)-1)*B-1/d*A/a/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2*ln(tan(1/
2*d*x+1/2*c)+1)*A*b+1/d/a*ln(tan(1/2*d*x+1/2*c)+1)*B+2/d*b^2/a^2/((a-b)*(a+
b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-2/d*b/a/((
a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87672, size = 1049, normalized size = 10.6

$$\left[\frac{\left(Bab - Ab^2\right)\sqrt{-a^2+b^2}\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(Ba^3-A\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((B*a*b - A*b^2)*sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c)
 + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*s
in(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
 + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) -
 (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c)), -1/2*(2
*(B*a*b - A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b
^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d
*x + c)*log(sin(d*x + c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x
 + c)*log(-sin(d*x + c) + 1) - 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^
```

```
2*b^2)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.36391, size = 236, normalized size = 2.38

$$\frac{\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}+\frac{2\left(Bab-Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan(\cdots}{\cdots}\right.\right.}{\sqrt{a^2-b^2}\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - (B*a - A*b)*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/a^2 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 - 1)*a) + 2*(B*a*b - A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2))/d
```

3.256 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{2b^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2A-2abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(Ab-aB)\tan(c+dx)}{a^2d}+\frac{A\tan}{}$$

```
[Out] (-2*b^2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
3*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2*A + 2*A*b^2 - 2*a*b*B)*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - ((A*b - a*B)*Tan[c + d*x])/(a^2*d) + (A*Sec[c + d*x]*Ta
n[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.490337, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2A-2abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(Ab-aB)\tan(c+dx)}{a^2d}+\frac{A\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*b^2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
3*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2*A + 2*A*b^2 - 2*a*b*B)*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - ((A*b - a*B)*Tan[c + d*x])/(a^2*d) + (A*Sec[c + d*x]*Ta
n[c + d*x])/(2*a*d)
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-2(Ab-aB)+aA\cos(c+dx)+Ab\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2a}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(a^2A+2Ab^2-2abB+aAb\,c\right.}{a+b\cos(c}}{2a^2}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(b^2(Ab-aB)\right)\int \frac{}{a+b\,c}}{a^3}$$

$$= \frac{\left(a^2A+2Ab^2-2abB\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(}{}$$

$$= -\frac{2b^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{\left(a^2A+2Ab^2-2abB\right)\tanh^{-1}(\sin(}{2a^3d}$$

Mathematica [B] time = 1.59362, size = 300, normalized size = 2.1

$$\frac{\frac{8b^2(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 2\left(a^2A-2abB+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\left(a^2A-2abB+2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((8*b^2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] - 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a*(-(A*b
) + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^2*A)/
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*(-(A*b) + a*B)*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*a^3*d)
```

Maple [B] time = 0.151, size = 410, normalized size = 2.9

$$\frac{A}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2} + \frac{A}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{Ab}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} - \frac{B}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} - \frac{A}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/2/d*A/a/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d*A/a/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2
/(tan(1/2*d*x+1/2*c)-1)*A*b-1/d/a/(tan(1/2*d*x+1/2*c)-1)*B-1/2/d*A/a*ln(tan
(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+1/d/a^2*ln(tan(1/
2*d*x+1/2*c)-1)*B*b-1/2/d*A/a/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d*A/a/(tan(1/2*d
*x+1/2*c)+1)+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*A*b-1/d/a/(tan(1/2*d*x+1/2*c)+1
)*B+1/2/d*A/a*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*A*b
^2-1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B*b-2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d*b^2/a^2/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 22.0896, size = 1334, normalized size = 9.33

$$\left[\frac{2\left(Bab^2 - Ab^3\right)\sqrt{-a^2+b^2}\cos\left(dx+c\right)^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(Aa^4 - \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*(B*a*b^2 - A*b^3)*sqrt(-a^2 + b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*
x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
```

```
+ b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 a^2)) + (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A
*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 + 2*(B*a
^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3
*b^2)*d*cos(d*x + c)^2), 1/4*(4*(B*a*b^2 - A*b^3)*sqrt(a^2 - b^2)*arctan(-(
a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (A*a^4
 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 + 2*(B*a^4 - A*a^3*b -
B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x
 + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

Giac [B] time = 1.64769, size = 363, normalized size = 2.54

$$\frac{\frac{(Aa^2-2Bab+2Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{(Aa^2-2Bab+2Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3} - \frac{4(Bab^2-Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\cdots\right.}{\cdots}\right.\right.}{\sqrt{a^2-b^2}a^3}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (
A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 4*(B*a*
b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
```

```
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*a^3) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3
 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*
d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*
a^2))/d
```

3.257 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{2b^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{3a^3d}-\frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2a^4d}$$

```
[Out] (2*b^3*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*Sqrt[a - b]*Sqrt[a + b]*d) - ((a^2 + 2*b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*
x]])/(2*a^4*d) + ((2*a^2*A + 3*A*b^2 - 3*a*b*B)*Tan[c + d*x])/(3*a^3*d) - (
(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*a*d)
```

Rubi [A] time = 0.769869, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{3a^3d}-\frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*b^3*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*Sqrt[a - b]*Sqrt[a + b]*d) - ((a^2 + 2*b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*
x]])/(2*a^4*d) + ((2*a^2*A + 3*A*b^2 - 3*a*b*B)*Tan[c + d*x])/(3*a^3*d) - (
(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*a*d)
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
```

```
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3(Ab-aB)+2aA\cos(c+dx)+2Ab\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}}{3a}$$

$$= -\frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(2\left(2a^2A\right.\right.}{}}{}$$

$$= \frac{\left(2a^2A+3Ab^2-3abB\right)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A}{}$$

$$= \frac{\left(2a^2A+3Ab^2-3abB\right)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A}{}$$

$$= -\frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{\left(2a^2A+3Ab^2-3abB\right)\tan(c+}{3a^3d}$$

$$= \frac{2b^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} - \frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+}{2a^4d}$$

Mathematica [B] time = 2.07219, size = 422, normalized size = 2.26

$$\frac{\frac{4a\left(2a^2A-3abB+3Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4a\left(2a^2A-3abB+3Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} + \frac{24b^3(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 6\left(a^2+2b^2\right)(aB-Ab)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((24*b^3*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/Sqrt[-a^2 + b^2] - 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])^3 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
```

```
Sin[(c + d*x)/2])^3 - (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]))/(12*a^4*d)
```

Maple [B] time = 0.175, size = 688, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*A*b+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)*B+1/2/
d*A/a/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a/(tan(1/2*d*x+1/2*c)+1)*B-1/2/d*A/a/(
tan(1/2*d*x+1/2*c)-1)^2-1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*A*b-2/d*b^3/a^3/
((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+
2/d*b^4/a^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))*A-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*A*b+1/2/d/a/(tan(1/2*d*x+1/2*c
)-1)^2*B-1/3/d*A/a/(tan(1/2*d*x+1/2*c)+1)^3-1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^
2*B-1/3/d*A/a/(tan(1/2*d*x+1/2*c)-1)^3-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A*b^2
+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B*b-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2*A*b+
1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A*b^3-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B*b
^2-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*A*b^2-1/d*A/a/(tan(1/2*d*x+1/2*c)-1)-1/2/
d/a*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d*A/a/(tan(1/2*d*x+1/2*c)+1)+1/2/d/a*ln(ta
n(1/2*d*x+1/2*c)+1)*B+1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B*b+1/2/d/a^2/(tan(1/2
*d*x+1/2*c)+1)^2*A*b-1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*A*b^3+1/d/a^3*ln(tan(
1/2*d*x+1/2*c)+1)*B*b^2+1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.02031, size = 1634, normalized size = 8.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(B*a*b^3 - A*b^4)*sqrt(-a^2 + b^2)*cos(d*x + c)^3*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)) + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)
*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*
a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2
*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B*a^4*b + A*a^3*b^2 + 3*B*a^2*b^3 - 3
*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), -1/12*(12*(B*a*
b^3 - A*b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*
sin(d*x + c)))*cos(d*x + c)^3 - 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3
- 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(B*a^5 - A*
a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) - 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B*a^4*b + A*a^3
*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3
*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x +
c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.73242, size = 556, normalized size = 2.97

$$\frac{3\left(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{3\left(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{12\left(Bab^3-Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\cdots\right.}{\sqrt{a^2-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^4 + 12*(B*a*b^3 - A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)
^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^2*tan(1
/2*d*x + 1/2*c)^3 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) -
3*A*a*b*tan(1/2*d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.258 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=263

$$\frac{\left(2a^2Ab-3a^3B+2ab^2B-Ab^3\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2Ab-3a^3B+4ab^2B-3Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-}{bd\left(a^2\right.}$$

```
[Out] -((4*a*A*b - 6*a^2*B - b^2*B)*x)/(2*b^4) + (2*a^2*(2*a^2*A*b - 3*A*b^3 - 3*
a^3*B + 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*A*b - A*b^3 - 3*a^3*B + 2*a*b^2*B
)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b - 3*a^2*B + b^2*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.658914, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(2a^2Ab-3a^3B+2ab^2B-Ab^3\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2Ab-3a^3B+4ab^2B-3Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-}{bd\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((4*a*A*b - 6*a^2*B - b^2*B)*x)/(2*b^4) + (2*a^2*(2*a^2*A*b - 3*A*b^3 - 3*
a^3*B + 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*A*b - A*b^3 - 3*a^3*B + 2*a*b^2*B
)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b - 3*a^2*B + b^2*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
```

```
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+b(Ab-aB)\cos(c+dx)+\left(\right.\right.}{a+b\cos(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(2aAb-3a^2B+b^2B\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c+dx)\sin}{b\left(a^2-b^2\right)d(a+b\cos(c+\right.}$$

$$= \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{\left(2aAb-3a^2B+b^2B\right)\cos(c+d}{2b^2\left(a^2-b^2\right)d}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)x}{2b^4} + \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{(2a}{}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)x}{2b^4} + \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{(2a}{}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)x}{2b^4} + \frac{2a^2\left(2a^2Ab-3Ab^3-3a^3B+4ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,t}{}\right.}{(a-b)^{3/2}b^4(a+b)^{3/2}d}$$

Mathematica [A] time = 0.982682, size = 184, normalized size = 0.7

$$\frac{2(c+dx)\left(6a^2B-4aAb+b^2B\right) - \frac{8a^2\left(-2a^2Ab+3a^3B-4ab^2B+3Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{4a^3b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + 4b(Ab-2a}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(-4*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - (8*a^2*(-2*a^2*A*b + 3*A*b^3 +
3*a^3*B - 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(3/2) + 4*b*(A*b - 2*a*B)*Sin[c + d*x] + (4*a^3*b*(A*b - a*B)*
```

```
Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*B*Sin[2*(c + d*x
)])/(4*b^4*d)
```

Maple [B] time = 0.145, size = 643, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-4/d/b^3/(1+tan(1/
2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*a-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)
^2*tan(1/2*d*x+1/2*c)^3*B+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/
2*c)*A-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B*a+1/d/b^2/(1
+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B-4/d/b^3*arctan(tan(1/2*d*x+1/
2*c))*A*a+6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*B*a^2+1/d/b^2*arctan(tan(1/2*d
*x+1/2*c))*B+2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b+a+b)*A-2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)*B+4/d*a^4/b^3/(a-b)/(a+b
)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*
A-6/d*a^2/b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)
/((a-b)*(a+b))^(1/2))*A-6/d*a^5/b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(
tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+8/d*a^3/b^2/(a-b)/(a+b)/((a
-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59442, size = 2120, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 -
 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2
+ 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*d*x - (3*B*a^6 - 2*A*a
^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 +
 3*A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*B
*a^6*b - 4*A*a^5*b^2 - 10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2*A*a*b^6
 - (B*a^4*b^3 - 2*B*a^2*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2 - 2*A*a^
4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(
d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^
6 + a*b^8)*d), 1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 +
 4*B*a^2*b^5 - 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7 - 4*A*a^6*b -
 11*B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*d*x - 2*
(3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*b - 2*A*a^4*b^2
 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^6*b - 4*A*a^5*b^2 -
10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2*A*a*b^6 - (B*a^4*b^3 - 2*B*a^2
*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*B*a^3*b^4 + 4
*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2
*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.62894, size = 456, normalized size = 1.73

$$\frac{\frac{4\left(3\,Ba^{5}-2\,Aa^{4}b-4\,Ba^{3}b^{2}+3\,Aa^{2}b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}-\frac{4\left(Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aa^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\left(a^{2}b^{3}-b^{5}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*(3*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 3*A*a^2*b^3)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - 4*(B
*a^4*tan(1/2*d*x + 1/2*c) - A*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (6*B*a^2 -
 4*A*a*b + B*b^2)*(d*x + c)/b^4 - 2*(4*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*t
an(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*tan(1/2*d*x + 1/
2*c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^2*b^3))/d
```

3.259 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x(Ab-2aB)}{b^3}+\frac{B\sin}{}$$

```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^
(3/2)*d) + (B*Sin[c + d*x])/(b^2*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.440118, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2988, 3023, 2735, 2659, 205}

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x(Ab-2aB)}{b^3}+\frac{B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```



```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^
(3/2)*d) + (B*Sin[c + d*x])/(b^2*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
```

```
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{ab(Ab-aB)+(a^2-b^2)(Ab-aB)\cos(c+dx)+b(a^2-b^2)B}{a+b\cos(c+dx)}}{b^2(a^2-b^2)}$$

$$= \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{ab^2(Ab-aB)+b(a^2-b^2)(Ab-2aB}{a+b\cos(c+dx)}}{b^3(a^2-b^2)}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{(a(a^2Ab-}{}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{(2a(a^2Ab}{}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} +$$

Mathematica [A] time = 0.781552, size = 147, normalized size = 0.95

$$\frac{\frac{2a(-a^2Ab+2a^3B-3ab^2B+2Ab^3)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{3/2}} + \frac{a^2b(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + (c+dx)(Ab-2aB) + bB\sin(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(c + d*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B
)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2)
+ b*B*Sin[c + d*x] + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(
a + b*Cos[c + d*x])))/(b^3*d)
```

Maple [B] time = 0.125, size = 445, normalized size = 2.9

$$2\frac{B\tan(1/2\,dx+c/2)}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)} + 2\frac{A\arctan(\tan(1/2\,dx+c/2))}{b^2d} - 4\frac{B\arctan(\tan(1/2\,dx+c/2))\,a}{db^3} - 2\frac{}{bd(a^2-b^2)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b^2*A*arctan(tan(
1/2*d*x+1/2*c))-4/d/b^3*B*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a^2/b/(a^2-b^2)*
tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)*A+2/
d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b+a+b)*B-2/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+4/d*a/(a-b)/(a+b)/((a-b)*(a+b))^(
1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+4/d*a^4/b^3/(a-
b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))*B-6/d*a^2/b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.43307, size = 1701, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A
*b^6)*d*x*cos(d*x + c) + 2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 +
 2*B*a^2*b^4 - A*a*b^5)*d*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^
3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a
^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-
a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x +
```

```
c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3
+ A*a^2*b^4 + B*a*b^5 + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(d*x + c))*sin
(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b
^5 + a*b^7)*d), -((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*
a*b^5 - A*b^6)*d*x*cos(d*x + c) + (2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^
3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*d*x - (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A
*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5 + (B*a^4*b^2
 - 2*B*a^2*b^4 + B*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 +
 b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30676, size = 502, normalized size = 3.24

$$-\frac{\frac{2\left(2\,Ba^4-Aa^3b-3\,Ba^2b^2+2\,Aab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{2\left(2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(2*B*a^3*
tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d
*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*b^3*tan(1/2*d*x + 1/2*c)
```

```
^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x + 1/2*c) + B*a^2*b*
tan(1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) - B*b^3*tan(1/2*d*x + 1
/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)) + (2*B*a - A*b)*(d*x + c)/b^3)/d
```

3.260 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=122

$$-\frac{2\left(a^3B-2ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Bx}{b^2}$$

```
[Out] (B*x)/b^2 - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) + (a*(A*b - a*B)*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.240552, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2968, 3021, 2735, 2659, 205}

$$-\frac{2\left(a^3B-2ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/b^2 - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) + (a*(A*b - a*B)*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{b(Ab-aB)-\left(a^2-b^2\right)B\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{Bx}{b^2} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(Ab^3+a\left(a^2-2b^2\right)B\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= \frac{Bx}{b^2} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(Ab^3+a\left(a^2-2b^2\right)B\right)\right)\text{Subst}\left(\int \frac{}{a+b}\right.}{b^2\left(a^2-b^2\right)}$$

$$= \frac{Bx}{b^2} - \frac{2\left(Ab^3+a^3B-2ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} + \frac{a(Ab-aB)\sin(c+}{b\left(a^2-b^2\right)d(a+b\cos(}$$

Mathematica [A] time = 0.52837, size = 119, normalized size = 0.98

$$\frac{-\frac{2\left(aB\left(a^2-2b^2\right)+Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{ab(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}+B(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*(c + d*x) - (2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (a*b*(A*b - a*B)*Sin[c + d*x]
)/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(b^2*d)
```

Maple [B] time = 0.131, size = 320, normalized size = 2.6

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)B}{b^2d}+2\,\frac{a\tan\left(1/2\,dx+c/2\right)A}{d\left(a^2-b^2\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a-\left(\tan\left(1/2\,dx+c/2\right)\right)^2b+a+b\right)}-2\,\frac{}{bd\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*B+2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)*A-2/d/b*a^2/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d*b/
(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))*A-2/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+
1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+4/d/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.29146, size = 1210, normalized size = 9.92

$$\left[\frac{2\left(Ba^4b-2\,Ba^2b^3+Bb^5\right)dx\cos\left(dx+c\right)+2\left(Ba^5-2\,Ba^3b^2+Bab^4\right)dx-\left(Ba^4-2\,Ba^2b^2+Aab^3+\left(Ba^3b-2\,Bab^3+Ab\right.\right.}{2\left(\left(a^4b^3-2\,a^2b^5+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(B*a^4*b - 2*B*a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + 2*(B*a^5 - 2*B*a
^3*b^2 + B*a*b^4)*d*x - (B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a*b
^3 + A*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a
^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5
 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((B*a^4*b - 2*B*
a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*d*x - (
B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a*b^3 + A*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)
)) - (B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^4*b^3 -
2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.60041, size = 269, normalized size = 2.2

$$\frac{\frac{2\left(Ba^3-2\,Bab^2+Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}+\frac{(dx+c)B}{b^2}-\frac{2\left(Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] (2*(B*a^3 - 2*B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + (d*x + c)*B/b^2 - 2*(B*a^2*tan
(1/2*d*x + 1/2*c) - A*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d
*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

3.261 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] (2*(a*A - b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - ((A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*
Cos[c + d*x]))
```

Rubi [A] time = 0.0885387, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2754, 12, 2659, 205}

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - ((A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*
Cos[c + d*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{-aA+bB}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}\\ &= -\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{(aA-bB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2}\\ &= -\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{(2(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\ &= \frac{2(aA-bB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.317253, size = 97, normalized size = 0.97

$$\frac{\frac{2(aA-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*(a*A - b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2
 + b^2)^(3/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c
 + d*x])))/d
```

Maple [B] time = 0.103, size = 234, normalized size = 2.3

$$-2\,{\frac{A\tan \left(1/2\,dx+c/2 \right) b}{d \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}b+a+b \right) }}+2\,{\frac{a\tan \left(1/2\,dx+c/2 \right) B}{d \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \left(1/2 \right. \right. \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)*A*b+2/d/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)*a*B+2/d*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-2/d/(a-b)/(a+b)/((a-b)*(a+b)
)^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28773, size = 846, normalized size = 8.46

$$\left[-\frac{\left(Aa^{2} - Bab + \left(Aab - Bb^{2}\right)\cos\left(dx + c\right)\right)\sqrt{-a^{2} + b^{2}}\log\left(\frac{2\,ab\cos(dx+c) + \left(2\,a^{2} - b^{2}\right)\cos(dx+c)^{2} + 2\,\sqrt{-a^{2} + b^{2}}\left(a\cos(dx+c) + b\right)\sin(dx+c) - a^{2} + 2}{b^{2}\cos(dx+c)^{2} + 2\,ab\cos(dx+c) + a^{2}}\right.}{2\left(\left(a^{4}b - 2\,a^{2}b^{3} + b^{5}\right)d\cos\left(dx + c\right) + \left(a^{5} - 2\,a^{3}b^{2} + ab^{4}\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(
(2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*
cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + a^2)) - 2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*sin(d*x + c))/(
(a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), ((
A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*co
s(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (B*a^3 - A*a^2*b - B*a*b^
2 + A*b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 -
 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29264, size = 215, normalized size = 2.15

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(Aa-Bb)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(A*a - B*b)/(a^2 -
 b^2)^(3/2) - (B*a*tan(1/2*d*x + 1/2*c) - A*b*tan(1/2*d*x + 1/2*c))/((a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

3.262 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$-\frac{2\left(2a^2Ab+a^3(-B)-Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] (-2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c + d*x]])/(a
^2*d) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.281664, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3000, 3001, 3770, 2659, 205}

$$-\frac{2\left(2a^2Ab+a^3(-B)-Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (-2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c + d*x]])/(a
^2*d) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
```

```
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{\left(A\left(a^2-b^2\right)-a(Ab-aB)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{A\int\sec(c+dx)\,dx}{a^2}-\frac{\left(2a^2Ab-Ab^3-a^3B\right)\ldots}{a^2\left(a^2-\ldots\right.}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\left(2\left(2a^2Ab-Ab^3-\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(2a^2Ab-Ab^3-a^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}+\ldots$$

Mathematica [A] time = 0.590359, size = 191, normalized size = 1.44

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(\frac{2\left(-2a^2Ab+a^3B+Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{ab(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{a^2d(A+B\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-2*a^2*A*b + A*b^3 + a^3*B)*ArcTanh
[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - A*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (a*b*(A*b - a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]
))))/(a^2*d*(A + B*Cos[c + d*x]))
```

Maple [B] time = 0.158, size = 342, normalized size = 2.6

$$-\frac{A}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{A}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\,\frac{b^2\tan(1/2\,dx+c/2)\,A}{da\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d*A/a^2*ln(tan(1/2*d*x+1/2*c)-1)+1/d*A/a^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/
a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b+a+b)*A-2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)*B-4/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(ta
n(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d/a^2/(a-b)/(a+b)/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^3+2/d/(
a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)
)^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 24.139, size = 1534, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^3*b - 2*A*a^2*b^2 + A*b^4)*cos(d*
x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^5 - 2*A*a^3*b^2 + A
*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1
) - (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*
x + c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b
^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*
a^5*b^2 + a^3*b^4)*d), 1/2*(2*(B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^3*b - 2*A
*a^2*b^2 + A*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b
)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4
*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) - (A*a^5 - 2*
A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(-si
n(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c
))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b
^4)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

<u>Giac</u> [A] time = 1.26783, size = 301, normalized size = 2.26

$$\frac{\frac{2\left(Ba^{3}-2\,Aa^{2}b+Ab^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}-a^{2}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] (2*(B*a^3 - 2*A*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/a^2 - A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 2*(B*a*b*tan(1/2*d*x
+ 1/2*c) - A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*
c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

3.263 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{2b\left(3a^2Ab-2a^3B+ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2A+abB-2Ab^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c}{ad\left(a^2-b^2\right)(a+b\cos}$$

```
[Out] (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*A*b - a*B
)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((a^2*A - 2*A*b^2 + a*b*B)*Tan[c + d*x])
/(a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b
*Cos[c + d*x]))
```

Rubi [A] time = 0.673164, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{2b\left(3a^2Ab-2a^3B+ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2A+abB-2Ab^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c}{ad\left(a^2-b^2\right)(a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*A*b - a*B
)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((a^2*A - 2*A*b^2 + a*b*B)*Tan[c + d*x])
/(a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b
*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
```

```
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2A-2Ab^2+abB-a(Ab-aB)\cos(c+dx)+b(Ab-aB)\cos}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2A-2Ab^2+abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-\left(a^2\right.\right.}{}}{}$$

$$= \frac{\left(a^2A-2Ab^2+abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(2Ab-}{}$$

$$= -\frac{(2Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(a^2A-2Ab^2+abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b}{a\left(a^2\right.}$$

$$= \frac{2b\left(3a^2Ab-2Ab^3-2a^3B+ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{(2Ab-aB)\tan}{a}$$

Mathematica [A] time = 1.7186, size = 240, normalized size = 1.27

$$-\frac{2b\left(-3a^2Ab+2a^3B-ab^2B+2Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{ab^2(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + aA\tan(c+dx) - aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \text{si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*b*(-3*a^2*A*b + 2*A*b^3 + 2*a^3*B - a*b^2*B)*ArcTanh[((a - b)*Tan[(c +
 d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 2*A*b*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - a*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*A*
b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a*B*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a +
b*Cos[c + d*x])) + a*A*Tan[c + d*x])/(a^3*d)
```

Maple [B] time = 0.174, size = 502, normalized size = 2.7

$$-\frac{A}{a^2 d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{\ln\left(\tan\left(1/2\,dx+c/2\right)-1\right)Ab}{da^3}-\frac{B}{a^2 d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{A}{a^2 d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d*A/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A*b-1/d/
a^2*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(
tan(1/2*d*x+1/2*c)+1)*A*b+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B-2/d*b^3/a^2/(a
^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a
+b)*A+2/d*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b+a+b)*B+6/d*b^2/a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(ta
n(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-4/d*b^4/a^3/(a-b)/(a+b)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-4/d/(
a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)
)^(1/2))*B*b+2/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x
+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 69.4434, size = 2419, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(((2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2
*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b
^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 +
b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2)) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a
^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b
^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(sin(d*x + c) +
1) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^
6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*
b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(A*a^6 - 2*A*a^4*
b^2 + A*a^2*b^4 + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^
5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c
)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)), -1/2*(2*((2*B*a^3*b^2 -
3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2*B*a^4*b - 3*A*a^3*b^2
- B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x +
 c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^
3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*
b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(si
n(d*x + c) + 1) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a
*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3
*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(A*a
^6 - 2*A*a^4*b^2 + A*a^2*b^4 + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b
^4 + 2*A*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*
d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos{(c + dx)})\sec^2{(c + dx)}}{(a + b\cos{(c + dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)
```

Giac [B] time = 1.40503, size = 545, normalized size = 2.88

$$\frac{\frac{2\left(2\,Ba^{3}b-3\,Aa^{2}b^{2}-Bab^{3}+2\,Ab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{5}-a^{3}b^{2}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\left(Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Aa^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Aa\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^3*b - 3*A*a^2*b^2 - B*a*b^3 + 2*A*b^4)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) - 2*(A*a^3*tan
(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d*x
+ 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^
3 + A*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*tan
(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) + (B*a - 2*A*b)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1))/a^3 - (B*a - 2*A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)
/d
```

3.264 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=270

$$-\frac{2b^2\left(4a^2Ab-3a^3B+2ab^2B-3Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(2a^2Ab+a^3(-B)+2ab^2B-3Ab^3\right)\tan(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{(a^2}{}$$

```
[Out] (-2*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a^3*B + 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2*A
+ 6*A*b^2 - 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - ((2*a^2*A*b - 3*A*b
^3 - a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*A - 3*A*b
^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(A*b -
a*B)*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.975468, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^2\left(4a^2Ab-3a^3B+2ab^2B-3Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(2a^2Ab+a^3(-B)+2ab^2B-3Ab^3\right)\tan(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{(a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (-2*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a^3*B + 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2*A
+ 6*A*b^2 - 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - ((2*a^2*A*b - 3*A*b
^3 - a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*A - 3*A*b
^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(A*b -
a*B)*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
```

```
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2A-3Ab^2+2abB-a(Ab-aB)\cos(c+dx)+2b(Ab-}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2A-3Ab^2+2abB\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec(c+dx)\tan(}{a\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2abB\right)\sec(c}{2a^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2abB\right)\sec(c}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)}{a^3\left(a^2-b^2\right)d}$$

$$= -\frac{2b^2\left(4a^2Ab-3Ab^3-3a^3B+2ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(a^2A+6A\right.}{}$$

Mathematica [A] time = 6.24621, size = 438, normalized size = 1.62

$$\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{a^3d(a-b)(a+b)(a+b\cos(c+dx))} - \frac{2b^2\left(-4a^2Ab+3a^3B-2ab^2B+3Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{a^4d\left(a^2-b^2\right)\sqrt{b^2-a^2}} + \frac{\left(a^2(-A)+4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(-4*a^2*A*b + 3*A*b^3 + 3*a^3*B - 2*a*b^2*B)*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^4*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((-(
```

```
a^2*A) - 6*A*b^2 + 4*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^
4*d) + ((a^2*A + 6*A*b^2 - 4*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
])/(2*a^4*d) + A/(4*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - A/(4*a
^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (-2*A*b*Sin[(c + d*x)/2] +
a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (-2*A
*b*Sin[(c + d*x)/2] + a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])) + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x])/(a^3*(a - b)*(
a + b)*d*(a + b*Cos[c + d*x]))
```

Maple [B] time = 0.198, size = 690, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d
/a^3/(tan(1/2*d*x+1/2*c)-1)*A*b-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B-1/2/d*A/a^
2*ln(tan(1/2*d*x+1/2*c)-1)-3/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+2/d/a^3*l
n(tan(1/2*d*x+1/2*c)-1)*B*b-1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d*A/a^
2/(tan(1/2*d*x+1/2*c)+1)+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A*b-1/d/a^2/(tan(1/
2*d*x+1/2*c)+1)*B+1/2/d*A/a^2*ln(tan(1/2*d*x+1/2*c)+1)+3/d/a^4*ln(tan(1/2*d
*x+1/2*c)+1)*A*b^2-2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B*b+2/d*b^4/a^3/(a^2-b^
2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)*A
-2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b+a+b)*B-8/d/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^3+6/d*b^5/a^4/(a-b)/(a+b)/((a-b)
*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+6/d*b^
2/a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))*B-4/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 109.609, size = 2952, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(2*((3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6)*cos(d*x + c)^3
+ (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c)^2)*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sq
rt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b
^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*
a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 +
6*A*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((A*a^6*b - 4*B*a^5*b^2
+ 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x +
 c)^3 + (A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B
*a^2*b^5 + 6*A*a*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(A*a^7 - 2
*A*a^5*b^2 + A*a^3*b^4 + 2*(B*a^6*b - 2*A*a^5*b^2 - 3*B*a^4*b^3 + 5*A*a^3*b
^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a
^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c
))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5
*b^4)*d*cos(d*x + c)^2), 1/4*(4*((3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3
*A*b^6)*cos(d*x + c)^3 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b
^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c))) + ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 + 8*B*a^3*b^4
 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*a^7 - 4*B*a^6*b
+ 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*cos(d
*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 +
8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*a^7 -
 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a
*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^7 - 2*A*a^5*b^2 + A*a
^3*b^4 + 2*(B*a^6*b - 2*A*a^5*b^2 - 3*B*a^4*b^3 + 5*A*a^3*b^4 + 2*B*a^2*b^5
 - 3*A*a*b^6)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b^2 + 6*A*a^4
*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a
```

```
^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x
+ c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.5163, size = 510, normalized size = 1.89

$$-\frac{\frac{4\left(3\,Ba^3b^2-4\,Aa^2b^3-2\,Bab^4+3\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{4\left(Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3 - 2*B*a*b^4 + 3*A*b^5)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) + 4*(
B*a*b^3*tan(1/2*d*x + 1/2*c) - A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - (A*a^2 -
4*B*a*b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 + (A*a^2 - 4*B*a*
b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*(A*a*tan(1/2*d*x +
1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 + A*
a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) - 4*A*b*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.265 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=398

$$\frac{\left(-11a^2Ab^3+6a^4Ab+21a^3b^2B-12a^5B-6ab^4B+2Ab^5\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-15a^2Ab^3+6a^4Ab+29a^3b^2B-12a^5B-2\right.}{b^5d(a-b)^{5/2}(}$$

```
[Out] -((6*a*A*b - 12*a^2*B - b^2*B)*x)/(2*b^5) + (a^2*(6*a^4*A*b - 15*a^2*A*b^3
+ 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*A
*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*B - 6*a*b^4*B)*Sin[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*A*b^3 - 6*a^4*B + 10*a^2*
b^2*B - b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) + (a*(A*b
 - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B)*Cos[c + d*x]^2*Sin[c
+ d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.72365, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2989, 3047, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(-11a^2Ab^3+6a^4Ab+21a^3b^2B-12a^5B-6ab^4B+2Ab^5\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-15a^2Ab^3+6a^4Ab+29a^3b^2B-12a^5B-2\right.}{b^5d(a-b)^{5/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((6*a*A*b - 12*a^2*B - b^2*B)*x)/(2*b^5) + (a^2*(6*a^4*A*b - 15*a^2*A*b^3
+ 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*A
*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*B - 6*a*b^4*B)*Sin[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*A*b^3 - 6*a^4*B + 10*a^2*
b^2*B - b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) + (a*(A*b
 - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B)*Cos[c + d*x]^2*Sin[c
+ d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-3a(Ab-aB)+2b(Ab-aB)\cos(c+dx)+2\right.}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2Ab-5Ab^3-4a^3B+7ab^2B\right)\cos^2(}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{\left(3a^3Ab-6aAb^3-6a^4B+10a^2b^2B-b^4B\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d} + \frac{a(Ab-a}{2b\left(a^2-\right.}$$

$$= \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} - \frac{\left(3a^3Ab\right.}{}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)x}{2b^5} + \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6\right.}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)x}{2b^5} + \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6\right.}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)x}{2b^5} + \frac{a^2\left(6a^4Ab-15a^2Ab^3+12Ab^5-12a^5B+29a^3b^2B\right.}{(a-b)^{5/2}b^5(a+b)}$$

Mathematica [A] time = 3.29314, size = 734, normalized size = 1.84

$$\frac{16ab\left(a^2-b^2\right)^2(c+dx)\left(12a^2B-6aAb+b^2B\right)\cos(c+dx)+4\left(b^3-a^2b\right)^2(c+dx)\left(12a^2B-6aAb+b^2B\right)\cos(2(c+dx))+48a^6Ab^2\sin(c+dx)+36a^5Ab^3\sin(2(c+dx))-84a^4Ab^4\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((16*a^2*(-6*a^4*A*b + 15*a^2*A*b^3 - 12*A*b^5 + 12*a^5*B - 29*a^3*b^2*B +
20*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b
^2)^(5/2) + (-48*a^7*A*b*c + 72*a^5*A*b^3*c - 24*a*A*b^7*c + 96*a^8*B*c - 1
36*a^6*b^2*B*c - 12*a^4*b^4*B*c + 48*a^2*b^6*B*c + 4*b^8*B*c - 48*a^7*A*b*d
```

```
*x + 72*a^5*A*b^3*d*x - 24*a*A*b^7*d*x + 96*a^8*B*d*x - 136*a^6*b^2*B*d*x -
 12*a^4*b^4*B*d*x + 48*a^2*b^6*B*d*x + 4*b^8*B*d*x + 16*a*b*(a^2 - b^2)^2*(
-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*
(-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[2*(c + d*x)] + 48*a^6*A*b^2*Sin
[c + d*x] - 84*a^4*A*b^4*Sin[c + d*x] + 8*a^2*A*b^6*Sin[c + d*x] + 4*A*b^8*
Sin[c + d*x] - 96*a^7*b*B*Sin[c + d*x] + 160*a^5*b^3*B*Sin[c + d*x] - 32*a^
3*b^5*B*Sin[c + d*x] - 8*a*b^7*B*Sin[c + d*x] + 36*a^5*A*b^3*Sin[2*(c + d*x
)] - 64*a^3*A*b^5*Sin[2*(c + d*x)] + 16*a*A*b^7*Sin[2*(c + d*x)] - 72*a^6*b
^2*B*Sin[2*(c + d*x)] + 130*a^4*b^4*B*Sin[2*(c + d*x)] - 48*a^2*b^6*B*Sin[2
*(c + d*x)] + 2*b^8*B*Sin[2*(c + d*x)] + 4*a^4*A*b^4*Sin[3*(c + d*x)] - 8*a
^2*A*b^6*Sin[3*(c + d*x)] + 4*A*b^8*Sin[3*(c + d*x)] - 8*a^5*b^3*B*Sin[3*(c
 + d*x)] + 16*a^3*b^5*B*Sin[3*(c + d*x)] - 8*a*b^7*B*Sin[3*(c + d*x)] + a^4
*b^4*B*Sin[4*(c + d*x)] - 2*a^2*b^6*B*Sin[4*(c + d*x)] + b^8*B*Sin[4*(c + d
*x)])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*b^5*d)
```

Maple [B] time = 0.132, size = 1504, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B-6/d/b^4*arctan(tan(
1/2*d*x+1/2*c))*A*a+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*ar
ctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-20/d*a^3/b/(a^4-2*a^2*
b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))*B-12/d*a^7/b^5/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)/(
(a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-1
5/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))*A-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*
d*x+1/2*c)*B*a+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)^3*B*a+10/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*a^3/b/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*
c)*A-1/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+10/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d*a^3
/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*a^6/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
```

```
1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-1/d*a^4/b^2/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*A+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-6/d*a^6/b
^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+4/d*a^5/b^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*A+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*B+2/d/b^3/(1+tan
(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A+12/d/b^5*arctan(tan(1/2*d*x+1/2
*c))*B*a^2-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+2/d/b^
3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.166, size = 4001, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")

[Out] [1/4*(2*(12*B*a^8*b^2 - 6*A*a^7*b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^
4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 + 6*A*a*b^9 - B*b^10)*d*x*cos(d*x + c)^2
 + 4*(12*B*a^9*b - 6*A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5
 - 18*A*a^4*b^6 - 9*B*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*d*x*cos(d*x + c) + 2
*(12*B*a^10 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A
*a^5*b^5 - 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*d*x + (12*B*a^9 - 6*A*a^8
```

```
*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B*a^5*b^4 - 12*A*a^4*b^5 + (12*B*a^7*
b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b
^7)*cos(d*x + c)^2 + 2*(12*B*a^8*b - 6*A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*
b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*b^3 + 17*A*a^6*b^4
+ 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^8 - (B*a^6*b^4 - 3*
B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10)*cos(d*x + c)^3 + 2*(2*B*a^7*b^3 - A*a^6*b
^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*
b^10)*cos(d*x + c)^2 + (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^
5*b^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c
))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^
2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5
 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d), 1/2*((12*B*a^8*b^2 - 6*A*a^7*b^3 -
 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*d*x*cos(d*x + c)^2 + 2*(12*B*a^9*b - 6*A*a^8*b^2 - 35*B
*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*b^7 + 6*A*a
^2*b^8 - B*a*b^9)*d*x*cos(d*x + c) + (12*B*a^10 - 6*A*a^9*b - 35*B*a^8*b^2
+ 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*a^4*b^6 + 6*A*a^3*b^7 -
B*a^2*b^8)*d*x - (12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B
*a^5*b^4 - 12*A*a^4*b^5 + (12*B*a^7*b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A
*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2 + 2*(12*B*a^8*b - 6*
A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(
d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin
(d*x + c))) - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*b^3 + 17*A*a^6*b^4 + 27*
B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^8 - (B*a^6*b^4 - 3*B*a^4
*b^6 + 3*B*a^2*b^8 - B*b^10)*cos(d*x + c)^3 + 2*(2*B*a^7*b^3 - A*a^6*b^4 -
6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*b^10)
*cos(d*x + c)^2 + (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^5*b^5
 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c))*si
n(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2
*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*
a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65749, size = 1821, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(12*B*a^7 - 6*A*a^6*b - 29*B*a^5*b^2 + 15*A*a^4*b^3 + 20*B*a^3*b^4 -
 12*A*a^2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-
(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b
^5 - 2*a^2*b^7 + b^9)*sqrt(a^2 - b^2)) - 2*(12*B*a^7*tan(1/2*d*x + 1/2*c)^7
 - 6*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 9
*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 9
*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 1
6*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 2
*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 13*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4
*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*A*b^
7*tan(1/2*d*x + 1/2*c)^7 + B*b^7*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^7*tan(1/2*
d*x + 1/2*c)^5 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^6*b*tan(1/2*d*x
 + 1/2*c)^5 + 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 67*B*a^5*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*B*a^4*b^3*tan(1/2*d*
x + 1/2*c)^5 - 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 26*B*a^3*b^4*tan(1/2*d
*x + 1/2*c)^5 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 5*B*a^2*b^5*tan(1/2*d
*x + 1/2*c)^5 + 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 4*B*a*b^6*tan(1/2*d*x +
1/2*c)^5 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^7*tan(1/2*d*x + 1/2*c)^5
+ 36*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 18*
B*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*B*
a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*B
*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 26*
B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 5*
B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*B*a
*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^3 + 3*B*b^7*tan(
1/2*d*x + 1/2*c)^3 + 12*B*a^7*tan(1/2*d*x + 1/2*c) - 6*A*a^6*b*tan(1/2*d*x
+ 1/2*c) + 18*B*a^6*b*tan(1/2*d*x + 1/2*c) - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*
c) - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c) + 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c) -
 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*
B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c) + 13*B*a^
2*b^5*tan(1/2*d*x + 1/2*c) - 4*A*a*b^6*tan(1/2*d*x + 1/2*c) + 4*B*a*b^6*tan
```

```
(1/2*d*x + 1/2*c) - 2*A*b^7*tan(1/2*d*x + 1/2*c) - B*b^7*tan(1/2*d*x + 1/2*
c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x
+ 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (12*B*a^2 - 6*A*a*b +
 B*b^2)*(d*x + c)/b^5)/d
```

3.266 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=280

$$-\frac{\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2Ab^3+2a^4Ab+15a^3b^2B-6a^5B-12ab^4B+6Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] ((A*b - 3*a*B)*x)/b^4 - (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 1
5*a^3*b^2*B - 12*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(a^2*A*b - 4*A*b^3 - 3*
a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]
))
```

Rubi [A] time = 1.22187, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3031, 3023, 2735, 2659, 205}

$$-\frac{\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2Ab^3+2a^4Ab+15a^3b^2B-6a^5B-12ab^4B+6Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((A*b - 3*a*B)*x)/b^4 - (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 1
5*a^3*b^2*B - 12*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(a^2*A*b - 4*A*b^3 - 3*
a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]
))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+2b(Ab-aB)\cos(c+dx)+\left(a\right.\right.}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right)\sin(c}{2b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2}{}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c+dx)\sin}{2b\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c+dx)\sin}{2b\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{a\left(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2B-12ab^4B\right)\tan^{-1}}{(a-b)^{5/2}b^4(a+b)^{5/2}d}$$

Mathematica [A] time = 2.05949, size = 232, normalized size = 0.83

$$\frac{\frac{a^2b\left(-3a^2Ab+5a^3B-8ab^2B+6Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2a\left(5a^2Ab^3-2a^4Ab-15a^3b^2B+6a^5B+12ab^4B-6Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{a^3b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(A*b - 3*a*B)*(c + d*x) - (2*a*(-2*a^4*A*b + 5*a^2*A*b^3 - 6*A*b^5 + 6*a
^5*B - 15*a^3*b^2*B + 12*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
```

```
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*B*Sin[c + d*x] + (a^3*b*(A*b - a*B)*S
in[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^2*b*(-3*a^2*A*b
+ 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*
Cos[c + d*x])))/(2*b^4*d)
```

Maple [B] time = 0.169, size = 1301, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b^3*A*arctan(tan(
1/2*d*x+1/2*c))-6/d/b^4*B*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a^4/b^2/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*A+1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*a^2/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*A+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*B-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*a^3/b/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*
d*x+1/2*c)*A+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/
(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a^4/b^2/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d
*x+1/2*c)*B-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)*
(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+5/d*a^3
/b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/
((a-b)*(a+b))^(1/2))*A-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arct
an(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+6/d*a^6/b^4/(a^4-2*a^2*b
^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(
1/2))*B-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.96437, size = 3380, normalized size = 12.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(4*(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6
 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*d*x*cos(d*x + c)^2 + 8*(3*B*a^8*b - A*a
^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^
7 + A*a*b^8)*d*x*cos(d*x + c) + 4*(3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^
6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*d*x - (6*B*a^8
 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^5 + (6
*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*
a*b^7)*cos(d*x + c)^2 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4
*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 +
13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(B*a^6*b^3 - 3*B*a^4*b^5 + 3*B
*a^2*b^7 - B*b^9)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^
4 + 9*A*a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin
(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*
(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^
6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(2*(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^
5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*d*x*co
s(d*x + c)^2 + 4*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a
^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*d*x*cos(d*x + c) + 2*(3*B*a^9
 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^
```

```
3*b^6 + A*a^2*b^7)*d*x - (6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3
+ 12*B*a^4*b^4 - 6*A*a^3*b^5 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 +
5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c)^2 + 2*(6*B*a^7*b - 2*A
*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c))) - (6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4
*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(B*a^6*b^3 - 3*B*a^4*b^5 + 3*B*a^2*b^7
 - B*b^9)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*
a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c
))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5
 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^6*b^6 +
3*a^4*b^8 - a^2*b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70252, size = 733, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((6*B*a^6 - 2*A*a^5*b - 15*B*a^4*b^2 + 5*A*a^3*b^3 + 12*B*a^2*b^4 - 6*A*a*
b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2
*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*B*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^5*b*
tan(1/2*d*x + 1/2*c)^3 - 5*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^2*tan
(1/2*d*x + 1/2*c)^3 - 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^3*b^3*tan(
1/2*d*x + 1/2*c)^3 + 8*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^4*tan(1
```

```
/2*d*x + 1/2*c)^3 + 4*B*a^6*tan(1/2*d*x + 1/2*c) - 2*A*a^5*b*tan(1/2*d*x +
1/2*c) + 5*B*a^5*b*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^2*tan(1/2*d*x + 1/2*c)
- 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 5*A*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*B
*a^3*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^3
 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 +
a + b)^2) + (3*B*a - A*b)*(d*x + c)/b^4 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

3.267 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$\frac{\left(a^2Ab^3+5a^3b^2B-2a^5B-6ab^4B+2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a\left(a^2Ab-3\right.}{2b^2d\left(\right.}$$

```
[Out] (B*x)/b^3 + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3*b^2*B - 6*a*b^4*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^
(5/2)*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2
*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.564418, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2988, 3021, 2735, 2659, 205}

$$\frac{\left(a^2Ab^3+5a^3b^2B-2a^5B-6ab^4B+2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a\left(a^2Ab-3\right.}{2b^2d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (B*x)/b^3 + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3*b^2*B - 6*a*b^4*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^
(5/2)*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2
*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
```

```
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{2ab(Ab-aB)+\left(a^2-2b^2\right)(Ab-aB)\cos(c+dx)+2b\left(a\right.}{(a+b\cos(c+dx))^2}}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right)\sin(c}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx)}$$

$$= \frac{Bx}{b^3} - \frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right)\mathrm{s}}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{Bx}{b^3} - \frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right)\mathrm{s}}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{Bx}{b^3} + \frac{\left(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \frac{}{2}$$

Mathematica [A] time = 1.30794, size = 204, normalized size = 0.97

$$\frac{\frac{ab\left(a^2Ab-3a^3B+6ab^2B-4Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{2\left(-a^2Ab^3-5a^3b^2B+2a^5B+6ab^4B-2Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{a^2b(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + 2B(c+}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*B*(c + d*x) + (2*(-(a^2*A*b^3) - 2*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 6*a*b
^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d
*x])^2) + (a*b*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/((a
- b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^3*d)
```

Maple [B] time = 0.125, size = 1023, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*B-1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*b/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^3/b/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*B+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^2/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-
4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a-b)^2
*tan(1/2*d*x+1/2*c)*A-2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-1/d*a^3/b/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+6/d
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/(a-b)^2*ta
n(1/2*d*x+1/2*c)*B+1/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d*b^2/(a^4-2*a^2*b^2+b^4)/
((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-
2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))*B+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-6/d*b/(a^4-2*
a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73315, size = 2462, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(4*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x + c)^2
+ 8*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 4*(B
*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x + (2*B*a^7 - 5*B*a^5*b^2
- A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^
2*b^5 + 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 -
A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos
(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + a^2)) - 2*(2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*a^3*b^5 -
 3*A*a^2*b^6 + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 6*B*a
^2*b^6 - 4*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a
^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^
10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2
*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x + c)^2 + 4*(B*
a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 2*(B*a^8 -
3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x - (2*B*a^7 - 5*B*a^5*b^2 - A*a^4
*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^2*b^5 +
 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 - A*a^3*b
^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^7*b - 7*B*a^5*b^3 +
3*A*a^4*b^4 + 5*B*a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^
4*b^4 + 5*A*a^3*b^5 + 6*B*a^2*b^6 - 4*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/
((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3
*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^
4*b^7 - a^2*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.6466, size = 614, normalized size = 2.91

$$-\frac{\frac{\left(2\,Ba^5-5\,Ba^3b^2-Aa^2b^3+6\,Bab^4-2\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^3-2\,a^2b^5+b^7\right)\sqrt{a^2-b^2}}-\frac{(dx+c)B}{b^3}+\frac{2\,Ba^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ba^4b\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((2*B*a^5 - 5*B*a^3*b^2 - A*a^2*b^3 + 6*B*a*b^4 - 2*A*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 -
 b^2)) - (d*x + c)*B/b^3 + (2*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^4*b*tan(
1/2*d*x + 1/2*c)^3 + A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 - 4*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^5*tan(1/2*d*x + 1
/2*c) + 3*B*a^4*b*tan(1/2*d*x + 1/2*c) - A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5
*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 6*B*a^
2*b^3*tan(1/2*d*x + 1/2*c) + 4*A*a*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*
a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b
)^2))/d
```

3.268 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a(Ab-aB)\sin}{2bd\left(a^2-b^2\right)(a+b}$$

```
[Out] -(((3*a*A*b - a^2*B - 2*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(A*b - a*B)*Sin[c + d*x])/(2*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a
*b^2*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.290082, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2754, 12, 2659, 205}

$$-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a(Ab-aB)\sin}{2bd\left(a^2-b^2\right)(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(((3*a*A*b - a^2*B - 2*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(A*b - a*B)*Sin[c + d*x])/(2*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a
*b^2*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b(Ab-aB)-\left(aAb+a^2B-2b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$
$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$
$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$
$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$
$$= -\frac{\left(3aAb-a^2B-2b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx)}$$

Mathematica [A] time = 0.784091, size = 172, normalized size = 0.96

$$\frac{\frac{\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\sin(c+dx)}{b(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(a^2B-3aAb+2b^2B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{a(Ab-aB)\sin(c+dx)}{b(a-b)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-3*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*(A*b - a*B)*Sin[c + d*x])/((a - b)*b*(
a + b)*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*S
in[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [B] time = 0.108, size = 886, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*A*b^2-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*B*a*b+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(
a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2*A-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*a*b+2/d/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*
tan(1/2*d*x+1/2*c)*A*b^2+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*B*a^2-4/d/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)*B*a*b-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*
x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+1/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d/(a^4-
2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(
a+b))^(1/2))*b^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.39819, size = 1616, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*((B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4)
*cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 +
3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 + 4*B*a*
b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((B*a^4 - 3*A*a
^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 2*(
B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*A*a^5 - 3*B*a^4*b -
 A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2
*b^3 + 4*B*a*b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58545, size = 528, normalized size = 2.93

$$\frac{\left(Ba^2-3\,Aab+2\,Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((B*a^2 - 3*A*a*b + 2*B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*B
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*b^2*
tan(1/2*d*x + 1/2*c)^3 - 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan(1/2*d
*x + 1/2*c) + B*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - 3
*B*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan(1/2*d*x + 1/2*c) - 4*B*a*b^2*ta
n(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.269 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(Ab-aB)\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx)}$$

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Sin[c + d*x])/(2*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sin[c + d*x
])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.188168, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2754, 12, 2659, 205}

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(Ab-aB)\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Sin[c + d*x])/(2*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sin[c + d*x
])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{\int \frac{-2(aA-bB)+(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}+\frac{\int \frac{2a^2A+Ab^2-3abB}{a+b\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}+\frac{\left(2a^2A+Ab^2-3ab\right.}{2\left(a^2\right.}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}+\frac{\left(2a^2A+Ab^2-3ab\right.}{}$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{\left(3aAb\right.}{2\left(a^2-\right.}$$

Mathematica [A] time = 0.607202, size = 157, normalized size = 0.96

$$\frac{\frac{\left(a^2B-3aAb+2b^2B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}-\frac{2\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+\frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a
+ b)*(a + b*Cos[c + d*x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sin[c + d*x])/
((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [B] time = 0.116, size = 886, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^2+2/d/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*B+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*a*b+2/d/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*b^2*B-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*a*b+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*b^2+2/d/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)
*tan(1/2*d*x+1/2*c)*B*a^2-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*B*a*b+2/d/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/
2*c)*b^2*B+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+1/d*b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(
a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-3/d*b/(a
^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
)*(a+b))^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.44302, size = 1616, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A*a^4 - 3*B*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)
*cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5 - 4*A*a^4*b - B*a^3*b^2 +
5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*
b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((2*A*a^4 - 3*B
*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 2*(
2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*B*a^5 - 4*A*a^4*b -
 B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^3*b^2 + B*a^2
*b^3 + 3*A*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60985, size = 527, normalized size = 3.21

$$\frac{\left(2\,Aa^{2}-3\,Bab+Ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,Aa^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ba^{2}b\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*B*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A*b^3*
tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d
*x + 1/2*c) - 4*A*a^2*b*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c)
 - 3*A*a*b^2*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) + A*b^3*ta
n(1/2*d*x + 1/2*c) + 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.270 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(-5a^2Ab^3+6a^4Ab-a^3b^2B-2a^5B+2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2Ab-3a^3B-2Ab^3\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b}{2ad\left(a\right.}$$

```
[Out] -(((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d))
+ (A*ArcTanh[Sin[c + d*x]])/(a^3*d) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B)*Sin
[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.705715, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-5a^2Ab^3+6a^4Ab-a^3b^2B-2a^5B+2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2Ab-3a^3B-2Ab^3\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b}{2ad\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d))
+ (A*ArcTanh[Sin[c + d*x]])/(a^3*d) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B)*Sin
[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
```

```
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2A\left(a^2-b^2\right)-2a(Ab-aB)\cos(c+dx)+b(Ab-aB)\cos^2(c+$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \int$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{A}{}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3\right.}{2a^2\left(a^2-b^2\right)^2 d(}$$

$$= -\frac{\left(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{A\tanh}{}$$

Mathematica [A] time = 1.24046, size = 269, normalized size = 1.26

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(\frac{ab\left(5a^2Ab-3a^3B-2Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(5a^2Ab^3-6a^4Ab+a^3b^2B+2a^5B-2Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{a^2b(}{(a-b)(a}\right.}{2a^3d(A+B\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((-2*(-6*a^4*A*b + 5*a^2*A*b^3 - 2*A*b^5
 + 2*a^5*B + a^3*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(-a^2 + b^2)^(5/2) - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*(A*b - a*B)*Sin[c + d*x])
/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b*(5*a^2*A*b - 2*A*b^3 - 3*a
```

```
^3*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]))))/(2*a^3*d*(
A + B*Cos[c + d*x]))
```

Maple [B] time = 0.173, size = 1045, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d*A/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/d*A/a^3*ln(tan(1/2*d*x+1/2*c)+1)+6/d/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*A*b^2+1/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d/a^2/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*A-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*a*b-1/d/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*b^2*B+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(
a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-2/d/a^2/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+
1/2*c)*A-4/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b
)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*a*b/(a^4-2*a^2*b^2
+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/
2))*A+5/d/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^3-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^5+2/d*a^2
/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))*B+1/d/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan
(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*b^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 128.467, size = 3032, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*a
^5*b^2 - 6*A*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 +
2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x +
 c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^
2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(A*a^8 - 3*A*a^6*b^2 + 3*
A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos
(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c
))*log(sin(d*x + c) + 1) - 2*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6
 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^
7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(-sin(d*x + c)
+ 1) - 2*(4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b^5 -
 3*A*a^2*b^6 + (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 - 2*A
*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3
*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos
(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), 1/2*((2*B*a^7 - 6*
A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 6*A*a^4*b^
3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*
a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2
)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^8 - 3
*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b
^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b
^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - (A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^
4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c
)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(-
sin(d*x + c) + 1) - (4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 +
B*a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^
3*b^5 - 2*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^
```

```
5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4
*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

Giac [B] time = 1.5023, size = 649, normalized size = 3.03

$$\frac{\left(2\,Ba^5-6\,Aa^4b+Ba^3b^2+5\,Aa^2b^3-2\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}+\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((2*B*a^5 - 6*A*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - A*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (4*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 - 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*b^3*tan(1/2*d
*x + 1/2*c)^3 - B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*A*a*b^4*tan(1/2*d*x +
1/2*c)^3 - 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^4*b*tan(1/2*d*x + 1/2*c)
- 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*A
*a^2*b^3*tan(1/2*d*x + 1/2*c) - B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a*b^4*
tan(1/2*d*x + 1/2*c) + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^
2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.271 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=299

$$\frac{b\left(-15a^2Ab^3+12a^4Ab+5a^3b^2B-6a^5B-2ab^4B+6Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2Ab^2+2a^4A+5a^3bB-2ab\right.}{2a^3d\left(a^2-b^2\right.}$$

```
[Out] (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a
 + b)^(5/2)*d) - ((3*A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((2*a^4*A
- 11*a^2*A*b^2 + 6*A*b^4 + 5*a^3*b*B - 2*a*b^3*B)*Tan[c + d*x])/(2*a^3*(a^2
 - b^2)^2*d) + (b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (b*(6*a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Tan[c + d*x])/(2
*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.76445, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{b\left(-15a^2Ab^3+12a^4Ab+5a^3b^2B-6a^5B-2ab^4B+6Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2Ab^2+2a^4A+5a^3bB-2ab\right.}{2a^3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a
 + b)^(5/2)*d) - ((3*A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((2*a^4*A
- 11*a^2*A*b^2 + 6*A*b^4 + 5*a^3*b*B - 2*a*b^3*B)*Tan[c + d*x])/(2*a^3*(a^2
 - b^2)^2*d) + (b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (b*(6*a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Tan[c + d*x])/(2
*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{\int \frac{\left(2a^2A-3Ab^2+abB-2a(Ab-aB)\cos(c+dx)+2b(Ab-aB\right.}{(a+b\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$=\frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{b\left(6a^2Ab-3Ab^3-4a^3B+ab^2B\right)\tan(c+}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}$$

$$=\frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d}+\frac{b(Ab-aB)\tan}{2a\left(a^2-b^2\right)d(a+b}$$

$$=\frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d}+\frac{b(Ab-aB)\tan}{2a\left(a^2-b^2\right)d(a+b}$$

$$=-\frac{(3Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3\right.}{2a^3\left(a^2-b^2\right)^2d}$$

$$=\frac{b\left(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a+b}}\right.}{a^4(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 5.5268, size = 352, normalized size = 1.18

$$\frac{\frac{ab^2\left(-7a^2Ab+5a^3B-2ab^2B+4Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}+\frac{a^2b^2(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}-\frac{2b\left(-15a^2Ab^3+12a^4Ab+5a^3b^2B-6a^5B-2ab^4B+6Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right.}{\sqrt{b^2-a^2}}\right.}{\left(b^2-a^2\right)^{5/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b
^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + 2*(3*A*b - a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-3*A*b
+ a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*A*Sin[(c + d*x)/2])/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*a*A*Sin[(c + d*x)/2])/(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) + (a^2*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b
)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b^2*(-7*a^2*A*b + 4*A*b^3 + 5*a^3*B
- 2*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*a
^4*d)
```

Maple [B] time = 0.187, size = 1358, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d*A/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A*b-1/d/
a^3*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(
tan(1/2*d*x+1/2*c)+1)*A*b+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B-8/d/a/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*A-1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+
a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d*b^5/a^3/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*A+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/
(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B+1/d*b^3/a/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*B-2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(
a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d/a/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d/a^2
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a+b)/(a-b)^2*ta
n(1/2*d*x+1/2*c)*A+4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+6/d/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-1/d*b^3/
a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1
/2*d*x+1/2*c)*B-2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a-
b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-15/d
*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))*A+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^
(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-6/d*b/(a^4-2*a
```

```
^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))*B*a+5/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)
/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

Giac [B] time = 1.5991, size = 775, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((6*B*a^5*b - 12*A*a^4*b^2 - 5*B*a^3*b^3 + 15*A*a^2*b^4 + 2*B*a*b^5 - 6*A*b
^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2
+ a^4*b^4)*sqrt(a^2 - b^2)) + (6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*A*a^3
*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^2*
b^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*A*a*b^5
*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^6*tan(1/
2*d*x + 1/2*c)^3 + 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b^3*tan(1/2*d
*x + 1/2*c) + 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 7*A*a^2*b^4*tan(1/2*d*x +
1/2*c) - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*A*a*b^5*tan(1/2*d*x + 1/2*c)
- 2*B*a*b^5*tan(1/2*d*x + 1/2*c) + 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*
a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^2) + (B*a - 3*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (B*a - 3*
A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

3.272 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{b^2\left(-29a^2Ab^3+20a^4Ab+15a^3b^2B-12a^5B-6ab^4B+12Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(-21a^2Ab^3+6a^4Ab+11a^3b\right.}{2}$$

```
[Out] -((b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*
a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(
5/2)*(a + b)^(5/2)*d)) + ((a^2*A + 12*A*b^2 - 6*a*b*B)*ArcTanh[Sin[c + d*x]
])/(2*a^5*d) - ((6*a^4*A*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2
*B - 6*a*b^4*B)*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*
b^2 + 6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a
^2 - b^2)^2*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)
*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.23645, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{b^2\left(-29a^2Ab^3+20a^4Ab+15a^3b^2B-12a^5B-6ab^4B+12Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(-21a^2Ab^3+6a^4Ab+11a^3b\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*
a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(
5/2)*(a + b)^(5/2)*d)) + ((a^2*A + 12*A*b^2 - 6*a*b*B)*ArcTanh[Sin[c + d*x]
])/(2*a^5*d) - ((6*a^4*A*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2
*B - 6*a*b^4*B)*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*
b^2 + 6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a
^2 - b^2)^2*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)
*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2A-2Ab^2+abB\right)-2a(Ab-aB)\cos(c+dx)+3b(Ab}{(a+b\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(7a^2Ab-4Ab^3-5a^3B+2ab^2B\right)\sec(c}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c}$$

$$= \frac{\left(a^4A-10a^2Ab^2+6Ab^4+6a^3bB-3ab^3B\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{b(Ab-a}{2a\left(a^2-\right.}$$

$$= -\frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{\left(a^4A\right.}{}$$

$$= -\frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{\left(a^4A\right.}{}$$

$$= \frac{\left(a^2A+12Ab^2-6abB\right)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a\right.}{2a^4\left(a\right.}$$

$$= -\frac{b^2\left(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(}{\sqrt{a-}}\right.}{a^5(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 2.70888, size = 507, normalized size = 1.26

$$\frac{\frac{16b^2\left(-29a^2Ab^3+20a^4Ab+15a^3b^2B-12a^5B-6ab^4B+12Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}-8\left(a^2A-6abB+12Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((16*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B -
6*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^
2)^(5/2) - 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] + (2*a*(4*a^7*A - 30*a^5*A*b^2 + 68*a^3*A*b^4 - 36*a*A*b^6 + 8*a^6*b
*B - 32*a^4*b^3*B + 18*a^2*b^5*B + (-16*a^6*A*b + 14*a^4*A*b^3 + 47*a^2*A*b
^5 - 36*A*b^7 + 8*a^7*B - 10*a^5*b^2*B - 25*a^3*b^4*B + 18*a*b^6*B)*Cos[c +
 d*x] + 2*a*b*(-11*a^4*A*b + 32*a^2*A*b^3 - 18*A*b^5 + 4*a^5*B - 16*a^3*b^2
*B + 9*a*b^4*B)*Cos[2*(c + d*x)] - 6*a^4*A*b^3*Cos[3*(c + d*x)] + 21*a^2*A*
b^5*Cos[3*(c + d*x)] - 12*A*b^7*Cos[3*(c + d*x)] + 2*a^5*b^2*B*Cos[3*(c + d
*x)] - 11*a^3*b^4*B*Cos[3*(c + d*x)] + 6*a*b^6*B*Cos[3*(c + d*x)])*Sec[c +
d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*a^5*d)
```

Maple [B] time = 0.206, size = 1551, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/2/d*A/a^3/(tan(1/2*d*x+1/2*c)-1)^2-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*B-1/2/d
*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*B-1/2/d*A/a^
3*ln(tan(1/2*d*x+1/2*c)-1)+1/2/d*A/a^3*ln(tan(1/2*d*x+1/2*c)+1)+12/d/(a^4-2
*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))*b^2*B-20/d/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan
(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^3+29/d/a^3/(a^4-2*a^2*b^2+b^
4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
*A*b^5+4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^
```

```
2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*A+4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)
/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-8/d*b^3/a/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*
c)*B+1/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*b^4/a^
2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*B-1/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d*b^5/a^3/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3*A+10/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+10/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*
A-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*
(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+6/d*b^6
/a^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))*B+1/2/d*A/a^3/(tan(1/2*d*x+1/2*c)-1)+1/2/d*A/a^3/(ta
n(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B*b+3/d/a^4/(tan(1/2*d
*x+1/2*c)-1)*A*b-6/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+3/d/a^4*ln(tan(1/2*
d*x+1/2*c)-1)*B*b+3/d/a^4/(tan(1/2*d*x+1/2*c)+1)*A*b+6/d/a^5*ln(tan(1/2*d*x
+1/2*c)+1)*A*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60215, size = 1883, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(12*B*a^5*b^2 - 20*A*a^4*b^3 - 15*B*a^3*b^4 + 29*A*a^2*b^5 + 6*B*a*
b^6 - 12*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(
-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^9
- 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) - 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 -
 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a^
6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5*
b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*a^4*
b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*
b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2*
b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6*
tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*tan(1/2*d*x + 1/2*c)^7 + 3*A*a^7*tan(1/2*
d*x + 1/2*c)^5 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^6*b*tan(1/2*d*x + 1
/2*c)^5 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*
c)^5 + 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 26*A*a^4*b^3*tan(1/2*d*x + 1/2
*c)^5 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*A*a^3*b^4*tan(1/2*d*x + 1/
```

2*c)^5 - 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^3 + A*a^7*tan(1/2*d*x + 1/2*c) + 2*B*a^7*tan(1/2*d*x + 1/2*c) - 4*A*a^6*b*tan(1/2*d*x + 1/2*c) + 4*B*a^6*b*tan(1/2*d*x + 1/2*c) - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c) + 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c) - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c) - 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c) + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c) - 18*A*a*b^6*tan(1/2*d*x + 1/2*c) + 6*B*a*b^6*tan(1/2*d*x + 1/2*c) - 12*A*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (A*a^2 - 6*B*a*b + 12*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 + (A*a^2 - 6*B*a*b + 12*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5)/d

3.273 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=409

$$-\frac{\left(3a^3Ab+23a^2b^2B-12a^4B-8aAb^3-6b^4B\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{a\left(-7a^4Ab^3+8a^2Ab^5+2a^6Ab+28a^5b^2B-35a^3b^4B-8a\right.}{b^5d(a-b)^{7/2}(a+}$$

```
[Out] ((A*b - 4*a*B)*x)/b^5 - (a*(2*a^6*A*b - 7*a^4*A*b^3 + 8*a^2*A*b^5 - 8*A*b^7
 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6*B)*ArcTan[(Sqrt[a - b]*
Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((3*a
^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b^4*B)*Sin[c + d*x])/(6*b^
4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^
2*B)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*a^5*B + 11*a^3*b^2*B - 12
*a*b^4*B)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 5.1748, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2989, 3047, 3031, 3023, 2735, 2659, 205}

$$-\frac{\left(3a^3Ab+23a^2b^2B-12a^4B-8aAb^3-6b^4B\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{a\left(-7a^4Ab^3+8a^2Ab^5+2a^6Ab+28a^5b^2B-35a^3b^4B-8a\right.}{b^5d(a-b)^{7/2}(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((A*b - 4*a*B)*x)/b^5 - (a*(2*a^6*A*b - 7*a^4*A*b^3 + 8*a^2*A*b^5 - 8*A*b^7
 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6*B)*ArcTan[(Sqrt[a - b]*
Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((3*a
^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b^4*B)*Sin[c + d*x])/(6*b^
4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^
2*B)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*a^5*B + 11*a^3*b^2*B - 12
*a*b^4*B)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-3a(Ab-aB)+3b(Ab-aB)\cos(c+dx)+(\right.}{(a+b\cos(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\cos^2(c}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\cos^2(c}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= -\frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} + \frac{a(Ab-aB)\cos^3(c}{3b\left(a^2-b^2\right)d(a}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} + \frac{a(A}{3b}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} + \frac{a(A}{3b}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{a\left(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7-8a^7B+28a^5b^2B-35a^3\right.}{(a-b)^{7/2}b^5(a+b)^{7/2}d}$$

Mathematica [B] time = 6.61236, size = 1278, normalized size = 3.12

$$\frac{96B(c+dx)a^{10}-24Ab(c+dx)a^9+288bB(c+dx)\cos(c+dx)a^9-96bB\sin(c+dx)a^9-144b^2B(c+dx)a^8-72Ab^2(c+a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((a*(-2*a^6*A*b + 7*a^4*A*b^3 - 8*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 28*a^5*b
^2*B + 35*a^3*b^4*B - 20*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(b^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d)) + (-24*a^9*A*b*(c + d*
x) + 36*a^7*A*b^3*(c + d*x) + 36*a^5*A*b^5*(c + d*x) - 84*a^3*A*b^7*(c + d*
x) + 36*a*A*b^9*(c + d*x) + 96*a^10*B*(c + d*x) - 144*a^8*b^2*B*(c + d*x) -
```

```
144*a^6*b^4*B*(c + d*x) + 336*a^4*b^6*B*(c + d*x) - 144*a^2*b^8*B*(c + d*x
) - 72*a^8*A*b^2*(c + d*x)*Cos[c + d*x] + 198*a^6*A*b^4*(c + d*x)*Cos[c + d
*x] - 162*a^4*A*b^6*(c + d*x)*Cos[c + d*x] + 18*a^2*A*b^8*(c + d*x)*Cos[c +
 d*x] + 18*A*b^10*(c + d*x)*Cos[c + d*x] + 288*a^9*b*B*(c + d*x)*Cos[c + d*
x] - 792*a^7*b^3*B*(c + d*x)*Cos[c + d*x] + 648*a^5*b^5*B*(c + d*x)*Cos[c +
 d*x] - 72*a^3*b^7*B*(c + d*x)*Cos[c + d*x] - 72*a*b^9*B*(c + d*x)*Cos[c +
d*x] - 36*a^7*A*b^3*(c + d*x)*Cos[2*(c + d*x)] + 108*a^5*A*b^5*(c + d*x)*Co
s[2*(c + d*x)] - 108*a^3*A*b^7*(c + d*x)*Cos[2*(c + d*x)] + 36*a*A*b^9*(c +
 d*x)*Cos[2*(c + d*x)] + 144*a^8*b^2*B*(c + d*x)*Cos[2*(c + d*x)] - 432*a^6
*b^4*B*(c + d*x)*Cos[2*(c + d*x)] + 432*a^4*b^6*B*(c + d*x)*Cos[2*(c + d*x)
] - 144*a^2*b^8*B*(c + d*x)*Cos[2*(c + d*x)] - 6*a^6*A*b^4*(c + d*x)*Cos[3*
(c + d*x)] + 18*a^4*A*b^6*(c + d*x)*Cos[3*(c + d*x)] - 18*a^2*A*b^8*(c + d*
x)*Cos[3*(c + d*x)] + 6*A*b^10*(c + d*x)*Cos[3*(c + d*x)] + 24*a^7*b^3*B*(c
 + d*x)*Cos[3*(c + d*x)] - 72*a^5*b^5*B*(c + d*x)*Cos[3*(c + d*x)] + 72*a^3
*b^7*B*(c + d*x)*Cos[3*(c + d*x)] - 24*a*b^9*B*(c + d*x)*Cos[3*(c + d*x)] +
 24*a^8*A*b^2*Sin[c + d*x] - 57*a^6*A*b^4*Sin[c + d*x] + 72*a^4*A*b^6*Sin[c
 + d*x] + 36*a^2*A*b^8*Sin[c + d*x] - 96*a^9*b*B*Sin[c + d*x] + 228*a^7*b^3
*B*Sin[c + d*x] - 135*a^5*b^5*B*Sin[c + d*x] - 90*a^3*b^7*B*Sin[c + d*x] +
18*a*b^9*B*Sin[c + d*x] + 30*a^7*A*b^3*Sin[2*(c + d*x)] - 90*a^5*A*b^5*Sin[
2*(c + d*x)] + 120*a^3*A*b^7*Sin[2*(c + d*x)] - 120*a^8*b^2*B*Sin[2*(c + d*
x)] + 336*a^6*b^4*B*Sin[2*(c + d*x)] - 300*a^4*b^6*B*Sin[2*(c + d*x)] + 18*
a^2*b^8*B*Sin[2*(c + d*x)] + 6*b^10*B*Sin[2*(c + d*x)] + 11*a^6*A*b^4*Sin[3
*(c + d*x)] - 32*a^4*A*b^6*Sin[3*(c + d*x)] + 36*a^2*A*b^8*Sin[3*(c + d*x)]
 - 44*a^7*b^3*B*Sin[3*(c + d*x)] + 125*a^5*b^5*B*Sin[3*(c + d*x)] - 114*a^3
*b^7*B*Sin[3*(c + d*x)] + 18*a*b^9*B*Sin[3*(c + d*x)] - 3*a^6*b^4*B*Sin[4*(
c + d*x)] + 9*a^4*b^6*B*Sin[4*(c + d*x)] - 9*a^2*b^8*B*Sin[4*(c + d*x)] + 3
*b^10*B*Sin[4*(c + d*x)])/(24*b^5*(-a^2 + b^2)^3*d*(a + b*Cos[c + d*x])^3)
```

Maple [B] time = 0.141, size = 2787, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+40/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*B-4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+35/d*a^4/b/(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
```

(a+b))^(1/2))*B+8/d*a^8/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+8/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-28/d*a^6/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d/b^4*A*arctan(tan(1/2*d*x+1/2*c))-24/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-18/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-12/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+5/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-1/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-4/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+1/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+44/3/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-5/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+12/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-18/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-12/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-116/3/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+7/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+20/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1

```
/2*d*x+1/2*c)^5*B+20/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a
+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-8/d/b^5*B*arctan
(tan(1/2*d*x+1/2*c))*a+2/d/b^4*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2
)-8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.16999, size = 5723, normalized size = 13.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(12*(4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5
*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*d*x
*cos(d*x + c)^3 + 36*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5
 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 -
 A*a*b^11)*d*x*cos(d*x + c)^2 + 36*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3
+ 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4
*B*a^3*b^9 - A*a^2*b^10)*d*x*cos(d*x + c) + 12*(4*B*a^12 - A*a^11*b - 16*B*
a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^
5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*d*x - 3*(8*B*a^11 - 2*A*a^10*b - 28*B*a^9*
b^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7
 + (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 -
 8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 -
 2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20
*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28
```

```
*B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*
a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(24*B*a
^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a
^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*a^3*b^9 + 6*(B*a^8*b^4 - 4*B*a^6
*b^6 + 6*B*a^4*b^8 - 4*B*a^2*b^10 + B*b^12)*cos(d*x + c)^3 + (44*B*a^9*b^3
- 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*b^6 + 239*B*a^5*b^7 - 68*A*a^4*b^
8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a*b^11)*cos(d*x + c)^2 + 3*(20*B*a
^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*A*a^7*b^5 + 110*B*a^6*b^6 - 35*A*
a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*B*a^2*b^10)*cos(d*x + c))*sin(d*x
 + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x +
 c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*
x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*
cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d
), -1/6*(6*(4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5
*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*d*x
*cos(d*x + c)^3 + 18*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5
 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 -
 A*a*b^11)*d*x*cos(d*x + c)^2 + 18*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3
+ 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4
*B*a^3*b^9 - A*a^2*b^10)*d*x*cos(d*x + c) + 6*(4*B*a^12 - A*a^11*b - 16*B*a
^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5
*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*d*x - 3*(8*B*a^11 - 2*A*a^10*b - 28*B*a^9*b
^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7
+ (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 -
8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 -
2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*
B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28*
B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*a
^3*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^
2 - b^2)*sin(d*x + c))) - (24*B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A
*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B
*a^3*b^9 + 6*(B*a^8*b^4 - 4*B*a^6*b^6 + 6*B*a^4*b^8 - 4*B*a^2*b^10 + B*b^12
)*cos(d*x + c)^3 + (44*B*a^9*b^3 - 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*
b^6 + 239*B*a^5*b^7 - 68*A*a^4*b^8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a
*b^11)*cos(d*x + c)^2 + 3*(20*B*a^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*
A*a^7*b^5 + 110*B*a^6*b^6 - 35*A*a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*
B*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12
 - 4*a^2*b^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^1
1 - 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6
*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a
^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49078, size = 1304, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(8*B*a^8 - 2*A*a^7*b - 28*B*a^6*b^2 + 7*A*a^5*b^3 + 35*B*a^4*b^4 -
8*A*a^3*b^5 - 20*B*a^2*b^6 + 8*A*a*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(a^2 - b^2)
) - (18*B*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 4
2*B*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 24
*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 11
7*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 -
24*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 -
105*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5
+ 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5
 + 36*B*a^9*tan(1/2*d*x + 1/2*c)^3 - 12*A*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 15
2*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 56*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 +
236*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^3
 - 120*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^2*b^7*tan(1/2*d*x + 1/2*c)
^3 + 18*B*a^9*tan(1/2*d*x + 1/2*c) - 6*A*a^8*b*tan(1/2*d*x + 1/2*c) + 42*B*
a^8*b*tan(1/2*d*x + 1/2*c) - 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a^7*b
^2*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c) - 117*B*a^6*b^3*
tan(1/2*d*x + 1/2*c) + 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^4*tan
(1/2*d*x + 1/2*c) + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*B*a^4*b^5*tan(1/
2*d*x + 1/2*c) - 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 60*B*a^3*b^6*tan(1/2*d
```

```
*x + 1/2*c) - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*
a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a +
b)^3) + 3*(4*B*a - A*b)*(d*x + c)/b^5 - 6*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*b^4))/d
```

3.274 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=301

$$-\frac{\left(3a^2Ab^5-7a^5b^2B+8a^3b^4B+2a^7B-8ab^6B+2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

```
[Out] (B*x)/b^4 - ((3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B -
 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7
/2)*b^4*(a + b)^(7/2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^2*(5*A*b^3 + 3*a^3*B - 8*a*b^2*
B)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - (a*(a^2*A
*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B + 34*a*b^4*B)*Sin[c + d*x])/(6*b^3
*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.20668, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3031, 3021, 2735, 2659, 205}

$$-\frac{\left(3a^2Ab^5-7a^5b^2B+8a^3b^4B+2a^7B-8ab^6B+2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] (B*x)/b^4 - ((3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B -
 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7
/2)*b^4*(a + b)^(7/2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^2*(5*A*b^3 + 3*a^3*B - 8*a*b^2*
B)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - (a*(a^2*A
*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B + 34*a*b^4*B)*Sin[c + d*x])/(6*b^3
*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+3b(Ab-aB)\cos(c+dx)-\right.}{(a+b\cos(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}$$

$$= \frac{Bx}{b^4} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\sin(c}{6b^3\left(a^2-b^2\right)^2d(a+b\cos(c+d}$$

$$= \frac{Bx}{b^4} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\sin(c}{6b^3\left(a^2-b^2\right)^2d(a+b\cos(c+d}$$

$$= \frac{Bx}{b^4} - \frac{\left(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}b^4(a+b)^{7/2}d}$$

Mathematica [B] time = 3.13096, size = 717, normalized size = 2.38

$$\frac{18a^5Ab^4\sin(c+dx)+2a^5Ab^4\sin(3(c+dx))+6a^4Ab^5\sin(2(c+dx))+39a^3Ab^6\sin(c+dx)-5a^3Ab^6\sin(3(c+dx))+54a^2Ab^7\sin(2(c+dx))-30a^7b^2B\sin(2(c+dx))+57a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b^
6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/
2) + (24*a^9*B*c - 36*a^7*b^2*B*c - 36*a^5*b^4*B*c + 84*a^3*b^6*B*c - 36*a*
b^8*B*c + 24*a^9*B*d*x - 36*a^7*b^2*B*d*x - 36*a^5*b^4*B*d*x + 84*a^3*b^6*B
*d*x - 36*a*b^8*B*d*x + 18*b*(a^2 - b^2)^3*(4*a^2 + b^2)*B*(c + d*x)*Cos[c
+ d*x] + 36*a*b^2*(a^2 - b^2)^3*B*(c + d*x)*Cos[2*(c + d*x)] + 6*a^6*b^3*B*
c*Cos[3*(c + d*x)] - 18*a^4*b^5*B*c*Cos[3*(c + d*x)] + 18*a^2*b^7*B*c*Cos[3
*(c + d*x)] - 6*b^9*B*c*Cos[3*(c + d*x)] + 6*a^6*b^3*B*d*x*Cos[3*(c + d*x)]
 - 18*a^4*b^5*B*d*x*Cos[3*(c + d*x)] + 18*a^2*b^7*B*d*x*Cos[3*(c + d*x)] -
6*b^9*B*d*x*Cos[3*(c + d*x)] + 18*a^5*A*b^4*Sin[c + d*x] + 39*a^3*A*b^6*Sin
[c + d*x] + 18*a*A*b^8*Sin[c + d*x] - 24*a^8*b*B*Sin[c + d*x] + 57*a^6*b^3*
B*Sin[c + d*x] - 72*a^4*b^5*B*Sin[c + d*x] - 36*a^2*b^7*B*Sin[c + d*x] + 6*
a^4*A*b^5*Sin[2*(c + d*x)] + 54*a^2*A*b^7*Sin[2*(c + d*x)] - 30*a^7*b^2*B*S
in[2*(c + d*x)] + 90*a^5*b^4*B*Sin[2*(c + d*x)] - 120*a^3*b^6*B*Sin[2*(c +
d*x)] + 2*a^5*A*b^4*Sin[3*(c + d*x)] - 5*a^3*A*b^6*Sin[3*(c + d*x)] + 18*a*
A*b^8*Sin[3*(c + d*x)] - 11*a^6*b^3*B*Sin[3*(c + d*x)] + 32*a^4*b^5*B*Sin[3
*(c + d*x)] - 36*a^2*b^7*B*Sin[3*(c + d*x)])/((a^2 - b^2)^3*(a + b*Cos[c +
d*x])^3))/(24*b^4*d)
```

Maple [B] time = 0.13, size = 2158, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5*A+12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^
2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-12/d*b/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*t
an(1/2*d*x+1/2*c)*B-24/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a
+b)^3*a^2/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-12/d*b/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^
5*A-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^6/(a^2+
2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+44/3/d/b/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^4/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*B+6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
```

```
b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+1/d*a^5/b^2
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+3/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^
5*A+6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-2/d*a^6/b^3/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(
1/2*d*x+1/2*c)^5*B+6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-1/d*a^5/b^2/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)*B-3/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d
*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*B+4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+2/d*B/b^4*arctan(tan(1/2*d*
x+1/2*c))+4/3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(
a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+7/d/b^2/(a^6-3*a^4*b^
2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))*B*a^5+8/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a-2/d/b^4/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)
/((a-b)*(a+b))^(1/2))*B*a^7-3/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*a^2-2/d*b^3
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))*A-8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+
b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56686, size = 4095, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(12*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11)*d*
x*cos(d*x + c)^3 + 36*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8
+ B*a*b^10)*d*x*cos(d*x + c)^2 + 36*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 -
 4*B*a^4*b^7 + B*a^2*b^9)*d*x*cos(d*x + c) + 12*(B*a^11 - 4*B*a^9*b^2 + 6*B
*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*d*x + 3*(2*B*a^10 - 7*B*a^8*b^2 + 8*B*a
^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 7*B*a^5*b
^5 + 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10)*cos(d*x + c)^3 + 3*(
2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a
*b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b
^6 - 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*c
os(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x
+ c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + a^2)) - 2*(6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*a^6*b^5 - 7
*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8 + (11*B*a^8*b^3 - 2*A*a^7*b^4 - 43
*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36*B*a^2*b^9 + 18*
A*a*b^10)*cos(d*x + c)^2 + 3*(5*B*a^9*b^2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B
*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 9*A*a^2*b^9)*cos(d*x + c))*sin(d*x
+ c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*cos(d*x + c
)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x
+ c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos
(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d),
1/6*(6*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11)*d*x*c
os(d*x + c)^3 + 18*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B
*a*b^10)*d*x*cos(d*x + c)^2 + 18*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*
B*a^4*b^7 + B*a^2*b^9)*d*x*cos(d*x + c) + 6*(B*a^11 - 4*B*a^9*b^2 + 6*B*a^7
*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*d*x - 3*(2*B*a^10 - 7*B*a^8*b^2 + 8*B*a^6*b
^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 7*B*a^5*b^5 +
 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2*B*
a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a*b^9
)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b^6 -
 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*
x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^10*b - 23*B*a^8*b^3 -
4*A*a^7*b^4 + 43*B*a^6*b^5 - 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8 + (1
1*B*a^8*b^3 - 2*A*a^7*b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*
A*a^3*b^8 - 36*B*a^2*b^9 + 18*A*a*b^10)*cos(d*x + c)^2 + 3*(5*B*a^9*b^2 - 2
```

```
0*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 9*A*a
^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*
a^2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4
*a^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9
- 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8
 - 4*a^5*b^10 + a^3*b^12)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72082, size = 1098, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(2*B*a^7 - 7*B*a^5*b^2 + 8*B*a^3*b^4 + 3*A*a^2*b^5 - 8*B*a*b^6 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^
4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) + 3*(d*x + c)*B/b^4 - (6*B*a^8*t
an(1/2*d*x + 1/2*c)^5 - 15*B*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^6*b^2*tan
(1/2*d*x + 1/2*c)^5 - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*B*a^5*b^3*tan
(1/2*d*x + 1/2*c)^5 + 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b^4*tan(
1/2*d*x + 1/2*c)^5 - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*b^5*tan(
1/2*d*x + 1/2*c)^5 + 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b^6*tan
(1/2*d*x + 1/2*c)^5 - 18*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^8*tan(1/2*
d*x + 1/2*c)^3 - 56*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^5*b^3*tan(1/2*
d*x + 1/2*c)^3 + 116*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*A*a^3*b^5*tan(1/
2*d*x + 1/2*c)^3 - 72*B*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^7*tan(1/2
```

```
*d*x + 1/2*c)^3 + 6*B*a^8*tan(1/2*d*x + 1/2*c) + 15*B*a^7*b*tan(1/2*d*x + 1
/2*c) - 6*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)
 - 45*B*a^5*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6
*B*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 60*B*a
^3*b^5*tan(1/2*d*x + 1/2*c) - 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*B*a^2*
b^6*tan(1/2*d*x + 1/2*c) - 18*A*a*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 + a + b)^3))/d
```

3.275 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=274

$$\frac{\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(a^2Ab-4a^3B+9ab^2\right.}{6b^2d\left(a^2-b^2\right)^2(a}$$

```
[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Sin[
c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*
b^3 - 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[
c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B +
 18*a*b^4*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.636047, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2988, 3021, 2754, 12, 2659, 205}

$$\frac{\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(a^2Ab-4a^3B+9ab^2\right.}{6b^2d\left(a^2-b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Sin[
c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*
b^3 - 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[
c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B +
 18*a*b^4*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
```

```
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{3ab(Ab-aB)+\left(a^2-3b^2\right)(Ab-aB)\cos(c+dx)+3b\left(a\right.}{(a+b\cos(c+dx))^3}}{3b^2\left(a^2-b^2\right)}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\sin(c}{6b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\sin(c}{6b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\sin(c}{6b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\sin(c}{6b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}$$

$$= \frac{\left(a^3A+4aAb^2-3a^2bB-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2(Ab-aB)\,\mathrm{s}}{3b^2\left(a^2-b^2\right)d(a+}$$

Mathematica [A] time = 1.19524, size = 251, normalized size = 0.92

$$\frac{\frac{2\sin(c+dx)\left(6a\left(-9a^2Ab^2+a^4A+a^3bB+9ab^3B-2Ab^4\right)\cos(c+dx)+\left(-10a^2Ab^3+a^4Ab-5a^3b^2B+2a^5B+18ab^4B-6Ab^5\right)\cos(2(c+dx))-14a^2Ab^3-25a^4Ab+17a^3b^2B+}{(a+b\cos(c+dx))^3}}{24d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-25*a^4*A*b - 14*a^2*A*b^
3 - 6*A*b^5 + 10*a^5*B + 17*a^3*b^2*B + 18*a*b^4*B + 6*a*(a^4*A - 9*a^2*A*b
^2 - 2*A*b^4 + a^3*b*B + 9*a*b^3*B)*Cos[c + d*x] + (a^4*A*b - 10*a^2*A*b^3
- 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 18*a*b^4*B)*Cos[2*(c + d*x)])*Sin[c + d
*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Maple [B] time = 0.142, size = 1726, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*A-2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(
1/2*d*x+1/2*c)^5*A*b^3+2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+3/d*b/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b
^2-28/3/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+
2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3*A*b^3+4/3/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a
+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+12/d/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B*a*b^2+1/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d*
a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^
2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*A-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^3+2/d*a^3/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x
+1/2*c)*B-3/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+6/d/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*B*a*b^2+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2
)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+4/d*a*b^2/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2))*A-3/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-2/d/(a^6-3*a^
4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
```

```
a-b)*(a+b))^(1/2))*B*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21513, size = 2692, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 + (A*a^3*b^3 - 3*B
*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4*b^2 - 3*B*a^3*b^3
 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b - 3*B*a^4*b^2 + 4*A
*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
 c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b
)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) - 2*(4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 +
 2*A*a^2*b^5 + (2*B*a^7 + A*a^6*b - 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b
^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*
b - 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos
(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b
^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2
*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d), 1/6*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 + (A*a^3*b^3 - 3
*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4*b^2 - 3*B*a^3*b
^3 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b - 3*B*a^4*b^2 + 4
*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
```

```
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (4*B*a^7 - 13*A*a^6*b + 7*B*a^5
*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 + 2*A*a^2*b^5 + (2*B*a^7 + A*a^6*b - 7*B
*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A*b^7
)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*b - 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^
3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*
a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61159, size = 930, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(A*a^3 - 3*B*a^2*b + 4*A*a*b^2 - 2*B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
 b^2)) + (3*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*tan(1/2*d*x + 1/2*c)^5 +
 12*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*
A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*
A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*
A*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^
5*tan(1/2*d*x + 1/2*c)^5 - 4*B*a^5*tan(1/2*d*x + 1/2*c)^3 + 28*A*a^4*b*tan(
1/2*d*x + 1/2*c)^3 - 32*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*A*a^2*b^3*tan
```

```
(1/2*d*x + 1/2*c)^3 + 36*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^5*tan(1/2*
d*x + 1/2*c)^3 - 3*A*a^5*tan(1/2*d*x + 1/2*c) - 6*B*a^5*tan(1/2*d*x + 1/2*c
) + 12*A*a^4*b*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b*tan(1/2*d*x + 1/2*c) + 27*A
*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 12*A*a^2
*b^3*tan(1/2*d*x + 1/2*c) - 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a*b^4*t
an(1/2*d*x + 1/2*c) - 18*B*a*b^4*tan(1/2*d*x + 1/2*c) + 6*A*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.276 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=263

$$-\frac{\left(4a^2Ab+a^3(-B)-4ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(2a^3Ab-10a^2b^2B+a^4B+13aAb^3-6b^4B\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+$$

```
[Out] -(((4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b - a*B)*Sin[c
 + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((2*a^2*A*b + 3*A*b^3
 + a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) + ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Sin[c +
d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.530179, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2754, 12, 2659, 205}

$$-\frac{\left(4a^2Ab+a^3(-B)-4ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(2a^3Ab-10a^2b^2B+a^4B+13aAb^3-6b^4B\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] -(((4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b - a*B)*Sin[c
 + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((2*a^2*A*b + 3*A*b^3
 + a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) + ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Sin[c +
d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{3b(Ab-aB)-\left(2aAb+a^2B-3b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= -\frac{\left(4a^2Ab+Ab^3-a^3B-4ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a(Ab-aB)\sin}{3b\left(a^2-b^2\right)d(a+b}$$

Mathematica [A] time = 1.05164, size = 252, normalized size = 0.96

$$\frac{\frac{2\sin(c+dx)\left(6\left(9a^2Ab^3+2a^4Ab-9a^3b^2B+a^5B-2ab^4B-Ab^5\right)\cos(c+dx)+b\left(2a^3Ab-10a^2b^2B+a^4B+13aAb^3-6b^4B\right)\cos(2(c+dx))+22a^3Ab^2+12a^5A-14a^2b^3B-25a^4b}{(a+b\cos(c+dx))^3}}{24d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(-4*a^2*A*b - A*b^3 + a^3*B + 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(12*a^5*A + 22*a^3*A*b^2 +
11*a*A*b^4 - 25*a^4*b*B - 14*a^2*b^3*B - 6*b^5*B + 6*(2*a^4*A*b + 9*a^2*A*b
^3 - A*b^5 + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B)*Cos[c + d*x] + b*(2*a^3*A*b +
 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Cos[2*(c + d*x)])*Sin[c + d*x
```

```
])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Maple [B] time = 0.138, size = 1883, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*A+6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1/d/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1
/2*d*x+1/2*c)^5*A*b^3-1/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-6/d*b/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a
*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b^
2-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*b^3+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*A+28/3/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-28/3/d*b/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a^2+2*a*b+b^2)/(a^2-2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*b^3+2/d
*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*A+6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-1/d/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*A*b^3+1/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d*b/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c)*B+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*a*b^2-2/d/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*
tan(1/2*d*x+1/2*c)*B*b^3-4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^
```

```
(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*a^2-1/d*b^3/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))*A+1/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a^3+4/d*b^2/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="maxi
ma")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15369, size = 2700, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fric
as")

[Out] [-1/12*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3 + (B*a^3*b^3 - 4*A*a
^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4
*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*a^3*b
^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*
(6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2
*b^5 + A*a*b^6 + (B*a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B
*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10
*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*
cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 +
 b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 +
a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 +
```

```
a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b
^8)*d), 1/6*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3 + (B*a^3*b^3 -
4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*b^2 - 4*A*a^3*b^
3 + 4*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*
a^3*b^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2
+ 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^6 + (B*a^6*b + 2*A*a^5*
b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7)*cos
(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b
^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^
3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2
 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b
 - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*
a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64286, size = 975, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(B*a^3 - 4*A*a^2*b + 4*B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b
^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 6
*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*
```

```
a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*A
*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*
A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^5
*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^5*tan(1/2
*d*x + 1/2*c)^3 - 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 + 16*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 28*A*a*b^4*tan(1/2*d
*x + 1/2*c)^3 + 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/2
*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B*
a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*b
^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 12*B*a^2*b^3*
tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2*
d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^5*tan(1/2*d*x + 1/2*c))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2
*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.277 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=237

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{\left(-2a^2B+\cdots\right.}{6d\left(a^2-\cdots\right.}$$

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b - a*B)*Sin[c +
d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^
2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*
b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b
*Cos[c + d*x]))
```

Rubi [A] time = 0.477634, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2754, 12, 2659, 205}

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{\left(-2a^2B+\cdots\right.}{6d\left(a^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b - a*B)*Sin[c +
d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^
2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*
b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b
*Cos[c + d*x]))
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
```

```
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}+\int \frac{2\left(3a^2A+2Ab^2-5a\right.}{}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}-\frac{\left(11a^2Ab+4Ab^3\right.}{6\left(a^2-b\right.}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}-\frac{\left(11a^2Ab+4Ab^3\right.}{6\left(a^2-b\right.}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}-\frac{\left(11a^2Ab+4Ab^3\right.}{6\left(a^2-b\right.}$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

Mathematica [A] time = 2.15806, size = 227, normalized size = 0.96

$$\frac{\frac{\left(-11a^2Ab+2a^3B+13ab^2B-4Ab^3\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))}+\frac{\left(2a^2B-5aAb+3b^2B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}+\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}+\frac{2(aB-Ab)}{(a-b)(a+b)(a+}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sin[c + d*x
])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*
B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + ((-11*a^2*A
*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a
+ b*Cos[c + d*x])))/(6*d)
```

Maple [B] time = 0.118, size = 1727, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -6/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-3/d*b^2/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*A-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-
b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^3+2/d*a^3/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*B+2/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^
2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b^2+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*b
^3-12/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/3/d/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3*A*b^3+4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+28/3/d/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B*a*b^2-6/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+3/
d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A
*b^3+2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d*b/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d
*x+1/2*c)*B+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*a*b^2-1/d/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c)*B*b^3+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2
)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+3/d*a*b^2/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2))*A-4/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-1/d/(a^6-3*a^
4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))*B*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19025, size = 2692, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 + (2*A*a^3*b^3 - 4
*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a^4*b^2 - 4*B*a^3*b
^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*b - 4*B*a^4*b^2 + 3
*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
 c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b
)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) - 2*(6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 -
 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (2*B*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b
^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*
A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos
(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b
^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2
*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d), 1/6*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 + (2*A*a^3*b^3 -
 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a^4*b^2 - 4*B*a^3
*b^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*b - 4*B*a^4*b^2 +
 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*B*a^7 - 18*A*a^6*b + 4*B*a^5
*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (2*B
*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7
)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 -
```

```
 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*
a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56153, size = 933, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
 b^2)) - (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^
5 - 6*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
+ 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
- 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 +
12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^
5*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 36*A*a^4*b*tan
(1/2*d*x + 1/2*c)^3 + 16*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*A*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^5*tan(1/2*
d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(1/2*d*x + 1/
2*c) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c) +
 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*
B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 12*B*a*b^
```

```
4*tan(1/2*d*x + 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.278 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=301

$$-\frac{\left(-8a^4Ab^3+7a^2Ab^5+8a^6Ab-3a^5b^2B-2a^7B-2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b\left(-17a^2Ab^3+26a^4Ab-4a^3b^2B-\right.}{6a^3d\left(a^2-b^2\right)^3(a+b}$$

```
[Out] -(((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a
 + b)^(7/2)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + (b*(A*b - a*B)*Sin[c
+ d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^
3 - 5*a^3*B)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) +
 (b*(26*a^4*A*b - 17*a^2*A*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B)*Sin[c +
d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.50886, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-8a^4Ab^3+7a^2Ab^5+8a^6Ab-3a^5b^2B-2a^7B-2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b\left(-17a^2Ab^3+26a^4Ab-4a^3b^2B-\right.}{6a^3d\left(a^2-b^2\right)^3(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] -(((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a
 + b)^(7/2)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + (b*(A*b - a*B)*Sin[c
+ d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^
3 - 5*a^3*B)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) +
 (b*(26*a^4*A*b - 17*a^2*A*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B)*Sin[c +
d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3A\left(a^2-b^2\right)-3a(Ab-aB)\cos(c+dx)+2b(Ab-aB)\cos^2(c}{(a+b\cos(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \int$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + b\,($$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + b\,($$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3\right.}{6a^2\left(a^2-b^2\right)^2 d(}$$

$$= -\frac{\left(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 1.60375, size = 368, normalized size = 1.22

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(-\frac{2ab\sin(c+dx)\left(6ab\left(15a^2Ab^3-20a^4Ab+a^3b^2B+9a^5B-5Ab^5\right)\cos(c+dx)+b^2\left(17a^2Ab^3-26a^4Ab+4a^3b^2B+11a^5B-6Ab^5\right.\right.}{\left(a^2-b^2\right)^3(a+b\cos(c+dx))^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((24*(-8*a^6*A*b + 8*a^4*A*b^3 - 7*a^2*A
*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/
Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 24*A*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 24*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*a*b*(-72*a^
6*A*b + 38*a^4*A*b^3 - 5*a^2*A*b^5 - 6*A*b^7 + 36*a^7*B + a^5*b^2*B + 8*a^3
*b^4*B + 6*a*b*(-20*a^4*A*b + 15*a^2*A*b^3 - 5*A*b^5 + 9*a^5*B + a^3*b^2*B)
*Cos[c + d*x] + b^2*(-26*a^4*A*b + 17*a^2*A*b^3 - 6*A*b^5 + 11*a^5*B + 4*a^
3*b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x]
)^3)))/(24*a^4*d*(A + B*Cos[c + d*x]))
```

Maple [B] time = 0.187, size = 2180, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*a*b^2-3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)
^5*B*a*b^2+4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^
6/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d/a/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^
3)*tan(1/2*d*x+1/2*c)^5*A-1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d
/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*A+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b
^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-44/3/d/a/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a^2+2*a*b+b^2)/(a^2-2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+24/d*
b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2+2*a*b+b^2)
/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*B-12/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a^
2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-6/d*b/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
```

```
n(1/2*d*x+1/2*c)^5*B+12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+12/d*b^2/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+
3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-7/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a
-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^5
+2/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^7+1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+
1)-1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-4/3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
*B*b^3-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*b^3+3/d*b^2/(a^6-3*a^4*b^2+3*a^2
*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))*B*a-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(
tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*a^2+4/d/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5*A*b^3-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*b^3-4/d/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)*A*b^3+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d/(a^6-3*
a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/
((a-b)*(a+b))^(1/2))*B*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxi
ma")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.89333, size = 1130, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - 7*A*a^2*b^5 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^2
 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) + 3*A*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - (18*B*a^7*b*tan(1
/2*d*x + 1/2*c)^5 - 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^6*b^2*tan(
1/2*d*x + 1/2*c)^5 + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5*b^3*tan(
1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*b^4*tan(1
/2*d*x + 1/2*c)^5 - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^3*b^5*tan(1
/2*d*x + 1/2*c)^5 + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 15*A*a*b^7*tan(1/2
*d*x + 1/2*c)^5 - 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^7*b*tan(1/2*d*x +
 1/2*c)^3 - 72*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 32*B*a^5*b^3*tan(1/2*d*x
+ 1/2*c)^3 + 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^3*b^5*tan(1/2*d*x
 + 1/2*c)^3 - 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^8*tan(1/2*d*x +
1/2*c)^3 + 18*B*a^7*b*tan(1/2*d*x + 1/2*c) - 36*A*a^6*b^2*tan(1/2*d*x + 1/2
```

```
*c) + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)
 + 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c) + 3*
B*a^4*b^4*tan(1/2*d*x + 1/2*c) + 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 6*B*a^
3*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c) - 15*A*a*b^7*
tan(1/2*d*x + 1/2*c) - 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*
a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^3))/d
```

3.279 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=420

$$\frac{b\left(-35a^4Ab^3+28a^2Ab^5+20a^6Ab+8a^5b^2B-7a^3b^4B-8a^7B+2ab^6B-8Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-65a^4Ab^2\right.}{}$$

```
[Out] (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 8*a^5*b^
2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - ((4*A*b - a*B)*ArcTanh[Sin[c
+ d*x]])/(a^5*d) + ((6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*
a^5*b*B - 17*a^3*b^3*B + 6*a*b^5*B)*Tan[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) +
 (b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
(b*(9*a^2*A*b - 4*A*b^3 - 6*a^3*B + a*b^2*B)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 -
6*a^5*B + 2*a^3*b^2*B - a*b^4*B)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rubi [A] time = 6.22096, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{b\left(-35a^4Ab^3+28a^2Ab^5+20a^6Ab+8a^5b^2B-7a^3b^4B-8a^7B+2ab^6B-8Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-65a^4Ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```



```
[Out] (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 8*a^5*b^
2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - ((4*A*b - a*B)*ArcTanh[Sin[c
+ d*x]])/(a^5*d) + ((6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*
a^5*b*B - 17*a^3*b^3*B + 6*a*b^5*B)*Tan[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) +
 (b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
(b*(9*a^2*A*b - 4*A*b^3 - 6*a^3*B + a*b^2*B)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 -
```

```
6*a^5*B + 2*a^3*b^2*B - a*b^4*B)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3a^2A-4Ab^2+abB-3a(Ab-aB)\cos(c+dx)+3b(Ab-a\right.}{(a+b\cos(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(9a^2Ab-4Ab^3-6a^3B+ab^2B\right)\tan(c+}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(9a^2Ab-4Ab^3-6a^3B+ab^2B\right)\tan(c+}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}$$

$$= \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B\right)\tan(c+}{6a^4\left(a^2-b^2\right)^3d}$$

$$= \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B\right)\tan(c+}{6a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(4Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+\right.}{6a^4\left(a^2\right.}$$

$$= \frac{b\left(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^5b^2B-7a^3b^4B+2ab^6B\right.}{a^5(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 3.02178, size = 549, normalized size = 1.31

$$\frac{2a\tan(c+dx)\left(6ab^2\left(-53a^4Ab^2+57a^2Ab^4+6a^6A-15a^3b^3B+20a^5bB+5ab^5B-20Ab^6\right)\cos(2(c+dx))+b\left(-438a^6Ab^2+305a^4Ab^4+28a^2Ab^6+72a^8A-50a^5b^3B-7a^3b^5B-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-48*b*(-20*a^6*A*b + 35*a^4*A*b^3 - 28*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 8*
a^5*b^2*B + 7*a^3*b^4*B - 2*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + 48*(4*A*b - a*B)*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + 48*(-4*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (2*a*(24*a^9*A - 36*a^7*A*b^2 - 246*a^5*A*b^4 + 318*a^3*A*b^6 - 120
*a*A*b^8 + 120*a^6*b^3*B - 90*a^4*b^5*B + 30*a^2*b^7*B + b*(72*a^8*A - 438*
a^6*A*b^2 + 305*a^4*A*b^4 + 28*a^2*A*b^6 - 72*A*b^8 + 144*a^7*b*B - 50*a^5*
b^3*B - 7*a^3*b^5*B + 18*a*b^7*B)*Cos[c + d*x] + 6*a*b^2*(6*a^6*A - 53*a^4*
A*b^2 + 57*a^2*A*b^4 - 20*A*b^6 + 20*a^5*b*B - 15*a^3*b^3*B + 5*a*b^5*B)*Co
s[2*(c + d*x)] + 6*a^6*A*b^3*Cos[3*(c + d*x)] - 65*a^4*A*b^5*Cos[3*(c + d*x
)] + 68*a^2*A*b^7*Cos[3*(c + d*x)] - 24*A*b^9*Cos[3*(c + d*x)] + 26*a^5*b^4
*B*Cos[3*(c + d*x)] - 17*a^3*b^6*B*Cos[3*(c + d*x)] + 6*a*b^8*B*Cos[3*(c +
d*x)])*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(48*a^5*d)
```

Maple [B] time = 0.194, size = 2844, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d*A/a^4/(tan(1/2*d*x+1/2*c)-1)-1/d*A/a^4/(tan(1/2*d*x+1/2*c)+1)+1/d/a^4*
ln(tan(1/2*d*x+1/2*c)+1)*B+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*
(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+12/d/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)*B*a*b^2+24/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*
a*b^2+12/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b^2-5/d/a/(tan(1/2*d*x+1/2*c
```

)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan
(1/2*d*x+1/2*c)^5*A+18/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d/a^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5*A+18/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+5/d/a/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
n(1/2*d*x+1/2*c)*A-8/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-35/d*b^4/a/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))*A+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*
(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-7/d*b^5
/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d*b^7/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6
)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*
B-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5*B-1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+2/d*b^6/a^3/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c)*B-44/3/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-
6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+116/3/d*b^5/a^2/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*A-12/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*b^5
/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+4/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*B-6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d*b^4/a/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
n(1/2*d*x+1/2*c)*B+2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-4/d/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)*B*b^3-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B-20/d/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^3-40/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^3
+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*b^3-20/d/(tan(1/2*d*x+1/2*c)^2*a-tan

```
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*A*b^3-4/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*A*b+4/d/a^5*ln(tan(1/2*d*x+1/2*c)
-1)*A*b+8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.71497, size = 1345, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(8*B*a^7*b - 20*A*a^6*b^2 - 8*B*a^5*b^3 + 35*A*a^4*b^4 + 7*B*a^3*b^5
 - 28*A*a^2*b^6 - 2*B*a*b^7 + 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2))
 + (36*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c)
^5 - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 105*A*a^5*b^4*tan(1/2*d*x + 1/2*
c)^5 - 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 117*A*a^3*b^6*tan(1/2*d*x + 1/
2*c)^5 - 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^2*b^7*tan(1/2*d*x + 1/
2*c)^5 - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 42*A*a*b^8*tan(1/2*d*x + 1/2
*c)^5 + 6*B*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 18*A*b^9*tan(1/2*d*x + 1/2*c)^5
+ 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^
3 - 116*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 236*A*a^4*b^5*tan(1/2*d*x + 1/2*
c)^3 + 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 152*A*a^2*b^7*tan(1/2*d*x + 1/
2*c)^3 - 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^9*tan(1/2*d*x + 1/2*c)^
3 + 36*B*a^7*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c) +
 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) - 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 6
*B*a^5*b^4*tan(1/2*d*x + 1/2*c) + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c) - 45*B*
a^4*b^5*tan(1/2*d*x + 1/2*c) + 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 6*B*a^3
*b^6*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c) + 15*B*a^2*b^
7*tan(1/2*d*x + 1/2*c) - 42*A*a*b^8*tan(1/2*d*x + 1/2*c) + 6*B*a*b^8*tan(1/
2*d*x + 1/2*c) - 18*A*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*
b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b
)^3) + 3*(B*a - 4*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(B*a - 4*
A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 - 6*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^4))/d
```

3.280 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=547

$$-\frac{b^2\left(-84a^4Ab^3+69a^2Ab^5+40a^6Ab+35a^5b^2B-28a^3b^4B-20a^7B+8ab^6B-20Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d(a-b)^{7/2}(a+b)^{7/2}}-\frac{(-146}{}$$

```
[Out] -((b^2*(40*a^6*A*b - 84*a^4*A*b^3 + 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35
*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ((a^2*A + 20*A*b^2 -
 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^6*d) - ((24*a^6*A*b - 146*a^4*A*b^3 +
 167*a^2*A*b^5 - 60*A*b^7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^
6*B)*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^
2*A*b^4 - 10*A*b^6 + 12*a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*Sec[c + d*x]*Ta
n[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3
- 7*a^3*B + 2*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a
 + b*Cos[c + d*x])^2) + (b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B
 + 20*a^3*b^2*B - 8*a*b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^
3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 7.30446, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{b^2\left(-84a^4Ab^3+69a^2Ab^5+40a^6Ab+35a^5b^2B-28a^3b^4B-20a^7B+8ab^6B-20Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d(a-b)^{7/2}(a+b)^{7/2}}-\frac{(-146}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] -((b^2*(40*a^6*A*b - 84*a^4*A*b^3 + 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35
*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ((a^2*A + 20*A*b^2 -
 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^6*d) - ((24*a^6*A*b - 146*a^4*A*b^3 +
 167*a^2*A*b^5 - 60*A*b^7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^
```

```
6*B)*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^
2*A*b^4 - 10*A*b^6 + 12*a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*Sec[c + d*x]*Ta
n[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3
- 7*a^3*B + 2*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a
 + b*Cos[c + d*x])^2) + (b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B
 + 20*a^3*b^2*B - 8*a*b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^
3*d*(a + b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{(3a^2A-5Ab^2+2abB-3a(Ab-aB)\cos(c+dx)+4b(A}{(a+b\cos(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B+2ab^2B\right)\sec}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B+2ab^2B\right)\sec}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{\left(a^6A-23a^4Ab^2+27a^2Ab^4-10Ab^6+12a^5bB-11a^3b^3B+4ab^5B\right)\sec(c+d}{2a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B+\right.}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= -\frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B+\right.}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)\tanh^{-1}(\sin(c+dx))}{2a^6d} - \frac{\left(24a^6Ab-146a^4Ab^3+167a^2\right.}{}$$

$$= -\frac{b^2\left(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-20a^7B+35a^5b^2B-28a^3b^4B+\right.}{a^6(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 4.93706, size = 781, normalized size = 1.43

$$\frac{\frac{96b^2\left(84a^4Ab^3-69a^2Ab^5-40a^6Ab-35a^5b^2B+28a^3b^4B+20a^7B-8ab^6B+20Ab^7\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - 48\left(a^2A-8abB+20Ab^2\right)\log\left(\cos\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((96*b^2*(-40*a^6*A*b + 84*a^4*A*b^3 - 69*a^2*A*b^5 + 20*A*b^7 + 20*a^7*B -
 35*a^5*b^2*B + 28*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2]
```

```
)/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 48*(a^2*A + 20*A*b^2 - 8*a*b*B)*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 48*(a^2*A + 20*A*b^2 - 8*a*b*B)*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*(24*a^10*A - 324*a^8*A*b^2 +
 1116*a^6*A*b^4 - 830*a^4*A*b^6 - 61*a^2*A*b^8 + 180*A*b^10 + 72*a^9*b*B -
438*a^7*b^3*B + 305*a^5*b^5*B + 28*a^3*b^7*B - 72*a*b^9*B + 6*a*(-20*a^8*A*
b - 9*a^6*A*b^3 + 309*a^4*A*b^5 - 400*a^2*A*b^7 + 150*A*b^9 + 8*a^9*B - 6*a
^7*b^2*B - 135*a^5*b^4*B + 163*a^3*b^6*B - 60*a*b^8*B)*Cos[c + d*x] + 12*b*
(-21*a^8*A*b + 85*a^6*A*b^3 - 55*a^4*A*b^5 - 19*a^2*A*b^7 + 20*A*b^9 + 6*a^
9*B - 36*a^7*b^2*B + 20*a^5*b^4*B + 8*a^3*b^6*B - 8*a*b^8*B)*Cos[2*(c + d*x
)] - 138*a^7*A*b^3*Cos[3*(c + d*x)] + 738*a^5*A*b^5*Cos[3*(c + d*x)] - 840*
a^3*A*b^7*Cos[3*(c + d*x)] + 300*a*A*b^9*Cos[3*(c + d*x)] + 36*a^8*b^2*B*Co
s[3*(c + d*x)] - 318*a^6*b^4*B*Cos[3*(c + d*x)] + 342*a^4*b^6*B*Cos[3*(c +
d*x)] - 120*a^2*b^8*B*Cos[3*(c + d*x)] - 24*a^6*A*b^4*Cos[4*(c + d*x)] + 14
6*a^4*A*b^6*Cos[4*(c + d*x)] - 167*a^2*A*b^8*Cos[4*(c + d*x)] + 60*A*b^10*C
os[4*(c + d*x)] + 6*a^7*b^3*B*Cos[4*(c + d*x)] - 65*a^5*b^5*B*Cos[4*(c + d*
x)] + 68*a^3*b^7*B*Cos[4*(c + d*x)] - 24*a*b^9*B*Cos[4*(c + d*x)])*Sec[c +
d*x]*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(96*a^6*d)
```

Maple [B] time = 0.219, size = 3042, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d*b^7/a^4/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*B-6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+24/d*b^8/a^5/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/2/d*A/a^4/(tan(1/2*d*x+1/2*c)-1)+1/2/d*A/
a^4/(tan(1/2*d*x+1/2*c)+1)-4/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*B*b+116/3/d*b^5
/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-12/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*B+12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)
^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+28/d*b^6/a^3/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-
b)/((a-b)*(a+b))^(1/2))*B+20/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)
*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-8/d*b^
```

8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-212/3/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+30/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-34/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-34/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+60/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+30/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+84/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^5-69/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^7-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)*B+1/2/d*A/a^4/(tan(1/2*d*x+1/2*c)-1)^2-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*B-1/2/d*A/a^4/(tan(1/2*d*x+1/2*c)+1)^2-5/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+18/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-3/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+18/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+3/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+5/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+1/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-40/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*b^3-20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*b^3+20/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a-20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*b^3+4/d/a^5/(tan(1/2*d*x+1/2*c)-1)*A*b-10/d/a^6*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+4/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)*B*b+4/d/a^5/(tan(1/2*d*x+1/2*c)+1)*A*b+10/d/a^6*ln(tan(1/2*d*x+1/2*c)+1)*A*b^2-35/d*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.78131, size = 1472, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(20*B*a^7*b^2 - 40*A*a^6*b^3 - 35*B*a^5*b^4 + 84*A*a^4*b^5 + 28*B*a
^3*b^6 - 69*A*a^2*b^7 - 8*B*a*b^8 + 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/
2*c))/sqrt(a^2 - b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(a^2
 - b^2)) + 2*(60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 90*A*a^6*b^4*tan(1/2*d*
x + 1/2*c)^5 - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 162*A*a^5*b^5*tan(1/2
*d*x + 1/2*c)^5 - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*b^6*tan(1/
2*d*x + 1/2*c)^5 + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 213*A*a^3*b^7*tan
(1/2*d*x + 1/2*c)^5 - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^2*b^8*ta
n(1/2*d*x + 1/2*c)^5 - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 81*A*a*b^9*tan
(1/2*d*x + 1/2*c)^5 + 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^10*tan(1/2
*d*x + 1/2*c)^5 + 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 - 180*A*a^6*b^4*tan(
1/2*d*x + 1/2*c)^3 - 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 + 392*A*a^4*b^6*t
an(1/2*d*x + 1/2*c)^3 + 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3 - 284*A*a^2*b^
8*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^9*tan(1/2*d*x + 1/2*c)^3 + 72*A*b^10*ta
n(1/2*d*x + 1/2*c)^3 + 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) - 90*A*a^6*b^4*tan
(1/2*d*x + 1/2*c) + 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 162*A*a^5*b^5*tan(
1/2*d*x + 1/2*c) - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c) + 48*A*a^4*b^6*tan(1/2
*d*x + 1/2*c) - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) + 213*A*a^3*b^7*tan(1/2*
d*x + 1/2*c) - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c) + 48*A*a^2*b^8*tan(1/2*d*x
 + 1/2*c) + 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c) - 81*A*a*b^9*tan(1/2*d*x + 1/
2*c) + 18*B*a*b^9*tan(1/2*d*x + 1/2*c) - 36*A*b^10*tan(1/2*d*x + 1/2*c))/((
a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c)^2 + a + b)^3) - 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(
1/2*d*x + 1/2*c) + 1))/a^6 + 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^6 - 6*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x
 + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B
*a*tan(1/2*d*x + 1/2*c) - 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 - 1)^2*a^5))/d
```

3.281 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}$$

```
[Out] (B*Sin[c + d*x])/d - (B*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.015511, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {21, 2633}

$$\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Sin[c + d*x])/d - (B*Sin[c + d*x]^3)/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^3(c+dx)\,dx$$
$$= -\frac{B\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d}$$
$$= \frac{B\sin(c+dx)}{d} - \frac{B\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0076623, size = 28, normalized size = 1.

$$B\left(\frac{\sin(c+dx)}{d} - \frac{\sin^3(c+dx)}{3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*(Sin[c + d*x]/d - Sin[c + d*x]^3/(3*d))
```

Maple [A] time = 0.051, size = 23, normalized size = 0.8

$$\frac{B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/3/d*B*(2+cos(d*x+c)^2)*sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79838, size = 61, normalized size = 2.18

$$\frac{\left(B\cos\left(dx+c\right)^{2}+2\,B\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(B*cos(d*x + c)^2 + 2*B)*sin(d*x + c)/d
```

Sympy [A] time = 1.6412, size = 56, normalized size = 2.

$$\begin{cases} \frac{2B\sin^{3}\left(c+dx\right)}{3d}+\frac{B\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ \frac{x(Ba+Bb\cos\left(c\right))\cos^{3}\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((2*B*sin(c + d*x)**3/(3*d) + B*sin(c + d*x)*cos(c + d*x)**2/d, Ne
(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**3/(a + b*cos(c)), True))
```

Giac [A] time = 1.47308, size = 34, normalized size = 1.21

$$-\frac{B\sin\left(dx+c\right)^{3}-3\,B\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/3*(B*sin(d*x + c)^3 - 3*B*sin(d*x + c))/d
```

3.282 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}$$

```
[Out] (B*x)/2 + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0145795, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 2635, 8}

$$\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*x)/2 + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^2(c+dx)\,dx$$
$$= \frac{B\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}B\int 1\,dx$$
$$= \frac{Bx}{2} + \frac{B\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0201968, size = 24, normalized size = 0.89

$$\frac{B(2(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.053, size = 28, normalized size = 1.

$$\frac{B}{d}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50298, size = 61, normalized size = 2.26

$$\frac{Bdx + B\cos(dx + c)\sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(B*d*x + B*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A] time = 1.14062, size = 68, normalized size = 2.52

$$\begin{cases} \frac{Bx\sin^2(c+dx)}{2} + \frac{Bx\cos^2(c+dx)}{2} + \frac{B\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\cos^2(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*x*sin(c + d*x)**2/2 + B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*c
os(c + d*x)/(2*d), Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**2/(a + b*cos(c)
), True))
```

Giac [A] time = 1.5728, size = 45, normalized size = 1.67

$$\frac{(dx + c)B + \frac{B\tan(dx+c)}{\tan(dx+c)^2+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/2*((d*x + c)*B + B*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

3.283 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{B\sin(c+dx)}{d}$$

```
[Out] (B*Sin[c + d*x])/d
```

Rubi [A] time = 0.0087709, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {21, 2637}

$$\frac{B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Sin[c + d*x])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \cos(c+dx)\,dx\\ &= \frac{B\sin(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.0073006, size = 23, normalized size = 2.09

$$B\left(\frac{\sin(c)\cos(dx)}{d}+\frac{\cos(c)\sin(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*((Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d)
```

Maple [A] time = 0.046, size = 12, normalized size = 1.1

$$\frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] B*sin(d*x+c)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40392, size = 24, normalized size = 2.18

$$\frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] B*sin(d*x + c)/d
```

Sympy [A] time = 0.746644, size = 31, normalized size = 2.82

$$\begin{cases} \frac{B\sin(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\cos(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*sin(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)/(a + b
*cos(c)), True))
```

Giac [A] time = 1.39754, size = 15, normalized size = 1.36

$$\frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] B*sin(d*x + c)/d
```

3.284 $\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=3

$$Bx$$

```
[Out] B*x
```

Rubi [A] time = 0.0011028, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {21, 8}

$$Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*x
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{aB + bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int 1\,dx \\ &= Bx \end{aligned}$$

Mathematica [A] time = 0.0002416, size = 3, normalized size = 1.

$$Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*x
```

Maple [A] time = 0.025, size = 4, normalized size = 1.3

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] B*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.1665, size = 7, normalized size = 2.33

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] B*x
```

Sympy [A] time = 0.158853, size = 2, normalized size = 0.67

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] B*x
```

Giac [C] time = 1.32111, size = 14, normalized size = 4.67

$$\frac{(dx+c)B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)*B/d
```

3.285 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=12

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0068244, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {21, 3770}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int \sec(c+dx)\,dx\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0030115, size = 12, normalized size = 1.

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.056, size = 20, normalized size = 1.7

$$\frac{B\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.41919, size = 81, normalized size = 6.75

$$\frac{B\log\left(\sin\left(dx+c\right)+1\right)-B\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*log(sin(d*x + c) + 1) - B*log(-sin(d*x + c) + 1))/d
```

Sympy [A] time = 4.50306, size = 39, normalized size = 3.25

$$\begin{cases} \frac{B\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x\left(Ba+Bb\cos\left(c\right)\right)\sec\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise(((B*log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*b*c
os(c))*sec(c)/(a + b*cos(c)), True))
```

Giac [B] time = 1.47797, size = 63, normalized size = 5.25

$$\frac{B\log\left(\left|\frac{1}{\sin\left(dx+c\right)}+\sin\left(dx+c\right)+2\right|\right)-B\log\left(\left|\frac{1}{\sin\left(dx+c\right)}+\sin\left(dx+c\right)-2\right|\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/4*(B*log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - B*log(abs(1/sin(d*x +
c) + sin(d*x + c) - 2)))/d
```

3.286 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{B\tan(c+dx)}{d}$$

```
[Out] (B*Tan[c + d*x])/d
```

Rubi [A] time = 0.011593, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 3767, 8}

$$\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Tan[c + d*x])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^2(c+dx)\,dx$$
$$= -\frac{B\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}$$
$$= \frac{B\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0043691, size = 11, normalized size = 1.

$$\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Tan[c + d*x])/d
```

Maple [A] time = 0.056, size = 12, normalized size = 1.1

$$\frac{B\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] B*tan(d*x+c)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31363, size = 45, normalized size = 4.09

$$\frac{B\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] B*sin(d*x + c)/(d*cos(d*x + c))
```

Sympy [A] time = 8.56322, size = 32, normalized size = 2.91

$$\begin{cases} \frac{B\tan(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\sec^2(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*tan(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*sec(c)**2/(a
+ b*cos(c)), True))
```

Giac [A] time = 1.53468, size = 15, normalized size = 1.36

$$\frac{B\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] B*tan(d*x + c)/d
```

3.287 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0197869, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 3768, 3770}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^3(c+dx)\,dx$$
$$= \frac{B\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}B\int \sec(c+dx)\,dx$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))}{2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0083923, size = 36, normalized size = 1.

$$B\left(\frac{\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
```

Maple [A] time = 0.066, size = 40, normalized size = 1.1

$$\frac{B\sec(dx+c)\tan(dx+c)}{2d} + \frac{B\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/2*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54512, size = 170, normalized size = 4.72

$$\frac{B\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-B\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\,B\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/4*(B*cos(d*x + c)^2*log(sin(d*x + c) + 1) - B*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*B*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] B*Integral(sec(c + d*x)**3, x)
```

Giac [A] time = 1.60417, size = 70, normalized size = 1.94

$$\frac{B\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-B\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,B\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/4*(B*log(abs(sin(d*x + c) + 1)) - B*log(abs(sin(d*x + c) - 1)) - 2*B*sin(
d*x + c)/(sin(d*x + c)^2 - 1))/d
```

3.288 $\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}$$

```
[Out] (B*Tan[c + d*x])/d + (B*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0155774, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {21, 3767}

$$\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Tan[c + d*x])/d + (B*Tan[c + d*x]^3)/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^4(c+dx)\,dx$$
$$= -\frac{B\,\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(c+dx)\right)}{d}$$
$$= \frac{B\tan(c+dx)}{d} + \frac{B\tan^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0371981, size = 24, normalized size = 0.86

$$\frac{B\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.063, size = 25, normalized size = 0.9

$$-\frac{B\tan(dx+c)}{d}\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/d*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36135, size = 84, normalized size = 3.

$$\frac{\left(2\,B\cos\left(dx+c\right)^{2}+B\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*cos(d*x + c)^2 + B)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [A] time = 114.743, size = 42, normalized size = 1.5

$$\begin{cases} \frac{B\left(\frac{\tan^{3}\left(c+dx\right)}{3}+\tan\left(c+dx\right)\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x\left(Ba+Bb\cos\left(c\right)\right)\sec^{4}\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*(tan(c + d*x)**3/3 + tan(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*b
*cos(c))*sec(c)**4/(a + b*cos(c)), True))
```

Giac [A] time = 1.56388, size = 34, normalized size = 1.21

$$\frac{B\tan\left(dx+c\right)^{3}+3\,B\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/3*(B*tan(d*x + c)^3 + 3*B*tan(d*x + c))/d
```

3.289 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$-\frac{2a^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{Bx\left(2a^2+b^2\right)}{2b^3}-\frac{aB\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] ((2*a^2 + b^2)*B*x)/(2*b^3) - (2*a^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*B*Sin[c + d*x])/(b^2*d
) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.21655, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {21, 2793, 3023, 2735, 2659, 205}

$$-\frac{2a^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{Bx\left(2a^2+b^2\right)}{2b^3}-\frac{aB\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```



```
[Out] ((2*a^2 + b^2)*B*x)/(2*b^3) - (2*a^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*B*Sin[c + d*x])/(b^2*d
) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```


Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
```

```
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{B\int \frac{a+b\cos(c+dx)-2a\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$
$$= -\frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{B\int \frac{ab+\left(2a^2+b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{2b^2}$$
$$= \frac{\left(2a^2+b^2\right)Bx}{2b^3} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\left(a^3B\right)\int \frac{1}{a+b\cos}}{b^3}$$
$$= \frac{\left(2a^2+b^2\right)Bx}{2b^3} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\left(2a^3B\right)\text{Subst}\,(}{}$$
$$= \frac{\left(2a^2+b^2\right)Bx}{2b^3} - \frac{2a^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+}{}$$

Mathematica [A] time = 0.228605, size = 98, normalized size = 0.86

$$\frac{B\left(2\left(2a^2+b^2\right)(c+dx)+\frac{8a^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-4ab\sin(c+dx)+b^2\sin(2(c+dx))\right)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*(2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/S
qrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*
x)]))/(4*b^3*d)
```

Maple [B] time = 0.122, size = 229, normalized size = 2.

$$-2\,\frac{(\tan(1/2\,dx+c/2))^3Ba}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - \frac{B}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} - 2\,\frac{B\tan(1/2\,dx+c/2)\,a}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*a-1/d/b/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^
2*tan(1/2*d*x+1/2*c)*B*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c
)*B+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*B*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c
))*B-2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60562, size = 759, normalized size = 6.66

$$\left[-\frac{\sqrt{-a^2+b^2}Ba^3\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(2\,Ba^4-Ba^2b^2-Bb^4\right)dx+\left(\right.}{2\left(a^2b^3-b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^4 - B*a^2*b^
2 - B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))
*sin(d*x + c))/((a^2*b^3 - b^5)*d), -1/2*(2*sqrt(a^2 - b^2)*B*a^3*arctan(-(
```

```
a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^4 - B*a^2*b^2
- B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^3 - b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46652, size = 250, normalized size = 2.19

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba^3}{\sqrt{a^2-b^2}\,b^3}-\frac{\left(2Ba^2+Bb^2\right)(dx+c)}{b^3}+\frac{2\left(2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Ba\tan\left(\cdots\right)\cdots\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^3/(sqrt(a^2
- b^2)*b^3) - (2*B*a^2 + B*b^2)*(d*x + c)/b^3 + 2*(2*B*a*tan(1/2*d*x + 1/2*
c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - B*b*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.290 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{2a^2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{aBx}{b^2}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] -((a*B*x)/b^2) + (2*a^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.131642, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {21, 2746, 12, 2735, 2659, 205}

$$\frac{2a^2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{aBx}{b^2}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*B*x)/b^2) + (2*a^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx\\
&= \frac{B\sin(c+dx)}{bd} - \frac{B\int \frac{a\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{B\sin(c+dx)}{bd} - \frac{(aB)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= -\frac{aBx}{b^2} + \frac{B\sin(c+dx)}{bd} + \frac{\left(a^2B\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}\\
&= -\frac{aBx}{b^2} + \frac{B\sin(c+dx)}{bd} + \frac{\left(2a^2B\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= -\frac{aBx}{b^2} + \frac{2a^2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d} + \frac{B\sin(c+dx)}{bd}
\end{aligned}
$$

Mathematica [A] time = 0.133713, size = 73, normalized size = 0.92

$$\frac{B\left(-\frac{2a^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-a(c+dx)+b\sin(c+dx)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*(-(a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 +
b^2]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x]))/(b^2*d)
```

Maple [A] time = 0.135, size = 105, normalized size = 1.3

$$2\,\frac{B\tan(1/2\,dx+c/2)}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,Ba}{b^2d}+2\,\frac{Ba^2}{b^2d\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan(1/2\,dx+c/2)\,(a}{\sqrt{(a-b)(a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*arctan(tan(1/2*
d*x+1/2*c))*B*a+2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57561, size = 612, normalized size = 7.75

$$\left[-\frac{\sqrt{-a^2+b^2}Ba^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\left(Ba^3-Bab^2\right)dx-2\left(Ba^2b-\right.}{2\left(a^2b^2-b^4\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(B*a^3 - B*a*b^2)
*d*x - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2 - b
^2)*B*a^2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (B
*a^3 - B*a*b^2)*d*x + (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48652, size = 173, normalized size = 2.19

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba^2}{\sqrt{a^2-b^2}b^2}-\frac{(dx+c)Ba}{b^2}+\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^2/(sqrt(a^2 - b^2
)*b^2) - (d*x + c)*B*a/b^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*b))/d
```

3.291 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$\frac{Bx}{b}-\frac{2aB\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (B*x)/b - (2*a*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[
a - b]*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0666067, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {21, 2735, 2659, 205}

$$\frac{Bx}{b}-\frac{2aB\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/b - (2*a*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[
a - b]*b*Sqrt[a + b]*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx \\
&= \frac{Bx}{b} - \frac{(aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b} \\
&= \frac{Bx}{b} - \frac{(2aB)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd} \\
&= \frac{Bx}{b} - \frac{2aB\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.0733948, size = 59, normalized size = 0.97

$$
\frac{B\left(\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+c+dx\right)}{bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*(c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2]))/(b*d)
```

Maple [A] time = 0.119, size = 69, normalized size = 1.1

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)B}{bd}-2\,\frac{aB}{bd\sqrt{\left(a-b\right)\left(a+b\right)}}\arctan\left(\frac{\tan\left(1/2\,dx+c/2\right)\left(a-b\right)}{\sqrt{\left(a-b\right)\left(a+b\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))*B-2/d/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*a*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56616, size = 505, normalized size = 8.28

$$\left[-\frac{\sqrt{-a^2+b^2}Ba\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\left(Ba^2-Bb^2\right)dx}{2\left(a^2b-b^3\right)d},-\frac{\sqrt{a^2-b^2}Ba}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2 - B*b^2)*d*x
)/((a^2*b - b^3)*d), -(sqrt(a^2 - b^2)*B*a*arctan(-(a*cos(d*x + c) + b)/(sq
```

```
rt(a^2 - b^2)*sin(d*x + c))) - (B*a^2 - B*b^2)*d*x)/((a^2*b - b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28204, size = 130, normalized size = 2.13

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba}{\sqrt{a^2-b^2}\,b}-\frac{(dx+c)B}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a/(sqrt(a^2 - b^2)
*b) - (d*x + c)*B/b)/d
```

3.292 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=50

$$\frac{2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Rubi [A] time = 0.0353222, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {21, 2659, 205}

$$\frac{2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{1}{a+b\cos(c+dx)}\,dx\\ &= \frac{(2B)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\ &= \frac{2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.0362841, size = 49, normalized size = 0.98

$$-\frac{2B\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*B*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^
2]*d)
```

Maple [A] time = 0.102, size = 45, normalized size = 0.9

$$2\,\frac{B}{d\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan(1/2\,dx+c/2)(a-b)}{\sqrt{(a-b)(a+b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51127, size = 409, normalized size = 8.18

$$\left[-\frac{\sqrt{-a^2+b^2}B\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2-b^2\right)d}, \frac{B\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)}{\sqrt{a^2-b^2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*B*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), B*arctan(
-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

[Out] Timed out

Giac [A] time = 1.39676, size = 105, normalized size = 2.1

$$\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)B}{\sqrt{a^2-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B/(sqrt(a^2 - b^2)*d)
```

3.293 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2bB\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (-2*b*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
Sqrt[a + b]*d) + (B*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rubi [A] time = 0.0805626, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {21, 2747, 3770, 2659, 205}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2bB\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
Sqrt[a + b]*d) + (B*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx \\
&= \frac{B\int \sec(c+dx)\,dx}{a} - \frac{(bB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a} \\
&= \frac{B\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(2bB)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad} \\
&= -\frac{2bB\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d} + \frac{B\tanh^{-1}(\sin(c+dx))}{ad}
\end{aligned}
$$

Mathematica [A] time = 0.0765383, size = 103, normalized size = 1.47

$$
\frac{B\left(\frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]))/(a*d)
```

Maple [A] time = 0.119, size = 91, normalized size = 1.3

$$-\frac{B}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{B}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\,\frac{Bb}{da\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan(1/2\,dx+c/2)(a-b)}{\sqrt{(a-b)(a+b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a*ln(tan(1/2*d*x+1/2*c)-1)*B+1/d/a*ln(tan(1/2*d*x+1/2*c)+1)*B-2/d*b/a/
((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94822, size = 670, normalized size = 9.57

$$\left[-\frac{\sqrt{-a^2+b^2}Bb\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(Ba^2-Bb^2\right)\log(\sin(dx+c)+}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (B*a^2 - B*b^2)*log(s
in(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d
), -1/2*(2*sqrt(a^2 - b^2)*B*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2
)*sin(d*x + c))) - (B*a^2 - B*b^2)*log(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*
log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int \frac{\sec (c+dx)}{a+b\cos (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Giac [B] time = 1.41802, size = 165, normalized size = 2.36

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb}{\sqrt{a^2-b^2}a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}+\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b/(sqrt(a^2 - b^2)
*a) - B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + B*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a)/d
```

3.294 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=88

$$\frac{2b^2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

```
[Out] (2*b^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]*d) - (b*B*ArcTanh[Sin[c + d*x]])/(a^2*d) + (B*Tan[c + d*x])/
(a*d)
```

Rubi [A] time = 0.144806, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {21, 2802, 12, 2747, 3770, 2659, 205}

$$\frac{2b^2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]*d) - (b*B*ArcTanh[Sin[c + d*x]])/(a^2*d) + (B*Tan[c + d*x])/
(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
```

```
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{B\int \frac{b\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{(bB)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{(bB)\int \sec(c+dx)\,dx}{a^2} + \frac{(b^2B)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2}$$
$$= -\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{B\tan(c+dx)}{ad} + \frac{(2b^2B)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,\right.}{a^2d}$$
$$= \frac{2b^2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} - \frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{B\tan(c+dx)}{ad}$$

Mathematica [A] time = 0.351039, size = 116, normalized size = 1.32

$$\frac{B\left(-\frac{2b^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + a\tan(c+dx) + b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}\right.\right.\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
 + b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]]) + a*Tan[c + d*x]))/(a^2*d)
```

Maple [A] time = 0.138, size = 139, normalized size = 1.6

$$-\frac{B}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{Bb}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) - \frac{B}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{Bb}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + 2\,\frac{}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a/(tan(1/2*d*x+1/2*c)-1)*B+1/d*B/a^2*b*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a/
(tan(1/2*d*x+1/2*c)+1)*B-1/d*B/a^2*b*ln(tan(1/2*d*x+1/2*c)+1)+2/d*B/a^2*b^2
/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99455, size = 925, normalized size = 10.51

$$\left[-\frac{\sqrt{-a^2+b^2}Bb^2\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(Ba^2b-Bb^3\right)\mathrm{co}}{2\left(a^4-a^2b^2\right)d\,\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*b^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (B*a^2
*b - B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^2*b - B*b^3)*cos(d*x
+ c)*log(-sin(d*x + c) + 1) - 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2
*b^2)*d*cos(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*B*b^2*arctan(-(a*cos(d*x + c)
 + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^2*b - B*b^3)*cos(
d*x + c)*log(sin(d*x + c) + 1) + (B*a^2*b - B*b^3)*cos(d*x + c)*log(-sin(d*
```

```
x + c) + 1) + 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x
+ c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int\frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.5398, size = 209, normalized size = 2.38

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^2}{\sqrt{a^2-b^2}a^2}-\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^2/(sqrt(a^2 - b^2
)*a^2) - B*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + B*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*a))/d
```

3.295 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{2b^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{B\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{bB\tan(c+dx)}{a^2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] (-2*b^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a -
 b]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*B*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b
*B*Tan[c + d*x])/(a^2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.346015, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {21, 2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{B\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{bB\tan(c+dx)}{a^2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a -
 b]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*B*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b
*B*Tan[c + d*x])/(a^2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
```

```
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \frac{B\int \frac{\left(-2b+a\cos(c+dx)+b\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2a}$$
$$= -\frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \frac{B\int \frac{\left(a^2+2b^2+ab\cos(c+dx)\right)\sec(c\cdots}{a+b\cos(c+dx)}}{2a^2}$$
$$= -\frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(b^3B\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^3} + \frac{(\cdots}{}$$
$$= \frac{\left(a^2+2b^2\right)B\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+\cdots}{2ad}$$
$$= -\frac{2b^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{\left(a^2+2b^2\right)B\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{bB\,\mathrm{ta}\cdots}{}$$

Mathematica [A] time = 0.962602, size = 239, normalized size = 1.94

$$\frac{B\left(\frac{8b^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\cdots\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
+ b^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2
```

```
] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a
*b*Tan[c + d*x]))/(4*a^3*d)
```

Maple [B] time = 0.154, size = 273, normalized size = 2.2

$$\frac{B}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{B}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{Bb}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{B}{2\,da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2*B+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)*B+1/d/a^2
/(tan(1/2*d*x+1/2*c)-1)*B*b-1/2/d/a*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^3*ln(t
an(1/2*d*x+1/2*c)-1)*B*b^2-1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^2*B+1/2/d/a/(tan(
1/2*d*x+1/2*c)+1)*B+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B*b+1/2/d/a*ln(tan(1/2*d
*x+1/2*c)+1)*B+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B*b^2-2/d*b^3/a^3/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.77612, size = 1121, normalized size = 9.11

$$\left[-\frac{2\,\sqrt{-a^2+b^2}Bb^3\cos\left(dx+c\right)^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(Ba^4+Ba^2b^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a^2 + b^2)*B*b^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (B
*a^4 + B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a^4 +
 B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4 - B*
a^2*b^2 - 2*(B*a^3*b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2
)*d*cos(d*x + c)^2), -1/4*(4*sqrt(a^2 - b^2)*B*b^3*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (B*a^4 + B*a^2*b^2 -
2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a^4 + B*a^2*b^2 - 2*B*b^
4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4 - B*a^2*b^2 - 2*(B*a^3*
b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int\frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.5661, size = 298, normalized size = 2.42

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^3}{\sqrt{a^2-b^2}a^3}-\frac{(Ba^2+2Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{(Ba^2+2Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right.}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^3/(sqrt(a^2
- b^2)*a^3) - (B*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + (B
*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(B*a*tan(1/2*d*x
 + 1/2*c)^3 + 2*B*b*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1/2*d*x + 1/2*c) - 2*B
*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.296 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=386

$$-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^3d}+\frac{2\left(24a^2Ab-16a^3B-36ab^2B+75Ab^3\right)\sin(c+dx)\sqrt{}}{315b^3d}$$

```
[Out] (2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*Sqrt[a +
 b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(24*a^2*A*b + 75*A*b^3 - 16*a^
3*B - 36*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(24*a^2*A*b + 75
*A*b^3 - 16*a^3*B - 36*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315
*b^3*d) - (2*(36*a*A*b - 24*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(315*b^3*d) + (2*(3*A*b - 2*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*B*Cos[c + d*x]^2*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.785479, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2990, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^3d}+\frac{2\left(24a^2Ab-16a^3B-36ab^2B+75Ab^3\right)\sin(c+dx)\sqrt{}}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*Sqrt[a +
 b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(24*a^2*A*b + 75*A*b^3 - 16*a^
3*B - 36*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(24*a^2*A*b + 75
*A*b^3 - 16*a^3*B - 36*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315
*b^3*d) - (2*(36*a*A*b - 24*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(315*b^3*d) + (2*(3*A*b - 2*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*
```

```
x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*B*Cos[c + d*x]^2*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2B\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9bd} + \frac{2\int \cos(c$$

$$= \frac{2(3Ab-2aB)\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{21b^2d} +$$

$$= -\frac{2\left(36aAb-24a^2B-49b^2B\right)(a+b\cos(c+dx))^{3/2}\sin(c+d}{315b^3d}$$

$$= \frac{2\left(24a^2Ab+75Ab^3-16a^3B-36ab^2B\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{si}}{315b^3d}$$

$$= \frac{2\left(24a^2Ab+75Ab^3-16a^3B-36ab^2B\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{si}}{315b^3d}$$

$$= \frac{2\left(24a^2Ab+75Ab^3-16a^3B-36ab^2B\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{si}}{315b^3d}$$

$$= \frac{2\left(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+147b^4B\right)\sqrt{a+b\,\mathrm{c}}}{315b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

<u>Mathematica</u> [A] time = 1.47992, size = 292, normalized size = 0.76

$$\frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2\left(6a^2Ab-4a^3B+111ab^2B+75Ab^3\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(24a^3Ab-24a^2b^2B-16a^4B+57aAb^3+147\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(6*a^2*A*b + 75*A*b^3 - 4*a^3*B
+ 111*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (24*a^3*A*b + 57*a*A
*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d*x)/2,
 (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) - b*(a + b*Cos[
c + d*x])*(-2*(-48*a^2*A*b + 345*A*b^3 + 32*a^3*B + 57*a*b^2*B)*Sin[c + d*x
] - b*((36*a*A*b - 24*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*(9*A*b +
 a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^4*d*Sqrt[a + b*
Cos[c + d*x]])
```

Maple [B] time = 4.869, size = 1635, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^5+640*B*a*b^4+2240*B
*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-432*A*a*b^4-1080*A*b^5+8*B*
a^2*b^3-960*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-1
2*A*a^2*b^3+432*A*a*b^4+840*A*b^5+8*B*a^3*b^2-8*B*a^2*b^3+728*B*a*b^4+952*B
*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(24*A*a^3*b^2+6*A*a^2*b^3-258
*A*a*b^4-240*A*b^5-16*B*a^4*b-4*B*a^3*b^2-24*B*a^2*b^3-204*B*a*b^4-168*B*b^
5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-51*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))+24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^4*b-24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+
57*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-57*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+16*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+20*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-36*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^4-16*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^5+16*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^4*b-24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3
*b^2+24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+14
7*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
```

```
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^4/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^4+A\cos(dx+c)^3\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

3.297 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=303

$$-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(14*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + (2*(a^2 - b^2)*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt
[a + b*Cos[c + d*x]]) - (2*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*A*b - 4*a*B)*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.541572, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-2*(14*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + (2*(a^2 - b^2)*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt
[a + b*Cos[c + d*x]]) - (2*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*A*b - 4*a*B)*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(7*b*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd} + \frac{2\int\sqrt{a+b}\cdots}{\cdots}$$

$$= \frac{2(7Ab-4aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d} + \frac{2B\cos(c+\cdots}{\cdots}$$

$$= -\frac{2\left(14aAb-8a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} + \cdots$$

$$= -\frac{2\left(14aAb-8a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} + \cdots$$

$$= -\frac{2\left(14aAb-8a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} + \cdots$$

$$= -\frac{2\left(14a^2Ab-63Ab^3-8a^3B-19ab^2B\right)\sqrt{a+b\cos(c+dx)}E\cdots}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.946124, size = 232, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(\left(-16a^2B+28aAb+115b^2B\right)\sin(c+dx)+3b(2(aB+7Ab)\sin(2(c+dx))+5bB\sin(3(c+dx))\right)\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(49*a*A*b + 2*a^2*B + 25*b^2*B)*
EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B +
 19*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((28*a*A*b - 16*a^2*B
 + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + a*B)*Sin[2*(c + d*x)] + 5*b*B*
Sin[3*(c + d*x)])))/(210*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.021, size = 1305, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4-144*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(112*A*a*b^3+168*A*b^4-4*B*a^2*b
^2+144*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A*a^
2*b^2-56*A*a*b^3-42*A*b^4+8*B*a^3*b+2*B*a^2*b^2-86*B*a*b^3-80*B*b^4)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b-14*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^3-14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^3*b+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+
63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-8*B*(sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^4-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3
*b+19*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-19*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^3/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.298 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=231

$$-\frac{2\left(a^2-b^2\right)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-2a^2B+5aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a
^2 - b^2)*(5*A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b -
2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*B*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.407589, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-2a^2B+5aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a
^2 - b^2)*(5*A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b -
2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*B*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int \sqrt{a+b\cos(c+dx)}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$= \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int\sqrt{a+b\cos(c+dx)}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2B(a+b\cos(c$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2B(a+b\cos(c$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2B(a+b\cos(c$$

$$= \frac{2\left(5aAb-2a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.823999, size = 179, normalized size = 0.77

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(-2a^2B+5aAb+9b^2B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(7aB+5Ab)F\left(\frac{1}{2}(c+dx)|\frac{2}{a-}\right.}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 7*a*B)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (5*a*A*b - 2*a^2*B + 9*b^2*B)*((a + b)*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2*b
```

```
*(a + b*Cos[c + d*x])*(5*A*b + a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x])/(15*
b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.45, size = 993, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3+16*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-2*B*a^2*b-8*B*a
*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2*b-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^2+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-
2*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^3)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.299 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$-\frac{2B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin(c+dx)\sqrt{a+}}{3d}$$

```
[Out] (2*(3*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*B*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```

Rubi [A] time = 0.219655, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin(c+dx)\sqrt{a+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```



```
[Out] (2*(3*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*B*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{1}{2}(3Ab+aB)\,\mathrm{c}}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} - \frac{\left(\left(a^2-b^2\right)B\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b} + \cdots$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left((3Ab+aB)\sqrt{a+b\cos(c+dx)}\right)\int}{3b\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

$$= \frac{2(3Ab+aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(a^2-b^2\right)B\sqrt{\frac{a+b\,\mathrm{c}}{}}}{3bd\sqrt{a+}}$$

Mathematica [A] time = 0.575338, size = 146, normalized size = 0.85

$$\frac{-2B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(a+b)(aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2bB\sin(c+dx)(a}{3bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b*Cos[c + d*x])*Sin[c +
 d*x])/(3*b*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.319, size = 600, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
```

```
c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*B*cos(1/2*d*x+1/
2*c)^3*a*b-6*B*cos(1/2*d*x+1/2*c)^3*b^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2+B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2
*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*B*cos(1/2*d*x+1/2*c)*a*b+2*B*cos(1/2*d*x+1
/2*c)*b^2)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

3.300 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=178

$$\frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.360206, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3002, 2655, 2653, 2803, 2663, 2661, 2807, 2805}

$$\frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = A\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx + B\int \sqrt{a+b\cos(c+$$

$$= (aA)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + (Ab)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,d$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)}{\sqrt{a+b\,\mathrm{co}}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F}{d\sqrt{a+b\cos}}$$

Mathematica [A] time = 2.37566, size = 107, normalized size = 0.6

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(A\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+B(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + A*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2,
(c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 4.004, size = 247, normalized size = 1.4

$$-2\,\frac{\sqrt{\left(2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+bd}}\sqrt{\frac{2\,b\,(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-a*A*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos (dx+c)+A)\sqrt {b\cos (dx+c)+a}\sec (dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\cos (c+dx))\sqrt {a+b\cos (c+dx)}\sec (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

3.301 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=213

$$\frac{(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos}}{d}$$

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a*A + 2*b*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + ((A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.606732, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2999, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```



```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a*A + 2*b*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + ((A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/d
```


Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
```

```
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{1}{2}(Ab+2aB)+bB\right.}{}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}A\int \sqrt{a+b\cos(c+}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}(-Ab-2aB)\int \frac{\text{s}}{\sqrt{a+}}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b\cos(c-}}{}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(aA+2bB)\sqrt{\frac{a+}{}}}{d\sqrt{a}}$$

Mathematica [C] time = 10.2451, size = 372, normalized size = 1.75

$$\frac{2(4aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4A\tan(c+dx)\sqrt{a+b\cos(c+dx)}-\frac{2iA\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\,s\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*
x]] - ((2*I)*A*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c
 + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 6.549, size = 746, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-2*(A*b+B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))+2*a*A*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
```

```
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^
2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.302 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=292

$$\frac{\left(4a^2A+4abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{(4aB+3Ab)\sqrt{}}{4d}$$

```
[Out] -((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*A*b + 4*a*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt
[a + b*Cos[c + d*x]]) + ((4*a^2*A - A*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*C
os[c + d*x]]) + ((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*
d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.954792, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2999, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2A+4abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{(4aB+3Ab)\sqrt{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*A*b + 4*a*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt
[a + b*Cos[c + d*x]]) + ((4*a^2*A - A*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*C
os[c + d*x]]) + ((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*
d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{\left(\frac{1}{2}(Ab\right.}{}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{A\sqrt{a+b\cos(c}{}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{A\sqrt{a+b\cos(c}{}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{A\sqrt{a+b\cos(c}{}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(Ab+4}{}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(3Ab+}{}$$

Mathematica [C] time = 4.12581, size = 420, normalized size = 1.44

$$\frac{\frac{2\left(8a^2A+4abB-3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}} - \frac{2i(4aB+Ab)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right.\right.\right.\right.}{a^2 \cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A - 3*A*b^2 + 4*a*b*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*Sq
rt[a + b*Cos[c + d*x]]) - ((2*I)*(A*b + 4*a*B)*Sqrt[-((b*(-1 + Cos[c + d*x]
))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a^2*b*Sqrt[-(a
```

```
+ b)^(-1)]) + (4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (A*b + 4*a*B)*Cos[c + d*
x])*Sec[c + d*x]*Tan[c + d*x])/a)/(16*d)
```

Maple [B] time = 7.812, size = 1290, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(A*b+B*a)*(-cos(1/2*d*x+1/2*c)/a*(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(
-2*b/(a-b))^(1/2)))+2*a*A*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2
*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
```

```
*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.303 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=378

$$\frac{\left(16a^2A+6abB-3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^2d}+\frac{\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\cos(c+dx)}}-\frac{(16\ldots}{}$$

```
[Out] -((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*
a^2*A - A*b^2 + 18*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A*b +
A*b^3 + 8*a^3*B - 2*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a
^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^2*d)
 + ((A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a
*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.33862, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2999, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(16a^2A+6abB-3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^2d}+\frac{\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\cos(c+dx)}}-\frac{(16\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*
a^2*A - A*b^2 + 18*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A*b +
A*b^3 + 8*a^3*B - 2*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a
^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^2*d)
 + ((A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a
*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \frac{\left(\frac{1}{2}(A\right.}{}$$

$$= \frac{(Ab+6aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12ad}+\frac{A}{}$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^2d}+$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^2d}+$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^2d}+$$

$$= -\frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{}{a}\right.}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{}{a}\right.}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.50926, size = 635, normalized size = 1.68

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+6abB\sin(c+dx)-3Ab^2\sin(c+dx)\right)}{24a^2}+\frac{\sec^2(c+dx)(6aB\sin(c+dx)+Ab\sin(c+dx))}{12a}+\frac{1}{3}A\tan(c+d\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*(4*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A*b + 9*A*
b^3 + 48*a^3*B - 18*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*a^2*
A*b + 3*A*b^3 - 6*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b
```

```
*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[
Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*El
lipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt
[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*
Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*C
os[c + d*x])^2)))/(96*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(
A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/(12*a) + (Sec[c + d*x]*(16*a^2*A*Si
n[c + d*x] - 3*A*b^2*Sin[c + d*x] + 6*a*b*B*Sin[c + d*x]))/(24*a^2) + (A*Se
c[c + d*x]^2*Tan[c + d*x])/3))/d
```

Maple [B] time = 11.474, size = 2213, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*A*(-1/3
*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*
cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-5/16/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
```

```
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^
3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*B*b*(-cos(1/2*d*x+1/2*c)/a*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2)))+2*(A*b+B*a)*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8/a^2*b^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
```

```
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

3.304 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=378

$$-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\left(18a^2Ab-8a^3B-39ab^2B-75Ab^3\right)\sin(c+dx)\sqrt{a}}{315b^2d}$$

```
[Out] (-2*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B - 147*b^4*B)*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*A*b - 75*A*b^3 - 8*a^
3*B - 39*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(18*a^2*A*b - 75
*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*
b^2*d) - (2*(18*a*A*b - 8*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(315*b^2*d) + (2*(9*A*b - 4*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(63*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(9*b*d)
```

Rubi [A] time = 0.733061, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\left(18a^2Ab-8a^3B-39ab^2B-75Ab^3\right)\sin(c+dx)\sqrt{a}}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B - 147*b^4*B)*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*A*b - 75*A*b^3 - 8*a^
3*B - 39*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(18*a^2*A*b - 75
*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*
b^2*d) - (2*(18*a*A*b - 8*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(315*b^2*d) + (2*(9*A*b - 4*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
```

```
 + d*x])/(63*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(9*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9bd} + \frac{2\int(a+$$

$$= \frac{2(9Ab-4aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63b^2d} + \frac{2B\cos(c$$

$$= -\frac{2\left(18aAb-8a^2B-49b^2B\right)(a+b\cos(c+dx))^{3/2}\sin(c+d}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315b^2d}$$

$$= -\frac{2\left(18a^3Ab-246aAb^3-8a^4B-33a^2b^2B-147b^4B\right)\sqrt{a+}}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.44465, size = 291, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(\left(72a^2Ab-32a^3B+804ab^2B+690Ab^3\right)\sin(c+dx)+b\left(2\left(6a^2B+144aAb+133b^2B\right)\sin(2(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(153*a^2*A*b + 75*A*b^3 + 2*a^3*
B + 186*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-18*a^3*A*b + 246
*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d*x)
/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*C
os[c + d*x])*((72*a^2*A*b + 690*A*b^3 - 32*a^3*B + 804*a*b^2*B)*Sin[c + d*x
] + b*(2*(144*a*A*b + 6*a^2*B + 133*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*(9*A*b
 + 10*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^3*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Maple [B] time = 4.509, size = 1635, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^5+1360*B*a*b^4+2240*
B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-936*A*a*b^4-1080*A*b^5-424
*B*a^2*b^3-2040*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
+(324*A*a^2*b^3+936*A*a*b^4+840*A*b^5-4*B*a^3*b^2+424*B*a^2*b^3+1568*B*a*b^
4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A*a^3*b^2-162*A*a
^2*b^3-384*A*a*b^4-240*A*b^5+8*B*a^4*b+2*B*a^3*b^2-282*B*a^2*b^3-444*B*a*b^
4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+18*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-93*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^4*b+18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3*b^2+246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*
b^3-246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-8*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*a*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^5-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^4*b+33*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3*b^2-33*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2*b^3+147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
```

```
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-1
47*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^3/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos(dx+c)^4+Aa\cos(dx+c)^2+(Ba+Ab)\cos(dx+c)^3\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^4 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c
)^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

3.305 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=297

$$\frac{2\left(-6a^2B+21aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(21*a^2*A*b + 63*A*b^3 - 6*a^3*B + 82*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) - (2*(a^2 - b^2)*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(35*b*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(7*b*d)
```

Rubi [A] time = 0.527524, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-6a^2B+21aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*(21*a^2*A*b + 63*A*b^3 - 6*a^3*B + 82*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) - (2*(a^2 - b^2)*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(35*b*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(7*b*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^{3/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)\,dx$$

$$= \frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2\int(a+b\cos(c+dx)}{}$$

$$= \frac{2(7Ab-2aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd} + \frac{2B(a+b}{}$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} +$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} +$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} +$$

$$= \frac{2\left(21a^2Ab+63Ab^3-6a^3B+82ab^2B\right)\sqrt{a+b\cos(c+dx)}E}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.00906, size = 233, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(\left(12a^2B+168aAb+115b^2B\right)\sin(c+dx)+3b(2(8aB+7Ab)\sin(2(c+dx))+5bB\sin(3(c+dx)))\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(84*a*A*b + 51*a^2*B + 25*b^2*B)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (21*a^2*A*b + 63*A*b^3 - 6*a^3*B +
 82*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((168*a*A*b + 12*a^2*
B + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + 8*a*B)*Sin[2*(c + d*x)] + 5*b
*B*Sin[3*(c + d*x)])))/(210*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.073, size = 1305, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4-312*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(252*A*a*b^3+168*A*b^4+108*B*a^2
*b^2+312*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-84*A*
a^2*b^2-126*A*a*b^3-42*A*b^4-6*B*a^3*b-54*B*a^2*b^2-128*B*a*b^3-80*B*b^4)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3*b+21*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^3+21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3*b-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2
*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+6*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
```

```
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3*b+82*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2
-82*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^2/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^3+Aa\cos(dx+c)+(Ba+Ab)\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.306 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=225

$$-\frac{2\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b + 3*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.352248, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b + 3*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}(5aA\right.$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2B(a+b\cos(c+dx))^{3/2}}{5d}$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2B(a+b\cos(c+dx))^{3/2}}{5d}$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2B(a+b\cos(c+dx))^{3/2}}{5d}$$

$$= \frac{2\left(20aAb+3a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2\right.}{}$$

Mathematica [A] time = 0.733757, size = 203, normalized size = 0.9

$$\frac{2\left(b\left(15a^2A+12abB+5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(3a^2B+20aAb+9b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+d\right.\right.\right.}{15bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*(15*a^2*A + 5*A*b^2 + 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)] + (20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
 - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]) + b*(a + b*Cos[c + d*x])*(5*A*b
 + 6*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Sqrt[a + b*Cos[c + d*
x]])
```

Maple [B] time = 4.003, size = 993, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3+36*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-12*B*a^2*b-18*B
*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b-20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3+3*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.307 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{2\left(a^2(-B)+3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(4aB+3Ab)\sqrt{a}}{}$$

```
[Out] (2*(3*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a*A*b - a^2*B + b^
2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.707682, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2990, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(a^2(-B)+3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(4aB+3Ab)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] (2*(3*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a*A*b - a^2*B + b^
2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
```

```
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\left(\frac{3a^2A}{2}+\frac{1}{2}\left(6aAb\right.\right.}{}$$

$$= \frac{2bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2\int \frac{\left(-\frac{3}{2}a^2Ab-\frac{1}{2}b\left(3aAb-a^2\right.\right.}{\sqrt{a+b}}}{}$$

$$= \frac{2bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \left(a^2A\right)\int \frac{\sec(c+}{\sqrt{a+b\cos(c}}$$

$$= \frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2bB\sqrt{}}{}$$

$$= \frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(3aA\right.}{}$$

Mathematica [C] time = 2.44486, size = 406, normalized size = 1.72

$$\frac{\frac{4\left(3a^2B+6aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(6a^2A+4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i(4aB+3Ab)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^2*A + 3*A
*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b + 4*a*B)*Sqrt[-
((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Cs
c[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(6*d)
```

Maple [B] time = 4.046, size = 738, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-3*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+2*B*cos(1/2*d*x+1/2*c)^3*a*b-6*B*cos(1/2*d*x+1/2*c)^3
*b^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+B*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2-4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-
2*B*cos(1/2*d*x+1/2*c)*a*b+2*B*cos(1/2*d*x+1/2*c)*b^2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.308 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=232

$$\frac{\left(a^2A+2abB+2Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(2aB+3A}{}$$

```
[Out] -(((a*A - 2*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2*A + 2*A*b^2 + 2*a*b*B
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.685552, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2989, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2A+2abB+2Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(2aB+3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```



```
[Out] -(((a*A - 2*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2*A + 2*A*b^2 + 2*a*b*B
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
```

```
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{1}{2}a(3Ab+2aB)+\right.}{}$$

$$= \frac{aA\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{\int \frac{\left(-\frac{1}{2}ab(3Ab+2aB)-\frac{1}{2}b\left(a^2\right.\right.}{\sqrt{}}}{}$$

$$= \frac{aA\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}(a(3Ab+2aB))\int$$

$$= -\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{aA\sqrt{a}}{}$$

$$= -\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(a^2A\right.}{}$$

Mathematica [C] time = 2.44529, size = 398, normalized size = 1.72

$$\frac{\frac{2\left(4a^2B+5aAb+2b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{8b(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i(2bB-aA)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)}{b-a}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*(A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(5*a*A*b + 4*a^2*B + 2*b^2
*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-(a*A) + 2*b*B)*Sqrt[-((b*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*
(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sq
rt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*
Sqrt[-(a + b)^(-1)]) + 4*a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 4.417, size = 1167, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*a*b*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a^2-2*A*a*b)*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2+2*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-A*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+A*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b-3*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*a*b+2*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+2*B*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*B*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))*a^2)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b+2*B*a*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*a^2)/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

3.309 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=295

$$\frac{(4a^2B+7aAb+8b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{(4a^2A+12abB+3Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+$$

```
[Out] -((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((7*a*A*b + 4*a^2*B + 8*b
^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 + 12*a*b*B)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4
*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*d) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rubi [A] time = 1.05807, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2989, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2B+7aAb+8b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{(4a^2A+12abB+3Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```


```
[Out] -((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((7*a*A*b + 4*a^2*B + 8*b
^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 + 12*a*b*B)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4
*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*d) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \frac{\left(\frac{1}{2}\right.}{}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+\frac{aA\sqrt{a+b}}{}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+\frac{aA\sqrt{a+b}}{}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+\frac{aA\sqrt{a+b}}{}$$

$$= -\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(5A}{}$$

$$= -\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(7a}{}$$

Mathematica [C] time = 4.79466, size = 422, normalized size = 1.43

$$\frac{\frac{2(8a^2A+20abB+Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{8b(aA+4bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(a*A + 4*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + A*b^2 + 20*a*b*
B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(5*A*b + 4*a*B)*Sqrt[-((b*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-
2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sq
```

```
rt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (5*A*b + 4*a*B)*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Maple [B] time = 9.067, size = 1403, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*b*(A*b+2*B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))+2*a*(2*A*b+B*a)*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+
2*a^2*A*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2
*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2
*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
```

```
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

3.310 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=375

$$\frac{\left(16a^2A+30abB+3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((16*a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a
^2*A + 17*A*b^2 + 42*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*A*b -
 A*b^3 + 8*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a^
2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) +
 ((7*A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d
) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.44994, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2989, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(16a^2A+30abB+3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] -((16*a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a
^2*A + 17*A*b^2 + 42*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*A*b -
 A*b^3 + 8*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a^
2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) +
 ((7*A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d
) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int \frac{\left(\frac{1}{2}a\right.}{}$$

$$= \frac{(7Ab+6aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d} +$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24ad}$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24ad}$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24ad}$$

$$= -\frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.59094, size = 634, normalized size = 1.69

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+30abB\sin(c+dx)+3Ab^2\sin(c+dx)\right)}{24a}+\frac{1}{12}\sec^2(c+dx)(6aB\sin(c+dx)+7Ab\sin(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*(28*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(56*a^2*A*b - 9*
A*b^3 + 48*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*a^2
```

```
*A*b - 3*A*b^3 - 30*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b +
 b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*
EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/
(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sq
rt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a +
b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b
*Cos[c + d*x])^2)))/(96*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(
7*A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*(16*a^2*A*Sin[
c + d*x] + 3*A*b^2*Sin[c + d*x] + 30*a*b*B*Sin[c + d*x]))/(24*a) + (a*A*Sec
[c + d*x]^2*Tan[c + d*x])/3))/d
```

Maple [B] time = 12.028, size = 2327, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*b*(A*b+2*B*a)*(-cos(1/2*d*x+1/2*c)/a*(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2)))+2*a^2*A*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3
+5/12/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(
1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
```

```
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+
1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16/a^2*b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2)))+2*a*(2*A*b+B*a)*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b
*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+
3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b
+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x
)
```

3.311 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=462

$$-\frac{2\left(-8a^2B+22aAb-81b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\left(110a^2Ab-40a^3B-335ab^2B-539Ab^3\right)\sin(c+dx}{3465b^2d}$$

```
[Out] (-2*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*B - 255*a^3*b^2*B -
 3705*a*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(110*a
^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a
^2*b^2*B - 675*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) -
 (2*(110*a^2*A*b - 539*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) - (2*(22*a*A*b - 8*a^2*B - 81*b^2*B)*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*A*b - 4*a*B)*(a +
 b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*B*Cos[c + d*x]*(a + b*
Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rubi [A] time = 0.931401, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2B+22aAb-81b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\left(110a^2Ab-40a^3B-335ab^2B-539Ab^3\right)\sin(c+dx}{3465b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-2*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*B - 255*a^3*b^2*B -
 3705*a*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(110*a
^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a
```

```
^2*b^2*B - 675*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) -
 (2*(110*a^2*A*b - 539*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) - (2*(22*a*A*b - 8*a^2*B - 81*b^2*B)*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*A*b - 4*a*B)*(a +
 b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*B*Cos[c + d*x]*(a + b*
Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{11bd} + \frac{2\int(a+\cdots}{\cdots}$$

$$= \frac{2(11Ab-4aB)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{99b^2d} + \frac{2B\cos(\cdots}{\cdots}$$

$$= -\frac{2\left(22aAb-8a^2B-81b^2B\right)(a+b\cos(c+dx))^{5/2}\sin(c+\cdots}{693b^2d}$$

$$= -\frac{2\left(110a^2Ab-539Ab^3-40a^3B-335ab^2B\right)(a+b\cos(c+\cdots}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-675b^4B\right)\cdots}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-675b^4B\right)\cdots}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-675b^4B\right)\cdots}{3465b^2d}$$

$$= -\frac{2\left(110a^4Ab-3069a^2Ab^3-1617Ab^5-40a^5B-255a^3b^2\cdots\right.}{3465b^3d\sqrt{\frac{a+b}{\cdots}}}$$

Mathematica [A] time = 1.98594, size = 357, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(\left(880a^3Ab+18660a^2b^2B-320a^4B+32868aAb^3+13050b^4B\right)\sin(c+dx)+b\left(4\left(1650a^2Ab+30a^3\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(1705*a^3*A*b + 2871*a*A*b^3 +
10*a^4*B + 3315*a^2*b^2*B + 675*b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B
+ 3705*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((880*a^3*A*b + 32
868*a*A*b^3 - 320*a^4*B + 18660*a^2*b^2*B + 13050*b^4*B)*Sin[c + d*x] + b*(
```

```
4*(1650*a^2*A*b + 1463*A*b^3 + 30*a^3*B + 3095*a*b^2*B)*Sin[2*(c + d*x)] +
5*b*((836*a*A*b + 452*a^2*B + 513*b^2*B)*Sin[3*(c + d*x)] + 7*b*((22*A*b +
46*a*B)*Sin[4*(c + d*x)] + 9*b*B*Sin[5*(c + d*x)]))))))/(27720*b^3*d*Sqrt[a
+ b*Cos[c + d*x]])
```

Maple [B] time = 4.483, size = 1983, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3465*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-245*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2+20160*B*b^6
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+40*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^6-40*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^6-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*b^6+675*B*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+(-12320*A*b^6-35840*B*a*b^5-50400*B*b^6)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x
+1/2*c)+(22880*A*a*b^5+24640*A*b^6+21920*B*a^2*b^4+71680*B*a*b^5+56880*B*b^
6)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-14960*A*a^2*b^4-34320*A*a*b^5-
22792*A*b^6-4640*B*a^3*b^3-32880*B*a^2*b^4-66160*B*a*b^5-34920*B*b^6)*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(3520*A*a^3*b^3+14960*A*a^2*b^4+26488*A*
a*b^5+10472*A*b^6-20*B*a^4*b^2+4640*B*a^3*b^3+25120*B*a^2*b^4+30320*B*a*b^5
+13860*B*b^6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-110*A*a^4*b^2-1760*
A*a^3*b^3-7326*A*a^2*b^4-7524*A*a*b^5-1848*A*b^6+40*B*a^5*b+10*B*a^4*b^2-32
10*B*a^3*b^3-7080*B*a^2*b^4-6690*B*a*b^5-2790*B*b^6)*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+1254*A*a*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))+110*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4
*b^2-3069*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4+
1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5-390*a^2
*b^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
```

```
(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+255*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2+3069*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3-110*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b-1364*A*a^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-255*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3*b^3+110*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a^5*b+3705*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^2*b^4-40*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^5*b-3705*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
^5)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^5 + Aa^2\cos(dx+c)^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^4 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)^3\right)\sqrt{b\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^5 + A*a^2*cos(d*x + c)^2 + (2*B*a*b + A*b^2)*c
os(d*x + c)^4 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

3.312 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=372

$$\frac{2\left(-10a^2B+45aAb+49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\left(45a^2Ab-10a^3B+114ab^2B+75Ab^3\right)\sin(c+dx)\sqrt{a}}{315bd}$$

```
[Out] (2*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*A*b + 75*A*b^3 - 10*
a^3*B + 114*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(45*a^2*A*b +
 75*A*b^3 - 10*a^3*B + 114*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(315*b*d) + (2*(45*a*A*b - 10*a^2*B + 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(315*b*d) + (2*(9*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin
[c + d*x])/(63*b*d) + (2*B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.796367, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-10a^2B+45aAb+49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\left(45a^2Ab-10a^3B+114ab^2B+75Ab^3\right)\sin(c+dx)\sqrt{a}}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*A*b + 75*A*b^3 - 10*
a^3*B + 114*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(45*a^2*A*b +
 75*A*b^3 - 10*a^3*B + 114*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(315*b*d) + (2*(45*a*A*b - 10*a^2*B + 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(315*b*d) + (2*(9*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin
[c + d*x])/(63*b*d) + (2*B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^{5/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$= \frac{2B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{2\int (a+b\cos(c+dx)}{}$$

$$= \frac{2(9Ab-2aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd} + \frac{2B(a+b}{}$$

$$= \frac{2\left(45aAb-10a^2B+49b^2B\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315bd}$$

$$= \frac{2\left(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B+147b^4B\right)\sqrt{a+}}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.48195, size = 291, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(2\left(540a^2Ab+20a^3B+747ab^2B+345Ab^3\right)\sin(c+dx)+b\left(\left(300a^2B+540aAb+266b^2B\right)\sin(2(\cdots\right.\right.}{\quad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(405*a^2*A*b + 75*A*b^3 + 155*a^
3*B + 261*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (45*a^3*A*b + 43
5*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d
*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a +
b*Cos[c + d*x])*(2*(540*a^2*A*b + 345*A*b^3 + 20*a^3*B + 747*a*b^2*B)*Sin[c
 + d*x] + b*((540*a*A*b + 300*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*
(9*A*b + 19*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^2*d*
Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.468, size = 1635, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^5+2080*B*a*b^4+2240*
B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1440*A*a*b^4-1080*A*b^5-13
60*B*a^2*b^3-3120*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1080*A*a^2*b^3+1440*A*a*b^4+840*A*b^5+320*B*a^3*b^2+1360*B*a^2*b^3+2408
*B*a*b^4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-270*A*a^3*b^2
-540*A*a^2*b^3-510*A*a*b^4-240*A*b^5-10*B*a^4*b-160*B*a^3*b^2-666*B*a^2*b^3
-684*B*a*b^4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-45*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-30*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
```

```
),(-2*b/(a-b))^(1/2))*a^4*b-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^3*b^2+435*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2*b^3-435*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a*b^4+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-124*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+114*a*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-10*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+10*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+279*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^3*b^2-279*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b^3+147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a*b^4-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^5)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^4 + Aa^2\cos(dx+c) + \left(2\,Bab + Ab^2\right)\cos(dx+c)^3 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)^2\right)\sqrt{b\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.313 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=288

$$\frac{2\left(15a^2B+56aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A*b + 5*a*B)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*d)
```

Rubi [A] time = 0.5163, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(15a^2B+56aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A*b + 5*a*B)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*d)
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{7}\int (a+b\cos(c+dx))^{3/2}\left(\frac{1}{2}(7a\right.$$

$$= \frac{2(7Ab+5aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+b\cos(c+dx))}{7d}$$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2(7Ab+}{}$$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2(7Ab+}{}$$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2(7Ab+}{}$$

$$= \frac{2\left(161a^2Ab+63Ab^3+15a^3B+145ab^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{105bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.01701, size = 254, normalized size = 0.88

$$\frac{b\sin(c+dx)(a+b\cos(c+dx))\left(90a^2B+6b(15aB+7Ab)\cos(c+dx)+154aAb+15b^2B\cos(2(c+dx))+65b^2B\right)+2b\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*b*(105*a^3*A + 119*a*A*b^2 + 135*a^2*b*B + 25*b^3*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*(161*a^2*A*b + 63
*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b
)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)]) + b*(a + b*Cos[c + d*x])*(154*a*A*b + 90*a^2*B + 65*b^2*B + 6*b*(7*
A*b + 15*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*
b*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.409, size = 1305, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4-480*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(392*A*a*b^3+168*A*b^4+360*B*a^2
*b^2+480*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-154*A
*a^2*b^2-196*A*a*b^3-42*A*b^4-90*B*a^3*b-180*B*a^2*b^2-170*B*a*b^3-80*B*b^4
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b-161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a*b^3-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*b^4-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+5
6*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+15*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-15*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+145*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-145*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a*b^3-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^4-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
))/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.314 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=292

$$\frac{2\left(10a^2Ab-8a^3B+8ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2B+35aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(10*
a^2*A*b + 5*A*b^3 - 8*a^3*B + 8*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*a^3*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)
/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rubi [A] time = 1.01367, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2990, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(10a^2Ab-8a^3B+8ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2B+35aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(10*
a^2*A*b + 5*A*b^3 - 8*a^3*B + 8*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*a^3*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)
/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\cos(c+}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bB(a+b\,c}{}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bB(a+b\,c}{}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bB(a+b\,c}{}$$

$$= \frac{2\left(35aAb+23a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(35aAb+23a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.80067, size = 453, normalized size = 1.55

$$\frac{\frac{4\left(45a^2Ab+15a^3B+17ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(30a^3A+23a^2bB+35aAb^2+9b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(23a^2B+35aAb\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*(45*a^2*A*b + 5*A*b^3 + 15*a^3*B + 17*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
+ (2*(30*a^3*A + 35*a*A*b^2 + 23*a^2*b*B + 9*b^3*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d
*x]] + ((2*I)*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[-((b*(-1 + Cos[c + d*x])
)/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
```

```
 b)^(-1)]) + 4*b*Sqrt[a + b*Cos[c + d*x]]*(5*A*b + 11*a*B + 3*b*B*Cos[c + d
*x])*Sin[c + d*x])/(30*d)
```

Maple [B] time = 4.432, size = 1067, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3+56*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-22*B*a^2*b-28*B
*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b-35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a*b^2-15*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+8*a
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+23*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b^3)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.315 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=296

$$\frac{\left(3a^3A+4a^2bB+12aAb^2+2b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{\left(3a^2A-14abB-6Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((3*a^2*A - 6*A*b^2 - 14*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^3*A
+ 12*a*A*b^2 + 4*a^2*b*B + 2*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*(5
*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*a*A - 2*b*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Tan[
c + d*x])/d
```

Rubi [A] time = 1.10906, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2989, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(3a^3A+4a^2bB+12aAb^2+2b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{\left(3a^2A-14abB-6Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```


```
[Out] -((3*a^2*A - 6*A*b^2 - 14*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^3*A
+ 12*a*A*b^2 + 4*a^2*b*B + 2*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*(5
*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*a*A - 2*b*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Tan[
c + d*x])/d
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \int \sqrt{a+b\cos(c+\ldots}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{aA(a+\ldots}{}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{aA(a+\ldots}{}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{aA(a+\ldots}{}$$

$$= -\frac{\left(3a^2A-6Ab^2-14abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\ldots\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(3a^2A-6Ab^2-14abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\ldots\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.79629, size = 442, normalized size = 1.49

$$\frac{4\tan(c+dx)\sqrt{a+b\cos(c+dx)}\left(3a^2A+2b^2B\cos(c+dx)\right) + \frac{8b\left(9a^2B+9aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(27a^2Ab+12a^3B\ldots\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*(9*a*A*b + 9*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(27*a^2*A*b
+ 6*A*b^3 + 12*a^3*B + 14*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-3
*a^2*A + 6*A*b^2 + 14*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[
(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-
2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
 + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt
```

```
[a + b*Cos[c + d*x]]*(3*a^2*A + 2*b^2*B*Cos[c + d*x])*Tan[c + d*x])/(12*d)
```

Maple [B] time = 4.425, size = 1563, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -1/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*A*a^2*b+8*B*a*b^2+16*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^3-6*A*a^2*b-4*B*a*b^2-4*B*b^3
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+12*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a*b^2-3*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+
3*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+6*A*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*A*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3-15*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2))*a^2*b+4*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+2*B
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+14*B*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-14*B*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a*b^2-6*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*a^3)*sin(1/2*d*x+1/2*c)^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^3+12*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+3*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+6*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+4*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+2*B*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
```

```
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
2*b-14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

3.316 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=315

$$\frac{\left(11a^2Ab+4a^3B+16ab^2B+8Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{\left(4a^2B+9aAb-8b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((9*a*A*b + 4*a^2*B - 8*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((11*a^2*A*
b + 8*A*b^3 + 4*a^3*B + 16*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(4*a
^2*A + 15*A*b^2 + 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(7*A*b +
 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a + b*Cos[c +
d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 1.05529, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2989, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(11a^2Ab+4a^3B+16ab^2B+8Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{\left(4a^2B+9aAb-8b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -((9*a*A*b + 4*a^2*B - 8*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((11*a^2*A*
b + 8*A*b^3 + 4*a^3*B + 16*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(4*a
^2*A + 15*A*b^2 + 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(7*A*b +
 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a + b*Cos[c +
d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \sqrt{a}$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{aA(a+b\,c}{}$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{aA(a+b\,c}{}$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{aA(a+b\,c}{}$$

$$= -\frac{\left(9aAb+4a^2B-8b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(9aAb+4a^2B-8b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 5.63626, size = 451, normalized size = 1.43

$$\frac{\frac{8b\left(a^2A+12abB+4Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(8a^3A+36a^2bB+21aAb^2+8b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(-4a^2B-9aAb+8b^2B\right)\csc(c+}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(a^2*A + 4*A*b^2 + 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^3*A +
21*a*A*b^2 + 36*a^2*b*B + 8*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-9
*a*A*b - 4*a^2*B + 8*b^2*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*a*Sqr
```

```
t[a + b*Cos[c + d*x]]*(2*a*A + (9*A*b + 4*a*B)*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x])/(16*d)
```

Maple [B] time = 9.44, size = 1742, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*b^2*(a
-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)
^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*B*a*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-2*B*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-6*a*b*(A
*b+B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a^2*(3*A*b+B*a)*(-cos(1/2*d*
x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A*a^3*(-1/2*cos(1/2*d*x+1/2*c)/a*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
```

```
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8/a^2*b^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*c
os(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*
sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

3.317 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=376

$$\frac{\left(16a^2A+54abB+33Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c-\right.}{24d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a^
3*A + 59*a*A*b^2 + 66*a^2*b*B + 48*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) +
((20*a^2*A*b + 5*A*b^3 + 8*a^3*B + 30*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d
*x]]) + ((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c +
d*x])/(24*d) + (a*(3*A*b + 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan
[c + d*x])/(4*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rubi [A] time = 1.43311, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2989, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(16a^2A+54abB+33Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c-\right.}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] -((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a^
3*A + 59*a*A*b^2 + 66*a^2*b*B + 48*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) +
((20*a^2*A*b + 5*A*b^3 + 8*a^3*B + 30*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d
*x]]) + ((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c +
d*x])/(24*d) + (a*(3*A*b + 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan
```

```
[c + d*x])/(4*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

qQ[a, 0])))

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int \sqrt{}$$

$$= \frac{a(3Ab+2aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{4d} +$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d}$$

$$= -\frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 5.80832, size = 486, normalized size = 1.29

$$\frac{\frac{8b\left(6a^2B+13aAb+24b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(104a^2Ab+48a^3B+126ab^2B-3Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\sec^2(c+dx)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((8*b*(13*a*A*b + 6*a^2*B + 24*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(104*a^2
*A*b - 3*A*b^3 + 48*a^3*B + 126*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))
]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*Elliptic
E[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
```

```
 + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) +
 4*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*a*(13*A*b + 6*a*B)*Sin[c + d*
x] + (8*a^2*A + (33*A*b^2)/2 + 27*a*b*B)*Sin[2*(c + d*x)] + 8*a^2*A*Tan[c +
 d*x]))/(96*d)
```

Maple [B] time = 12.175, size = 2438, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*b^2*(A*b+3*B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))+6*a*b*(A*b+B*a)*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*c
os(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
)+2*A*a^3*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12/a^2*b*cos(1/2*d*x+
1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2
```

```
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-5/16/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*
d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^
2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a^2*(3*A*b+B*a
)*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*
d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*
cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)
)*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x
)
```

3.318 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=465

$$\frac{(284a^2Ab+128a^3B+264ab^2B+15Ab^3)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{(356a^2Ab+128a^3B+472ab^2B+133Ab^3)\sqrt{\ldots}}{192d\sqrt{a+b\cos(c+\ldots}}$$

```
[Out] -((284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + ((356*a^2*A*b + 133*A*b^3 + 128*a^3*B + 472*a*b^2*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*Sq
rt[a + b*Cos[c + d*x]]) + ((48*a^4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*b*
B + 40*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((284*a^2*A*b + 15*A
*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192
*a*d) + ((36*a^2*A + 59*A*b^2 + 104*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]*Tan[c + d*x])/(96*d) + (a*(11*A*b + 8*a*B)*Sqrt[a + b*Cos[c + d*x]]*S
ec[c + d*x]^2*Tan[c + d*x])/(24*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c
+ d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 1.84537, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2989, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(284a^2Ab+128a^3B+264ab^2B+15Ab^3)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{(356a^2Ab+128a^3B+472ab^2B+133Ab^3)\sqrt{\ldots}}{192d\sqrt{a+b\cos(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```


```
[Out] -((284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + ((356*a^2*A*b + 133*A*b^3 + 128*a^3*B + 472*a*b^2*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*Sq
rt[a + b*Cos[c + d*x]]) + ((48*a^4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*b*
B + 40*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
```

```
2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((284*a^2*A*b + 15*A
*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192
*a*d) + ((36*a^2*A + 59*A*b^2 + 104*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]*Tan[c + d*x])/(96*d) + (a*(11*A*b + 8*a*B)*Sqrt[a + b*Cos[c + d*x]]*S
ec[c + d*x]^2*Tan[c + d*x])/(24*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c
+ d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int \surd$$

$$= \frac{a(11Ab+8aB)\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{24d}$$

$$= \frac{\left(36a^2A+59Ab^2+104abB\right)\sqrt{a+b\cos(c+dx)}\sec(c+dx}{96d}$$

$$= \frac{\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)\sqrt{a+b\cos(c+d}}{192ad}$$

$$= \frac{\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)\sqrt{a+b\cos(c+d}}{192ad}$$

$$= \frac{\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)\sqrt{a+b\cos(c+d}}{192ad}$$

$$= -\frac{\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)\sqrt{a+b\cos(c+}}{192ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)\sqrt{a+b\cos(c+}}{192ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.72104, size = 729, normalized size = 1.57

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{96}\sec^2(c+dx)\left(36a^2A\sin(c+dx)+104abB\sin(c+dx)+59Ab^2\sin(c+dx)\right)+\frac{\sec(c+dx)\left(284a^2Ab\sin(c\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((2*(144*a^3*A*b + 236*a*A*b^3 + 416*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (
2*(288*a^4*A + 436*a^2*A*b^2 - 45*A*b^4 + 832*a^3*b*B - 24*a*b^3*B)*Sqrt[(a
```

```
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt
[a + b*Cos[c + d*x]] - ((2*I)*(-284*a^2*A*b^2 - 15*A*b^4 - 128*a^3*b*B - 26
4*a*b^3*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(
a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ellip
ticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (
a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x
]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)
/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))
)/(768*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^3*(17*a*A*b*Sin[c +
d*x] + 8*a^2*B*Sin[c + d*x]))/24 + (Sec[c + d*x]^2*(36*a^2*A*Sin[c + d*x] +
 59*A*b^2*Sin[c + d*x] + 104*a*b*B*Sin[c + d*x]))/96 + (Sec[c + d*x]*(284*a
^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 128*a^3*B*Sin[c + d*x] + 264*
a*b^2*B*Sin[c + d*x]))/(192*a) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/4))/d
```

Maple [B] time = 16.885, size = 3548, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*a^3*(-1/4*cos(1/2*d*x+1/2*c)/a*(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^4+7/24/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3-1/96*(36*a^2+35*b^2)
/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5/192*b*(20*a^2+21*b^2)/a^4*cos(1/2*
d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-35/384*b
^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+25/96/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
```

))^(1/2))-25/96/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35/128/a^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^3-35/128*b^4/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-3/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/16/a^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))*b^2-35/128/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^4)+
2*a^2*(3*A*b+B*a)*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12/a^2*b*cos(
1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2
*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-5/16/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*
cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5
/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x
+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*b^2*(A
*b+3*B*a)*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4

```
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)
^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+6*a*b*(A*b+B*a)
*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*c
os(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5, x
)
```

3.319 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=320

$$-\frac{2\left(-24a^2B+28aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^3d}-\frac{2\left(56a^3Ab-32a^2b^2B-48a^4B+49aAb^3-25b^4B\right)\sqrt{\frac{a}{}}}{105b^4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(56*a^2*A*b + 63*A*b^3 - 48*a^3*B - 44*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(56*a^3*A*b + 49*a*A*b^3 - 48*a^4*B - 32*a^2*b^2*B - 25*b^
4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(105*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(28*a*A*b - 24*a^2*B - 25*b^2
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^3*d) + (2*(7*A*b - 6*a*B)
*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[
c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.619089, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-24a^2B+28aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^3d}-\frac{2\left(56a^3Ab-32a^2b^2B-48a^4B+49aAb^3-25b^4B\right)\sqrt{\frac{a}{}}}{105b^4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*(56*a^2*A*b + 63*A*b^3 - 48*a^3*B - 44*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(56*a^3*A*b + 49*a*A*b^3 - 48*a^4*B - 32*a^2*b^2*B - 25*b^
4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(105*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(28*a*A*b - 24*a^2*B - 25*b^2
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^3*d) + (2*(7*A*b - 6*a*B)
*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[
c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*b*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos(c+dx)\left(2aB+\frac{5}{2}bB\cos(c+dx)+\cdots\right)}{\sqrt{a+b\cos(c+\cdots}}}{7b}$$

$$= \frac{2(7Ab-6aB)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35b^2d} + \frac{2B\cos^2(c+dx)\sqrt{a\cdots}}{}$$

$$= -\frac{2\left(28aAb-24a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^3d} + \frac{2(7Ab-6aB)\,\mathrm{c}\cdots}{}$$

$$= -\frac{2\left(28aAb-24a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^3d} + \frac{2(7Ab-6aB)\,\mathrm{c}\cdots}{}$$

$$= -\frac{2\left(28aAb-24a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^3d} + \frac{2(7Ab-6aB)\,\mathrm{c}\cdots}{}$$

$$= \frac{2\left(56a^2Ab+63Ab^3-48a^3B-44ab^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \cdots$$

Mathematica [A] time = 1.0154, size = 230, normalized size = 0.72

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))\left(48a^2B+6b(7Ab-6aB)\cos(c+dx)-56aAb+15b^2B\cos(2(c+dx))+65b^2B\right)+4\sqrt{\frac{a+\cdots}{}}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(14*a*A*b - 12*a^2*B + 25*b^2*B)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-56*a^2*A*b - 63*A*b^3 + 48*a^3*B
 + 44*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(-56*a*A*b + 48*a
^2*B + 65*b^2*B + 6*b*(7*A*b - 6*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c + d*
x)])*Sin[c + d*x])/(210*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.258, size = 1305, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4+24*B*a*b^3-360*B*b^4
)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-28*A*a*b^3+168*A*b^4+24*B*a^2*b
^2-24*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(56*A*a^2*
b^2+14*A*a*b^3-42*A*b^4-48*B*a^3*b-12*B*a^2*b^2-44*B*a*b^3-80*B*b^4)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b-49*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^3+56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^3*b-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+
63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+48*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+32*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-48*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^4+48*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3*b-44*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+4
4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^4/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.320 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{2\left(10a^2Ab-8a^3B-7ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (-2*(10*a*A*b - 8*a^2*B - 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(10*a^2*A*b + 5*A*b^3 - 8*a^3*B - 7*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d)
 + (2*B*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.429668, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(10a^2Ab-8a^3B-7ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(10*a*A*b - 8*a^2*B - 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(10*a^2*A*b + 5*A*b^3 - 8*a^3*B - 7*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d)
 + (2*B*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
```

x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2752

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} + \frac{2\int \frac{aB+\frac{3}{2}bB\cos(c+dx)+\frac{1}{2}(5Ab-4aB)\,c}{\sqrt{a+b\cos(c+dx)}}}{5b}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)\sqrt{a+b\cos(c+d}}{5bd}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)\sqrt{a+b\cos(c+d}}{5bd}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)\sqrt{a+b\cos(c+d}}{5bd}$$

$$= -\frac{2\left(10aAb-8a^2B-9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(10a^2Ab+\right.}{}$$

Mathematica [A] time = 0.89092, size = 180, normalized size = 0.73

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(8a^2B-10aAb+9b^2B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(2aB+5Ab)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 2*a*B)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*((a + b)*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2
*b*(a + b*Cos[c + d*x])*(5*A*b - 4*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x])/
(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.907, size = 993, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3-4*B*a*b^2+24*B*b^3)*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3+8*B*a^2*b+2*B*a*
b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a*b^2-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
3-7*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-8*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.321 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=183

$$-\frac{2\left(-2a^2B+3aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin}{}$$

```
[Out] (2*(3*A*b - 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a*A*b - 2*a^2*
B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.291797, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2968, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-2a^2B+3aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```



```
[Out] (2*(3*A*b - 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a*A*b - 2*a^2*
B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{bB}{2}+\frac{1}{2}(3Ab-2aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{(3Ab-2aB)\int \sqrt{a+b\cos(c+dx)}\,dx}{3b^2} - \frac{(3}{}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\left((3Ab-2aB)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}}}{3b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(3aAb-2a^2B-b^2B\right)\sqrt{}}{3b^2d\sqrt{a+}}$$

Mathematica [A] time = 0.656256, size = 154, normalized size = 0.84

$$\frac{2\left(2a^2B-3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a+b)(2aB-3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2bB\sin}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(a + b)*(-3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)] + 2*(-3*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b*Co
s[c + d*x])*Sin[c + d*x])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.369, size = 671, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*cos(1/2*d*x+1/2*c)^3*a*b+6*B*cos(
1/2*d*x+1/2*c)^3*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2
*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
2-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b+2*B*cos(1/2*d*x+1/2*c)*a*b-2*B*cos(1/2*d*x+1/2*c)*b^2)/b^
2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos(dx+c)^2+A\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.322 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=130

$$\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]])
```

Rubi [A] time = 0.126202, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= \frac{B\int\sqrt{a+b\cos(c+dx)}\,dx}{b}+\frac{(Ab-aB)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}\\
&= \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left((Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{b\sqrt{a+b\cos(c+dx)}}\\
&= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 3.12133, size = 93, normalized size = 0.72

$$
\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((Ab-aB)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+B(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + (A*b - a*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [A] time = 3.184, size = 249, normalized size = 1.9

$$-2\,\frac{\sqrt{\left(2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+bd}}\sqrt{\frac{2\,b}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\cos\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

3.323 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.312866, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3002, 2663, 2661, 2807, 2805}

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx \\
&= \frac{\left(A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} + \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} \\
&= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.182617, size = 81, normalized size = 0.69

$$
\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(A\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+BF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(B*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] + A*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c + d
*x]])
```

Maple [A] time = 3.668, size = 194, normalized size = 1.6

$$2\,\frac{\sqrt{\left(2\,b\,(\cos(1/2\,dx+c/2))^2+a-b\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}}{\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^4+(a+b)\,(\sin(1/2\,dx+c/2))^2}\,\sin(1/2\,dx+c/2)\,\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,b+a+b}d}\sqrt{\frac{2\,b\,(\mathrm{co}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(A*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2)))/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx + c) + A)\sec(dx + c)}{\sqrt{b\cos(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.324 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=216

$$-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-$$

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - ((A*
b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/(a*d)
```

Rubi [A] time = 0.655118, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3000, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - ((A*
b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/(a*d)
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
```

```
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} + \frac{\int \frac{\left(\frac{1}{2}(-Ab+2aB)-\frac{1}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} - \frac{A\int \sqrt{a+b\cos(c+dx)}\,dx}{2a} - \frac{\int \frac{\left(\frac{1}{2}b(Ab-2aB)-\right.}{\sqrt{}}}{}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} + \frac{1}{2}A\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx - \frac{(Ab-2aB)}{}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} +$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} -$$

Mathematica [C] time = 6.36639, size = 320, normalized size = 1.48

$$\frac{\frac{2(4aB-3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4A\tan(c+dx)\sqrt{a+b\cos(c+dx)}-\frac{2iA\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a}\right.\right.\right.}{}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((2*(-3*A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*A*Sqrt[-((b*(-1 +
 Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x
]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*
b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d)
```

Maple [B] time = 5.221, size = 639, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
```

```
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.325 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=299

$$\frac{\left(4a^2A-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3Ab-4aB)\cdots}{}$$

```
[Out] ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b - 4*a*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 - 4*a*b*B)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*
x])/(4*a^2*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d)
```

Rubi [A] time = 0.954973, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2A-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3Ab-4aB)\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b - 4*a*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 - 4*a*b*B)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*
x])/(4*a^2*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d)
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(\frac{1}{2}(-3Ab+4aB)+aA\cos(c+dx)+\frac{1}{2}A\right.}{\sqrt{a+b\cos(c+}}}{2a}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+}{2ad}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+}{2ad}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+}{2ad}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(3Ab-4aB)\sqrt{a+b\cos(c}}{4a^2d}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{4ad\sqrt{a+b\cos(}}$$

Mathematica [C] time = 5.75284, size = 420, normalized size = 1.4

$$\frac{\frac{2\left(8a^2A-12abB+9Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}((4aB-3Ab)\cos(c+dx)+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((8*a*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + 9*A*b^2 - 12*a*b*B)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sq
rt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b - 4*a*B)*Sqrt[-((b*(-1 + Cos[c + d*x
]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
```

```
 b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (-3*A*b + 4*a*B)*Cos[c + d
*x])*Sec[c + d*x]*Tan[c + d*x])/(16*a^2*d)
```

Maple [B] time = 7.652, size = 1182, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/2*c
os(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*
B*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.326 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}+\frac{2\left(20a^2A\right.}{}$$

```
[Out] (-2*(40*a^3*A*b - 25*a*A*b^3 - 48*a^4*B + 24*a^2*b^2*B + 9*b^4*B)*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*(a^2 - b^2)*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*A*b + 5*A*b^3 - 48*a^3*B
 - 12*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Cos[c
+ d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*
a^2*A*b - 5*A*b^3 - 24*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*A*b - 6*a^2*B + b^2*B)*Cos[c + d*x]*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.727122, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2989, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}+\frac{2\left(20a^2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```



```
[Out] (-2*(40*a^3*A*b - 25*a*A*b^3 - 48*a^4*B + 24*a^2*b^2*B + 9*b^4*B)*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*(a^2 - b^2)*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*A*b + 5*A*b^3 - 48*a^3*B
 - 12*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Cos[c
+ d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*
a^2*A*b - 5*A*b^3 - 24*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*A*b - 6*a^2*B + b^2*B)*Cos[c + d*x]*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+\frac{1}{2}b(Ab-aB)\cos(c+dx\right.}{\sqrt{a+b\cos(c+\cdots}}}{b\left(a^2-b^2\cdots\right.}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\left(5aAb-6a^2B+b^2B\right)\cos(c+dx)\sqrt{a\cdots}}{5b^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2Ab-5Ab^3-24a^3B+9ab^2B\right)\cdots}{15b^3\left(a^2-b^2\cdots\right.}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2Ab-5Ab^3-24a^3B+9ab^2B\right)\cdots}{15b^3\left(a^2-b^2\cdots\right.}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2Ab-5Ab^3-24a^3B+9ab^2B\right)\cdots}{15b^3\left(a^2-b^2\cdots\right.}$$

$$= -\frac{2\left(40a^3Ab-25aAb^3-48a^4B+24a^2b^2B+9b^4B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\cdots\right.}{15b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.71575, size = 304, normalized size = 0.79

$$\frac{\frac{30a^3b(aB-Ab)\sin(c+dx)}{b^2-a^2} + \frac{2b^2\left(-10a^2Ab+12a^3B+3ab^2B-5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a-b)(a+b)} + \frac{2\left(-40a^3Ab-24a^2b^2B+48a^4B+25aAb^3-9b^4B\right)\sqrt{\frac{a+b\cos(c+d\cdots}{a+b}}\cdots}{(a-b)(a+b)}}{15b^4d\sqrt{a+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((2*b^2*(-10*a^2*A*b - 5*A*b^3 + 12*a^3*B + 3*a*b^2*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/((a - b)*(a + b)) + (
2*(-40*a^3*A*b + 25*a*A*b^3 + 48*a^4*B - 24*a^2*b^2*B - 9*b^4*B)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a
*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/((a - b)*(a + b)) + (30*a^3*b*(-(A
*b) + a*B)*Sin[c + d*x])/(-a^2 + b^2) + 2*b*(5*A*b - 9*a*B)*(a + b*Cos[c +
d*x])*Sin[c + d*x] + 3*b^2*B*(a + b*Cos[c + d*x])*Sin[2*(c + d*x)])/(15*b^4
```

```
*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 12.509, size = 1308, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*B/b*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8/b
^2*(A*b-B*a-3*B*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x
+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^4*(A*a*b+2*A*b^2-B*a^2-2*B*a*b-
3*B*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2)))+2*(A*a^2*b+A*a*b^2+A*b^3-B*a^3-B*a^2*b-B*a*b^2-B*b
^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*a^3*(A*b-B*a)/b^4/sin(1/2*d*x+1
/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b
+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\cos(dx+c)^4+A\cos(dx+c)^3\right)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

3.327 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+6aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2Ab-8a^3B+5ab^2\right.}{3b^3}$$

```
[Out] (2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - (2*(6*a*A*b - 8*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.486215, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2988, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+6aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2Ab-8a^3B+5ab^2\right.}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - (2*(6*a*A*b - 8*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
```

```
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}ab(Ab-aB)+\frac{1}{2}\left(2a^2-b^2\right)(Ab-aB)\cos(c+dx)+\frac{1}{2}b(a}{\sqrt{a+b\cos(c+dx)}}}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{4\int \frac{1}{4}}{}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2d} - \frac{(6aAl}{}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{((6a^2}{}$$

$$= \frac{2\left(6a^2Ab-3Ab^3-8a^3B+5ab^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(6a}{}$$

Mathematica [A] time = 1.38617, size = 189, normalized size = 0.72

$$\frac{2\left(b\sin(c+dx)\left(\frac{a\left(-4a^2B+3aAb+b^2B\right)}{b^2-a^2}+bB\cos(c+dx)\right)+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a-b)\left(8a^2B-6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(6a^2Ab-8a^3B+5ab^2B-3Ab^3\right)\right.}{a-b}}{3b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (2*((Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5
*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (a - b)*(-6*a*A*b + 8*a^2
*B + b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b) + b*((a*(3*a*A*
b - 4*a^2*B + b^2*B))/(-a^2 + b^2) + b*B*Cos[c + d*x])*Sin[c + d*x]))/(3*b^
```

```
3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 10.478, size = 954, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^3*(4*
b^2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*B*a*b-2*B*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*B*a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a*b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a^2*(A*b-B*a)/b^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

3.328 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=204

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-2aB)\sqrt{\frac{a+b\cos(c\text{-}}{a+b}}}{b^2d\sqrt{a+b\,\mathrm{c}}}$$

```
[Out] (-2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 (2*(A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.331767, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2968, 3021, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-2aB)\sqrt{\frac{a+b\cos(c\text{-}}{a+b}}}{b^2d\sqrt{a+b\,\mathrm{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 (2*(A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}b(Ab-aB)+\frac{1}{2}\left(aAb-2a^2B+b^2B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{(Ab-2aB)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b^2} - \frac{\left(aAb-2a^2B\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\left(aAb-2a^2B+b^2B\right)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{}}{b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{2\left(aAb-2a^2B+b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(Ab-2aB)\sqrt{\frac{a+}{}}}{b^2d\sqrt{a}}$$

Mathematica [A] time = 0.787943, size = 170, normalized size = 0.83

$$-\frac{2\left(\left(a^2-b^2\right)(2aB-Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a+b)\left(2a^2B-aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\right.}{b^2d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(-((a + b)*(-(a*A*b) + 2*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*(-(A*b) + 2*a*B)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a
*b*(-(A*b) + a*B)*Sin[c + d*x]))/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c +
d*x]])
```

Maple [B] time = 7.861, size = 515, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*a*(A*b-B*a)/b^2/sin(
1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\cos(dx+c)^{2}+A\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{b^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.329 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a
+ b*Cos[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```

Rubi [A] time = 0.230544, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a
+ b*Cos[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b\cos(c+dx)}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left((Ab-aB)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}} - $$

Mathematica [A] time = 0.541017, size = 151, normalized size = 0.82

$$\frac{2\left(B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b(aB-Ab)\sin(c+dx)-(a+b)(aB-Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{bd(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-((a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-(A*b) + a*B)*Sin[c + d*x]))
/((a - b)*b*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 7.684, size = 428, normalized size = 2.3

$$-\frac{1}{d}\sqrt{-\left(-2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^2-a+b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,\frac{B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}}{b\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+2*(A*b-B*a)/b/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(b^2*cos(d*x + c)^2
+ 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.330 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(a*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.50845, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3000, 3059, 2655, 2653, 12, 2807, 2805}

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(a*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
```

```
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{1}{2}A\left(a^2-b^2\right)-\frac{1}{2}a(Ab-aB)\cos(c+dx)-\frac{1}{2}b(Ab-aB)\cos^2(\right.}{\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int -\frac{Ab\left(a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab\left(a^2-b^2\right)} - \frac{(Ab-aB)\int \sqrt{a+}}{a\left(a^2\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a} - \frac{\left((Ab-aB)\sqrt{a+b\cos}\right.}{a\left(a^2-\right.}$$

$$= -\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+}}$$

$$= -\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{ad\sqrt{a+b\cos(c+d}}$$

Mathematica [C] time = 3.78656, size = 460, normalized size = 2.42

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(\frac{4b(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} - \frac{\frac{2\left(2a^2A+abB-3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{4a(aB-Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{\sqrt{a+b\cos(c+dx)}}}{}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*(-(((4*a*(-(A*b) + a*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
+ d*x]] + (2*(2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*(A*b - a*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b))) +
 (4*b*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2
*a*d*(A + B*Cos[c + d*x]))
```

Maple [A] time = 7.651, size = 429, normalized size = 2.3

$$-\frac{1}{d}\sqrt{-\left(-2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-a+b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\,\frac{\left(-Ab+aB\right)\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin(1/2\right.}}{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\left(-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a
)/a/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*A/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx + c) + A)\sec(dx + c)}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.331 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{b\left(a^2A+2abB-3Ab^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3Ab-2aB)\sqrt{\frac{a+b\cos}{a+}}}{a^2d\sqrt{a+}}$$

```
[Out] -(((a^2*A - 3*A*b^2 + 2*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]))
 + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 2*a*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b
*Cos[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(a^2*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c +
 d*x]])
```

Rubi [A] time = 0.990968, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3056, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(a^2A+2abB-3Ab^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3Ab-2aB)\sqrt{\frac{a+b\cos}{a+}}}{a^2d\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((a^2*A - 3*A*b^2 + 2*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]))
 + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 2*a*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b
*Cos[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(a^2*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c +
 d*x]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-3Ab+2aB)+\frac{1}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{a}$$

$$= \frac{b\left(a^2A-3Ab^2+2abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(-\frac{1}{4}\left(a^2-b^2\right)(3\right.}{}}{}$$

$$= \frac{b\left(a^2A-3Ab^2+2abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\left(\frac{1}{4}b\left(a^2-b^2\right)(3\right.}{}}{}$$

$$= \frac{b\left(a^2A-3Ab^2+2abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{A\int \frac{1}{\sqrt{a+b\cos(c+}}}{2a}$$

$$= -\frac{\left(a^2A-3Ab^2+2abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\left(a^2A-3Ab\right.}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2A-3Ab^2+2abB\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{\frac{a+b\cos(c+d}{a+b}}}{ad\sqrt{a+}}$$

Mathematica [C] time = 5.4687, size = 482, normalized size = 1.59

$$\frac{\frac{4\tan(c+dx)\left(b\left(a^2A+2abB-3Ab^2\right)\cos(c+dx)+aA\left(a^2-b^2\right)\right)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} + \frac{\frac{2\left(-7a^2Ab+4a^3B-6ab^2B+9Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(a^2A+2abB-3Ab^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos}{}}}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (((-8*a*b*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-7*a^2*A*b + 9*A*b^3
 + 4*a^3*B - 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(a^2*A - 3*A*b
^2 + 2*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c
 + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
```

```
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((a - b)*(a + b)) + (4
*(a*A*(a^2 - b^2) + b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Cos[c + d*x])*Tan[c + d*x
])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

Maple [B] time = 10.223, size = 908, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)
*b/a^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*(-A*b+B
*a)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A/a*(-cos(1/2*d*x+1/2*c)/a*(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx + c) + A)\sec(dx + c)^2}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

3.332 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=398

$$-\frac{b\left(7a^2Ab-4a^3B+12ab^2B-15Ab^3\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(7a^2Ab-4a^3B+12ab^2B-15Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] ((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - ((5*A*b - 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((
4*a^2*A + 15*A*b^2 - 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(
7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(4*a^3*(a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 4*a*B)*Tan[c + d*x])/(4*a^2*d*Sq
rt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*C
os[c + d*x]])
```

Rubi [A] time = 1.43139, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(7a^2Ab-4a^3B+12ab^2B-15Ab^3\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(7a^2Ab-4a^3B+12ab^2B-15Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - ((5*A*b - 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((
4*a^2*A + 15*A*b^2 - 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(
7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(4*a^3*(a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 4*a*B)*Tan[c + d*x])/(4*a^2*d*Sq
rt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*C
```

```
os[c + d*x]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
```

```
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+4aB)+aA\cos(c+dx)+\frac{3}{2}Ab\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,d}{2a}$$

$$= -\frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}\left(4a^2A+15Ab^2-12ab\right.\right.}{}}{}$$

$$= -\frac{b\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}}$$

$$= -\frac{b\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}}$$

$$= -\frac{b\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b\left(7\right.}{}$$

$$= \frac{\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(5A}{}$$

Mathematica [C] time = 6.84121, size = 678, normalized size = 1.7

$$\frac{\sqrt{a+b\cos(c+dx)}\left(-\frac{2\left(ab^3B\sin(c+dx)-Ab^4\sin(c+dx)\right)}{a^3\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\sec(c+dx)(4aB\sin(c+dx)-7Ab\sin(c+dx))}{4a^3}+\frac{A\tan(c+dx)\sec(c+dx)}{2a^2}\right)}{d} - \frac{2\left(4a^3Ab+16a^2b^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] -((2*(4*a^3*A*b - 20*a*A*b^3 + 16*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(
8*a^4*A + 29*a^2*A*b^2 - 45*A*b^4 - 28*a^3*b*B + 36*a*b^3*B)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*
Cos[c + d*x]] - ((2*I)*(7*a^2*A*b^2 - 15*A*b^4 - 4*a^3*b*B + 12*a*b^3*B)*Sq
rt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[
2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((
a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2
 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(16*a^3*(-a
 + b)*(a + b)*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(-7*A*b*Sin[c +
 d*x] + 4*a*B*Sin[c + d*x]))/(4*a^3) - (2*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*
Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*Tan
[c + d*x])/(2*a^2)))/d
```

Maple [B] time = 12.836, size = 1564, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a
)*b^2/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*(A*b
-B*a)/a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A/a*(-1/2*cos(1/2*d*x+1/
2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
```

```
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/
8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*(-A*b+B*a)/a^
2*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

3.333 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=550

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(5a^2Ab-8a^3B+12ab^2B-9Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(30a^3Ab+71a\right.}{}$$

```
[Out] (-2*(80*a^5*A*b - 140*a^3*A*b^3 + 40*a*A*b^5 - 128*a^6*B + 212*a^4*b^2*B -
55*a^2*b^4*B - 9*b^6*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(2*(80*a^4*A*b - 80*a^2*A*b^3 - 5*A*b^5 - 128*a^5*B + 116*a^3*b^2*B + 17*a*
b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(15*b^5*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Co
s[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) +
 (2*a*(5*a^2*A*b - 9*A*b^3 - 8*a^3*B + 12*a*b^2*B)*Cos[c + d*x]^2*Sin[c + d
*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(40*a^4*A*b - 65
*a^2*A*b^3 + 5*A*b^5 - 64*a^5*B + 98*a^3*b^2*B - 14*a*b^4*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d) - (2*(30*a^3*A*b - 50*a*A
*b^3 - 48*a^4*B + 71*a^2*b^2*B - 3*b^4*B)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.18714, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2989, 3047, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(5a^2Ab-8a^3B+12ab^2B-9Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(30a^3Ab+71a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*(80*a^5*A*b - 140*a^3*A*b^3 + 40*a*A*b^5 - 128*a^6*B + 212*a^4*b^2*B -
55*a^2*b^4*B - 9*b^6*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(2*(80*a^4*A*b - 80*a^2*A*b^3 - 5*A*b^5 - 128*a^5*B + 116*a^3*b^2*B + 17*a*
b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(15*b^5*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Co
```

```
s[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) +
 (2*a*(5*a^2*A*b - 9*A*b^3 - 8*a^3*B + 12*a*b^2*B)*Cos[c + d*x]^2*Sin[c + d
*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(40*a^4*A*b - 65
*a^2*A*b^3 + 5*A*b^5 - 64*a^5*B + 98*a^3*b^2*B - 14*a*b^4*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d) - (2*(30*a^3*A*b - 50*a*A
*b^3 - 48*a^4*B + 71*a^2*b^2*B - 3*b^4*B)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]

Rule 2752

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\left(-3a(Ab-aB)+\frac{3}{2}b(Ab-aB)\cos(c\cdots\right.}{(a+b\cos(c\cdots}}{3b\left(a^2-\cdots\right.}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+12ab^2B\right)\cdots}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+12ab^2B\right)\cdots}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+12ab^2B\right)\cdots}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+12ab^2B\right)\cdots}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+12ab^2B\right)\cdots}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}\cdots}}$$

$$= -\frac{2\left(80a^5Ab-140a^3Ab^3+40aAb^5-128a^6B+212a^4b^2B-55a^2b^4B-9b^6B\right)\cdots}{15b^5\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\cdots}$$

Mathematica [A] time = 3.89872, size = 372, normalized size = 0.68

$$\frac{b\left(\frac{10a^4(aB-Ab)\sin(c+dx)}{a^2-b^2} - \frac{10a^3\left(-8a^2Ab+11a^3B-15ab^2B+12Ab^3\right)\sin(c+dx)(a+b\cos(c+dx))}{\left(a^2-b^2\right)^2} + 2(5Ab-14aB)\sin(c+dx)(a+b\cos(c+d\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((-2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(20*a^4*A*b - 35*a^2*A*b^3 -
 5*A*b^5 - 32*a^5*B + 44*a^3*b^2*B + 8*a*b^4*B)*EllipticF[(c + d*x)/2, (2*b
)/(a + b)] - (-80*a^5*A*b + 140*a^3*A*b^3 - 40*a*A*b^5 + 128*a^6*B - 212*a^
4*b^2*B + 55*a^2*b^4*B + 9*b^6*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)) + b*
((10*a^4*(-(A*b) + a*B)*Sin[c + d*x])/(a^2 - b^2) - (10*a^3*(-8*a^2*A*b + 1
2*A*b^3 + 11*a^3*B - 15*a*b^2*B)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 -
b^2)^2 + 2*(5*A*b - 14*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x] + 3*b*B*(a
+ b*Cos[c + d*x])^2*Sin[2*(c + d*x)]))/(15*b^5*d*(a + b*Cos[c + d*x])^(3/2)
)
```

Maple [B] time = 21.393, size = 1746, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*B/b^2*(-
1/10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8
/b^3*(A*b-2*B*a-3*B*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^5*(2*A*a*b+2*A*b^2-3*B*a^2-
4*B*a*b-3*B*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(3*A*a^2*b+2*A*a*b^2+A*b^3-4*B*a^3-3*B*a
^2*b-2*B*a*b^2-B*b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
```

```
2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*a^3/b^5*(4*A*
b-5*B*a)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*a^4*(
A*b-B*a)/b^5*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8
/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(
1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*
a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^5+A\cos(dx+c)^4\right)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^5 + A*cos(d*x + c)^4)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.334 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=413

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2a^2\left(3a^2Ab-6a^3B+10ab^2B-7Ab^3\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sin}{3b^3d\left(\right.}$$

```
[Out] (2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*
B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(
a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^3*A*b - 9*a*A*
b^3 - 16*a^4*B + 16*a^2*b^2*B + b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*
a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(
a^2 - b^2)*d)
```

Rubi [A] time = 0.804646, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2989, 3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2a^2\left(3a^2Ab-6a^3B+10ab^2B-7Ab^3\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sin}{3b^3d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*
B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(
a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^3*A*b - 9*a*A*
b^3 - 16*a^4*B + 16*a^2*b^2*B + b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*
a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(
```

```
a^2 - b^2)*d)
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+\frac{3}{2}b(Ab-aB)\cos(c+dx\right.}{(a+b\cos(c+dx)}}{3b\left(a^2-b^2\right.}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+10ab^2B\right)s}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+10ab^2B\right)s}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+10ab^2B\right)s}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+10ab^2B\right)s}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= \frac{2\left(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B\right)\sqrt{a+b\cos(c+dx)}E}{3b^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.73851, size = 334, normalized size = 0.81

$$\frac{2\left(\frac{b\sin(c+dx)\left(2ab\left(-5a^3Ab-16a^2b^2B+10a^4B+9aAb^3+2b^4B\right)\cos(c+dx)+16a^3Ab^3-8a^5Ab+B\left(b^3-a^2b\right)^2\cos(2(c+dx))-25a^4b^2B+16a^6B+b^6B\right)}{2\left(a^2-b^2\right)^2}+\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)}{}\right.}{3b^4d(a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(2*a^3*A*b - 6*a*A*b^3 - 4*a
^4*B + 7*a^2*b^2*B + b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-8*a^4
*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*((a +
b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])))/((a - b)^2*(a + b)) + (b*(-8*a^5*A*b + 16*a^3*A*b^3 + 16*a^6*B -
 25*a^4*b^2*B + b^6*B + 2*a*b*(-5*a^3*A*b + 9*a*A*b^3 + 10*a^4*B - 16*a^2*b
^2*B + 2*b^4*B)*Cos[c + d*x] + (-(a^2*b) + b^3)^2*B*Cos[2*(c + d*x)])*Sin[c
```

```
+ d*x])/(2*(a^2 - b^2)^2)))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 19.001, size = 1389, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^4*(4*
b^2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*B*a*b-2*B*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+17*B*a
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a*b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a^2/b^4*(3*A*b-4*B*a)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^
2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2)-2*a^3*(A*b-B*a)/b^4*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x
+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*
a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1
/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/
(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*
```

```
c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^4+A\cos(dx+c)^3\right)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.335 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2Ab-8a^3B+9ab^2B-3\right.}{3b^3d\left(a^2-b^2\right.}$$

```
[Out] (-2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*A*b - 3*A*b^3 - 8*a^3*B + 9
*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*(A*b - a*B
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*a*(2*
a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.552717, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2988, 3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2Ab-8a^3B+9ab^2B-3\right.}{3b^3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*A*b - 3*A*b^3 - 8*a^3*B + 9
*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*(A*b - a*B
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*a*(2*
a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2\int \frac{\frac{3}{2}ab(Ab-aB)+\frac{1}{2}\left(2a^2-3b^2\right)(Ab-aB)\cos(c+dx)\cdot}{(a+b\cos(c+dx))^{3/2}}}{3b^2\left(a^2-b^2\right)}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\sin}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+d}}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\sin}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+d}}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\sin}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+d}}$$

$$= -\frac{2\left(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.16107, size = 274, normalized size = 0.83

$$\frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(a^2Ab+2a^3B-6ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(-2a^3Ab-15a^2b^2B+8a^4B+6aAb^3+3b^4B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{(a-b)^2(a+b)}-\frac{a}{}\right.}{3b^3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(a^2*A*b + 3*A*b^3 + 2*a^3*B
 - 6*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b
^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b))
 - (a*b*(a*(-(a^2*A*b) + 5*A*b^3 + 4*a^3*B - 8*a*b^2*B) + b*(-2*a^2*A*b + 6
*A*b^3 + 5*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(
3*b^3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 15.549, size = 950, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3*B*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*a/b^3*(2*A*b-3*B*a)/
sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*a^2*(A*b-B*a)/
b^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(
1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1
/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b
^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1
/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.336 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=307

$$\frac{2\left(a^2Ab+2a^3B-6ab^2B+3Ab^3\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2B+aAb-3b^2B\right)\sqrt{\frac{a+b\cos(c+}{a+b}}}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos}}$$

```
[Out] (-2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]) + (2*(a*A*b + 2*a^2*B - 3*b^2*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*
Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.471744, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2Ab+2a^3B-6ab^2B+3Ab^3\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2B+aAb-3b^2B\right)\sqrt{\frac{a+b\cos(c+}{a+b}}}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]) + (2*(a*A*b + 2*a^2*B - 3*b^2*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*
Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}b(Ab-aB)-\frac{1}{2}\left(aAb+2a^2B-3b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\sin(c+ d}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\sin(c+ d}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\sin(c+ d}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= -\frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(aA\right.}{}$$

Mathematica [A] time = 1.85905, size = 224, normalized size = 0.73

$$\frac{2\left(\frac{b\sin(c+dx)\left(b\left(a^2Ab+2a^3B-6ab^2B+3Ab^3\right)\cos(c+dx)+a\left(2a^2Ab+a^3B-5ab^2B+2Ab^3\right)\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2Ab+2a^3B-6ab^2B+3Ab^3\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a-\right.}{(a-b)^2}\right.}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2*A*b + 3*A*b^3 + 2*a^3*B -
 6*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(a*A*b + 2*a^2*
B - 3*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (b*(a*(2*
a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B) + b*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6
*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos[c
 + d*x])^(3/2))
```

Maple [B] time = 14.194, size = 860, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2/b^2*(A*b-2*B*a)/sin(1/2*d*x+1/2*c)^2/(-2*s
in(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*a*(A*b-B*a)/b^2*(1/6/b/(a-b)/(a+b)*cos(1
/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+
b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*
x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^3*
cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.337 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b -
a*B)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(4*a*A
*b - a^2*B - 3*b^2*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d
*x]])
```

Rubi [A] time = 0.373145, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b -
a*B)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(4*a*A
*b - a^2*B - 3*b^2*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d
*x]])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2A+Ab^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{(Ab-aB)\int \frac{\cdots}{\sqrt{\cdots}}}{3b\left(a^2 \cdots\right.}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\left(4aAb-a^2B \cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(4aAb-a^2B-3b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\cdots\right.}{3b\left(a^2-b^2\right)d\sqrt{a+b\,\mathrm{c}\cdots}}$$

Mathematica [A] time = 1.52271, size = 193, normalized size = 0.7

$$\frac{2\left(\frac{\sin(c+dx)\left(b\left(a^2B-4aAb+3b^2B\right)\cos(c+dx)-5a^2Ab+2a^3B+2ab^2B+Ab^3\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2B-4aAb+3b^2B\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a-b)(aB-Ab)F\left(\frac{1}{2}(c+dx)|\cdots\right.\right.}{b(a-b)^2}\cdots\right.}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*E
llipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)]))/((a - b)^2*b)) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B
+ 2*a*b^2*B + b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a
^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 13.244, size = 750, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b/sin(1
/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(A*b-B*a)/b*(1/6/b/
(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2
*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(b^3*cos(d*x + c)^3
+ 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.338 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=349

$$\frac{2b\left(7a^2Ab-4a^3B-3Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(7*a^2*A*b - 3*A*b^3 - 4*
a^3*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.09905, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b\left(7a^2Ab-4a^3B-3Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(7*a^2*A*b - 3*A*b^3 - 4*
a^3*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}A\left(a^2-b^2\right)-\frac{3}{2}a(Ab-aB)\cos(c+dx)+\frac{1}{2}b(Ab-aB)\cos}{(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} +$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} -$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} +$$

$$= -\frac{2\left(7a^2Ab-3Ab^3-4a^3B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b(Ab}{3a\left(a^2-b^2\right)}$$

$$= -\frac{2\left(7a^2Ab-3Ab^3-4a^3B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(Ab-aB)\sqrt{}}{3a\left(a^2-\right.}$$

Mathematica [C] time = 6.72774, size = 743, normalized size = 2.13

$$\frac{\cos(c+dx)\sqrt{a+b\cos(c+dx)}(A\sec(c+dx)+B)\left(-\frac{2\left(abB\sin(c+dx)-Ab^2\sin(c+dx)\right)}{3a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{2\left(-7a^2Ab^2\sin(c+dx)+4a^3bB\sin(c+dx)+3Ab^4\sin(c+\right.}{3a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right.}{d(A+B\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-12*a^3*A*b + 4*a*A*b^3 + 6*a^4*B +
 2*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4*A - 19*a^2*A*b^2 + 9*A*b^
4 + 4*a^3*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-7*a^2*A*b^2 + 3*A*b^4
 + 4*a^3*b*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x]
)/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)
```

```
^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]]
, (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c +
d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])
^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^
2))))/(6*a^2*(a - b)^2*(a + b)^2*d*(A + B*Cos[c + d*x])) + (Cos[c + d*x]*Sq
rt[a + b*Cos[c + d*x]]*(B + A*Sec[c + d*x])*((-2*(-(A*b^2*Sin[c + d*x]) + a
*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-7*a^2*A
*b^2*Sin[c + d*x] + 3*A*b^4*Sin[c + d*x] + 4*a^3*b*B*Sin[c + d*x]))/(3*a^2*
(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/(d*(A + B*Cos[c + d*x]))
```

Maple [B] time = 14.143, size = 854, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A*b/a^2/
sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*A/a^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))+2*(-A*b+B*a)/a*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+
1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*co
s(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.339 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=437

$$\frac{b\left(-26a^2Ab^2+3a^4A+14a^3bB-6ab^3B+15Ab^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2A+2abB-5Ab^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2A+2abB-\right.}{3a^2d\left(\right.}$$

```
[Out] -((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 2*a*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt[a
 + b*Cos[c + d*x]]) + (b*(3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(3*a^2
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4*A - 26*a^2*A*b^2 + 1
5*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt
[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 1.48093, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3000, 3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(-26a^2Ab^2+3a^4A+14a^3bB-6ab^3B+15Ab^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2A+2abB-5Ab^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2A+2abB-\right.}{3a^2d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] -((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 2*a*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt[a
 + b*Cos[c + d*x]]) + (b*(3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(3*a^2
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4*A - 26*a^2*A*b^2 + 1
5*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt
```

[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3/2))

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

qQ[a, 0])))

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+2aB)+\frac{3}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{a}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(-\frac{3}{4}\left(a^2-b^2\right.\right.}{}}{}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= -\frac{\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.09293, size = 750, normalized size = 1.72

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(ab^2B\sin(c+dx)-Ab^3\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{2\left(-10a^2Ab^3\sin(c+dx)+7a^3b^2B\sin(c+dx)-3ab^4B\sin(c+dx)+6Ab^5\sin(c+dx)\right)}{3a^3\left(a^2-b^2\right)^2(a+b\cos(c+dx))} + \frac{A\tan(c+dx}{a^3}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((2*(36*a^3*A*b^2 - 20*a*A*b^4 - 24*a^4*b*B + 8*a^2*b^3*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
```

```
+ d*x]] + (2*(-33*a^4*A*b + 86*a^2*A*b^3 - 45*A*b^5 + 12*a^5*B - 38*a^3*b^2
*B + 18*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-3*a^4*A*b + 26*a^2*
A*b^3 - 15*A*b^5 - 14*a^3*b^2*B + 6*a*b^4*B)*Sqrt[(b - b*Cos[c + d*x])/(a +
 b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-
(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c
 + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c +
d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(12*a^3*(-a + b)^2*(a + b)^2*d) + (Sqrt
[a + b*Cos[c + d*x]]*((2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3
*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-10*a^2*A*b^3*Sin[c + d*x] +
 6*A*b^5*Sin[c + d*x] + 7*a^3*b^2*B*Sin[c + d*x] - 3*a*b^4*B*Sin[c + d*x]))
/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (A*Tan[c + d*x])/a^3))/d
```

Maple [B] time = 18.753, size = 1341, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*(2*A*b-
B*a)/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*(-2*A
*b+B*a)/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(A*b-B*a)*b/a^2*(1/6/b/(
a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*
c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
```

```
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/a^2*A*(-cos(1/2*
d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.340 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=532

$$-\frac{b\left(-170a^2Ab^3+33a^4Ab+104a^3b^2B-12a^5B-60ab^4B+105Ab^5\right)\sin(c+dx)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{b\left(27a^2Ab-12a^3B+20ab^2B-35Ab^3\right)\ldots}{12a^3d\left(a^2-b^2\right)(a+b\cos(c+\ldots}$$

```
[Out] ((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*
b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(12*
a^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((27*a^2*A*b - 35
*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(12*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]) + ((4*a^2*A + 35*A*b^2 - 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^4*d*Sqrt[a + b*Cos[c + d*x]]
) - (b*(27*a^2*A*b - 35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sin[c + d*x])/(12*a^
3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (b*(33*a^4*A*b - 170*a^2*A*b^
3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Sin[c + d*x])/(12*a^
4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((7*A*b - 4*a*B)*Tan[c + d*x]
)/(4*a^2*d*(a + b*Cos[c + d*x])^(3/2)) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 1.93422, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(-170a^2Ab^3+33a^4Ab+104a^3b^2B-12a^5B-60ab^4B+105Ab^5\right)\sin(c+dx)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{b\left(27a^2Ab-12a^3B+20ab^2B-35Ab^3\right)\ldots}{12a^3d\left(a^2-b^2\right)(a+b\cos(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2),x]
```



```
[Out] ((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*
b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(12*
a^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((27*a^2*A*b - 35
*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(12*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]) + ((4*a^2*A + 35*A*b^2 - 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
```

llipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^4*d*Sqrt[a + b*Cos[c + d*x]]
) - (b*(27*a^2*A*b - 35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sin[c + d*x])/(12*a^
3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (b*(33*a^4*A*b - 170*a^2*A*b^
3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Sin[c + d*x])/(12*a^
4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((7*A*b - 4*a*B)*Tan[c + d*x]
)/(4*a^2*d*(a + b*Cos[c + d*x])^(3/2)) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a
d(a + b*Cos[c + d*x])^(3/2))

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-7Ab+4aB)+aA\cos(c+dx)+\frac{5}{2}Ab\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}}{2a}$$

$$= -\frac{(7Ab-4aB)\tan(c+dx)}{4a^2d(a+b\cos(c+dx))^{3/2}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{4}\left(4a^2A+35Ab^2-20\right.\right.}{}}{}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{(7Ab-4aB)\tan(c+}{4a^2d(a+b\cos(c+dx}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(33a^4Ab-170a^2Ab\right.}{1}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(33a^4Ab-170a^2Ab\right.}{1}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(33a^4Ab-170a^2Ab\right.}{1}$$

$$= \frac{\left(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B\right)\sqrt{a+b\cos(c}}{12a^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{\left(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B\right)\sqrt{a+b\cos(c}}{12a^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.74322, size = 820, normalized size = 1.54

$$\frac{\frac{2\left(12Aba^5+144b^2Ba^4-216Ab^3a^3-80b^4Ba^2+140Ab^5a\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(24Aa^6-132bBa^5+195Ab^2a^4+344b^3Ba^3-566Ab^4a^2-180b^5Ba+315Ab\right.}{\sqrt{a+b\cos(c+dx)}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((2*(12*a^5*A*b - 216*a^3*A*b^3 + 140*a*A*b^5 + 144*a^4*b^2*B - 80*a^2*b^4*
B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/Sqrt[a + b*Cos[c + d*x]] + (2*(24*a^6*A + 195*a^4*A*b^2 - 566*a^2*A*b^4 +
 315*A*b^6 - 132*a^5*b*B + 344*a^3*b^3*B - 180*a*b^5*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
 + d*x]] - ((2*I)*(33*a^4*A*b^2 - 170*a^2*A*b^4 + 105*A*b^6 - 12*a^5*b*B +
104*a^3*b^3*B - 60*a*b^5*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b +
b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*E
llipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*
Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqr
t[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b
*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*
Cos[c + d*x])^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[a + b*Cos[c + d*x
]]*((Sec[c + d*x]*(-11*A*b*Sin[c + d*x] + 4*a*B*Sin[c + d*x]))/(4*a^4) - (2
*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*
Cos[c + d*x])^2) - (2*(-13*a^2*A*b^4*Sin[c + d*x] + 9*A*b^6*Sin[c + d*x] +
10*a^3*b^3*B*Sin[c + d*x] - 6*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(
a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a^3)))/d
```

Maple [B] time = 23.7, size = 2000, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x)
```

[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2*(3*A
*b-2*B*a)/a^4/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2
)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*
b*(3*A*b-2*B*a)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*(A*b-B*a)*b^2/a^
3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/
2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2
*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2*(-2*A*
b+B*a)/a^3*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A/a^2*(-1/2*
cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-
b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c

```
),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*
d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/
sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.341 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.0447992, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {21, 2663, 2661}

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}$$
$$= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.0478468, size = 58, normalized size = 1.

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [C] time = 0.404, size = 76, normalized size = 1.3

$$2\,{\frac{B}{d\sqrt{2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b}}\sqrt{{\frac{2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b}{a+b}}}{\rm InverseJacobiAM}\left(1/2\,dx+c/2,{\frac{\sqrt{2}\sqrt{b}}{\sqrt{a+b}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*B/d/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/
(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(B/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.342 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.146201, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 2807, 2805}

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx\\ &= \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}\\ &= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0709969, size = 59, normalized size = 1.

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 3.125, size = 167, normalized size = 2.8

$$2\,\frac{\sqrt{\left(2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+bd}}\sqrt{\frac{2\,b\,(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2),
 x)
```

3.343 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2bB\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2
- b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*B*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.0819928, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {21, 2664, 2655, 2653}

$$\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2bB\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2
- b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*B*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = B\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(2B)\int \frac{-\frac{a}{2}-\frac{1}{2}b\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \sqrt{a+b\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.208368, size = 84, normalized size = 0.78

$$\frac{B\left(2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)\right)}{d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*(2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] - 2*b*Sin[c + d*x]))/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Maple [A] time = 3.993, size = 218, normalized size = 2.

$$-2\,{\frac {B}{ \left(a-b \right) \left(a+b \right) \sin \left(1/2\,dx+c/2 \right) \sqrt{-2\, \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}b+a+b}d} \left(\sqrt{ \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sqrt{-2\,{\frac { \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}{a-b}}} \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}B}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.344 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$\frac{2b^2B\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2bB\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*b*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*
(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b
*Cos[c + d*x]]) + (2*b^2*B*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
+ d*x]])
```

Rubi [A] time = 0.423576, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {21, 2802, 3059, 2655, 2653, 12, 2807, 2805}

$$\frac{2b^2B\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2bB\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*
(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b
*Cos[c + d*x]]) + (2*b^2*B*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
+ d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = B\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{(2B)\int \frac{\left(\frac{1}{2}\left(a^2-b^2\right)-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}b^2\cos^2(c+dx)\right)\sec}{\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(2B)\int -\frac{b\left(a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab\left(a^2-b^2\right)} - \frac{(bB)\int\sqrt{a+b}}{a\left(a^2\right.}$$

$$= \frac{2b^2B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a} - \frac{\left(bB\sqrt{a+b\cos(c+d\right.}}{a\left(a^2-b^2\right.}$$

$$= -\frac{2bB\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

$$= -\frac{2bB\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{ad\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 4.80388, size = 403, normalized size = 2.25

$$\frac{B\left(\frac{4b^2\sin(c+dx)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} - \frac{\frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\right.\right.}{}}{(b-a)(}\right.}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (B*(-(((-4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*C
os[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 +
 Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b
))) + (4*b^2*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2*a*d)
```

Maple [A] time = 4.299, size = 377, normalized size = 2.1

$$2\,{\frac {B}{ \left(a+b \right) \left(a-b \right) a\sin \left(1/2\,dx+c/2 \right) \sqrt{-2\, \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}b+a+b}d}\left(\sqrt{ \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sqrt{-2\,{\frac { \left(\sin \left(1/2\,dx+c/2 \right) \right) }{a-b}}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))*b^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/a/(a-b
)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c) + Ba)\sec(dx+c)}{(b\cos(dx+c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c) + Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2),
 x)
```

3.345 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=170

$$\frac{10(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2(9aA+7bB)\,\mathrm{si}}{}$$

```
[Out] (2*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(A*b + a*B)*Elli
pticF[(c + d*x)/2, 2])/(21*d) + (10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*(9*a*A + 7*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) +
 (2*(A*b + a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*B*Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.207367, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2639, 2641}

$$\frac{10(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2(9aA+7bB)\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```



```
[Out] (2*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(A*b + a*B)*Elli
pticF[(c + d*x)/2, 2])/(21*d) + (10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*(9*a*A + 7*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) +
 (2*(A*b + a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*B*Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{5}{2}}(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+a\right.$$

$$= \frac{2bB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}(9aA+\right.$$

$$= \frac{2bB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + (Ab+aB)\int \cos^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2(9aA+7bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2(Ab+aB)\cos^{\frac{5}{2}}(c}{7a}$$

$$= \frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10(Ab+aB)\sqrt{\cos(c+dx)}\,\mathrm{s}}{21d}$$

$$= \frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{21d}$$

Mathematica [A] time = 1.26723, size = 125, normalized size = 0.74

$$\frac{300(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(7(36aA+43bB)\cos(c+dx)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2] + 300*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a*A + 43*b*B)*Cos[c + d*x] + 5*(
78*A*b + 78*a*B + 18*(A*b + a*B)*Cos[2*(c + d*x)] + 7*b*B*Cos[3*(c + d*x)])
)*Sin[c + d*x])/(630*d)
```

Maple [B] time = 3.349, size = 451, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b+720*B*a+2240*B*b)*sin(1/2*
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*a-1080*A*b-1080*B*a-2072*B*b)*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A*a+840*A*b+840*B*a+952*B*b)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A*a-240*A*b-240*B*a-168*B*b)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+75*A*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b+75*a*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^4+Aa\cos(dx+c)^2+(Ba+Ab)\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^4 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c
)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.346 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{2(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(7aA+5bB)\sin}{}$$

```
[Out] (6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a*A + 5*b*B)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
b*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.183915, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(7aA+5bB)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```


```
[Out] (6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a*A + 5*b*B)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
b*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+a\right.$$

$$= \frac{2bB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}(7aA+\right.$$

$$= \frac{2bB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (Ab+aB)\int \cos^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(Ab+aB)\cos^{\frac{3}{2}}(c}{5a}$$

$$= \frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Mathematica [A] time = 0.822617, size = 103, normalized size = 0.74

$$\frac{10(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+126(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(42(aB+Ab)\cos(c+dx)+}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (126*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + 10*(7*a*A + 5*b*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*a*A + 65*b*B + 42*(A*b + a*B)*Cos[c
 + d*x] + 15*b*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 3.103, size = 413, normalized size = 3.

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,Bb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-168\,Ab-168\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+35*a*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+25*B*b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos\left(dx+c\right)^{3}+Aa\cos\left(dx+c\right)+\left(Ba+Ab\right)\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)^
2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.347 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bB\sin(c+dx)\,\mathrm{c}}{5d}$$

```
[Out] (2*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*b*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.168331, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bB\sin(c+dx)\,\mathrm{c}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```



```
[Out] (2*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*b*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c\right.$$

$$= \frac{2bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}(5aA\right.$$

$$= \frac{2bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (Ab+aB)\int \cos^{\frac{3}{2}}(c+dx)$$

$$= \frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)\sqrt{\cos(c+dx)}}{3d}$$

$$= \frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [A] time = 0.384445, size = 86, normalized size = 0.8

$$\frac{2\left(5(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(5aB+5Ab+3bB\cos(c+dx)\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(3*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*Si
n[c + d*x]))/(15*d)
```

Maple [B] time = 3.474, size = 371, normalized size = 3.4

$$-\frac{2}{15d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,Bb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(20\,Ab+20\,aB+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+
5*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(cos(d*x
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.348 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.146305, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2748, 2641, 2639}

$$\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{3}{2}(Ab+aB)\cos(c+ \ldots}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + (Ab+aB)\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}(3aA+ \ldots$$

$$= \frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2bB\sqrt{\cos(\ldots}}{}$$

Mathematica [A] time = 0.213341, size = 67, normalized size = 0.89

$$\frac{2\left((3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+bB\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (3*a*A + b*B)*EllipticF[(c +
d*x)/2, 2] + b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 3.522, size = 326, normalized size = 4.4

$$-\frac{2}{3d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,Bb\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^{4}+3\,aA\sqrt{(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*b+B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a-2*B*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb\cos(dx+c)^{2}+Aa+(Ba+Ab)\cos(dx+c)}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/sqrt(cos(d*x
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.349 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c
+ d*x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.153949, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3021, 2748, 2641, 2639}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c
+ d*x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}(Ab+aB)-\frac{1}{2}(aA-bB)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (Ab+aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + (-aA+bB)\int \sqrt{\cos(}$$

$$= -\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aA\sin(c}{d\sqrt{\cos(c}}$$

Mathematica [A] time = 0.32913, size = 64, normalized size = 0.9

$$\frac{2\left((aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(bB-aA)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{aA\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*((-(a*A) + b*B)*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*EllipticF[(c + d
*x)/2, 2] + (a*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [B] time = 3.262, size = 244, normalized size = 3.4

$$-2\frac{Ab\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)+A\mathrm{EllipticE}\left(\cos(1/2\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a-2*A*a*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/cos(d*x + c)
^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.350 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.169901, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}(Ab+aB)+\frac{1}{2}(aA+3bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (Ab+aB)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(aA+3bB)\int \frac{}{\sqrt{c}}$$

$$= \frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin}{3d\cos^{\frac{3}{2}}}$$

Mathematica [A] time = 0.445769, size = 107, normalized size = 1.04

$$\frac{2\left((aA+3bB)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) - 3(aB+Ab)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + aA\tan(c+dx) + 3aB\sin(c+\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(-3*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (a*A + 3*
b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin[c + d*x] + 3*
a*B*Sin[c + d*x] + a*A*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 7.556, size = 428, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
```

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(A*
b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)+2*a*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/cos(d*x + c)
^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.351 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aA+5bB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a}{5a}$$

```
[Out] (-2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a*A + 5*b*
B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.188476, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aA+5bB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (-2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a*A + 5*b*
B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}(Ab+aB)+\frac{1}{2}(3aA+5bB)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (Ab+aB)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{5}(3aA+5bB)\int \frac{}{\cos}$$
$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(3aA+5bB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} +$$
$$= -\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin}{5d\cos^{\frac{5}{2}}}$$

Mathematica [A] time = 0.771732, size = 134, normalized size = 0.96

$$\frac{10(aB+Ab)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 6(3aA+5bB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 9aA\sin(2(c+dx)) + 6aA\tan}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*a*A + 5*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(A*b
+ a*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x] +
 10*a*B*Sin[c + d*x] + 9*a*A*Sin[2*(c + d*x)] + 15*b*B*Sin[2*(c + d*x)] + 6
*a*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 9.989, size = 663, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a*A/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/cos(d*x + c)
^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=264

$$\frac{2\left(9a^2A+14abB+7Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2A+14abB+7Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10\left(11a(aB+2Ab)\right.}{}$$

```
[Out] (2*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(
9*b^2*B + 11*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*(9*b
^2*B + 11*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(9
*a^2*A + 7*A*b^2 + 14*a*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(
9*b^2*B + 11*a*(2*A*b + a*B))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*
b*(11*A*b + 13*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c
+ d*x]^(7/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.381274, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3023, 2748, 2635, 2639, 2641}

$$\frac{2\left(9a^2A+14abB+7Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2A+14abB+7Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10\left(11a(aB+2Ab)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(
9*b^2*B + 11*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*(9*b
^2*B + 11*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(9
*a^2*A + 7*A*b^2 + 14*a*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(
9*b^2*B + 11*a*(2*A*b + a*B))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*
b*(11*A*b + 13*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c
+ d*x]^(7/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(11*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
```

```
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{11d}+\frac{2}{11}\int \mathrm{c}$$
$$= \frac{2b(11Ab+13aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d}+\frac{2bB\cos^{\frac{7}{2}}(c+}{}$$
$$= \frac{2b(11Ab+13aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d}+\frac{2bB\cos^{\frac{7}{2}}(c+}{}$$
$$= \frac{2\left(9a^2A+7Ab^2+14abB\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{2\left(9\right.}{}$$
$$= \frac{2\left(9a^2A+7Ab^2+14abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10\left(9b^2B+1\right.}{}$$
$$= \frac{2\left(9a^2A+7Ab^2+14abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10\left(9b^2B+1\right.}{}$$

Mathematica [A] time = 1.71103, size = 196, normalized size = 0.74

$$\frac{1200\left(11a^2B+22aAb+9b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3696\left(9a^2A+14abB+7Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3696*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2] + 1200*(22*a
*A*b + 11*a^2*B + 9*b^2*B)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]
*(154*(36*a^2*A + 43*A*b^2 + 86*a*b*B)*Cos[c + d*x] + 180*(22*a*A*b + 11*a^
2*B + 16*b^2*B)*Cos[2*(c + d*x)] + 770*b*(A*b + 2*a*B)*Cos[3*(c + d*x)] + 1
5*(1144*a*A*b + 572*a^2*B + 531*b^2*B + 21*b^2*B*Cos[4*(c + d*x)]))*Sin[c +
 d*x])/(27720*d)
```

Maple [B] time = 3.161, size = 666, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*b^2*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*A*b^2-24640*B*a*b-50400*
B*b^2)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(15840*A*a*b+24640*A*b^2+79
20*B*a^2+49280*B*a*b+56880*B*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-5544*A*a^2-23760*A*a*b-22792*A*b^2-11880*B*a^2-45584*B*a*b-34920*B*b^2)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(5544*A*a^2+18480*A*a*b+10472*A*b^2+9
240*B*a^2+20944*B*a*b+13860*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-1386*A*a^2-5280*A*a*b-1848*A*b^2-2640*B*a^2-3696*B*a*b-2790*B*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1617*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*b^2+1650*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3234*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b+825*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675*b^2*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^5+Aa^2\cos(dx+c)^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^4+\left(Ba^2+2\,Aab\right)\cos(dx+c)^3\right)\sqrt{\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^5 + A*a^2*cos(d*x + c)^2 + (2*B*a*b + A*b^2)*c
os(d*x + c)^4 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=223

$$\frac{2\left(7a^2A+10abB+5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2A+10abB+5Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2\left(9a(aB+2Ab)+7\right.}{15}$$

```
[Out] (2*(7*b^2*B + 9*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*
a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a^2*A
 + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*b^2*
B + 9*a*(2*A*b + a*B))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*A*
b + 11*a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(
5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.330271, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(7a^2A+10abB+5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2A+10abB+5Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2\left(9a(aB+2Ab)+7\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```



```
[Out] (2*(7*b^2*B + 9*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*
a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a^2*A
 + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*b^2*
B + 9*a*(2*A*b + a*B))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*A*
b + 11*a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(
5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
```

```
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{3}{2}}$$

$$= \frac{2b(9Ab+11aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bB\cos^{\frac{5}{2}}(c+d}{}$$

$$= \frac{2b(9Ab+11aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bB\cos^{\frac{5}{2}}(c+d}{}$$

$$= \frac{2\left(7a^2A+5Ab^2+10abB\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2\left(7b\right.}{}$$

$$= \frac{2\left(7b^2B+9a(2Ab+aB)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2\left(7a^2A+5Ab\right.}{}$$

Mathematica [A] time = 1.37078, size = 167, normalized size = 0.75

$$\frac{60\left(7a^2A+10abB+5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(9a^2B+18aAb+7b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(18*a*A*b + 9*a^2*B + 7*b^2*B)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^2*A
+ 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(72
*a*A*b + 36*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(84*a^2*A + 78*A*b^2 + 156*a
*b*B + 18*b*(A*b + 2*a*B)*Cos[2*(c + d*x)] + 7*b^2*B*Cos[3*(c + d*x)]))*Sin
[c + d*x])/(630*d)
```

Maple [B] time = 3.724, size = 610, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^2+1440*B*a*b+2240*B*b^2)
```

```
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1008*A*a*b-1080*A*b^2-504*B*a^2-
2160*B*a*b-2072*B*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a^2+1
008*A*a*b+840*A*b^2+504*B*a^2+1680*B*a*b+952*B*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-210*A*a^2-252*A*a*b-240*A*b^2-126*B*a^2-480*B*a*b-168*B*
b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^2*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+75*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+1
50*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^4+Aa^2\cos(dx+c)+\left(2\,Bab+Ab^2\right)\cos(dx+c)^3+\left(Ba^2+2\,Aab\right)\cos(dx+c)^2\right)\sqrt{\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

3.354 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=182

$$\frac{2\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)\sin(}{21d}$$

```
[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*b
^2*B + 7*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*b^2*B +
 7*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(7*A*b +
 9*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*b*B*Cos[c + d*x]^(3/2)
*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.31587, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)\sin(}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*b
^2*B + 7*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*b^2*B +
 7*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(7*A*b +
 9*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*b*B*Cos[c + d*x]^(3/2)
*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
```

```
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{7d}+\frac{2}{7}\int \sqrt{}$$

$$= \frac{2b(7Ab+9aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2bB\cos^{\frac{3}{2}}(c+}{}$$

$$= \frac{2b(7Ab+9aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2bB\cos^{\frac{3}{2}}(c+}{}$$

$$= \frac{2\left(5a^2A+3Ab^2+6abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5b^2B+7a\right.}{}$$

$$= \frac{2\left(5a^2A+3Ab^2+6abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5b^2B+7a\right.}{}$$

Mathematica [A] time = 1.06837, size = 139, normalized size = 0.76

$$\frac{10\left(7a^2B+14aAb+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(5\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (42*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + 10*(14*a*A*b
+ 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*b*(
A*b + 2*a*B)*Cos[c + d*x] + 5*(28*a*A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*Cos
[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] time = 3.138, size = 548, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^2-336*B*a*b-360*B*b^2)*sin
```

```
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*B*a
*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*b^2-
70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+35*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.355 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2\left(3a^2A+2abB+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a(aB+2Ab)+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(7aB+5Ab)\sin(c+dx)\sqrt{\cos}}{15d}$$

```
[Out] (2*(3*b^2*B + 5*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3*a
^2*A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(5*A*b + 7*
a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a
 + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.26649, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2990, 3023, 2748, 2641, 2639}

$$\frac{2\left(3a^2A+2abB+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a(aB+2Ab)+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(7aB+5Ab)\sin(c+dx)\sqrt{\cos}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (2*(3*b^2*B + 5*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3*a
^2*A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(5*A*b + 7*
a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a
 + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bB\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{1}{2}a(5aA+bB)$$

$$= \frac{2b(5Ab+7aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bB\sqrt{\cos(c+dx)}(a+b\cos}{5d}$$

$$= \frac{2b(5Ab+7aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bB\sqrt{\cos(c+dx)}(a+b\cos}{5d}$$

$$= \frac{2\left(3b^2B+5a(2Ab+aB)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(3a^2A+Ab^2+2abB\right)F}{3d}$$

Mathematica [A] time = 0.575285, size = 106, normalized size = 0.76

$$\frac{2\left(5\left(3a^2A+2abB+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3\left(5a^2B+10aAb+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\sqrt{\cos(c+dx)}(10}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (2*(3*(10*a*A*b + 5*a^2*B + 3*b^2*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*A
 + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(5*A*b
 + 10*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 3.194, size = 487, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*b^2*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c) + A)(b\cos(dx+c) + a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

3.356 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{2\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+d}{3}$$

```
[Out] (-2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b +
3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.245916, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2988, 3023, 2748, 2641, 2639}

$$\frac{2\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+d}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b +
3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2\int \frac{-\frac{1}{2}a(2Ab+aB)+\frac{1}{2}\left(a^2A-Ab^2-2abB\right)\cos(c}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{4}{3}\int \frac{\frac{1}{4}\left(-b^2B-3a\right.}{}$$

$$= \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \left(a^2A-Ab^2-2abB\right.$$

$$= -\frac{2\left(a^2A-Ab^2-2abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(6aAb+3a^2B+b^2B\right)F\left(\frac{1}{2}\right.}{3d}$$

Mathematica [A] time = 0.605016, size = 102, normalized size = 0.84

$$\frac{2\left(\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(-3a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(3a^2A+b^2B\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (2*((-3*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3
*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + ((3*a^2*A + b^2*B*Cos[c + d*x])
*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)
```

Maple [B] time = 3.398, size = 404, normalized size = 3.3

$$-\frac{2}{3d}\left(4b^2B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+6\,Aab\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*b^2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2-6*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*B*a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

3.357 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{2\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(aB+2Ab)\text{si}}{d\sqrt{\cos(c+}}$$

```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*
A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(3*d*Cos[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

Rubi [A] time = 0.305041, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2988, 3021, 2748, 2641, 2639}

$$\frac{2\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(aB+2Ab)\text{si}}{d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*
A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(3*d*Cos[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
```

```
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}a(2Ab+aB)-\frac{1}{2}\left(a^2A+3Ab^2+6abB\right)\cos}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{4}{3}\int \frac{\frac{1}{4}\left(-a^2A-3Ab^2\right.}{}$$

$$= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{3}\left(-a^2A-3Ab^2-6a\right.$$

$$= -\frac{2\left(2aAb+a^2B-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2A+3Ab^2+6abB\right)F\left(\right.}{3d}$$

Mathematica [A] time = 1.11881, size = 105, normalized size = 0.83

$$\frac{2\left(\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)(3(aB+2Ab)\cos(c+dx)+aA)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (2*(-3*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b
^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c +
 d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 7.309, size = 677, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*B*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a*(2*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)+2*a^2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
```

```
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

3.358 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(a^2B+2aAb+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2A+10abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2A+10abB+5Ab^2\right)\sin(}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2
*a*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```

Rubi [A] time = 0.350534, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2988, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(a^2B+2aAb+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2A+10abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2A+10abB+5Ab^2\right)\sin(}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (-2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2
*a*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```


Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
```

```
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2Ab+aB) - \frac{1}{2}\left(3a^2A+5Ab^2+10abB\right)\cdots}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}\left(3a^2A+5A\cdots\right.}{\cdots}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}\left(-3a^2A-5Ab^2-1\cdots\right.$$

$$= \frac{2\left(2aAb+a^2B+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab\cdots}{3d\cdots}$$

$$= -\frac{2\left(3a^2A+5Ab^2+10abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(2aAb+a^2B+3b^2B\right)\cdots}{3d}$$

Mathematica [A] time = 1.05638, size = 175, normalized size = 1.02

$$\frac{10\left(a^2B+2aAb+3b^2B\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 6\left(3a^2A+10abB+5Ab^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 9a^2A\cdots}{15d\,\mathrm{co}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (-6*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2
, 2] + 10*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x
)/2, 2] + 20*a*A*b*Sin[c + d*x] + 10*a^2*B*Sin[c + d*x] + 9*a^2*A*Sin[2*(c
+ d*x)] + 15*A*b^2*Sin[2*(c + d*x)] + 30*a*b*B*Sin[2*(c + d*x)] + 6*a^2*A*T
an[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 10.05, size = 750, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b+2*B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)-2/5*a^2*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin
(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

3.359 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=305

$$\frac{2\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(27a^2Ab+9a^3B+21ab^2B+7Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b}{}$$

```
[Out] (2*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2, 2])
/(15*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(27*a^2*A*b + 7*A*b^3 + 9*
a^3*B + 21*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(33*a*A*
b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*b^2*(1
1*A*b + 15*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c + d*
x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.5446, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(27a^2Ab+9a^3B+21ab^2B+7Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2, 2])
/(15*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(27*a^2*A*b + 7*A*b^3 + 9*
a^3*B + 21*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(33*a*A*
b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*b^2*(1
1*A*b + 15*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c + d*
x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{2}{11}\int \mathrm{co}$$

$$= \frac{2b^2(11Ab+15aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2bB\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{2b\left(33aAb+26a^2B+9b^2B\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{77d} + \frac{2b^2}{}$$

$$= \frac{2b\left(33aAb+26a^2B+9b^2B\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{77d} + \frac{2b^2}{}$$

$$= \frac{2\left(77a^3A+165aAb^2+165a^2bB+45b^3B\right)\sqrt{\cos(c+dx)}\sin}{231d}$$

$$= \frac{2\left(27a^2Ab+7Ab^3+9a^3B+21ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2\,(}{}$$

Mathematica [A] time = 1.90529, size = 235, normalized size = 0.77

$$\frac{240\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3696\left(27a^2Ab+9a^3B+21ab^2B+7Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3696*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2,
2] + 240*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c + d
*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(154*(108*a^2*A*b + 43*A*b^3 + 36*a^3*B +
```

```
129*a*b^2*B)*Cos[c + d*x] + 180*b*(33*a*A*b + 33*a^2*B + 16*b^2*B)*Cos[2*(c
 + d*x)] + 770*b^2*(A*b + 3*a*B)*Cos[3*(c + d*x)] + 15*(616*a^3*A + 1716*a*
A*b^2 + 1716*a^2*b*B + 531*b^3*B + 21*b^3*B*Cos[4*(c + d*x)]))*Sin[c + d*x]
)/(27720*d)
```

Maple [B] time = 3.362, size = 825, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*A*b^3-36960*B*a*b^2-5040
0*B*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(23760*A*a*b^2+24640*A*b^
3+23760*B*a^2*b+73920*B*a*b^2+56880*B*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x
+1/2*c)+(-16632*A*a^2*b-35640*A*a*b^2-22792*A*b^3-5544*B*a^3-35640*B*a^2*b-
68376*B*a*b^2-34920*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(4620*A*
a^3+16632*A*a^2*b+27720*A*a*b^2+10472*A*b^3+5544*B*a^3+27720*B*a^2*b+31416*
B*a*b^2+13860*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310*A*a^3-4
158*A*a^2*b-7920*A*a*b^2-1848*A*b^3-1386*B*a^3-7920*B*a^2*b-5544*B*a*b^2-27
90*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1155*A*a^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+2475*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6237*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a^2*b-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+2475*a^2*b*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+675*B*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^3-4851*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^5+Aa^3\cos(dx+c)+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^4+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^3+\left(Ba^3+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^5 + A*a^3*cos(d*x + c) + (3*B*a*b^2 + A*b^3)*c
os(d*x + c)^4 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^3 + (B*a^3 + 3*A*a^2*b)*
cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.360 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=255

$$\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(22\right.}{}$$

```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c +
d*x)/2, 2])/(21*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*A*b + 13*a*B)*Cos[c + d*
x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.496418, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(22\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```


```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c +
d*x)/2, 2])/(21*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*A*b + 13*a*B)*Cos[c + d*
x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(9*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
```

```
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{9d}+\frac{2}{9}\int \sqrt{c$$

$$= \frac{2b^2(9Ab+13aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2bB\cos^{\frac{3}{2}}(c+$$

$$= \frac{2b\left(27aAb+22a^2B+7b^2B\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{2b^2$$

$$= \frac{2b\left(27aAb+22a^2B+7b^2B\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{2b^2$$

$$= \frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2$$

$$= \frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2$$

Mathematica [A] time = 1.15175, size = 197, normalized size = 0.77

$$\frac{60\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2
] + 60*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c + d*x)/2,
 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c + d*
x] + 5*(252*a^2*A*b + 78*A*b^3 + 84*a^3*B + 234*a*b^2*B + 18*b^2*(A*b + 3*a
*B)*Cos[2*(c + d*x)] + 7*b^3*B*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 3.348, size = 745, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^3+2160*B*a*b^2+2240*B*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-1512*B
*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12
60*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+952*B
*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b^2-240
*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-315*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-567*A*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-567*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b-147*B*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

3.361 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=205

$$\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^2Ab+5a^3B+9ab^2B+3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(18\cdots}{\cdots}$$

```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d)
```

Rubi [A] time = 0.477762, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^2Ab+5a^3B+9ab^2B+3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(18\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```



```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
```

```
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bB\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \frac{(a+b\cos(c+$$

$$= \frac{2b^2(7Ab+11aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bB\sqrt{\cos(c+dx)}(a+b\,c}{7a}$$

$$= \frac{2b\left(21aAb+18a^2B+5b^2B\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b^2(7Ab+11}{}$$

$$= \frac{2b\left(21aAb+18a^2B+5b^2B\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b^2(7Ab+11}{}$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(21a^3A+21a\right.}{}$$

Mathematica [A] time = 1.26794, size = 158, normalized size = 0.77

$$\frac{10\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\left(15a^2Ab+5a^3B+9ab^2B+3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (42*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2]
+ 10*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*x)/2,
2] + b*Sqrt[Cos[c + d*x]]*(42*b*(A*b + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b +
42*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] time = 3.418, size = 664, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^3-504*B*a*b^2-360*B*b^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*b+5
04*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a*b^2
-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*b^3+105*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+1
05*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa^2b\right)\cos}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

3.362 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=202

$$\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b\left(6a^2A-\right.}{}$$

```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) - (2*b*(6*a^2*A - A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*d) - (2*b^2*(5*a*A - b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) +
 (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.463768, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b\left(6a^2A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```



```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) - (2*b*(6*a^2*A - A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*d) - (2*b^2*(5*a*A - b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) +
 (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
```

```
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+dx))\left(\frac{1}{2}a(5Ab\right.}{}$$

$$= -\frac{2b^2(5aA-bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aA(a+b\cos(c+dx))^2\sin}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2b\left(6a^2A-Ab^2-3abB\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(5aA-bB)\,\mathrm{co}}{}$$

$$= -\frac{2b\left(6a^2A-Ab^2-3abB\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(5aA-bB)\,\mathrm{co}}{}$$

$$= -\frac{2\left(5a^3A-15aAb^2-15a^2bB-3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(9a^2Ab+A\right.}{}$$

Mathematica [A] time = 1.08531, size = 150, normalized size = 0.74

$$\frac{10\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(-30a^3A+90a^2bB+90aAb^2+18b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)(}{}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((-30*a^3*A + 90*a*A*b^2 + 90*a^2*b*B + 18*b^3*B)*EllipticE[(c + d*x)/2, 2]
 + 10*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2] +
 ((10*b^2*(A*b + 3*a*B)*Cos[c + d*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2*(c
 + d*x)]))*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(15*d)
```

Maple [B] time = 3.497, size = 867, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*b^2*(5*A*b+15*B*a+6*B*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^3+5*A*b
^3+15*B*a*b^2+3*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+45*A*a^2*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3-45*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*B*a*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-45*B*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2*b-9*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\cos}{\cos(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

3.363 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2Ab+a^3B-3ab^2B-Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2(3aB+7Ab}{3d\sqrt{\cos(c}}$$

```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/d +
(2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*a^2*(7*A*b + 3*a*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(
a*A - b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.465498, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2Ab+a^3B-3ab^2B-Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2(3aB+7Ab}{3d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/d +
(2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*a^2*(7*A*b + 3*a*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(
a*A - b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
```

```
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(c+dx))\left(\frac{1}{2}a(7A\right.}{}$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} -$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aA-bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aA-bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

$$= -\frac{2\left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^3A+9aAb^2\right.}{}$$

Mathematica [A] time = 1.03276, size = 165, normalized size = 0.86

$$\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(3a^2Ab+a^3B-3ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 18*a^2*A*b*Sin[c + d*x] + 6*a^3*B*Sin[c + d*x
] + b^3*B*Sin[2*(c + d*x)] + 2*a^3*A*Tan[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 9.1, size = 1212, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(8*B*b^3*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*
x+1/2*c)^2+18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+18*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-6*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*b^3*sin(1/2*d*x+1/2*c)^2-36*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+2*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*s
in(1/2*d*x+1/2*c)^2-18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^
2-12*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-8*B*b^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*a*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3+2*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*A*a^2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+9*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b^2+6*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*B*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\mathrm{co}}{\cos(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

3.364 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(3a^2A\right.}{}$$

```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2
)) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
)
```

Rubi [A] time = 0.48152, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3031, 3021, 2748, 2641, 2639}

$$\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(3a^2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2
)) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
)
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
```

```
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c+dx))\left(\frac{1}{2}a(9A\right.}{}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{}{1}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(3a^2A+14Ab^2+15abB\right)\sin(c+dx}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(3a^2A+14Ab^2+15abB\right)\sin(c+dx}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\left(3a^3A+15aAb^2+15a^2bB-5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(3a^2Ab+\right.}{}$$

Mathematica [A] time = 2.08235, size = 176, normalized size = 0.86

$$\frac{9a\left(a^2A+5abB+5Ab^2\right)\sin(2(c+dx))+10\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(3a^3A+\right.}{15d\cos^{\frac{3}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (-6*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Cos[c + d*x]^(3/2)*Ellipt
icE[(c + d*x)/2, 2] + 10*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*Cos[c +
d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*(3*A*b + a*B)*Sin[c + d*x] +
9*a*(a^2*A + 5*A*b^2 + 5*a*b*B)*Sin[2*(c + d*x)] + 6*a^3*A*Tan[c + d*x])/(1
5*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 10.698, size = 997, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
6*a*b*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1)+2*a^2*(3*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2
/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*
c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3\cos\left(dx+c\right)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+3\left(Ba^2b+Aab^2\right)\cos\left(dx+c\right)^2+\left(Ba^3+3\,Aa^2b\right)\text{co}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right)+a)^3}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

3.365 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=182

$$\frac{2\left(3a^2+b^2\right)(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}-\frac{2a^3(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\right.}{b^4d(a+b)}$$

```
[Out] (-2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2
*(3*a^2 + b^2)*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) - (2*a^3*(A
*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(
A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.819539, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(3a^2+b^2\right)(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}-\frac{2a^3(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\right.}{b^4d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```


```
[Out] (-2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2
*(3*a^2 + b^2)*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) - (2*a^3*(A
*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(
A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(5*b*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
```

```
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aB}{2}+\frac{3}{2}bB\cos(c+dx)+\frac{5}{2}(Ab-aB)\cos^2(c+dx\right.}{a+b\cos(c+dx)}}{5b}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{4\int \frac{\frac{5}{4}a(}{}}{}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} - \frac{4\int \frac{-\frac{5}{4}}{}}{}$$

$$= -\frac{2\left(5aAb-5a^2B-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+}{3b^2d}$$

$$= -\frac{2\left(5aAb-5a^2B-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2\left(3a^2+b^2\right)(Ab-aB)F\left(\frac{1}{2}(c+\right.}{3b^4d}$$

Mathematica [A] time = 2.37185, size = 264, normalized size = 1.45

$$\frac{\frac{2b^2\left(5a^2B-5aAb+9b^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6\left(5a^2B-5aAb+3b^2B\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right.}{a\sqrt{\sin^2(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*b^2*(-5*a*A*b + 5*a^2*B + 9*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b) + 2*b^2*(5*A*b + 4*a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*b^2*Sqrt[Cos[c + d*
x]]*(5*A*b - 5*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x] + (6*(-5*a*A*b + 5*a^
2*B + 3*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
 b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(
```

```
b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^
2]))/(30*b^4*d)
```

Maple [B] time = 4.116, size = 1074, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*B*a*b^3
+24*B*b^4)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*a*b^3-20*A*b^4-20*
B*a^2*b^2+44*B*a*b^3-24*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10
*A*a*b^3+10*A*b^4+10*B*a^2*b^2-16*B*a*b^3+6*B*b^4)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2*b^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-5*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^3*b-15*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^4+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2*b^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-15*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^3*b+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b^3+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))*a^4)/b^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.366 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{2\left(-3a^2B+3aAb-b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\,\mathrm{si}}{}$$

```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B -
 b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) + (2*a^2*(A*b - a*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.513183, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(-3a^2B+3aAb-b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B -
 b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) + (2*a^2*(A*b - a*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*b*d)
```


Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{aB}{2}+\frac{1}{2}bB\cos(c+dx)+\frac{3}{2}(Ab-aB)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{2\int \frac{-\frac{1}{2}abB+\frac{1}{2}\left(3aAb-3a^2B-b^2B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b^2} + \frac{(Ab-$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\left(a^2(Ab-aB)\right)\int$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} - \frac{2\left(3aAb-3a^2B-b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d} + \frac{2a^2($$

Mathematica [A] time = 1.41444, size = 209, normalized size = 1.53

$$\frac{-\frac{3(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}} + \frac{(3Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(\right.}{a+b}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (((3*A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + B*(2*E
llipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b)) + 2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (3*(A*b - a*B)*(2*a*b*Elli
pticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[C
os[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/(3*b*d)
```

Maple [B] time = 3.556, size = 786, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-4*B*a*b^2+4*
B*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(2*B*a*b^2-2*B*b^3)*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*a*b^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*b^3-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b-3*a^3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+3*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+B*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))*a^3)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.367 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) - (2*a*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(b^2*(a + b)*d)
```

Rubi [A] time = 0.209688, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3002, 2639, 2803, 2641, 2805}

$$\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) - (2*a*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(b^2*(a + b)*d)
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{(-Ab+aB)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}$$

$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{\cos(c+dx)}(a}}{b^2}$$

$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} - \frac{2a(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c}{b^2(a+b)d}$$

Mathematica [A] time = 0.862844, size = 131, normalized size = 1.47

$$\frac{Ab\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)-\frac{2B\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-a\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+bE\left(\sin^{-1}\left(\sqrt{\mathrm{co}}\right.\right.\right.}{\sqrt{\sin^2(c+dx)}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (A*b*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) - (2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - a*EllipticPi[-(b/a), -ArcS
in[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(b^2*d)
```

Maple [A] time = 3.138, size = 295, normalized size = 3.3

$$-2\,{\frac {\sqrt {\left(2\, \left(\cos \left(1/2\,dx+c/2\right)\right)^{2}-1\right) \left(\sin \left(1/2\,dx+c/2\right)\right)^{2}}\sqrt {\left(\sin \left(1/2\,dx+c/2\right)\right)^{2}}\sqrt {-2\, \left(\cos \left(1/2\,dx+c/2\right)\right)^{2}+1}}{\left(a-b\right){b}^{2}\sqrt {-2\, \left(\sin \left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin \left(1/2\,dx+c/2\right)\right)^{2}}\sin \left(1/2\,dx+c/2\right)\sqrt {2\, \left(\cos \left(1/2\,dx+c/2\right)\right)^{2}-1}d}} \Big(A{\rm EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-A*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+B*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.368 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=61

$$\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(b*(a + b)*d)
```

Rubi [A] time = 0.143713, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3002, 2641, 2805}

$$\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(b*(a + b)*d)
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{(-Ab+aB)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b}$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}$$

Mathematica [A] time = 0.210523, size = 58, normalized size = 0.95

$$\frac{2\left((Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)+B(a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*((a + b)*B*EllipticF[(c + d*x)/2, 2] + (A*b - a*B)*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]))/(b*(a + b)*d)
```

Maple [A] time = 3.542, size = 217, normalized size = 3.6

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{b\,(a-b)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\Big(A\mathrm{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*Elli
```

```
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))*a)/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

3.369 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=86

$$-\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a*(a + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c
+ d*x]])
```

Rubi [A] time = 0.309194, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3000, 3059, 2639, 12, 2805}

$$-\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a*(a + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c
+ d*x]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx &= \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{1}{2}aA\cos(c+dx)-\frac{1}{2}Ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}\\
&= \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{A\int\sqrt{\cos(c+dx)}\,dx}{a}-\frac{2\int \frac{b(Ab-aB)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{ab}\\
&= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}\\
&= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 2.40571, size = 210, normalized size = 2.44

$$-\frac{-\frac{2A\sin(c+dx)\left((2a^2-b^2)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{2(2aB-3Ab)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\right.}{a+b}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*(-3*A*b + 2*a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (
2*a*A*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(a + b)))/b + (4*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (2*A*(-2*a*
b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[
Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a*d)
```

Maple [B] time = 6.233, size = 327, normalized size = 3.8

$$-\frac{1}{d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,\frac{(-Ab+aB)\,b\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+}}{a\left(-2\,ab+2\,b^2\right)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b+B*a)/a
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

3.370 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=150

$$\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2A}{3ad}$$

```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*b*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*(A
*b - a*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.770002, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2A}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```


```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*b*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*(A
*b - a*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-\frac{3}{2}(Ab-aB)+\frac{1}{2}aA\cos(c+dx)+\frac{1}{2}Ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(a^2A+3Ab^2-3abB\right)+\frac{1}{4}a(4Ab-3aB)}{\sqrt{\cos(c+dx)}(a+}}{3a^2}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b\left(a^2A+3Ab^2-3abB\right)-\frac{1}{4}aAb^2\cos(}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3a^2b}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{A}{-}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{a^2(a+b)d}$$

Mathematica [A] time = 2.20994, size = 262, normalized size = 1.75

$$\frac{\frac{2a\left(2a^2A-9abB+9Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{6(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\right.}{b\sqrt{\sin^2(c+dx)}}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*a*(2*a^2*A + 9*A*b^2 - 9*a*b*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + (a*(8*a*A*b - 6*a^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (4*a^2*A*Sin[c + d*x
])/Cos[c + d*x]^(3/2) + (12*a*(-(A*b) + a*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x
]] - (6*(A*b - a*B)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*
(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c +
d*x]^2]))/(6*a^3*d)
```

Maple [B] time = 9.544, size = 468, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b-B*a)*b^
2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a^2*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

3.371 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$-\frac{\left(9a^3Ab+16a^2b^2B-15a^4B-12aAb^3+2b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}+\frac{\left(3a^2Ab-5a^3B+4ab^2B-2Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\ldots$$

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^
3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b
^4*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^4*(a + b)^2*d) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.933082, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2989, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(9a^3Ab+16a^2b^2B-15a^4B-12aAb^3+2b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}+\frac{\left(3a^2Ab-5a^3B+4ab^2B-2Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^
3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b
^4*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^4*(a + b)^2*d) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(Ab-aB)+b(Ab-aB)\cos(c+dx)+\right.}{a+b\cos(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{b\left(a^2-b^2\right)d(a+b\cos(}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{b\left(a^2-b^2\right)d(a+b\cos(}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{\left(3aAb-5a^2B+2b^2B\right)\sqrt{\mathrm{c}}}{3b^2\left(a^2-\right.}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{\left(9a^3Ab-12aAb^3-15a^4B\right.}{3b}$$

Mathematica [A] time = 3.0681, size = 322, normalized size = 1.06

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3a^2(aB-Ab)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2B\right) - \frac{\frac{2\left(-3a^2Ab+5a^3B-8ab^2B+6Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(2a^2B-3aAb+b^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\right.\right.}{a+b}}{}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*B + (3*a^2*(-(A*b) + a*B))/((a^2 - b^2)*(a + b*Cos
[c + d*x])))*Sin[c + d*x] - ((2*(-3*a^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B
)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-3*a*A*b + 2*a^2
*B + b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + (6*(-3*a^2*A*b + 2*A*b^3 + 5*a^3*B - 4*a*b^2*
B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Elliptic
F[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a
 - b)*(a + b)))/(12*b^2*d)
```

Maple [B] time = 11.879, size = 1066, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*B*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-9*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b-
B*a)/b^4*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
```

```
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

3.372 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=224

$$\frac{\left(a^2Ab-3a^3B+4ab^2B-2Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-3a^2B+aAb+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(a^2Ab-3a^3B+5ab^2B-\right.}{b^3d(a-}$$

```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*
d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B + 4*a*b^2*B)*EllipticF[(c + d*x)/2, 2])
/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a
*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.627027, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2Ab-3a^3B+4ab^2B-2Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-3a^2B+aAb+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(a^2Ab-3a^3B+5ab^2B-\right.}{b^3d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```



```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*
d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B + 4*a*b^2*B)*EllipticF[(c + d*x)/2, 2])
/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a
*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
```

```
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}a(Ab-aB)+b(Ab-aB)\cos(c+dx)+\frac{1}{2}\left(aAb-3a^2B \cdots\right.}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}ab(Ab-aB)+\frac{1}{2}\left(a^2Ab-2Ab^3-3a^3B+4ab^2B\right)\cos(\cdots}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{\left(a^2Ab-2Ab^3-3a^3B+4ab^2B\right)F\left(\frac{1}{2} \cdots\right.}{b^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 2.58955, size = 284, normalized size = 1.27

$$\frac{\frac{\frac{2\left(a^2B+aAb-2b^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{2\left(3a^2B-aAb-2b^2B\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}}+\cdots}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((-4*a*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*
Cos[c + d*x])) + ((2*(a*A*b + a^2*B - 2*b^2*B)*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(a + b) + (8*(-(A*b) + a*B)*((a + b)*EllipticF[(c + d*x)/2,
2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(-(a*A*b) +
 3*a^2*B - 2*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[
c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Maple [B] time = 9.332, size = 849, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-2
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-
B*a)/b^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

3.373 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=198

$$\frac{\left(a^2B+aAb-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(a^2Ab+a^3B-3ab^2B+Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

```
[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B
 - 2*b^2*B)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A*
b^3 + a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b
)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.540471, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2999, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2B+aAb-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(a^2Ab+a^3B-3ab^2B+Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B
 - 2*b^2*B)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A*
b^3 + a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b
)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(Ab-aB)-(aA-bB)\cos(c+dx)-\frac{1}{2}(Ab-aB)\,\text{co}}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{-a^2+b^2}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}b(Ab-aB)+\frac{1}{2}\left(aAb+a^2B-2b^2B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b\left(a^2-b^2\right)}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(aAb+a\right.}{}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(aAb+a^2B-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{\left(a^2Ab\right.}{}$$

Mathematica [A] time = 2.29557, size = 262, normalized size = 1.32

$$\frac{\frac{4(aB-Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} - \frac{-\frac{2(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}} + \frac{2(aB-}{}}{(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((4*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos
[c + d*x])) - ((2*(-(A*b) + a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b) + ((4*a*A - 4*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (2*(A*b - a*B)*(2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/
(4*d)
```

Maple [B] time = 8.609, size = 808, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-1/a*b^2/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2
-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

3.374 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2Ab+a^3(-B)-ab^2B-Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}+\frac{b(A}{a}$$

```
[Out] -(((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)
*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*B
 - a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x]))
```

Rubi [A] time = 0.618138, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3000, 3059, 2639, 3002, 2641, 2805}

$$-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2Ab+a^3(-B)-ab^2B-Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}+\frac{b(A}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -(((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)
*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*B
 - a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
```

```
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2A-Ab^2-abB\right)-a(Ab-aB)\cos(c+dx)-\frac{1}{2}b(A}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}b\left(2a^2A-Ab^2-abB\right)+\frac{1}{2}ab(Ab-aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{ab\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(Ab-a}{2}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(3a^2Ab-Ab^3-a\right.}{a(}$$

Mathematica [A] time = 2.55624, size = 276, normalized size = 1.38

$$\frac{\frac{4b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(4a^2A-abB-3Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\right.}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] ((4*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[
c + d*x])) + ((2*(4*a^2*A - 3*A*b^2 - a*b*B)*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2])/(a + b) + (4*a*(-(A*b) + a*B)*(2*EllipticF[(c + d*x)/2, 2] - (
2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(A*b - a*B)
*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[
Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b
)*(a + b)))/(4*a*d)
```

Maple [B] time = 7.84, size = 721, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

---

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

3.375 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=256

$$\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{\left(5a^2Ab-3a^3B+ab^2B-3Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{a^2d(a-b)(a+b)^2}$$

```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*
d)) + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - ((5*a^2*A
*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.922918, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{\left(5a^2Ab-3a^3B+ab^2B-3Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```


```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*
d)) + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - ((5*a^2*A
*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
```

```
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2A-3Ab^2+abB\right)-a(Ab-aB)\cos}{\cos^{\frac{3}{2}}(c+dx)(a+b}}{a\left(a^2-\right.}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$= -\frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{(5a^2}{}$$

Mathematica [A] time = 3.96245, size = 320, normalized size = 1.25

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{b^2(Ab-aB)\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}+2A\tan(c+dx)\right) - \frac{\frac{2\left(-10a^2Ab+4a^3B-3ab^2B+9Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{8a\left(a^2A+abB-2Ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-\right.}{b(a+b)}}{}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] (-(((2*(-10*a^2*A*b + 9*A*b^3 + 4*a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(a + b) - (8*a*(a^2*A - 2*A*b^2 + a*b*B)*((a + b)*Elli
pticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a
 + b)) - (2*(2*a^2*A - 3*A*b^2 + a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2
*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*
x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]*
((b^2*(A*b - a*B)*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])) + 2*A*T
an[c + d*x]))/(4*a^2*d)
```

Maple [B] time = 10.801, size = 883, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^2/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2/a^2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b+B*a)/a*(-1/a*b^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos
(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*
```

```
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

3.376 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=345

$$\frac{\left(2a^2A+3abB-5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{\left(4a^2Ab-2a^3B+3ab^2B-5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{b\left(7a^2Ab-5a^3B+3ab^2\cdots\right.}{a^3d(\cdots}$$

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(a^
3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*EllipticF[(c + d*x)/2, 2]
)/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3
/2)) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(a^3*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.29213, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(2a^2A+3abB-5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{\left(4a^2Ab-2a^3B+3ab^2B-5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{b\left(7a^2Ab-5a^3B+3ab^2\cdots\right.}{a^3d(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(a^
3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*EllipticF[(c + d*x)/2, 2]
)/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3
/2)) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(a^3*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2A-5Ab^2+3abB\right)-a(Ab-aB)\cos(\ldots}{\cos^{\frac{5}{2}}(c+dx)(a+b\,c\ldots}}{a\left(a^2-\ldots\right.}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+\ldots}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sin(c+\ldots}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sin(c+\ldots}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2+3abB\right)\sin\ldots}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c\ldots}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2+3abB\right)F\left(\ldots\right.}{3a^2\left(a^2-b^2\right)\ldots}$$

Mathematica [A] time = 6.31555, size = 367, normalized size = 1.06

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{3b^3(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2\tan(c+dx)(aA\sec(c+dx)+3aB-6Ab)\right)+\frac{\frac{2\left(44a^2Ab^2+4a^4A-30a^3bB+27ab^3B-45Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(\ldots\right.}{a+b}}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] (((2*(4*a^4*A + 44*a^2*A*b^2 - 45*A*b^4 - 30*a^3*b*B + 27*a*b^3*B)*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*a*(-7*a^2*A*b + 10*A*b^3 +
3*a^3*B - 6*a*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) - (6*(-4*a^2*A*b + 5*A*b^3 + 2*a^3
*B - 3*a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-
```

```
(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*
x]^2]))/((a - b)*(a + b)) + 4*Sqrt[Cos[c + d*x]]*((3*b^3*(A*b - a*B)*Sin[c
+ d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) + 2*(-6*A*b + 3*a*B + a*A*Sec[c
+ d*x])*Tan[c + d*x]))/(12*a^3*d)
```

Maple [B] time = 16.085, size = 1031, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(2*A*b-B
*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-2*A*b+B*a)/a^3*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(A*b-B*a)*b/a^2*
(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-
b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^2*(-1/6*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

3.377 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=367

$$\frac{\left(-5a^2Ab^3+3a^4Ab+33a^3b^2B-15a^5B-24ab^4B+8Ab^5\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{\left(3a^3Ab+29a^2b^2B-15a^4B-9aAb^3-8b^4B\right.}{4b^3d\left(a^2-b^2\right)^2}$$

```
[Out] -((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE[(c
+ d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5
 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4*B)*EllipticF[(c + d*x)/2, 2])/(4*b^4*
(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a
^3*b^2*B - 35*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b
)^2*b^4*(a + b)^3*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B +
11*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
Cos[c + d*x]))
```

Rubi [A] time = 1.01286, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2989, 3047, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-5a^2Ab^3+3a^4Ab+33a^3b^2B-15a^5B-24ab^4B+8Ab^5\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{\left(3a^3Ab+29a^2b^2B-15a^4B-9aAb^3-8b^4B\right.}{4b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE[(c
+ d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5
 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4*B)*EllipticF[(c + d*x)/2, 2])/(4*b^4*
(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a
^3*b^2*B - 35*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b
)^2*b^4*(a + b)^3*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B +
11*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
Cos[c + d*x]))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(Ab-aB)+2b(Ab-aB)\cos(c+dx)\right.}{(a+b\cos(c+dx)}}{2b\left(a^2-b^2\right.}$$

$$= \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2B\right)\sqrt{\cos}}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2B\right)\sqrt{\cos}}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} + \frac{a(Ab-aB)\,\mathrm{c}}{2b\left(a^2-b^2\right.}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} + \frac{\left(3a^4Ab-5a\right.}{}$$

Mathematica [A] time = 4.6716, size = 394, normalized size = 1.07

$$\frac{\frac{\left(-a^3Ab-7a^2b^2B+5a^4B-5aAb^3+8b^4B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(-3a^3Ab-29a^2b^2B+15a^4B+9aAb^3+8b^4B\right)\sin(c+d}{}}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((-2*a*Sqrt[Cos[c + d*x]]*(a*(-(a^2*A*b) + 7*A*b^3 + 5*a^3*B - 11*a*b^2*B)
+ b*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x
])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((-(a^3*A*b) - 5*a*A*b^3 + 5*a
^4*B - 7*a^2*b^2*B + 8*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
 + b) + (8*(a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*((a + b)*EllipticF[(c +
d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((-3*a
^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8
*b^2*d)
```

Maple [B] time = 14.966, size = 1977, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-3
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+12*a/b^3*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2/b^4*(3
*A*b-4*B*a)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(si
```

n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a^3*(A
*b-B*a)/b^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*
a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-
b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x
)
```

3.378 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=344

$$\frac{\left(a^3Ab-5a^2b^2B+3a^4B-7aAb^3+8b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(-10a^2\right.}{}$$

```
[Out] -((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*b
^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7*a*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4
*B)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*a^2
*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + (a*(A*b - a*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((
a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.989675, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2989, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^3Ab-5a^2b^2B+3a^4B-7aAb^3+8b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(-10a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*b
^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7*a*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4
*B)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*a^2
*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + (a*(A*b - a*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((
a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a(Ab-aB)+2b(Ab-aB)\cos(c+dx)-\frac{1}{2}\left(aAb+3\right.}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos}}{4b\left(a^2-b^2\right)^2 d(a+b\cos(}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos}}{4b\left(a^2-b^2\right)^2 d(a+b\cos(}$$

$$= -\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{a(Ab-aB)\sqrt{\cos(c+dx}}{2b\left(a^2-b^2\right)d(a+b\cos}$$

$$= -\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(a^3Ab-7aAb^3+3a^4B\right.}{4b^3}$$

Mathematica [A] time = 3.38592, size = 364, normalized size = 1.06

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)\cos(c+dx)+a\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\frac{\left(-5a^2Ab+a^3B+5ab^2B-Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{8\left(a^2B-3aAb+2b^2B\right)}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(a*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B) + b*(a^
2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) - (((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(-3*a*A*b + a^2*B
+ 2*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b),
 (c + d*x)/2, 2]))/(a + b) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^
2*(a + b)^2))/(8*b*d)
```

Maple [B] time = 14.346, size = 1937, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a/b^3*(2*A*b-3*B*a)*(-1/a*b
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*
```

```
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/
a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*a^2*(A*b-B*a)/b^3*(-1/2/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b
^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^
2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^
2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^
2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^
2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2
))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

3.379 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=337

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(5a^2Ab+a^3(-B)-5ab^2B+Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(10a^2Ab^3+3a^4\right.}{}$$

```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a*b*
(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*EllipticF[(c
+ d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
+ a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*a^2*A*b + A*b^3 - a
^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a
+ b*Cos[c + d*x]))
```

Rubi [A] time = 0.919423, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2999, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(5a^2Ab+a^3(-B)-5ab^2B+Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(10a^2Ab^3+3a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```



```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a*b*
(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*EllipticF[(c
+ d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
+ a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*a^2*A*b + A*b^3 - a
^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a
+ b*Cos[c + d*x]))
```


Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(Ab-aB)-2(aA-bB)\cos(c+dx)+\frac{1}{2}(Ab-aB)\cos}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)\sqrt{\cos(c}}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)\sqrt{\cos(c}}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} - \frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(}{2\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7ab\right.}{4b^2\left(a^2-b^2\right.}$$

Mathematica [A] time = 4.19748, size = 369, normalized size = 1.09

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(-5a^2Ab+a^3B+5ab^2B-Ab^3\right)\cos(c+dx)+a\left(-7a^2Ab+3a^3B+3ab^2B+Ab^3\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\frac{2\left(-9a^2Ab+5a^3B+ab^2B+3Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16a\left(2a^2A-3\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(a*(-7*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B) + b*(-
5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^
2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*a*(2*a^2*A +
A*b^2 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]))/(b*(a + b)) - (2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a
*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ell
ipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -
ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2])
)/((a - b)^2*(a + b)^2))/(16*a*d)
```

Maple [B] time = 14.49, size = 1850, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+
1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2)))-2*a*(A*b-B*a)/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+
1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+
a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2
)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^
2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

3.380 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{\left(-6a^2Ab^3+15\right.}{}$$

```
[Out] -((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a
^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*EllipticF[
(c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 + 3*A
*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(9*a^2*A*b
- 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2
 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.06, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{\left(-6a^2Ab^3+15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```



```
[Out] -((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a
^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*EllipticF[
(c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 + 3*A
*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(9*a^2*A*b
- 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2
 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2A-3Ab^2-abB\right)-2a(Ab-aB)\cos(c+dx)+\ldots}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx\ldots}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\text{co}\ldots}}{4a^2\left(a^2-b^2\right)^2d(a+b\cos\ldots}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\text{co}\ldots}}{4a^2\left(a^2-b^2\right)^2d(a+b\cos\ldots}$$

$$= -\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx\ldots}}{2a\left(a^2-b^2\right)d(a+b\,\text{co}\ldots}$$

$$= -\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d} - \frac{\left(7a^2Ab-Ab^3-3a^3B-\ldots\right.}{4ab\left(a^2\ldots\right.}$$

Mathematica [A] time = 4.67539, size = 387, normalized size = 1.12

$$\frac{\frac{\frac{(-19a^2Ab^2+16a^4A-9a^3bB+3ab^3B+9Ab^4)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a(-4a^2Ab+2a^3B+ab^2B+Ab^3)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}+\frac{(-9a^2Ab+5a^3B+ab^2B+3Ab^3)\sin(c+dx)\left((2a^2-\cdots\right.}{\cdots}}{(a-b)^2(a+b)^2}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] ((-2*b*Sqrt[Cos[c + d*x]]*(a*(-11*a^2*A*b + 5*A*b^3 + 7*a^3*B - a*b^2*B) +
b*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((
a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4
 - 9*a^3*b*B + 3*a*b^3*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b
) + (8*a*(-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*((a + b)*EllipticF[(c + d
*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((-
9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (
2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d
*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*a^2*d)
```

Maple [B] time = 13.454, size = 1744, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-1/a*b^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
```

```
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/
(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(A*b-B*a)/b*(-1/2/a*b^2/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*
cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b
^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

3.381 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=420

$$\frac{\left(11a^2Ab-7a^3B+ab^2B-5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2Ab^2+8a^4A+9a^3bB-3ab^3B+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-$$

```
[Out] -((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE[(c
+ d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B +
a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A*b
- 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A -
 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^
3 - 7*a^3*B + a*b^2*B)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*
x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.47385, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(11a^2Ab-7a^3B+ab^2B-5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2Ab^2+8a^4A+9a^3bB-3ab^3B+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```


```
[Out] -((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE[(c
+ d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B +
a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A*b
- 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A -
 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^
3 - 7*a^3*B + a*b^2*B)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*
```

```
x]]*(a + b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2A-5Ab^2+abB\right)-2a(Ab-aB)\,c}{\cos^{\frac{3}{2}}(c+dx)(a+}}{2a\left(a^2\right.}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+\right.}{4a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} + \frac{b(Ab-}{2a\left(a^2-b^2\right)d\sqrt{\cos}}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} + \frac{b(Ab-}{2a\left(a^2-b^2\right)d\sqrt{\cos}}$$

$$= -\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2d} + \frac{\left(8a^4A-29a\right.}{}$$

$$= -\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2d} + \frac{\left(11a^2Ab-5\right.}{}$$

Mathematica [A] time = 4.98334, size = 462, normalized size = 1.1

$$\frac{\frac{\sqrt{\cos(c+dx)}\left(b^2\left(-29a^2Ab^2+8a^4A+9a^3bB-3ab^3B+15Ab^4\right)\sin(2(c+dx))+2ab\left(-47a^2Ab^2+16a^4A+11a^3bB-5ab^3B+25Ab^4\right)\sin(c+dx)+16A\left(a^3-ab^2\right)^2\tan(c+dx)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] (-((((56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B - 9*a*
b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^4*A -
 10*a^2*A*b^2 + 5*A*b^4 + 4*a^3*b*B - a*b^3*B)*((a + b)*EllipticF[(c + d*x)
/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((8*a^
4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*(-2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d
```

```
*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]],
-1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + (Sq
rt[Cos[c + d*x]]*(2*a*b*(16*a^4*A - 47*a^2*A*b^2 + 25*A*b^4 + 11*a^3*b*B -
5*a*b^3*B)*Sin[c + d*x] + b^2*(8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*
B - 3*a*b^3*B)*Sin[2*(c + d*x)] + 16*A*(a^3 - a*b^2)^2*Tan[c + d*x]))/((a^2
 - b^2)^2*(a + b*Cos[c + d*x])^2))/(8*a^3*d)
```

Maple [B] time = 17.036, size = 2002, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2/a^3*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2*A*b/a^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*
d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b
^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2)))+2*(-A*b+B*a)/a*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x
+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2
+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
```

```
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^
2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)
*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a
^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

3.382 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=523

$$\frac{\left(-61a^2Ab^2+8a^4A+33a^3bB-15ab^3B+35Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{\left(-65a^2Ab^3+24a^4Ab+29a^3b^2B-8a^5B-15ab^4B\right.}{4a^4d\left(a^2-b^2\right)^2}$$

```
[Out] ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*
B)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4*A - 61*a^2*
A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*EllipticF[(c + d*x)/2, 2])/(12*
a^3*(a^2 - b^2)^2*d) + (b*(63*a^4*A*b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B
+ 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*
a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b
*B - 15*a*b^3*B)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
- ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^
4*B)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a
*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(4*a^2
*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.95256, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-61a^2Ab^2+8a^4A+33a^3bB-15ab^3B+35Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{\left(-65a^2Ab^3+24a^4Ab+29a^3b^2B-8a^5B-15ab^4B\right.}{4a^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```



```
[Out] ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*
B)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4*A - 61*a^2*
A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*EllipticF[(c + d*x)/2, 2])/(12*
a^3*(a^2 - b^2)^2*d) + (b*(63*a^4*A*b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B
+ 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*
a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b
*B - 15*a*b^3*B)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
```

```
- ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^
4*B)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a
*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(4*a^2
*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2A-7Ab^2+3abB\right)-2a(Ab-aB}{\cos^{\frac{5}{2}}(c+dx)(}}{2a(}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{b\left(13a^2Ab-7Ab^3-9a^3B+\right.}{4a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{b(}{2a\left(a^2-b^2\right)d}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(24a^4Ab-65a\right.}{}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(24a^4Ab-65a\right.}{}$$

$$= \frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d} +$$

$$= \frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d} +$$

Mathematica [A] time = 7.16701, size = 574, normalized size = 1.1

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{2a^3\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{17a^2Ab^4\sin(c+dx)-13a^3b^3B\sin(c+dx)+7ab^5B\sin(c+dx)-11Ab^6\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2(a+b\cos(c+dx))} + \frac{2\sec(c+dx)(aB\sin(c+d}{a^4}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] ((2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 315*A*b^6 - 168*a^5*b*B + 2
85*a^3*b^3*B - 135*a*b^5*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a +
```

```
b) + ((160*a^5*A*b - 512*a^3*A*b^3 + 280*a*A*b^5 - 48*a^6*B + 240*a^4*b^2*
B - 120*a^2*b^4*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(a + b)))/b + (2*(72*a^4*A*b^2 - 195*a^2*A*b^4 + 105
*A*b^6 - 24*a^5*b*B + 87*a^3*b^3*B - 45*a*b^5*B)*Cos[2*(c + d*x)]*(-2*a*b*E
llipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c
 + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c
 + d*x]^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c
 + d*x]*(-3*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/a^4 + (A*b^4*Sin[c + d*x]
 - a*b^3*B*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (17*a
^2*A*b^4*Sin[c + d*x] - 11*A*b^6*Sin[c + d*x] - 13*a^3*b^3*B*Sin[c + d*x] +
 7*a*b^5*B*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*
Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 27.283, size = 2158, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(3*A*b-B
*a)/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-3*A*b+B*a)/a^4*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*b*(2*A*b-B*a)/a^
3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^
2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^
(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
```

```
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^3*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)*b/a^2*(-1/2/a*b^2/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*co
s(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x
+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b
)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9
/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a
*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
```

```
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

3.383 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)
```

Rubi [A] time = 0.0248216, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2635, 2639}

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}(3B)\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

Mathematica [A] time = 0.0465736, size = 41, normalized size = 0.93

$$\frac{B\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\sqrt{\cos(c+dx)}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (B*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*
d)
```

Maple [B] time = 3.126, size = 203, normalized size = 4.6

$$-\frac{2B}{5d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+8\left(\sin(1/2\,dx+c/2)\right)^4 c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3
```

```
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(B\cos(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c) + Ba)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

3.384 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)
```

Rubi [A] time = 0.0236041, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2635, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.0428446, size = 37, normalized size = 0.84

$$\frac{2B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 2.984, size = 180, normalized size = 4.1

$$-\frac{2B}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(B\cos(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

3.385 $\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=17

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0109797, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21, 2639}

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A] time = 0.0230858, size = 17, normalized size = 1.

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d
```

Maple [B] time = 2.138, size = 134, normalized size = 7.9

$$2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{EllipticE}\left(\cos\left(\right.\right.}{\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(B\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

3.386 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=17

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0112141, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))}\,dx = B\int \frac{1}{\sqrt{\cos(c + dx)}}\,dx$$
$$= \frac{2BF\left(\frac{1}{2}(c + dx)\middle|2\right)}{d}$$

Mathematica [A] time = 0.0266889, size = 17, normalized size = 1.

$$\frac{2BF\left(\frac{1}{2}(c + dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Maple [C] time = 0.037, size = 19, normalized size = 1.1

$$2\,\frac{B\mathrm{InverseJacobiAM}\left(1/2\,dx + c/2, \sqrt{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2*B/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

3.387 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=40

$$\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]])
```

Rubi [A] time = 0.0216924, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2636, 2639}

$$\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - B\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.0589035, size = 40, normalized size = 1.

$$B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
),x]
```

```
[Out] B*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]))
```

Maple [A] time = 3.253, size = 102, normalized size = 2.6

$$-2\,\frac{B\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-2\,(\sin(1/2\,dx+c/2))^2\cos(1}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

3.388 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=44

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))
```

Rubi [A] time = 0.0229834, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2636, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx &= B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] time = 0.0596195, size = 37, normalized size = 0.84

$$
\frac{2B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])
),x]
```

```
[Out] (2*B*(EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 2.872, size = 214, normalized size = 4.9

$$
-\frac{2B}{3d}\left(-2\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\left(\sin(1/2\,dx+c/2)\right)^2+\sqrt{2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*B*((2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$\frac{2B\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

```
[Out] (-2*a*B*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*B*EllipticF[(
c + d*x)/2, 2])/(3*b^3*d) - (2*a^3*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.402893, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {21, 2793, 3059, 2639, 3002, 2641, 2805}

$$\frac{2B\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*B*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*B*EllipticF[(
c + d*x)/2, 2])/(3*b^3*d) - (2*a^3*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
```

```
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{(2B)\int \frac{\frac{a}{2}+\frac{1}{2}b\cos(c+dx)-\frac{3}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{3b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{(2B)\int \frac{-\frac{ab}{2}-\frac{1}{2}\left(3a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{3b^2} - \frac{(aB)\int \sqrt{c}}{}$$

$$= -\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{\left(a^3B\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a}}{b^3}$$

$$= -\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2\left(3a^2+b^2\right)BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d} - \frac{2a^3B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c}{b^3(a+b)d}$$

Mathematica [A] time = 1.57009, size = 161, normalized size = 1.39

$$\frac{B\left(\frac{6\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b^2\sqrt{\sin^2(c+dx)}} - \frac{6a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + 4\right.}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (B*(4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (6*(2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2])))/(6*b*d)
```

Maple [B] time = 3.52, size = 517, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*((4*a*b^2-4*
b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a*b^2+2*b^3)*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*a^3-3*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a*b^2-b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*
b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^
2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^
2, x)
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$\frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticF[(c + d*x)/2, 2])/(
b^2*d) + (2*a^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d
)
```

Rubi [A] time = 0.166554, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {21, 2804, 2639, 2803, 2641, 2805}

$$\frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticF[(c + d*x)/2, 2])/(
b^2*d) + (2*a^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
```

```
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx\\
&= \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{(aB)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{(aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2} + \frac{\left(a^2B\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b^2}\\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d}
\end{aligned}
$$

Mathematica [A] time = 0.102399, size = 85, normalized size = 1.09

$$-\frac{2B\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-a\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+bE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right.\right.\right.}{b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (-2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] - a*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x
]]], -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

Maple [A] time = 3.178, size = 228, normalized size = 2.9

$$2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{\left(a-b\right)b^{2}\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(\mathrm{EllipticF}\left(\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
2, x)
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(b*(a + b)*d)
```

Rubi [A] time = 0.102977, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 2803, 2641, 2805}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(b*(a + b)*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx\\ &= \frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}-\frac{(aB)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b}\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.0514915, size = 49, normalized size = 0.89

$$\frac{B\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (B*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b)))/(b*d)
```

Maple [A] time = 3.593, size = 189, normalized size = 3.4

$$-2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{b\left(a-b\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c) + Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
2, x)
```

3.392 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=30

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] (2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rubi [A] time = 0.0487697, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21, 2805}

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Mathematica [A] time = 0.0647051, size = 30, normalized size = 1.

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] (2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Maple [B] time = 2.624, size = 151, normalized size = 5.

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{\left(a-b\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\mathrm{EllipticPi}\Big(\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c) + a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c
))), x)
```

3.393 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$-\frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*B*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2])/(a*(a + b)*d) + (2*B*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.246779, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {21, 2802, 3059, 2639, 12, 2805}

$$-\frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*B*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2])/(a*(a + b)*d) + (2*B*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
```

```
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(2B)\int \frac{-\frac{b}{2}-\frac{1}{2}a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{B\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{(2B)\int \frac{b^2}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{ab}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(bB)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 2.60504, size = 200, normalized size = 2.5

$$-\frac{B\left(\frac{2\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{6b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\right.}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] -(B*((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2*Elli
pticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
+ b)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2])))/(2*a*d)
```

Maple [B] time = 4.01, size = 355, normalized size = 4.4

$$-2\,\frac{B}{(a-b)\,a\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(-2\sqrt{-2\,(\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*B*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^
(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/a/(a-b)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/
2)), x)
```

3.394 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*b*B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*B*EllipticF[(c + d*x)/2, 2])
/(3*a*d) + (2*b^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)
*d) + (2*B*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*B*Sin[c + d*x])/
(a^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.559779, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {21, 2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (2*b*B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*B*EllipticF[(c + d*x)/2, 2])
/(3*a*d) + (2*b^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)
*d) + (2*B*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*B*Sin[c + d*x])/
(a^2*d*Sqrt[Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{(2B)\int \frac{-\frac{3b}{2}+\frac{1}{2}a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(4B)\int \frac{\frac{1}{4}\left(a^2+3b^2\right)+ab\cos(c+dx)+\frac{3}{4}b^2\cos^2(c+dx}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3a^2}$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(4B)\int \frac{-\frac{1}{4}b\left(a^2+3b^2\right)-\frac{1}{4}ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{3a^2b} + \frac{(b}{}$$

$$= \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{3a}$$

$$= \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d} + \frac{2B}{3ad}$$

Mathematica [A] time = 4.06872, size = 215, normalized size = 1.62

$$\frac{B\left(\frac{2(2a^2+9b^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{6\sin(c+dx)\left((2a^2-b^2)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|\right.\right.}{a\sqrt{\sin^2(c+dx)}}\right.}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] (B*((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) +
 8*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*El
lipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a),
-ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2])))/
(6*a^2*d)
```

Maple [B] time = 8.419, size = 452, normalized size = 3.4

$$-2\,{\frac{\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}B}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}}\left(-2\,{\frac{b^{3}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}}}{a^{2}\left(-2\,ab+2\,b^{2}\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,d\right.\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-2/a^2*b^3/
(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-2*b/(a-b),2^(1/2))-1/a^2*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+1/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
```

```
(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/
2)), x)
```

3.395 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=560

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\left(-3a^2B+6aAb+16b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c}{a+b}}}{24ab^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(24*a*b^2*d) + (Sqrt[a + b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt[a + b]*(2*a^2*A*b - 8
*A*b^3 - a^3*B - 4*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(8*b^3*d) + ((6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*A*b - a*B)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d) + (B*Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 1.50734, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\left(-3a^2B+6aAb+16b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c}{a+b}}}{24ab^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(24*a*b^2*d) + (Sqrt[a + b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
```

```
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt[a + b]*(2*a^2*A*b - 8
*A*b^3 - a^3*B - 4*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(8*b^3*d) + ((6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*A*b - a*B)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d) + (B*Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{\int \frac{\sqrt{a+b\,c}}{}}{}$$

$$= \frac{(2Ab-aB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd} + $$

$$= \frac{\left(6aAb-3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^2d\sqrt{\cos(c+dx)}} + $$

$$= \frac{\left(6aAb-3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^2d\sqrt{\cos(c+dx)}} + $$

$$= \frac{\sqrt{a+b}\left(2a^2Ab-8Ab^3-a^3B-4ab^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\right.}{8b}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(6aAb-3a^2B+16b^2B\right)\cot(c+dx)E\left(\sin^{-}\right.}{24ab}$$

Mathematica [C] time = 6.30503, size = 1224, normalized size = 2.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] -((-4*a*(-18*a*A*b + a^2*B - 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
```

```
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-
24*A*b^2 - 28*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[
-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])) + 2*(-6*a*A*b + 3*a^2*B - 16*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]],
 (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sq
rt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(48*b*d) + (Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]*(((6*A*b + a*B)*Sin[c + d*x])/(12*b) + (B*Sin[2*(
c + d*x)])/6))/d
```

Maple [B] time = 0.501, size = 2949, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(-12*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*A*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
```

x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*B*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a*b^2+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b-28*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+16*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^2+18*A*cos(d*x+c)^3*a*b^2+6*A*cos(d*x+c)^2*a^2*b-
6*A*cos(d*x+c)^2*a*b^2-6*A*cos(d*x+c)*a^2*b-12*A*cos(d*x+c)*a*b^2+10*B*cos(
d*x+c)^4*a*b^2-B*cos(d*x+c)^3*a^2*b+3*B*cos(d*x+c)^2*a^2*b+6*B*cos(d*x+c)^2
*a*b^2-2*B*cos(d*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+12*A*cos(d*x+c)^4*b^3-12*
A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^3-3*B*cos(d*x+c)
^2*a^3-16*B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)*a^3-3*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48
*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*b^3-24*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*B*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^3+16*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*A*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+6*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b+6*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2+24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b

```
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-28*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-
24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^3+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a^3-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)/b^2/cos(d*x+c)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.396 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=473

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{(aB+}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(4*A*b + (a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*A*b + a*B)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.04152, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3003, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{(aB+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(4*A*b + (a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*Elliptic
```

```
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*A*b + a*B)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{4}\int \frac{aB+\cdots}{\cdots}$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+d\cdots}}{\cdots}$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+d\cdots}}{\cdots}$$

$$= -\frac{\sqrt{a+b}\left(4aAb-a^2B+4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\cdots\right.\right.}{4b^2d\cdots}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab+aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c\cdots}}{\sqrt{a+b}\sqrt{\cos(c\cdots}}\right.\right.}{4abd\cdots}$$

Mathematica [C] time = 21.0191, size = 1175, normalized size = 2.48

$$-\frac{4a(4Ab+3aB)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c\cdots\right.}{a}}\cdots}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cdots}}$$

$$\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*a
*(4*A*b + 3*a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
```

```
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a*A + 4*b*B)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b + a*B)*
((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]
) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Co
s[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8
*d)
```

Maple [B] time = 0.417, size = 2055, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/4/d/(a+b*cos(d*x+c))^(1/2)*(-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-3*B*cos(d*x+c)^3*a*
b+B*cos(d*x+c)^2*a*b+2*B*cos(d*x+c)*a*b-4*A*cos(d*x+c)^2*a*b+4*A*cos(d*x+c)
*a*b-4*A*cos(d*x+c)^3*b^2+4*A*cos(d*x+c)^2*b^2-2*B*cos(d*x+c)^4*b^2+2*B*cos
(d*x+c)^2*b^2-B*cos(d*x+c)^2*a^2+B*cos(d*x+c)*a^2-4*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*B*
```

sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*A*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-8*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-B
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*B*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^2-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/sin(d*x+c)/b/cos(d*x+c)
^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

3.397 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt{a+b}(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(aB+2Ab)\cot(c+d}{}$$

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (Sqrt[a + b]*(2*A*b
 + a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (B*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.713293, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3003, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(aB+2Ab)\cot(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (Sqrt[a + b]*(2*A*b
 + a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (B*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{2}\int \frac{-aB+2aA\cos(c+dx)+(2Ab\ \cdots}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos\cdots}}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{2}\int \frac{-aB+2aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\cdots$$

$$= -\frac{\sqrt{a+b}(2Ab+aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\cdots}}{bd}$$

$$= -\frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+\cdots}{a+b}}}{ad}$$

Mathematica [B] time = 17.8859, size = 3054, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] ((1 + Cos[c + d*x])^(3/2)*((A*Sqrt[a + b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
+ B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])*Sec[(c + d*x)/2]^2*(2*(a +
```

b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(A*b + a*(-A + B))*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] - 8*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
4*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*B*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*B*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*B*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Tan[(c + d*x)/2]))/(4*d*Sqrt[a + b*Cos[c + d*x]]*((b*(1 + Cos[c
+ d*x])^(3/2)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*(2*(a + b)*B*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)] - 4*(A*b + a*(-A + B))*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
] - 8*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*a*B*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + b*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c
+ d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Tan[(c + d*x)/2] - b*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*
x)/2]))/(8*(a + b*Cos[c + d*x])^(3/2)) - (3*Sqrt[1 + Cos[c + d*x]]*Sec[(c +
d*x)/2]^2*Sin[c + d*x]*(2*(a + b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4
*(A*b + a*(-A + B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 8*A*b*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] - 4*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + b*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c +
d*x))/2] + 2*a*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] -
b*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*Sqrt[a + b*
Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]*(2*(a + b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(A*b + a*(-A + B))*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] - 8*A*b*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)] - 4*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*B*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*B*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*B*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*Sqrt[a + b*Cos[c + d*x]]) + ((1
+ Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*((3*b*B*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Cos[(3*(c + d*x))/2]*Sec[(c + d*x)/2])/2 + a*B*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2 - (b*B*Sqrt[Cos[c + d*x]/(1 + Cos

```
[c + d*x])]*Sec[(c + d*x)/2]^2)/2 + ((a + b)*B*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))
 + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (2*(A*b + a*(-A + B)
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a +
b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))] - (4*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])
*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))] - (2*a*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) +
((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*B*Sec[(c + d*x)/2]*(
(Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
+ d*x]))*Sin[(3*(c + d*x))/2])/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) +
(a*B*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 +
Cos[c + d*x]))*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b
*B*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Co
s[c + d*x]))*Tan[(c + d*x)/2])/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) +
(b*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x
))/2]*Tan[(c + d*x)/2])/2 - (2*(A*b + a*(-A + B))*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (4*A*b*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c + d*
x)/2]^2)/(a + b)]) + (2*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*B*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt
[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))
/(4*Sqrt[a + b*Cos[c + d*x]])))
```

Maple [B] time = 0.573, size = 1693, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(2*A*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c
```

```
))/(1+cos(d*x+c)))^(1/2)*a-2*A*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+4*A*cos(d*x+c)^2*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+4*
A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*a-4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+2*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a-2*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*b+
4*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*b+B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+2*B*
cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+2
*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a+B*cos(d*x+c)^4*b+B*cos(d*x+c)^3*a-B*cos(d*x+c)^3*b-B*
cos(d*x+c)^2*a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx + c) + A)\sqrt{b\cos(dx + c) + a}}{\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

3.398 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{2\sqrt{a+b}(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2A(a-b)\sqrt{a+b}\,\mathrm{co}}{}$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt
[a + b]*(A*b - a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqr
t[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rubi [A] time = 0.504136, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2991, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2A(a-b)\sqrt{a+b}\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt
[a + b]*(A*b - a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqr
t[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rule 2991

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(B*d
)/b^2, Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = (bB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{aA+(Ab+aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{d}$$

$$= \frac{2A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{ad}$$

Mathematica [A] time = 12.9246, size = 275, normalized size = 0.78

$$\frac{2(a(A+B)+b(A-B))\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+\frac{2A\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{\sqrt{\cos(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (-2*A*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2
*(b*(A - B) + a*(A + B))*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] - 4*b*B*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + (2*A*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]])/(d*S
qrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.446, size = 1687, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a-B*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b+2*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*b+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+4*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^2*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a-A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
2*b+B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*a-B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*b+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b-A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*b+A*cos(d*x+c)^3*b+A*cos(d*x+c)^2*a-A*cos(d*x+c)^2*b-A*c
os(d*x+c)*a)/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

3.399 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2(a-b)\sqrt{a+b}(A}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*(a - b)*Sqrt[a + b]*(A - 3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(3*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2))
```

Rubi [A] time = 0.50524, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2999, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2(a-b)\sqrt{a+b}(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*(a - b)*Sqrt[a + b]*(A - 3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(3*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2))
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)+\frac{1}{2}(aA+3bB)\,\text{co}}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}((a-b)(A-3B))\int \frac{}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2(a-b)\sqrt{a+b}(Ab+3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle| -\frac{a+b}{a-b}\right)\sqrt{}}{3a^2d}$$

Mathematica [A] time = 14.0066, size = 407, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(3aB\sin(c+dx)+Ab\sin(c+dx))}{3a}+\frac{2}{3}A\tan(c+dx)\sec(c+dx)\right)}{d} + \frac{4\left(\frac{\cos(c+dx)}{\cos(c+dx)+1}\right)^{3/2}\sqrt{\text{c}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[
1 + Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(A*b +
3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + 2*a*(a + b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Cos[c + d*x]^(5/2)*Sqrt[a +
b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a) + (2*A*Sec[c + d*x]*Ta
n[c + d*x])/3))/d
```

Maple [B] time = 0.406, size = 1729, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3/d*(-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)^2*sin(d*x+c)*a*b+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^2-A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-3*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+
c)*a^2-3*B*cos(d*x+c)^3*a*b+3*B*cos(d*x+c)^2*a*b-A*cos(d*x+c)^2*a*b+2*A*cos
(d*x+c)*a*b-A*cos(d*x+c)^2*a^2+3*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b-3*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*
b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^2*sin(d*x+c)*a*b-A*cos(d*x+c)^3*b^2+A*cos(d*x+c)^2*b^2-3*B*cos(d*x+c)
^2*a^2+3*B*cos(d*x+c)*a^2-A*cos(d*x+c)^3*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+a^2*A+A*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+
3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2)/(a+b*cos(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

3.400 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=350

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+5abB-2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a-}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.824912, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2999, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+5abB-2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(Ab+5aB)+\frac{1}{2}(3aA+5bB)\,c}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(}{15ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(}{15ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A-2Ab^2+5abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{15a^3d}$$

Mathematica [C] time = 6.34229, size = 1315, normalized size = 3.76

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] -((-4*a*(2*a^2*A*b - 2*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
```

```
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(9*a^3*A - 2*a*A*b^2 + 5*a^2*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b - 2*A*b^3 + 5*a*b^2*B)*((
I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)])
+ (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
)/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(15*
a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(A
*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a) + (2*Sec[c + d*x]*(9*a^2*A*Si
n[c + d*x] - 2*A*b^2*Sin[c + d*x] + 5*a*b*B*Sin[c + d*x]))/(15*a^2) + (2*A*
Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.474, size = 2479, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+5*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-5*B*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
```

```
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-5*B*sin
(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a*b^2+7*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin
(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+5*B*sin(d*x+c)*cos(d*x+c)^3*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-5*B*sin
(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a^2*b-5*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+7*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin
(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*A*cos
(d*x+c)^3*a^2*b+9*A*cos(d*x+c)^4*a^2*b+A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^
4*a^2*b-5*B*cos(d*x+c)^3*a*b^2-2*A*cos(d*x+c)^3*a*b^2+A*cos(d*x+c)^2*a*b^2-
4*A*cos(d*x+c)*a^2*b+5*B*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^3*a^2*b-10*B*cos
(d*x+c)^2*a^2*b+5*B*cos(d*x+c)^3*a^3+9*A*cos(d*x+c)^3*a^3+2*A*cos(d*x+c)^3*
b^3-6*A*cos(d*x+c)^2*a^3-2*A*cos(d*x+c)^4*b^3-5*B*cos(d*x+c)*a^3+9*A*sin(d*
x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3+2*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+5*B*sin(d*x+c)*c
os(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*a^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
```

```
+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3
+5*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a^3)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

3.401 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\text{co}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2
 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^3*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*
x]^(7/2)) + (2*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d
*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.17564, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2999, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2
 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^3*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*
```

```
x]^(7/2)) + (2*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d
*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(Ab+7aB)+\frac{1}{2}(5aA+7bB}{\cos^{\frac{7}{2}}(c+dx)\sqrt{}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(19a^2Ab+8Ab^3+63a^3B-14ab^2B\right)\cot(c+dx)E\left(\sin^{-}\right.}{105a^4d}$$

Mathematica [C] time = 6.43009, size = 1408, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] ((-4*a*(25*a^4*A - 17*a^2*A*b^2 - 8*A*b^4 - 14*a^3*b*B + 14*a*b^3*B)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*A*b - 8*a*A*b^3 - 63*a^4*B + 14*a^2*b^
2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-19*a^2*A*b^2 - 8*A*b^4 - 63*a^3*b*B + 14*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(A*b*Sin[c + d*x] + 7*a
*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] - 4*A*b
^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]))/(105*a^2) + (2*Sec[c + d*x]*(19*a^
2*A*b*Sin[c + d*x] + 8*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] - 14*a*b^
2*B*Sin[c + d*x]))/(105*a^3) + (2*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.611, size = 3427, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(19*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-15*A*a^4+25*A*cos(d*x+c)^5*a^3*b+19*A*c
os(d*x+c)^5*a^2*b^2-4*A*cos(d*x+c)^5*a*b^3+19*A*cos(d*x+c)^4*a^3*b-20*A*cos
(d*x+c)^4*a^2*b^2+8*A*cos(d*x+c)^4*a*b^3-26*A*cos(d*x+c)^3*a^3*b-4*A*cos(d*
x+c)^3*a*b^3+A*cos(d*x+c)^2*a^2*b^2-18*A*cos(d*x+c)*a^3*b+7*B*cos(d*x+c)^3*
a^2*b^2-28*B*cos(d*x+c)^2*a^3*b+63*B*cos(d*x+c)^5*a^3*b+7*B*cos(d*x+c)^5*a^
2*b^2-14*B*cos(d*x+c)^5*a*b^3-35*B*cos(d*x+c)^4*a^3*b-14*B*cos(d*x+c)^4*a^2
*b^2+14*B*cos(d*x+c)^4*a*b^3+8*A*cos(d*x+c)^5*b^4-8*A*cos(d*x+c)^4*b^4+25*A
*cos(d*x+c)^4*a^4+25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*sin(d
*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-10*A*cos(d*x+c)^2*a^4+63*B*cos(d*x+c)
^4*a^4-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+c)*a^4+25*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4-8*
A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^4+63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^4*a^3*b+14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^2+14*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3+1
9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
```

+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3-19*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b-19*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^3+49*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^2-63*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^3*a^3*b+14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^2+14*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+2*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^4*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3-19*A*sin(d*x+c)*cos(d*x+c)^4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-19*A*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+49*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b-14*
B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^
(7/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

3.402 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=670

$$\frac{\left(-3a^2B+8aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{\left(24a^2Ab-9a^3B+156ab^2B+128Ab^3\right)\sin(c+}{192b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d) - (Sqrt[a + b]*(9*a^3*B - 6*a^
2*b*(4*A + B) - 8*b^3*(16*A + 9*B) - 4*a*b^2*(28*A + 39*B))*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 - 3*
a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(64*b^3*d) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((8*a*A*b
- 3*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(32*b*d) + ((8*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(24*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)*Sin[c + d*x])/(4*b*d)
```

Rubi [A] time = 2.0961, antiderivative size = 670, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2B+8aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{\left(24a^2Ab-9a^3B+156ab^2B+128Ab^3\right)\sin(c+}{192b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
```

```
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d) - (Sqrt[a + b]*(9*a^3*B - 6*a^
2*b*(4*A + B) - 8*b^3*(16*A + 9*B) - 4*a*b^2*(28*A + 39*B))*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 - 3*
a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(64*b^3*d) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((8*a*A*b
- 3*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(32*b*d) + ((8*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(24*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)*Sin[c + d*x])/(4*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd} + \frac{\int \frac{(a+b\,c}{}}{}$$

$$= \frac{(8Ab-3aB)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24bd}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{32bd}$$

$$= \frac{\left(24a^2Ab+128Ab^3-9a^3B+156ab^2B\right)\sqrt{a+b\cos(c+dx)}}{192b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(24a^2Ab+128Ab^3-9a^3B+156ab^2B\right)\sqrt{a+b\cos(c+dx)}}{192b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(8a^3Ab-96aAb^3-3a^4B-24a^2b^2B-48b^4B\right)\cot(}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24a^2Ab+128Ab^3-9a^3B+156ab^2B\right)\cot}{}$$

Mathematica [C] time = 6.39177, size = 1284, normalized size = 1.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] -((-4*a*(-136*a^2*A*b - 128*A*b^3 + 3*a^3*B - 228*a*b^2*B)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(-416*a*A*b^2 - 228*a^2*b*B - 144*b^3*B)*((Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-24*a^2*A*b - 128*A*b
^3 + 9*a^3*B - 156*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*E
llipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*S
ec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(b*Sqrt[Cos[c + d*x]])))/(384*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]]*(((56*a*A*b + 3*a^2*B + 42*b^2*B)*Sin[c + d*x])/(96*b) + ((8*A*b +
9*a*B)*Sin[2*(c + d*x)])/48 + (b*B*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.616, size = 4048, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/192/d/(a+b*cos(d*x+c))^(1/2)*(72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+176*A
*cos(d*x+c)^4*a*b^3-24*A*cos(d*x+c)^2*a^2*b^2-24*A*cos(d*x+c)*a^3*b+9*B*cos
(d*x+c)^2*a^3*b+120*B*cos(d*x+c)^5*a*b^3+78*B*cos(d*x+c)^4*a^2*b^2+64*A*cos
(d*x+c)^5*b^4+48*B*cos(d*x+c)^6*b^4+24*B*cos(d*x+c)^4*b^4-72*B*cos(d*x+c)^2
*b^4+64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b^4-9*B*cos(d*x+c)^2*a^4+78*B
*cos(d*x+c)^2*a^2*b^2-156*B*cos(d*x+c)^2*a*b^3-6*B*cos(d*x+c)*a^3*b-156*B*c
os(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+136*A*cos(d*x+c)^3*a^2*b^2+24*A*cos
(d*x+c)^2*a^3*b-48*A*cos(d*x+c)^2*a*b^3-3*B*cos(d*x+c)^3*a^3*b+108*B*cos(d*
x+c)^3*a*b^3-112*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*b^3+9*B*cos(d*x+c)
*a^4+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*b^4-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+18*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+
288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*b^4-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+24*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+24*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b
^2+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*a*b^3-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+576*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+112*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-416*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*a*b^3-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
```

```
)*sin(d*x+c)*a^3*b+156*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+156*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+144*B*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+6*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^3*b-228*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a^2*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+128*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-48
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*sin(d*x+c)*a^3*b+576*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+112*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a
^2*b^2-416*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^3-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+156*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a
^2*b^2+156*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^3+144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+6*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a^3*b-228*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+128*A*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4-9
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
```

```
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^4+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^4+288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4)/sin(d*x+c)/b^2/cos(d*x
+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.403 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=566

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(3a^2B+30aAb+14abB+12Ab^2+16b^2B\right)\cot(c+dx}{24}$$

```
[Out] -((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14
*a*b*B + 16*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (Sqrt[a +
b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(8*b^2*d) + ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + ((6*A*b + 7*a*B)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b*B*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.66311, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(3a^2B+30aAb+14abB+12Ab^2+16b^2B\right)\cot(c+dx}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14
*a*b*B + 16*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
```

```
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (Sqrt[a +
b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(8*b^2*d) + ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + ((6*A*b + 7*a*B)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b*B*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{bB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int$$

$$= \frac{(6Ab+7aB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(6a^2Ab+8Ab^3-a^3B+12ab^2B\right)\cot(c+dx)\Pi}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(30aAb+3a^2B+16b^2B\right)\cot(c+dx)E\left(\right.}{2}$$

Mathematica [C] time = 6.30445, size = 1227, normalized size = 2.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] ((-4*a*(42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(
```

```
48*a^2*A + 24*A*b^2 + 52*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(30*a*A*b + 3*a^2*B + 16*b^2*B)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(48*d) + (Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((6*A*b + 7*a*B)*Sin[c + d*x])/12 + (b*
B*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.505, size = 3139, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(36*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-48*A*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a^2*b+12*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+30*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
```

cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*A*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2+72*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+14*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-52*B*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b+16*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+42*A*cos(d*x+c)^3*a*b
^2+30*A*cos(d*x+c)^2*a^2*b-30*A*cos(d*x+c)^2*a*b^2-30*A*cos(d*x+c)*a^2*b-12
*A*cos(d*x+c)*a*b^2+22*B*cos(d*x+c)^4*a*b^2+17*B*cos(d*x+c)^3*a^2*b-3*B*cos
(d*x+c)^2*a^2*b-6*B*cos(d*x+c)^2*a*b^2-14*B*cos(d*x+c)*a^2*b-16*B*cos(d*x+c
)*a*b^2+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3+8*
B*cos(d*x+c)^3*b^3+3*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-3*B*cos(d*x+c
)*a^3-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2*b+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*A*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*A*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*b^3-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^3+36*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a^2*b+12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+30*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-

```
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+72*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*a*b^2+14*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b-52*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*A*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-6*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^3)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.404 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=472

$$-\frac{\sqrt{a+b}\left(3a^2B+12aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{(5aB}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*d) + (Sqrt[a + b]*(8*a*A + 4*A*b + 5*a*B + 2*b*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(4*d) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + ((4*A*b + 5*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.14696, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2B+12aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{(5aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*d) + (Sqrt[a + b]*(8*a*A + 4*A*b + 5*a*B + 2*b*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(4*d) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
```

Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + ((4*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{bB\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\frac{1}{2}a(4aA+bB)}{}$$

$$= \frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos(c+dx)}\sqrt{a+b}}{2}$$

$$= \frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos(c+dx)}\sqrt{a+b}}{2}$$

$$= -\frac{\sqrt{a+b}\left(12aAb+3a^2B+4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{4bd}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab+5aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid -\frac{a}{a}\right.}{4ad}$$

<u>Mathematica</u> [C] time = 6.33076, size = 1198, normalized size = 2.54

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (b*B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4
*a*(8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(16*a*A*
b + 8*a^2*B + 4*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
```

```
[c + d*x]])) + 2*(4*A*b^2 + 5*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(8*d)
```

Maple [B] time = 0.585, size = 2430, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-8*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2
+7*B*cos(d*x+c)^3*a*b-5*B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c
)^2*a*b-4*A*cos(d*x+c)*a*b+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+2*B*co
s(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2+5*B*cos(d*x+c)^2*a^2-5*B*cos(d*x+c)*a^2
+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+5*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-16*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
```

(1/2))*a*b+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-16*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24*A*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a*b+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/s
in(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\cos(dx+c)^{2}+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

3.405 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=449

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}(2a(A-B)-b(4A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F}{d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A + B))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d - (Sqrt[a + b]*(2*A*b + 3*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) - ((2*a*A - b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]])
```

Rubi [A] time = 1.17581, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}(2a(A-B)-b(4A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A + B))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d - (Sqrt[a + b]*(2*A*b + 3*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
```

a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*a*A - b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
```

```
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a(2Ab+aB)-\frac{1}{2}\left(a^2A-\right.}{\sqrt{}}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aA-bB)\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aA-bB)\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}(2Ab+3aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{d}$$

$$= \frac{(a-b)\sqrt{a+b}(2aA-bB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{ad}$$

Mathematica [C] time = 6.32715, size = 1196, normalized size = 2.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*
a*(-2*a*A*b - 2*a^2*B - b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(2*a^2*A -
 2*A*b^2 - 4*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-
(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
```

```
+ d*x]])) - 2*(2*a*A*b - b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(2*d)
```

Maple [B] time = 0.419, size = 2185, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a^2+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+B*cos(d*x+c)^3*b^2+2*A*cos(
d*x+c)*a^2+B*cos(d*x+c)^2*a*b-B*cos(d*x+c)*a*b+2*A*cos(d*x+c)^2*a*b-2*A*cos
(d*x+c)*a*b-B*cos(d*x+c)^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-2*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
```

```
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-2*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^
2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*a^2-2*a^2*A+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*B*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+B*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
```

```
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

3.406 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=419

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*d) + (2*Sqrt[a + b]*(3*A*b^2 + a^2*(A - 3*B) - a*(4*A*b - 6*b*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (2*b*Sqrt[a + b]*B*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.858342, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*d) + (2*Sqrt[a + b]*(3*A*b^2 + a^2*(A - 3*B) - a*(4*A*b - 6*b*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (2*b*Sqrt[a + b]*B*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
```

```
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a(4Ab+3aB)+\frac{1}{2}\left(a^2A+\right.}{\cos^{\frac{3}{2}}(}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a(4Ab+3aB)+\frac{1}{2}\left(a^2A+\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{2b\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}(4Ab+3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right.}{3ad}$$

Mathematica [C] time = 6.34377, size = 1236, normalized size = 2.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] ((-4*a*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-4*a*A*
b - 3*a^2*B + 3*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])) + 2*(-4*A*b^2 - 3*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]*(4*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/3
 + (2*a*A*Sec[c + d*x]*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.41, size = 2318, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

[Out] -2/3/d*(4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b+4*A*cos(d*x+c)^2*a*b-5*A*cos(d*x+c)*a*b+A*cos(d*x+c)^2*a^2-3*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+3*B*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)*a^2+A*cos(d*x+c)^3*a*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-a^2*A-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

```
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\cos(dx+c)^{2}+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

3.407 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=353

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}+2($$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B +
15*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (2*a*A*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*A*b + 5*a*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.924391, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2989, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}+2($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B +
15*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (2*a*A*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*A*b + 5*a*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a(6Ab+5aB)+\frac{1}{2}\left(3a^2A\right.}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+d}}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+d}}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+3Ab^2+20abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{15a^2d}$$

Mathematica [C] time = 6.41754, size = 1314, normalized size = 3.72

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] -((-4*a*(-3*a^2*A*b + 3*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
```

```
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - 4*a*(9*a^3*A + 3*a*A*b^2 + 20*a^2*b*B)*((Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 3*A*b^3 + 20*a*b^2*B)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
15*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(
6*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/15 + (2*Sec[c + d*x]*(9*a^2*A*Sin
[c + d*x] + 3*A*b^2*Sin[c + d*x] + 20*a*b*B*Sin[c + d*x]))/(15*a) + (2*a*A*
Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.441, size = 2666, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+20*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-20*B*sin(d*x+c)*cos(d*x+c)^2*El
```

lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-20*B*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+12*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-3*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+20*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-20*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-20*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+12*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+15*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2+9*A*cos(d*x+c)^4*a^2*b+6*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2*b-20*B*cos(d*x+c)^3*a*b^2+3*A*cos(d*x+c)^3*a*b^2-9*A*cos(d*x+c)^2*a*b^2-9*A*cos(d*x+c)*a^2*b+20*B*cos(d*x+c)^4*a*b^2+20*B*cos(d*x+c)^3*a^2*b-25*B*cos(d*x+c)^2*a^2*b+5*B*cos(d*x+c)^3*a^3+9*A*cos(d*x+c)^3*a^3-3*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*a^3+3*A*cos(d*x+c)^4*b^3-5*B*cos(d*x+c)*a^3+9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3-3*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-

```
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*b^3+5*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3+9*A*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+5*B*sin(d*x+c)*cos(d*x+c)^2*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3)/(a+b*c
os(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2
), x)
```

3.408 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\left(a^2(-(25A-63B))+3ab(19A-7B)+6Ab^2\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2
 - a^2*(25*A - 63*B) + 3*a*b*(19*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(105*a^2*d) + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
 d*x]^(7/2)) + (2*(8*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.34616, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\left(a^2(-(25A-63B))+3ab(19A-7B)+6Ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2
 - a^2*(25*A - 63*B) + 3*a*b*(19*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(105*a^2*d) + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
```

```
d*x]^(7/2)) + (2*(8*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}a(8Ab+7aB)+\frac{1}{2}\left(5a^2A\right.}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(82a^2Ab-6Ab^3+63a^3B+21ab^2B\right)\cot(c+dx)E\left(\sin\right.}{105a^3d}$$

Mathematica [C] time = 6.50158, size = 1407, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 31*a^2*A*b^2 + 6*A*b^4 + 21*a^3*b*B - 21*a*b^3*B)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*A*b + 6*a*A*b^3 - 63*a^4*B - 21*a^2*b^
2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-82*a^2*A*b^2 + 6*A*b^4 - 63*a^3*b*B - 21*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^2*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(8*A*b*Sin[c + d*x] + 7
*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 3*A*b^2
*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x]))/(105*a) + (2*Sec[c + d*x]*(82*a^2*A
*b*Sin[c + d*x] - 6*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] + 21*a*b^2*B
*Sin[c + d*x]))/(105*a^2) + (2*a*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.565, size = 3413, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(82*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-15*A*a^4+25*A*cos(d*x+c)^5*a^3*b+82*A*c
os(d*x+c)^5*a^2*b^2+3*A*cos(d*x+c)^5*a*b^3+82*A*cos(d*x+c)^4*a^3*b-55*A*cos
(d*x+c)^4*a^2*b^2-6*A*cos(d*x+c)^4*a*b^3-68*A*cos(d*x+c)^3*a^3*b+3*A*cos(d*
x+c)^3*a*b^3-27*A*cos(d*x+c)^2*a^2*b^2-39*A*cos(d*x+c)*a^3*b-63*B*cos(d*x+c
)^3*a^2*b^2-63*B*cos(d*x+c)^2*a^3*b+63*B*cos(d*x+c)^5*a^3*b+42*B*cos(d*x+c)
^5*a^2*b^2+21*B*cos(d*x+c)^5*a*b^3+21*B*cos(d*x+c)^4*a^2*b^2-21*B*cos(d*x+c
)^4*a*b^3-6*A*cos(d*x+c)^5*b^4+6*A*cos(d*x+c)^4*b^4+25*A*cos(d*x+c)^4*a^4+2
5*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4+6*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^4-10*A*cos(d*x+c)^2*a^4+63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x
+c)^3*a^4-21*B*cos(d*x+c)*a^4+25*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4+6*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
4+63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^3*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b
-21*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^4*a^2*b^2-21*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3+82*A*sin(d*x+c)*cos(
```

d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos (d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))* a^3*b+51*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+ b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x +c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-82*A*sin(d*x+c)* cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1 +cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/ 2))*a^3*b-82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1 /(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin (d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+ c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+84*B*sin(d*x +c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)) ^(1/2))*a^3*b+21*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+ c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^2-63*B*(cos(d*x+c)/(1+cos(d*x+c)))^ (1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x +c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3*b-21*B*(c os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)* cos(d*x+c)^3*a^2*b^2-21*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+51*A*(cos(d*x+c)/(1+cos(d*x+ c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^2- 6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+ c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d *x+c)*cos(d*x+c)^4*a*b^3-82*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d* x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+ cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*sin(d*x+c)*cos(d*x+ c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x +c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2* b^2+6*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)* (a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c) ,(-(a-b)/(a+b))^(1/2))*a*b^3+84*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+ c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos (d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))* a^2*b^2)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(7/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2
), x)
```

3.409 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos}}{315ad\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d) + (2*(a - b)*Sqr
t[a + b]*(8*A*b^3 - a^3*(147*A - 75*B) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A
 - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*a*A*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(10*A*b + 9*a*B)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*
a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d
*Cos[c + d*x]^(5/2)) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.88458, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos}}{315ad\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d) + (2*(a - b)*Sqr
t[a + b]*(8*A*b^3 - a^3*(147*A - 75*B) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A
```

```
 - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*a*A*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(10*A*b + 9*a*B)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*
a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d
*Cos[c + d*x]^(5/2)) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{1}{2}a(10Ab+9aB)+\frac{1}{2}\left(7a^2\right.}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+}}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+}}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+}}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+}}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB-18ab^3B\right)\mathrm{co}}{31}$$

Mathematica [C] time = 6.58595, size = 1515, normalized size = 2.9

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] -((-4*a*(-39*a^4*A*b + 31*a^2*A*b^3 + 8*A*b^5 - 75*a^5*B + 93*a^3*b^2*B - 1
8*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 33*a^3*A*b^2 + 8*
a*A*b^4 + 246*a^4*b*B - 18*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
```

```
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 33*a^2*A*b^3 + 8*A*b^5 + 246
*a^3*b^2*B - 18*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[
c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x
])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b
*Sqrt[Cos[c + d*x]])))/(315*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]*((2*Sec[c + d*x]^4*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/63 +
(2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 72*a*b*B*
Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]^2*(88*a^2*A*b*Sin[c + d*x] - 4*A*b
^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] + 9*a*b^2*B*Sin[c + d*x]))/(315*a^2
) + (2*Sec[c + d*x]*(147*a^4*A*Sin[c + d*x] + 33*a^2*A*b^2*Sin[c + d*x] + 8
*A*b^4*Sin[c + d*x] + 246*a^3*b*B*Sin[c + d*x] - 18*a*b^3*B*Sin[c + d*x]))/
(315*a^3) + (2*a*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 0.741, size = 4391, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(186*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
```

*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+246*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+153*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-18*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-246*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+186*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+33*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-33*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+153*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-18*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1

```
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^4*b-246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+18*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+18*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^4-35*A*a^5-81*B*cos(d*x+c)^3*a^3*b^2-117*B*cos(d*x+c)^2*a^4
*b+75*B*cos(d*x+c)^6*a^4*b+246*B*cos(d*x+c)^6*a^3*b^2+9*B*cos(d*x+c)^6*a^2*
b^3-18*B*cos(d*x+c)^6*a*b^4+246*B*cos(d*x+c)^5*a^4*b-165*B*cos(d*x+c)^5*a^3
*b^2-18*B*cos(d*x+c)^5*a^2*b^3+18*B*cos(d*x+c)^5*a*b^4-204*B*cos(d*x+c)^4*a
^4*b+9*B*cos(d*x+c)^4*a^2*b^3-85*A*cos(d*x+c)*a^4*b-4*A*cos(d*x+c)^6*a*b^4-
10*A*cos(d*x+c)^5*a^4*b+33*A*cos(d*x+c)^5*a^3*b^2-34*A*cos(d*x+c)^5*a^2*b^3
+8*A*cos(d*x+c)^5*a*b^4-68*A*cos(d*x+c)^4*a^3*b^2-4*A*cos(d*x+c)^4*a*b^4-52
*A*cos(d*x+c)^3*a^4*b+A*cos(d*x+c)^3*a^2*b^3-53*A*cos(d*x+c)^2*a^3*b^2+147*
A*cos(d*x+c)^6*a^4*b+88*A*cos(d*x+c)^6*a^3*b^2+33*A*cos(d*x+c)^6*a^2*b^3+8*
A*cos(d*x+c)^6*b^5+147*A*cos(d*x+c)^5*a^5-8*A*cos(d*x+c)^5*b^5-98*A*cos(d*x
+c)^4*a^5-14*A*cos(d*x+c)^2*a^5+75*B*cos(d*x+c)^5*a^5-30*B*cos(d*x+c)^3*a^5
-45*B*cos(d*x+c)*a^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d
*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-8*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^5+33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*
x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^4-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*c
os(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
b^2+75*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^5+147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*sin(d*x+c)*cos(d
*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
```

```
^5-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*b^5+75*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5)/(a+b*cos(d*x+c))^(1/2)/a
^3/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\cos(dx+c)^{2}+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/
2), x)
```

3.410 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=779

$$\frac{\left(-15a^2B+50aAb+64b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{\left(50a^2Ab-15a^3B+172ab^2B+120Ab^3\right)\mathrm{s}}{32}$$

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2
*B + 1024*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d) - (Sqrt[
a + b]*(45*a^4*B - 30*a^3*b*(5*A + B) - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355
*A + 193*B) - 4*a^2*b^2*(295*A + 423*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(1920*b^2*d) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a^5
*B - 40*a^3*b^2*B - 240*a*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(128*b^3*d) + ((150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B
 + 1024*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sqrt[Cos[
c + d*x]]) + ((50*a^2*A*b + 120*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) + ((50*a*A*b - 15*
a^2*B + 64*b^2*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(240*b*d) + ((10*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(40*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(7/
2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 3.08251, antiderivative size = 779, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-15a^2B+50aAb+64b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{\left(50a^2Ab-15a^3B+172ab^2B+120Ab^3\right)\mathrm{s}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2
*B + 1024*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d) - (Sqrt[
a + b]*(45*a^4*B - 30*a^3*b*(5*A + B) - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355
*A + 193*B) - 4*a^2*b^2*(295*A + 423*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(1920*b^2*d) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a^5
*B - 40*a^3*b^2*B - 240*a*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(128*b^3*d) + ((150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B
 + 1024*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sqrt[Cos[
c + d*x]]) + ((50*a^2*A*b + 120*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) + ((50*a*A*b - 15*
a^2*B + 64*b^2*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(240*b*d) + ((10*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(40*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(7/
2)*Sin[c + d*x])/(5*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd} + \frac{\int \frac{(a+b}{}}{}$$

$$= \frac{(10Ab-3aB)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+}{40bd}$$

$$= \frac{\left(50aAb-15a^2B+64b^2B\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx}{240bd}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a}}{320bd}$$

$$= \frac{\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B\right.}{1920b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B\right.}{1920b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5B-40a^3b^2B\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2\right.}{}$$

<u>Mathematica</u> [C] time = 6.50658, size = 1353, normalized size = 1.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] -((-4*a*(-1330*a^3*A*b - 3560*a*A*b^3 + 15*a^4*B - 3236*a^2*b^2*B - 1024*b^
4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
```

```
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6440*a^2*A*b^2 - 1440*A*b^4 - 2292
*a^3*b*B - 4624*a*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])) + 2*(-150*a^3*A*b - 2840*a*A*b^3 + 45*a^4*B - 1692*a^2*b^2*B
 - 1024*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*Ar
cSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/
(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c +
 d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[
c + d*x]])))/(3840*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((5
90*a^2*A*b + 420*A*b^3 + 15*a^3*B + 898*a*b^2*B)*Sin[c + d*x])/(960*b) + ((
170*a*A*b + 93*a^2*B + 88*b^2*B)*Sin[2*(c + d*x)])/480 + (b*(10*A*b + 21*a*
B)*Sin[3*(c + d*x)])/160 + (b^2*B*Sin[4*(c + d*x)])/40))/d
```

Maple [B] time = 0.894, size = 5164, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{5}{2}}\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos{(dx+c)}^4+Aa^2\cos{(dx+c)}+\left(2\,Bab+Ab^2\right)\cos{(dx+c)}^3+\left(Ba^2+2\,Aab\right)\cos{(dx+c)}^2\right)\sqrt{b\cos{(dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sq
rt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.411 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=664

$$\frac{\left(5a^2B+24aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\left(264a^2Ab+15a^3B+284ab^2B+128Ab^3\right)\sin(c}{192bd\sqrt{\cos(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d) + (Sqrt[a + b]*(15*a^3*B + 8*b
^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59*B) + 4*a*b^2*(52*A + 71*B))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b^
3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(64*b^2*d) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + ((2
4*a*A*b + 5*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(32*d) + ((8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(24*d) + (b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 2.20587, antiderivative size = 664, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(5a^2B+24aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\left(264a^2Ab+15a^3B+284ab^2B+128Ab^3\right)\sin(c}{192bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
```

```
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d) + (Sqrt[a + b]*(15*a^3*B + 8*b
^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59*B) + 4*a*b^2*(52*A + 71*B))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b^
3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(64*b^2*d) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + ((2
4*a*A*b + 5*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(32*d) + ((8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(24*d) + (b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{1}{4}\int \sqrt{}$$

$$= \frac{(8Ab+11aB)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+d}{24d}$$

$$= \frac{\left(24aAb+5a^2B+12b^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32d}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{a+b\cos(c+}}{192bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{a+b\cos(c+}}{192bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)}{}$$

Mathematica [C] time = 6.38703, size = 1287, normalized size = 1.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] ((-4*a*(472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(384*a^3*A + 608*a*A*b^2 + 644*a^2*b*B + 144*b^3*B)*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(264*a^2*A*
b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)
/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a
+ b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(384*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*(((104*a*A*b + 59*a^2*B + 42*b^2*B)*Sin[c + d*x])/96 + (b
*(8*A*b + 17*a*B)*Sin[2*(c + d*x)])/48 + (b^2*B*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.661, size = 4238, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

[Out] -1/192/d/(a+b*cos(d*x+c))^(1/2)*(72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+264*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+272*
A*cos(d*x+c)^4*a*b^3-264*A*cos(d*x+c)^2*a^2*b^2-264*A*cos(d*x+c)*a^3*b-15*B
*cos(d*x+c)^2*a^3*b+184*B*cos(d*x+c)^5*a*b^3+254*B*cos(d*x+c)^4*a^2*b^2+64*
A*cos(d*x+c)^5*b^4+48*B*cos(d*x+c)^6*b^4+24*B*cos(d*x+c)^4*b^4-72*B*cos(d*x
+c)^2*b^4+64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b^4+15*B*cos(d*x+c)^2*a^
4+30*B*cos(d*x+c)^2*a^2*b^2-284*B*cos(d*x+c)^2*a*b^3-118*B*cos(d*x+c)*a^3*b
-284*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+472*A*cos(d*x+c)^3*a^2*b^2+
264*A*cos(d*x+c)^2*a^3*b-144*A*cos(d*x+c)^2*a*b^3+133*B*cos(d*x+c)^3*a^3*b+
172*B*cos(d*x+c)^3*a*b^3-208*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*b^3-15
*B*cos(d*x+c)*a^4-384*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+128*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+15*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a^4-30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4-144*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*b^4+264*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^3*b+264*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+128*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+240*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b
+960*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a*b^3+208*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-608*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+15*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+284*B*(cos(d*

x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b
^2+284*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*a*b^3+720*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+118*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-644*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+264*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a^2*b^2+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+240*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+96
0*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*a*b^3+208*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-608*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*
b^3+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*a^3*b+284*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+284*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a
*b^3+720*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+118*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-644*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a^2*b^2+72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-384*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+128*A*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+

```
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4+15*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4-3
0*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^4+288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*b^4-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.412 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=564

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(a^2(48A+33B)+a(54Ab+26bB)+4b^2(3A+4B)\right)}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(24*a*d) + (Sqrt[a + b]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*
B) + a*(54*A*b + 26*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) - (Sqrt
[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(8*b*d) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (b*(2*A*b + 3
*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*
B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.6984, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(a^2(48A+33B)+a(54Ab+26bB)+4b^2(3A+4B)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(24*a*d) + (Sqrt[a + b]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*
B) + a*(54*A*b + 26*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
```

```
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) - (Sqrt
[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(8*b*d) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (b*(2*A*b + 3
*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*
B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{bB\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\sqrt{a+b\cos(}}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{bB\sqrt{\cos}}{}$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{b(2Ab+}{}$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{b(2Ab+}{}$$

$$= -\frac{\sqrt{a+b}\left(30a^2Ab+8Ab^3+5a^3B+20ab^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-}\right.}{8bd}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(54aAb+33a^2B+16b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{24ad}$$

Mathematica [C] time = 6.47815, size = 1251, normalized size = 2.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] ((-4*a*(48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - 4*a*(144*a^2*A*b + 24*A*b^3 + 48*a^3*B + 76*a*b^2*B)*((Sqrt[((a +
```

```
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(54*a*A*b^2 + 33*a
^2*b*B + 16*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[
I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*
x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec
[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[
Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b*
(6*A*b + 13*a*B)*Sin[c + d*x])/12 + (b^2*B*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.68, size = 3512, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*(180*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-144*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+12*A*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^2+54*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
```

(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+54*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+120*B*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*a*b^2+26*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+33*B*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+16*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+66*A*cos(d*x+c)^3*a*b^2+54*A*cos(d*x+c)^
2*a^2*b-54*A*cos(d*x+c)^2*a*b^2-54*A*cos(d*x+c)*a^2*b-12*A*cos(d*x+c)*a*b^2
+34*B*cos(d*x+c)^4*a*b^2+59*B*cos(d*x+c)^3*a^2*b-33*B*cos(d*x+c)^2*a^2*b-18
*B*cos(d*x+c)^2*a*b^2-26*B*cos(d*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+12*A*cos(
d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^
3+33*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-33*B*cos(d*x+c)*a^3-144*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a
^2*b+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*A*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^3+30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a^3+33*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+1
80*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*a^2*b+12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^2+54*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+54*A*sin(d*x+c)*(cos(d*x+c)/(1+c

```
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+120*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2
6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b-76*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^2+48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*B*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+48*A*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*b^3-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^3+30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+48*A*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

3.413 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=547

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\cot(c+dx}{4}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*a*d) - (Sqrt[a + b]*(8*a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*
(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(20*a*
A*b + 15*a^2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*d) - ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.66511, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2989, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\cot(c+dx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*a*d) - (Sqrt[a + b]*(8*a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*
(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
```

```
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(20*a*
A*b + 15*a^2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*d) - ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}a(\right.}{}$$

$$= -\frac{b(4aA-bB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{2aA(a+}{}$$

$$= -\frac{\left(8a^2A-4Ab^2-9abB\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{b(4aA-b}{}$$

$$= -\frac{\left(8a^2A-4Ab^2-9abB\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{b(4aA-b}{}$$

$$= -\frac{\sqrt{a+b}\left(20aAb+15a^2B+4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(8a^2A-4Ab^2-9abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{4ad}$$

Mathematica [C] time = 6.47292, size = 1241, normalized size = 2.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] ((4*a*(-16*a^2*A*b - 4*A*b^3 - 8*a^3*B - 11*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
```

```
*x]]) + 4*a*(8*a^3*A - 24*a*A*b^2 - 24*a^2*b*B - 4*b^3*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(8*a^2*A*b - 4*A*b^3 -
9*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSin
h[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x
])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
d*x]])))/(8*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b^2*B*Sin[c
 + d*x])/2 + 2*a^2*A*Tan[c + d*x]))/d
```

Maple [B] time = 0.454, size = 3270, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(-8*A*a^3+2*B*cos(d*x+c)^4*b^3+24*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-24*A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
```

cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*B*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^2+9*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*B*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+
11*B*cos(d*x+c)^3*a*b^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+40*A*sin(d*x+c)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+30*B*sin(d*x+
c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*a^2*b+8*A*cos(d*x+c)^2*a^2*b+4*A*cos(d*x+c)^2*a*b^2-8*A*cos(d*x+c)
*a^2*b-4*A*cos(d*x+c)*a*b^2+9*B*cos(d*x+c)^2*a^2*b-9*B*cos(d*x+c)^2*a*b^2-9
*B*cos(d*x+c)*a^2*b-2*B*cos(d*x+c)*a*b^2+4*A*cos(d*x+c)^3*b^3-4*A*cos(d*x+c
)^2*b^3-2*B*cos(d*x+c)^2*b^3+8*A*cos(d*x+c)*a^3+24*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+9*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*A
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^3-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*b^3+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c

```
),-1,(-(a-b)/(a+b))^(1/2))*b^3+30*B*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-8*A*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3+4*A*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*b^3-4*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+8*B*sin(d*x+c)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+40*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b
^2+8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*A*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)/(a+b*cos(
d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

3.414 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=536

$$-\frac{(6a^2B+14aAb-3b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B))\cot(c}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3
*b^2*(6*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (b*Sqrt[a + b]*(
2*A*b + 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(2*A
*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (
(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.66887, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(6a^2B+14aAb-3b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B))\cot(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3
*b^2*(6*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
```

```
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (b*Sqrt[a + b]*(
2*A*b + 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(2*A
*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (
(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
```

```
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3}{2}a(\right.}{}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA(a+b\cos(c+dx}{3d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(14aAb+6a^2B-3b^2\right.}{3a}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(14aAb+6a^2B-3b^2\right.}{3a}$$

$$= -\frac{b\sqrt{a+b}(2Ab+5aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|-\frac{a}{a}}{d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(14aAb+6a^2B-3b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{3ad}$$

Mathematica [C] time = 6.47071, size = 1269, normalized size = 2.37

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] ((-4*a*(2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
```

```
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(-14*a^2*A*b + 6*A*b^3 - 6*a^3*B + 18*a*b^2*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-14*a*A*b^2 - 6*a^2*b*
B + 3*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(6*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(7*a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x]))/3 + (2*a^2*A*Sec[c + d
*x]*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.475, size = 3204, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*(-2*A*a^3+18*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-6*B*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
```

x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+3*B*sin (d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1 /2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x +c)))^(1/2)*a*b^2+14*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^ (1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x +c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+18*A*sin(d*x+c)*cos(d*x+c)^2*(c os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-14*A* sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos (d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b) /(a+b))^(1/2))*a^2*b+3*B*cos(d*x+c)^4*b^3-14*A*sin(d*x+c)*cos(d*x+c)^2*(cos (d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/ 2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+14*A*co s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos (d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))* sin(d*x+c)*a^2*b+18*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/ 2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-14*A*cos(d*x+c)*sin(d*x+c)*(cos(d* x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)* EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-14*A*cos(d *x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)) ^(1/2))*a*b^2+30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)* (1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/ sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+18*B*cos(d*x+c)*sin(d*x+c)*(cos(d *x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2) *EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-18*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)* (1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c) /(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-18*B*EllipticF ((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d *x+c)*a*b^2+2*A*cos(d*x+c)^3*a^2*b+3*B*cos(d*x+c)^3*a*b^2+14*A*cos(d*x+c)^3 *a*b^2+14*A*cos(d*x+c)^2*a^2*b-14*A*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2* b+6*B*cos(d*x+c)^3*a^2*b-6*B*cos(d*x+c)^2*a^2*b-3*B*cos(d*x+c)^2*a*b^2+2*A* cos(d*x+c)^2*a^3+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d *x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a- b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-3*B*cos(d*x+c)^3*b^3+6*B*cos (d*x+c)^2*a^3-6*B*cos(d*x+c)*a^3-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d *x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+12*A*(cos(d*x+c)/(1

```
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*b^3-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+12*A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^3-6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b^3-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+3*B*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+2*A*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^3+6*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+6*B*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

3.415 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=493

$$\frac{2\sqrt{a+b}\left(a^2b(17A-35B)+a^3(-(9A-5B))-ab^2(23A-45B)+15Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-}\right.}{15ad}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a*d) + (2*Sqrt[a + b]*(15*A*b^3 - a*b^2*(23*A - 45*B) +
a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a*d) - (2*b^2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/d + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Co
s[c + d*x]^(3/2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2))
```

Rubi [A] time = 1.24664, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2989, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2b(17A-35B)+a^3(-(9A-5B))-ab^2(23A-45B)+15Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-}\right.}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a*d) + (2*Sqrt[a + b]*(15*A*b^3 - a*b^2*(23*A - 45*B) +
a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
```

qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) - (2*b^2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))

Rule 2989

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3047

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

Rule 3053

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}a(\right.}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+d}{5d\cos^{\frac{5}{2}}(c}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+d}{5d\cos^{\frac{5}{2}}(c}$$

$$= -\frac{2b^2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+23Ab^2+35abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\sqrt{}}\right.\right.}{15ad}$$

Mathematica [C] time = 6.51808, size = 1319, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] ((4*a*(-8*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) + 4*a*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B - 15*b^3*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(9*a^2*A*b + 23*A*b^3 +
```

```
 35*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(15*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c
+ d*x]^2*(11*a*A*b*Sin[c + d*x] + 5*a^2*B*Sin[c + d*x]))/15 + (2*Sec[c + d*
x]*(9*a^2*A*Sin[c + d*x] + 23*A*b^2*Sin[c + d*x] + 35*a*b*B*Sin[c + d*x]))/
15 + (2*a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.491, size = 3274, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+35*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-35*B*sin(d*x+c)*cos(d*x+c)^2*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-35*B*
sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*a*b^2+17*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+23*A*sin(d*x+c)*cos(d*x+c)^3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*
A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a^2*b-23*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
```

+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+35*B*sin(d*x+c)*cos(d*x+c)
^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-
35*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a^2*b-35*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+17*A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b+23*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-23*A*sin(d*x+c)*cos(d
x+c)^2(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^2+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^3*a*b^2+45*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2+5*A*cos(d*x+c)^3*a
^2*b+9*A*cos(d*x+c)^4*a^2*b+11*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2*b-
35*B*cos(d*x+c)^3*a*b^2+23*A*cos(d*x+c)^3*a*b^2-34*A*cos(d*x+c)^2*a*b^2-14*
A*cos(d*x+c)*a^2*b+35*B*cos(d*x+c)^4*a*b^2+35*B*cos(d*x+c)^3*a^2*b-40*B*cos
(d*x+c)^2*a^2*b+15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3-15*B*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*b^3+30*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*b^3-15*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*b^3+30*B*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*b^3+5*B*co
s(d*x+c)^3*a^3+9*A*cos(d*x+c)^3*a^3-23*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*
a^3+23*A*cos(d*x+c)^4*b^3-5*B*cos(d*x+c)*a^3+15*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+9*A*si
n(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+

```
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3-23*A*sin(d*x+c)*cos(d*x+c)^3*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+5*B*sin(d*x
+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-23*A*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^3+5*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2
), x)
```

3.416 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=434

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)-8ab(15A-7B)+15b^2(A\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(a - b)*Sqrt[a + b]*(a^2*
(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*a*b*(15*A - 7*B))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(105*a*d) + (2*a*(10*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*
A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.37185, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)-8ab(15A-7B)+15b^2(A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(a - b)*Sqrt[a + b]*(a^2*
(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*a*b*(15*A - 7*B))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(105*a*d) + (2*a*(10*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*S
```

```
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*
A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0]))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}a(\right.}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+}{7d\cos^{\frac{7}{2}}(}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2A+45Ab^2\right.}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2A+45Ab^2\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(145a^2Ab+15Ab^3+63a^3B+161ab^2B\right)\cot(c+dx)E}{105a^2d}$$

Mathematica [C] time = 6.57998, size = 1409, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 10*a^2*A*b^2 - 15*A*b^4 + 56*a^3*b*B - 56*a*b^3*B)*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - 4*a*(-145*a^3*A*b - 15*a*A*b^3 - 63*a^4*B - 161*a^
2*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
```

```
 2*(-145*a^2*A*b^2 - 15*A*b^4 - 63*a^3*b*B - 161*a*b^3*B)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a*d) + (Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(15*a*A*b*Sin[c +
d*x] + 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x]
 + 45*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x]))/105 + (2*Sec[c + d*x]*(1
45*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] + 1
61*a*b^2*B*Sin[c + d*x]))/(105*a) + (2*a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/7
))/d
```

Maple [B] time = 0.563, size = 3628, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-15*A*a^4+25*A*cos(d*x+c)^5*a^3*b+145*A
*cos(d*x+c)^5*a^2*b^2+45*A*cos(d*x+c)^5*a*b^3+145*A*cos(d*x+c)^4*a^3*b-55*A
*cos(d*x+c)^4*a^2*b^2+15*A*cos(d*x+c)^4*a*b^3-110*A*cos(d*x+c)^3*a^3*b-60*A
*cos(d*x+c)^3*a*b^3-90*A*cos(d*x+c)^2*a^2*b^2-60*A*cos(d*x+c)*a^3*b-238*B*c
os(d*x+c)^3*a^2*b^2-98*B*cos(d*x+c)^2*a^3*b+63*B*cos(d*x+c)^5*a^3*b+77*B*co
s(d*x+c)^5*a^2*b^2+161*B*cos(d*x+c)^5*a*b^3+35*B*cos(d*x+c)^4*a^3*b+161*B*c
os(d*x+c)^4*a^2*b^2-161*B*cos(d*x+c)^4*a*b^3+15*A*cos(d*x+c)^5*b^4-15*A*cos
(d*x+c)^4*b^4+25*A*cos(d*x+c)^4*a^4+25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*A*sin(d*x+c)
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
```

1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-10*A*cos(d*x+c)^2*a^4+63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+c)*a^4+25*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4-15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b-161*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^2-161*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3+145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+135*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+119*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+161*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^2-63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3*b-161*B*(cos(d*x+c)/(1+cos(d*x+c)))

```
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^2-161*
B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^3+135*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^2+15*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b
^3-145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*A*sin(d*x+c)*
cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3+119*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b+161*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+105*B*cos
(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^3+105*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3)/(a+b*cos(d*x+c))^(1/2)/a/sin(
d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2
), x)
```

3.417 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sqrt{a+}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*S
qrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A - 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b
^2*(11*A - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a*(4*A
*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)
) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 13
5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/
2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/
2))
```

Rubi [A] time = 1.93889, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sqrt{a+}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*S
```

```
qrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A - 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b
^2*(11*A - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a*(4*A
*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)
) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 13
5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/
2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/
2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3}{2}a(\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+d}{9d\cos^{\frac{9}{2}}(c}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2+\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2+\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2+\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4A+279a^2Ab^2-10Ab^4+435a^3bB+45ab^3B\right)}{3}$$

Mathematica [C] time = 6.68505, size = 1517, normalized size = 2.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] -((-4*a*(-114*a^4*A*b + 124*a^2*A*b^3 - 10*A*b^5 - 75*a^5*B + 30*a^3*b^2*B
+ 45*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 279*a^3*A*b^2
- 10*a*A*b^4 + 435*a^4*b*B + 45*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
```

```
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 279*a^2*A*b^3 - 10*A*b^
5 + 435*a^3*b^2*B + 45*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]]))))/(315*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]^4*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin[c + d
*x]))/63 + (2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*x]
 + 135*a*b*B*Sin[c + d*x]))/315 + (2*Sec[c + d*x]^2*(163*a^2*A*b*Sin[c + d*
x] + 5*A*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] + 135*a*b^2*B*Sin[c + d*x
]))/(315*a) + (2*Sec[c + d*x]*(147*a^4*A*Sin[c + d*x] + 279*a^2*A*b^2*Sin[c
 + d*x] - 10*A*b^4*Sin[c + d*x] + 435*a^3*b*B*Sin[c + d*x] + 45*a*b^3*B*Sin
[c + d*x]))/(315*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 0.735, size = 4392, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(261*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
```

2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+435*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+405*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+45*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-435*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+261*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-279*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+435*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+405*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+45*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-435*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d

*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^4*b-435*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-45*B*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3
-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^4-35*A*a^5-270*B*cos(d*x+c)^3*a^3*b^2-180*B*cos(d*
x+c)^2*a^4*b+75*B*cos(d*x+c)^6*a^4*b+435*B*cos(d*x+c)^6*a^3*b^2+135*B*cos(d
*x+c)^6*a^2*b^3+45*B*cos(d*x+c)^6*a*b^4+435*B*cos(d*x+c)^5*a^4*b-165*B*cos(
d*x+c)^5*a^3*b^2+45*B*cos(d*x+c)^5*a^2*b^3-45*B*cos(d*x+c)^5*a*b^4-330*B*co
s(d*x+c)^4*a^4*b-180*B*cos(d*x+c)^4*a^2*b^3-130*A*cos(d*x+c)*a^4*b+5*A*cos(
d*x+c)^6*a*b^4+65*A*cos(d*x+c)^5*a^4*b+279*A*cos(d*x+c)^5*a^3*b^2-199*A*cos
(d*x+c)^5*a^2*b^3-10*A*cos(d*x+c)^5*a*b^4-272*A*cos(d*x+c)^4*a^3*b^2+5*A*co
s(d*x+c)^4*a*b^4-82*A*cos(d*x+c)^3*a^4*b-80*A*cos(d*x+c)^3*a^2*b^3-170*A*co
s(d*x+c)^2*a^3*b^2+147*A*cos(d*x+c)^6*a^4*b+163*A*cos(d*x+c)^6*a^3*b^2+279*
A*cos(d*x+c)^6*a^2*b^3-10*A*cos(d*x+c)^6*b^5+147*A*cos(d*x+c)^5*a^5+10*A*co
s(d*x+c)^5*b^5-98*A*cos(d*x+c)^4*a^5-14*A*cos(d*x+c)^2*a^5+75*B*cos(d*x+c)^
5*a^5-30*B*cos(d*x+c)^3*a^5-45*B*cos(d*x+c)*a^5+147*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos
(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-14
7*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^5+10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+279*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(
d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2
+155*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^4*b-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+75*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*sin(d*x+
c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^5-147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s

```
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+10*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+75*B*sin(d*x+c)
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^5)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/
2), x)
```

3.418 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=622

$$\frac{2\left(1025a^2Ab^2+675a^4A+1793a^3bB+55ab^3B-20Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3465a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(1145a^2Ab+539a^3B+82\right.}{3}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*
a^4*d) + (2*(a - b)*Sqrt[a + b]*(40*A*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b
*(505*A - 209*B) + 15*a^2*b^2*(19*A - 121*B) + 10*a*b^3*(3*A - 11*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*(81*a^2*A +
113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Cos[c
+ d*x]^(7/2)) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*a*d*Cos[c + d*x]^(5/2)) + (2*(675*
a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Cos[c + d*x]^(3/2)) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 2.6248, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(1025a^2Ab^2+675a^4A+1793a^3bB+55ab^3B-20Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3465a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(1145a^2Ab+539a^3B+82\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x
]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
```

```
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*
a^4*d) + (2*(a - b)*Sqrt[a + b]*(40*A*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b
*(505*A - 209*B) + 15*a^2*b^2*(19*A - 121*B) + 10*a*b^3*(3*A - 11*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*(81*a^2*A +
113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Cos[c
+ d*x]^(7/2)) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*a*d*Cos[c + d*x]^(5/2)) + (2*(675*
a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Cos[c + d*x]^(3/2)) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}a(\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+}{11d\cos^{\frac{11}{2}}}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3705a^4Ab+255a^2Ab^3+40Ab^5+1617a^5B+3069a^3b\right.}{}$$

Mathematica [C] time = 6.80217, size = 1640, normalized size = 2.64

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
3/2),x]
```

```
[Out] ((-4*a*(675*a^6*A - 390*a^4*A*b^2 - 245*a^2*A*b^4 - 40*A*b^6 + 1254*a^5*b*B
 - 1364*a^3*b^3*B + 110*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3705*a
```

```
^5*A*b - 255*a^3*A*b^3 - 40*a*A*b^5 - 1617*a^6*B - 3069*a^4*b^2*B + 110*a^2
*b^4*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
2*(-3705*a^4*A*b^2 - 255*a^2*A*b^4 - 40*A*b^6 - 1617*a^5*b*B - 3069*a^3*b^3
*B + 110*a*b^5*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I
*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x
])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[
c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[C
os[c + d*x]]))))/(3465*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*((2*Sec[c + d*x]^5*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c + d*x]))/99 + (
2*Sec[c + d*x]^4*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x] + 209*a*b*
B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(1145*a^2*A*b*Sin[c + d*x] + 15*A*
b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin[c + d*x]))/(346
5*a) + (2*Sec[c + d*x]^2*(675*a^4*A*Sin[c + d*x] + 1025*a^2*A*b^2*Sin[c + d
*x] - 20*A*b^4*Sin[c + d*x] + 1793*a^3*b*B*Sin[c + d*x] + 55*a*b^3*B*Sin[c
+ d*x]))/(3465*a^2) + (2*Sec[c + d*x]*(3705*a^4*A*b*Sin[c + d*x] + 255*a^2*
A*b^3*Sin[c + d*x] + 40*A*b^5*Sin[c + d*x] + 1617*a^5*B*Sin[c + d*x] + 3069
*a^3*b^2*B*Sin[c + d*x] - 110*a*b^4*B*Sin[c + d*x]))/(3465*a^3) + (2*a^2*A*
Sec[c + d*x]^5*Tan[c + d*x])/11))/d
```

Maple [B] time = 0.94, size = 5373, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(13/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/
2), x)
```

3.419 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=418

$$-\frac{B(a-3b)\sqrt{a+b}\left(2a^2-ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2B(a-b)}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d) - ((a - 3*b)*Sqrt[a + b]*(2*a^2 - a*b + 3*b^2)*B*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d) - (b*Sqrt[a + b]*(5*a + (3*b^2)/a)*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.949508, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {2989, 2991, 2809, 2998, 2816, 2994}

$$-\frac{B(a-3b)\sqrt{a+b}\left(2a^2-ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2B(a-b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Cos[c + d
*x]^(5/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d) - ((a - 3*b)*Sqrt[a + b]*(2*a^2 - a*b + 3*b^2)*B*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
```

```
]))/(a - b)])/(a*d) - (b*Sqrt[a + b]*(5*a + (3*b^2)/a)*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(d*Cos[c + d*x]^(3/2))
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2991

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(B*d
)/b^2, Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3}{2}\left(a\right.\right.}{}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{3}{2}a\left(a^2+3b^2\right)B+\left(\frac{3}{2}b\left(a\right.\right.}{\cos^{\frac{3}{2}}(c}$$

$$= -\frac{b\sqrt{a+b}\left(5a+\frac{3b^2}{a}\right)B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ad}$$

Mathematica [C] time = 19.4089, size = 1236, normalized size = 2.96

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Cos
[c + d*x]^(5/2),x]
```

```
[Out] -(B*((-4*a*(-5*a^3*b - 3*a*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(2*a^4 +
a^2*b^2 - 3*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/
b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]])) + 2*(2*a^3*b + 6*a*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)
]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
 + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(b*Sqrt[Cos[c + d*x]]))))/(2*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*(Sec[c + d*x]*(2*a^2*B*Sin[c + d*x] + 7*b^2*B*Sin[c + d*x]) + a*b
*B*Sec[c + d*x]*Tan[c + d*x]))/d
```

Maple [B] time = 0.549, size = 2346, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

[Out] -B/a/d*(6*a^2*b^2*cos(d*x+c)^2+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*cos(d*x+c)*a^2*b^2+2*cos(d*x+c)^3*a^2*b^2+6*cos(d*x+c)^3*a*b^3-7*cos(d*x+c)^2*a*b^3+2*cos(d*x+c)^3*a^3*b-cos(d*x+c)^2*a^3*b+cos(d*x+c)^4*a*b^3-b*a^3-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-2*a^4*cos(d*x+c)+2*cos(d*x+c)^2*a^4+7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+9*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+10*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2

```
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+10*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*a^2*b^2-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos
(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\int \frac{\left(2\,B\cos\left(dx+c\right)+\frac{3\,Bb}{a}\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="maxima")
```

```
[Out] 1/2*integrate((2*B*cos(d*x + c) + 3*B*b/a)*(b*cos(d*x + c) + a)^(5/2)/cos(d
*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(2\,Bab^2\cos\left(dx+c\right)^3+3\,Ba^2b+\left(4\,Ba^2b+3\,Bb^3\right)\cos\left(dx+c\right)^2+2\left(Ba^3+3\,Bab^2\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+\right.}}{2\,a\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*B*a*b^2*cos(d*x + c)^3 + 3*B*a^2*b + (4*B*a^2*b + 3*B*b^3)*
cos(d*x + c)^2 + 2*(B*a^3 + 3*B*a*b^2)*cos(d*x + c))*sqrt(b*cos(d*x + c) +
a)/(a*cos(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,B\cos\left(dx+c\right)+\frac{3\,Bb}{a}\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{2\,\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="giac")
```

```
[Out] integrate(1/2*(2*B*cos(d*x + c) + 3*B*b/a)*(b*cos(d*x + c) + a)^(5/2)/cos(d
*x + c)^(5/2), x)
```

3.420 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=479

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}+\frac{(4Ab}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*b^2*d) + (Sqrt[a + b]*(4*A*b - 3*a*B + 2*b*B)*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^2*d) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Cot[c + d*x
]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*A*b - 3*a*B)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 1.07636, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}+\frac{(4Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*b^2*d) + (Sqrt[a + b]*(4*A*b - 3*a*B + 2*b*B)*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^2*d) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Cot[c + d*x
]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
```

```
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*A*b - 3*a*B)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} + \frac{\int \frac{\frac{aB}{2}+bB\cos(c+dx)+\frac{1}{2}(4Ab-3aB)\cos^2}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}$$

$$= \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}{2bd}$$

$$= \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}{2bd}$$

$$= \frac{\sqrt{a+b}\left(4aAb-3a^2B-4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{4b^3d}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab-3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{4ab^2d}$$

<u>Mathematica</u> [C] time = 12.3764, size = 1175, normalized size = 2.45

$$-\frac{4a(4Ab-aB)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx\right.}{a}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

$$\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} + \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) + ((-4
*a*(4*A*b - a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
```

```
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a*b*B*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b - 3*a*B)*((I*Cos[(
c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/S
qrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*
((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))))/b + (
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*b*d)
```

Maple [B] time = 0.479, size = 1871, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)^3*a*b
+3*B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c
)*a*b+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+2*B*cos(d*x+c)^4*b^2-2*B*co
s(d*x+c)^2*b^2-3*B*cos(d*x+c)^2*a^2+3*B*cos(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
```

```
)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a*b+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/sin(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

3.421 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=427

$$-\frac{\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+dx)\sqrt{\mathrm{co}}}{d\sqrt{a+b\cos(c}}$$

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) - (Sqrt[a + b]*(2*A*b -
 a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c +
 d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.08826, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3003, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+dx)\sqrt{\mathrm{co}}}{d\sqrt{a+b\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) - (Sqrt[a + b]*(2*A*b -
 a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c +
```

```
 d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} + \frac{1}{2}\int \frac{aB+2aA\cos(c+dx)+(2Ab-aB)\cos^2(c}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{abB+(2aAb-a(2Ab-aB))\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{2b} + \frac{(2Ab-aB)\int}{}$$

$$= -\frac{\sqrt{a+b}(2Ab-aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a-}}}{b^2d}$$

$$= -\frac{\sqrt{a+b}(2Ab-aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a-}}}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{abd}$$

Mathematica [C] time = 17.7297, size = 4017, normalized size = 9.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] ((1 + Cos[c + d*x])^(3/2)*((A*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]
+ (B*Cos[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]])*Sec[(c + d*x)/2]^2*((2*I
)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] +
 (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]
], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*
Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)
/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(
c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[a + b*Cos[c + d*x]]*(((1 +
Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*B*Sqrt[(
```

a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*Sqrt[(a - b)/(a + b)]*(a + b*Cos[c + d*x])^(3/2)) - (3*Sqrt[1 + Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*b*Sqrt[(a - b)/(a + b)]*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*((2*I)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*((3*b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Cos[(3*(c + d*x))/2]*Sec[(c + d*x)/2])/2 + a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2 - (b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2)/2 + (I*(a - b)*B*EllipticE[I*A

```
rcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))] + ((2*I)*(A*b - a*B)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] -
 ((4*I)*A*b*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
)^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((2*I)*a*B
*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/
2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))
+ ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*Sqrt[(a - b)/(a +
b)]*B*Sec[(c + d*x)/2]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
Sin[c + d*x]/(1 + Cos[c + d*x]))*Sin[(3*(c + d*x))/2])/(2*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]) + (a*Sqrt[(a - b)/(a + b)]*B*((Cos[c + d*x]*Sin[c + d
*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/
2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*Sqrt[(a - b)/(a + b)]*B*((Co
s[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d
*x]))*Tan[(c + d*x)/2])/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) + (b*Sqrt
[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*
Sin[(3*(c + d*x))/2]*Tan[(c + d*x)/2])/2 - (2*Sqrt[(a - b)/(a + b)]*(A*b -
a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 + ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]) - ((a - b)*Sqrt[(a - b)/(a + b)]*B*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2])/S
qrt[1 + ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (4*A*b*Sqrt[(a - b)/(a + b)
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^
2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 + ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]) - (2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 + ((a - b)*Tan[(c + d*x)/
2]^2)/(a + b)])))/(4*b*Sqrt[(a - b)/(a + b)]*Sqrt[a + b*Cos[c + d*x]])))
```

Maple [B] time = 0.53, size = 1005, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d/(a+b*cos(d*x+c))^(1/2)*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b-4*A*cos(d*x+c)*sin(d*x+c)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*B*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*a-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-4*A*sin(d*x+c)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+2*B
*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*a-B*sin(d*x+c)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a-B*sin(d*x+c)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)^3*b-B*cos(d*x+c)^2*a+b*B*cos(d*x+c)^2+B*c
os(d*x+c)*a)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

3.422 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=228

$$\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{}$$

```
[Out] (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*
B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rubi [A] time = 0.273712, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3006, 2809, 2816}

$$\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*
B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```


Rule 3006

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[B/d, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist
[(B*c - A*d)/d, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx + B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2A\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c}{a-b}}}{ad}$$

Mathematica [A] time = 1.48001, size = 146, normalized size = 0.64

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left((A-B)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2B\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\right.}{d\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*((A - B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
```

```
 2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2])
```

Maple [A] time = 0.464, size = 197, normalized size = 0.9

$$2\,\frac{(\sin(dx+c))^2}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))\,\sqrt{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\left(A\mathrm{EllipticF}\left(\frac{-1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(A*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2)))/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos{(dx + c)} + A}{\sqrt{b\cos{(dx + c)} + a}\sqrt{\cos{(dx + c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

3.423 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=230

$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(A-B)\cot(c+d}{}$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rubi [A] time = 0.31596, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2998, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(A-B)\cot(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + (-A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

$$= \frac{2A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a}{}}}{a^2d}$$

Mathematica [A] time = 12.8641, size = 299, normalized size = 1.3

$$\frac{2\left(A\sin(c+dx)(a+b\cos(c+dx)) - \frac{2\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\left(-2a(A+B)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-}{a+}\right.\right.}{}\right.}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (2*(A*(a + b*Cos[c + d*x])*Sin[c + d*x] - (2*Sqrt[2]*(Cos[(c + d*x)/2]^2)^(
3/2)*(2*A*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(A + B)*Cos[(c + d*x)/2]^2*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*Cos[c
 + d*x]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2)))/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.388, size = 935, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(-B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a-2*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^2*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a-B*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a+A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a-A*cos(d*x+c)^3*b-A*cos(d*x+c)^2*a+A*cos(d*x+
c)^2*b+A*cos(d*x+c)*a)/a/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

3.424 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=290

$$\frac{2\sqrt{a+b}(a(A-3B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d} - \frac{2(a-b)\sqrt{a+b}(2}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*A*b + a*(A - 3*B))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos
[c + d*x]^(3/2))
```

Rubi [A] time = 0.5236, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3000, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a(A-3B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d} - \frac{2(a-b)\sqrt{a+b}(2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*A*b + a*(A - 3*B))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos
[c + d*x]^(3/2))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-2Ab+3aB)+\frac{1}{2}aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{(2Ab+a(A-3B))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a}$$

$$= -\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{3a^3d}$$

Mathematica [A] time = 16.2928, size = 416, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(3aB\sin(c+dx)-2Ab\sin(c+dx))}{3a^2}+\frac{2A\tan(c+dx)\sec(c+dx)}{3a}\right)}{d} + \frac{8\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\cos^2\left(\frac{1}{2}(c+d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*A*b + a
*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^(3/
2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^2) + (2*
A*Sec[c + d*x]*Tan[c + d*x])/(3*a)))/d
```

Maple [B] time = 0.397, size = 1536, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)*a*b+2*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^2+A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2
-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^2*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+3*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin
(d*x+c)*a^2+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)^2*a*b+
A*cos(d*x+c)*a*b+A*cos(d*x+c)^2*a^2-3*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b+2*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+
c)*a*b-2*A*cos(d*x+c)^3*b^2+2*A*cos(d*x+c)^2*b^2+3*B*cos(d*x+c)^2*a^2-3*B*c
os(d*x+c)*a^2+A*cos(d*x+c)^3*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-
2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b-a^2*A+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/(a+b*
cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^4+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

3.425 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=363

$$-\frac{2\sqrt{a+b}\left(a^2(9A-5B)-2ab(A+5B)+8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*
b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*A*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b - 5*a*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.862124, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\left(a^2(9A-5B)-2ab(A+5B)+8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*
b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*A*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b - 5*a*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-4Ab+5aB)+\frac{3}{2}aA\cos(c+dx)+Ab\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+8Ab^2-10abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{15a^4d}$$

Mathematica [C] time = 6.38858, size = 1319, normalized size = 3.63

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] -((-4*a*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - 4*a*(9*a^3*A + 8*a*A*b^2 - 10*a^2*b*B)*((Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 8*A*b^3 - 10*a*b^2*B)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
15*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2
*(-4*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a^2) + (2*Sec[c + d*x]*(9*
a^2*A*Sin[c + d*x] + 8*A*b^2*Sin[c + d*x] - 10*a*b*B*Sin[c + d*x]))/(15*a^3
) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(5*a)))/d
```

Maple [B] time = 0.467, size = 2480, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3-10*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
```

a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+10*B*sin(d*x+c)*cos(d*x+c)^2*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+10*B*
sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*a*b^2+2*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*
sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-10*B*sin(d*x+c)*cos(d*x+c)^3*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+10*
B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a^2*b+10*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+2*A*sin(d*x+c)*cos(d*x+c)^
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-1
0*A*cos(d*x+c)^3*a^2*b+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2+5*B*co
s(d*x+c)^4*a^2*b+10*B*cos(d*x+c)^3*a*b^2+8*A*cos(d*x+c)^3*a*b^2-4*A*cos(d*x
+c)^2*a*b^2+A*cos(d*x+c)*a^2*b-10*B*cos(d*x+c)^4*a*b^2-10*B*cos(d*x+c)^3*a^
2*b+5*B*cos(d*x+c)^2*a^2*b+5*B*cos(d*x+c)^3*a^3+9*A*cos(d*x+c)^3*a^3-8*A*co
s(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*a^3+8*A*cos(d*x+c)^4*b^3-5*B*cos(d*x+c)*a^3
+9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+5*B*s
in(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d

```
*x+c)))^(1/2)*a^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^3+5*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x+c)/c
os(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^5+a\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^5 + a*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/2)
), x)
```

3.426 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=500

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(-3a^2B+2aAb+b\right.}{}$$

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt
[a + b]*d) - ((2*A*b - (3*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2
*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]])
```

Rubi [A] time = 1.28697, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(-3a^2B+2aAb+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt
[a + b]*d) - ((2*A*b - (3*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2
```

```
*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)+\frac{1}{2}b(Ab-aB)\cos(c+dx)+\frac{1}{2}(2aA}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c}}}{b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+b\cos(c+}}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx}}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+b\cos(c+}}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx}}$$

$$= -\frac{\sqrt{a+b}(2Ab-3aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-se}{a}}}{b^3d}$$

$$= \frac{\left(2aAb-3a^2B+b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{ab^2\sqrt{a+b}d}$$

Mathematica [C] time = 6.39673, size = 1234, normalized size = 2.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x]))/(b*(-a^
2 + b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*B - b^2*B)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-2*A*b^2 + 2*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
```

```
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-2*a*A*b + 3*a^2*B - b^2*B)*((I*Cos[(c
 + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*(
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(2*(a - b)*b
*(a + b)*d)
```

Maple [B] time = 0.427, size = 2881, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(-4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*B*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
```

```
a-b)/(a+b))^(1/2))*a*b^2-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*co
s(d*x+c)^2*a^2*b-2*A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)*a^2*b+2*A*cos(d*x+c)
*a*b^2-B*cos(d*x+c)^3*a^2*b+3*B*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2-2*B
*cos(d*x+c)*a^2*b-B*cos(d*x+c)*a*b^2+B*cos(d*x+c)^3*b^3-3*B*cos(d*x+c)^2*a^
3-B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)*a^3-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*A*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*b^3-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*B*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^3-4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-
6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*a*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-2*A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
```

```
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a
^3-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b^2/(a^2-b^2)/cos(d
*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

3.427 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{abd\sqrt{a+b}}$$

```
[Out] (-2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) + (2
*(A*b - a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) - (2*Sq
rt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (2*a*(
A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])
```

Rubi [A] time = 0.61189, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2992, 2809, 2794, 2795, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{abd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) + (2
*(A*b - a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) - (2*Sq
rt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (2*a*(
```

```
A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])
```

Rule 2992

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{(Ab-aB)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= -\frac{2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= -\frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(}{}}}{ab\sqrt{a+b}d}$$

Mathematica [C] time = 17.9859, size = 1012, normalized size = 2.43

$$\frac{2\sqrt{\cos(c+dx)}(aB\sin(c+dx)-Ab\sin(c+dx))}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\left(\frac{i\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\cos(c+dx)}E\left(i\sinh^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\middle|-\frac{2a}{-a-b}\right)\sec(c+dx)}{b\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{\frac{(a+b\cos(c+dx))\sec(c+dx)}{a+b}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x]))/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) - (-4*a*(a*A - b*B)*((Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(A*b - a*B)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
```

```
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/((-a + b)*(a + b)*d)
```

Maple [B] time = 0.411, size = 2013, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(A*cos(d*x+c)*b^2+B*cos(d*x+c)^2*a*b-B*cos(d*x+
c)*a*b+A*cos(d*x+c)^2*a*b-A*cos(d*x+c)*a*b-A*cos(d*x+c)^2*b^2-B*cos(d*x+c)^
2*a^2+B*cos(d*x+c)*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*a^2-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-B*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-
A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
```

```
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-2*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2
-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*b^2)/sin(d*x+c)/b/(a^2-b^2)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sqrt{\cos(c + dx)}}{(a + b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx + c) + A)\sqrt{\cos(dx + c)}}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

3.428 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{a^2d\sqrt{a+b}}$$

```
[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*
(A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*(A*b - a*
B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rubi [A] time = 0.511618, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2993, 2998, 2816, 2994}

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{a^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*
(A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*(A*b - a*
B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
```

```
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{Ab-aB+(aA-bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{(A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}}{a+b}$$

$$= \frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{a^2\sqrt{a+b}d}$$

<u>Mathematica</u> [C] time = 6.3602, size = 1223, normalized size = 4.31

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(a*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*A - A*b^2)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-(a*A*b) + a^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-(A*b^2) + a*b*B)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d
*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
```

```
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a*(a - b)*(a + b)*d)
```

Maple [B] time = 0.475, size = 1633, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d
*x+c)*a^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(
d*x+c)*a^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-A*sin(d*x+c)*(cos(d*x
```

```
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+A*cos(d*x+c)^2*a*b-A*cos(d*x+c)^2*b^2-B*cos(d*x+c)^2*a^2
+B*cos(d*x+c)^2*a*b-A*cos(d*x+c)*a*b+A*cos(d*x+c)*b^2+B*cos(d*x+c)*a^2-B*co
s(d*x+c)*a*b)/(a^2-b^2)/a/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^3+2\,ab\cos(dx+c)^2+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

3.429 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\right.\right.}{a^3d\sqrt{a+b}}$$

```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
 + b]*d) - (2*(2*A*b + a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.61453, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3000, 2998, 2816, 2994}

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\right.\right.}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
 + b]*d) - (2*(2*A*b + a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2A-2Ab^2+abB\right)-\frac{1}{2}a(Ab-aB)\,\mathrm{c}\cdots}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+d\cdots}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{((a-b)(2Ab+a(A-B)))\int \cdots}{a\left(a^2-b\cdots\right.}$$

$$= \frac{2\left(a^2A-2Ab^2+abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle| -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c\cdots}{a+b\cdots}}\cdots}{a^3\sqrt{a+b}d}$$

Mathematica [C] time = 6.47248, size = 1281, normalized size = 4.2

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] ((-4*a*(2*a^2*A*b - 2*A*b^3 - a^3*B + a*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- 4*a*(a^3*A - 2*a*A*b^2 + a^2*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])) + 2*(a^2*A*b - 2*A*b^3 + a*b^2*B)*((I*Cos[(c + d*
x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Co
s[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
```

```
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^2*(-a + b)*(a
+ b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b^3*Sin[c +
 d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2
*A*Tan[c + d*x])/a^2))/d
```

Maple [B] time = 0.472, size = 2280, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(-2*A*cos(d*x+c)*b^3+A*a^3+A*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+
2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*cos(d*x+c)
^2*a^2*b-A*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*a^2*b+2*A*cos(d*x+c)*a*b^2+B*cos
(d*x+c)^2*a^2*b-B*cos(d*x+c)^2*a*b^2-B*cos(d*x+c)*a^2*b+B*cos(d*x+c)*a*b^2-
A*a*b^2+2*A*cos(d*x+c)^2*b^3-A*cos(d*x+c)*a^3+A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
```

```
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2
+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+A*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3-2
*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*b^3-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-B*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)/
a^2/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
```

```
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^4+2\,ab\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

3.430 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=393

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5a^2Ab-3a^3B\right.}{}$$

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(a + 2*b)*(4*A*b + a*(A - 3*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(
a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.980115, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5a^2Ab-3a^3B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(a + 2*b)*(4*A*b + a*(A - 3*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(
a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a
```

```
^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2A-4Ab^2+3abB\right)-\frac{1}{2}a(Ab-aB)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}}{a\left(a^2-\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a+}}{3a^2\left(a^2-b^2\right)d\,\mathrm{c}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a+}}{3a^2\left(a^2-b^2\right)d\,\mathrm{c}}$$

$$= -\frac{2\left(5a^2Ab-8Ab^3-3a^3B+6ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{3a^4\sqrt{a+b}d}$$

Mathematica [C] time = 6.64411, size = 1357, normalized size = 3.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] ((-4*a*(a^4*A + 7*a^2*A*b^2 - 8*A*b^4 - 6*a^3*b*B + 6*a*b^3*B)*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - 4*a*(5*a^3*A*b - 8*a*A*b^3 - 3*a^4*B + 6*a^2*b^2*B)*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(5*a^2*A*b^
2 - 8*A*b^4 - 3*a^3*b*B + 6*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-5*A*b*Sin[c + d*x] + 3*
a*B*Sin[c + d*x]))/(3*a^3) - (2*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*
x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x
])/(3*a^2)))/d
```

Maple [B] time = 0.498, size = 3334, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-A*a^4+5*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin
(d*x+c)*a^3*b+A*cos(d*x+c)^3*a^3*b-4*A*cos(d*x+c)^3*a*b^3+4*A*cos(d*x+c)^2*
a^2*b^2+4*A*cos(d*x+c)*a^3*b+3*B*cos(d*x+c)^3*a^2*b^2-3*B*cos(d*x+c)^2*a^3*
b+A*a^2*b^2+8*A*cos(d*x+c)^3*b^4-8*A*cos(d*x+c)^2*b^4+3*B*cos(d*x+c)^2*a^4-
6*B*cos(d*x+c)^2*a^2*b^2+6*B*cos(d*x+c)^2*a*b^3+3*B*cos(d*x+c)*a^2*b^2-5*A*
cos(d*x+c)^3*a^2*b^2-5*A*cos(d*x+c)^2*a^3*b+8*A*cos(d*x+c)^2*a*b^3+3*B*cos(
d*x+c)^3*a^3*b-6*B*cos(d*x+c)^3*a*b^3-4*A*cos(d*x+c)*a*b^3-3*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^2*a^3*b-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+5*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*a^2*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+2*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^2*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+A*cos(d*x+
c)^2*a^4-3*B*cos(d*x+c)*a^4-5*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-8*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4-3*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*a^4+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
```

```
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^4+A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^2*a^4-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a^4+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4+A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+5*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*a^2*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+8*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+6*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2)/(a+b*cos(d*
x+c))^(1/2)/a^3/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^5+2\,ab\cos(dx+c)^4+a^2\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

3.431 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=674

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(6a^3Ab+26a^2\right.}{}$$

```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((6*a^2*A*b + 2*a
*A*b^2 - 12*A*b^3 - 15*a^3*B - 5*a^2*b*B + 21*a*b^2*B + 3*b^3*B)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^3*(a + b)^(3/2)*d) - (Sqrt[a + b]*(2*
A*b - 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d) + (2*a*
(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
- ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 2.18903, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(6a^3Ab+26a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
```

```
[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((6*a^2*A*b + 2*a
*A*b^2 - 12*A*b^3 - 15*a^3*B - 5*a^2*b*B + 21*a*b^2*B + 3*b^3*B)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^3*(a + b)^(3/2)*d) - (Sqrt[a + b]*(2*
A*b - 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d) + (2*a*
(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
- ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(Ab-aB)+\frac{3}{2}b(Ab-aB)\cos(c\cdots\right.}{(a+b\cos(c+\cdots}}{3b\left(a^2-\cdots\right.}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\sqrt{\cdots}}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\sqrt{\cdots}}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\sqrt{\cdots}}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos\cdots}}$$

$$= -\frac{\sqrt{a+b}(2Ab-5aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec\cdots}{a\cdots}}}{b^4d}$$

$$= \frac{\left(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+\cdots}}{\sqrt{a+b}\sqrt{\cos(c+\cdots}}\right.\right.}{3a(a-b)b^3(a+b)^{3/2}d}$$

Mathematica [C] time = 6.6546, size = 1396, normalized size = 2.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(a^2*A*b*Sin[c + d*x])
+ a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-3
*a^3*A*b*Sin[c + d*x] + 7*a*A*b^3*Sin[c + d*x] + 6*a^4*B*Sin[c + d*x] - 10*
a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d +
((-4*a*(-2*a^3*A*b + 2*a*A*b^3 + 5*a^4*B - 8*a^2*b^2*B + 3*b^4*B)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - 4*a*(2*a^2*A*b^2 + 6*A*b^4 + 4*a^3*b*B - 12*a*b^3*B)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^3
*A*b + 14*a*A*b^3 + 15*a^4*B - 26*a^2*b^2*B + 3*b^4*B)*((I*Cos[(c + d*x)/2]
*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
 d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(6*(a - b)^2*b^2*(a +
b)^2*d)
```

Maple [B] time = 0.592, size = 8611, normalized size = 12.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\cos(dx+c)^{3}+A\cos(dx+c)^{2}\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^{3}\cos(dx+c)^{3}+3\,ab^{2}\cos(dx+c)^{2}+3\,a^{2}b\cos(dx+c)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sqr
t(cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(
d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

3.432 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=545

$$-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2bB-3a^3B+aAb^2+\right.}{}$$

```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*A*b^2 - 3*A*b^3 - 3*a^3*B - a^2*b*B +
6*a*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2
)*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3
*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.40189, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2989, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2bB-3a^3B+aAb^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*A*b^2 - 3*A*b^3 - 3*a^3*B - a^2*b*B +
6*a*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
```

```
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2
)*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3
*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
```

```
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)+\frac{3}{2}b(Ab-aB)\cos(c+dx)-\frac{3}{2}\left(a^2 \cdots\right.}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx)) \cdots}}{3b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ab(Ab-aB)+\left(\frac{3}{2}a\left(a^2-b^2\right)B+\frac{3}{2}b^2(Ab-aB) \cdots\right.}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}}{3b^2\left(a^2-b^2\right)}$$

$$= -\frac{2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{b^3d}$$

$$= -\frac{2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{b^3d}$$

$$= \frac{2\left(4Ab^3+3a^3B-7ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s\cdots}{\cdots}}}{3a(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [C] time = 6.49095, size = 1342, normalized size = 2.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(a*A*b*Sin[c + d*x]) + a
^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(4*A*b^3
*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*x]))/(3*b*(-a^2
+ b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B - a
*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(4*a*A*b^2 - a^2*b*B - 3*b^3*B)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
```

```
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b^
3 + 3*a^3*B - 7*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[
c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x
])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b
*Sqrt[Cos[c + d*x]])))/(3*(a - b)^2*b*(a + b)^2*d)
```

Maple [B] time = 0.521, size = 5751, normalized size = 10.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*
x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

3.433 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4abB+Ab^2\right)\cot(c+$$

```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(
a - b)*(a + b)^(3/2)*d) + (2*(3*a*A - A*b + a*B - 3*b*B)*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(A*b - a*B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*
a^2*A + A*b^2 - 4*a*b*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.868569, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2999, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4abB+Ab^2\right)\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(
a - b)*(a + b)^(3/2)*d) + (2*(3*a*A - A*b + a*B - 3*b*B)*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(A*b - a*B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*
a^2*A + A*b^2 - 4*a*b*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
```

```
]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))]]/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(Ab-aB)-\frac{3}{2}(aA-bB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2A+Ab^2-4abB\right)\sin(c-}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\,c}}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2A+Ab^2-4abB\right)\sin(c-}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\,c}}$$

$$= -\frac{2\left(3a^2A+Ab^2-4abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{3a^2(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.42196, size = 1335, normalized size = 3.41

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b*Sin[c + d*x]) + a*B
*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*A*b*Sin[
c + d*x] + A*b^3*Sin[c + d*x] - 4*a*b^2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2
*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B - a*b^2*B)*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
```

```
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3*A + a*A*b^2 - 4*a^2*b*B)*((Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*A*b + A*
b^3 - 4*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*
ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x]
)/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
 + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Co
s[c + d*x]])))/(3*a*(a - b)^2*(a + b)^2*d)
```

Maple [B] time = 0.473, size = 4237, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(3/2)*(-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2-3*A*cos(d*x+c)*a
^4-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^2*a*b^3+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+2*A*cos(d*x+c)^3*a^3*b+2
```

```
*A*cos(d*x+c)^3*a*b^3+4*A*cos(d*x+c)^2*a^2*b^2+4*A*cos(d*x+c)*a^3*b-5*B*cos
(d*x+c)^3*a^2*b^2-4*B*cos(d*x+c)^2*a^3*b-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+3*B*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a*b^3
-A*cos(d*x+c)^3*b^4+A*cos(d*x+c)^2*b^4+8*B*cos(d*x+c)^2*a^2*b^2-4*B*cos(d*x
+c)^2*a*b^3+4*B*cos(d*x+c)*a^3*b-3*B*cos(d*x+c)*a^2*b^2-3*A*cos(d*x+c)^3*a^
2*b^2-6*A*cos(d*x+c)^2*a^3*b-2*A*cos(d*x+c)^2*a*b^3+4*B*cos(d*x+c)^3*a*b^3-
A*cos(d*x+c)*a^2*b^2+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+4*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+3*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^3*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-3*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^3*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3+3*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a^4+3*A*cos(d*x+c)^2*a^4+B*cos(d*x+c)^3*a^4-B*cos(d*x+c)*a^4-7
*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*a^3*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*a^2*b^2-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
```

/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^3*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+3*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)*a^4+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^2*b^4-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+4*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+
2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a*b^3-5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-4*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a^3*b-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+5*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(
d*x+c)*a^3*b+7*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^4-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^3*b)/sin(d*x+c)/a/(a-b)^2/(a+b)^2/cos(d*x+c)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

3.434 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=429

$$-\frac{2\left(6a^2Ab-3a^3B-ab^2B-2Ab^3\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2(A+B)+ab(3A+\right.}{}$$

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3
*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*
d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x])^(3/2)) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*S
in[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])
```

Rubi [A] time = 0.985463, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 2993, 2998, 2816, 2994}

$$-\frac{2\left(6a^2Ab-3a^3B-ab^2B-2Ab^3\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2(A+B)+ab(3A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```


```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3
*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*
d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x])^(3/2)) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*S
```

```
in[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2A-2Ab^2-abB\right)-\frac{3}{2}a(Ab-aB)\cos(c+}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(6a^2Ab-2Ab^3-3a^3B-ab^2B\right)}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(6a^2Ab-2Ab^3-3a^3B-ab^2B\right)}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

$$= \frac{2\left(6a^2Ab-2Ab^3-3a^3B-ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{3a^3(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.52494, size = 1384, normalized size = 3.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(A*b^2*Sin[c + d*x]) +
a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-6*a^2*
A*b^2*Sin[c + d*x] + 2*A*b^4*Sin[c + d*x] + 3*a^3*b*B*Sin[c + d*x] + a*b^3*
B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3
*a^4*A - 5*a^2*A*b^2 + 2*A*b^4 - a^3*b*B + a*b^3*B)*Sqrt[((a + b)*Cot[(c +
```

```
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(-6*a^3*A*b + 2*a*A*b^3 + 3*a^4*B + a^2*b^2*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^2*A*b^2 + 2*A*b^4
 + 3*a^3*b*B + a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c
 + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x]
)*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*
Sqrt[Cos[c + d*x]])))/(3*a^2*(a - b)^2*(a + b)^2*d)
```

Maple [B] time = 0.856, size = 5203, normalized size = 12.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^4+3\,ab^2\cos(dx+c)^3+3\,a^2b\cos(dx+c)^2+a^3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

3.435 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=456

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3A+B\right.}{}$$

```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (2*(8*A*b^3 - 3*a^3
*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*b*(3*A + B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x]
)/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (2*b*
(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.16091, antiderivative size = 456, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3A+B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```


```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (2*(8*A*b^3 - 3*a^3
*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*b*(3*A + B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x]
```

```
)/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (2*b*
(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2A-4Ab^2+abB\right)-\frac{3}{2}a(A}{\cos^{\frac{3}{2}}(c+}}{3}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{2\left(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{3a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.68392, size = 1431, normalized size = 3.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] -((-4*a*(9*a^4*A*b - 17*a^2*A*b^3 + 8*A*b^5 - 3*a^5*B + 5*a^3*b^2*B - 2*a*b
^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^5*A - 15*a^3*A*b^2 + 8*a*A*b^4
 + 6*a^4*b*B - 2*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]])) + 2*(3*a^4*A*b - 15*a^2*A*b^3 + 8*A*b^5 + 6*a^3*b^2*B - 2
*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh
[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x]
)/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d
*x]])))/(3*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*((2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2 -
 b^2)*(a + b*Cos[c + d*x])^2) + (2*(-9*a^2*A*b^3*Sin[c + d*x] + 5*A*b^5*Sin
[c + d*x] + 6*a^3*b^2*B*Sin[c + d*x] - 2*a*b^4*B*Sin[c + d*x]))/(3*a^3*(a^2
 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/a^3))/d
```

Maple [B] time = 1.06, size = 6498, normalized size = 14.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^5+3\,ab^2\cos(dx+c)^4+3\,a^2b\cos(dx+c)^3+a^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^5 + 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^3*
cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

3.436 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=567

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(10a^2Ab-7a^3B+3ab^2B-6Ab^3\right)\mathrm{si}}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{co}}}$$

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a + b)^(3/2)*d) - (2
*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*
b^2*(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2
 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]
^(3/2)*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B +
 3*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b
^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c +
 d*x]^(3/2))
```

Rubi [A] time = 1.87687, antiderivative size = 567, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(10a^2Ab-7a^3B+3ab^2B-6Ab^3\right)\mathrm{si}}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a + b)^(3/2)*d) - (2
*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*
```

```
b^2*(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2
 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]
^(3/2)*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B +
 3*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b
^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c +
 d*x]^(3/2))
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}\left(a^2A-2Ab^2+abB\right)-\frac{3}{2}a(Ab-}{\cos^{\frac{5}{2}}(c+dx}}{3a}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+}$$

$$= -\frac{2\left(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B\right)\cot(c+dx)E\left(\sin\right.}{3a^5(a-b)(a+b)^{3/2}}$$

Mathematica [C] time = 6.92098, size = 1499, normalized size = 2.64

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] ((-4*a*(a^6*A + 15*a^4*A*b^2 - 32*a^2*A*b^4 + 16*A*b^6 - 9*a^5*b*B + 17*a^3
*b^3*B - 8*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a^5*A*b - 28*a^3*A
*b^3 + 16*a*A*b^5 - 3*a^6*B + 15*a^4*b^2*B - 8*a^2*b^4*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
```

```
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*A*b^2 - 28*a^2*A
*b^4 + 16*A*b^6 - 3*a^5*b*B + 15*a^3*b^3*B - 8*a*b^5*B)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^4*(a - b)^2*(a +
 b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-
8*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^4) - (2*(-(A*b^4*Sin[c + d*x
]) + a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2
*(-12*a^2*A*b^4*Sin[c + d*x] + 8*A*b^6*Sin[c + d*x] + 9*a^3*b^3*B*Sin[c + d
*x] - 5*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) +
 (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 0.74, size = 8093, normalized size = 14.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^6+3\,ab^2\cos(dx+c)^5+3\,a^2b\cos(dx+c)^4+a^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^6 + 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^3*
cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

3.437 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=419

$$\frac{aB\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+$$

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (a*Sqrt[a + b]*B*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c + d*x])/(b*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.788881, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {21, 2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{aB\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2
),x]
```


```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (a*Sqrt[a + b]*B*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
```

```
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c + d*x])/(b*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
```

```
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{(aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{a\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{abd}$$

<u>Mathematica</u> [C] time = 1.43994, size = 480, normalized size = 1.15

$$\frac{B\sqrt{\cos(c+dx)}\left(2a\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)+b\sqrt{\frac{a-b}{a+b}}\sin\left(\frac{3}{2}(c+dx)\right)\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
 -((a + b)/(a - b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
```

```
[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(
a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*
Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2
]))/(2*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.611, size = 623, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -B/d/b*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b-2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c
)+b*cos(d*x+c)^3+a*cos(d*x+c)^2-b*cos(d*x+c)^2-cos(d*x+c)*a)/(a+b*cos(d*x+c
))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c) + Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos\left(dx+c\right)+Ba\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

3.438 $\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

```
[Out] (-2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rubi [A] time = 0.0845342, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {21, 2809}

$$-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2
),x]
```


```
[Out] (-2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```


Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= -\frac{2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{bd}$$

Mathematica [A] time = 0.138341, size = 133, normalized size = 1.14

$$-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*EllipticPi
[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.479, size = 160, normalized size = 1.4

$$-2\,\frac{B\left(\sin\left(dx+c\right)\right)^{2}}{d\sqrt{a+b\cos\left(dx+c\right)}\left(-1+\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}\left(\mathrm{EllipticF}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-2*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(a+b*cos(d
*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

3.439 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rubi [A] time = 0.0868031, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {21, 2816}

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
```

```
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a+b}B\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{ad}$$

Mathematica [A] time = 0.881904, size = 171, normalized size = 1.55

$$-\frac{4B(a+b)\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{a-b}}\sqrt{\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\cos(c+dx)}{a(\cos(c+dx)-1)}}\right)|\frac{2a}{a-b}\right)}{ad\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
^(3/2)),x]
```

```
[Out] (-4*(a + b)*B*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a - b
))]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a -
 b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a))^(3/2))
```

Maple [A] time = 0.565, size = 124, normalized size = 1.1

$$-2\,\frac{B(\sin(dx+c))^4}{d\sqrt{a+b\cos(dx+c)}(\cos(dx+c))^{3/2}(-1+\cos(dx+c))^2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a+b*cos(d*x+c))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x
 + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}B\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 +
a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x
 + c))), x)
```

3.440 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{2B(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rubi [A] time = 0.268329, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {21, 2801, 2816, 2994}

$$\frac{2B(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\left(B\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx\right) + B\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos}}$$

$$= \frac{2(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2 d}$$

Mathematica [A] time = 2.11003, size = 212, normalized size = 0.94

$$\frac{2B\left(\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+a\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b\cdots}{a\cdots}\right.\right.}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^(3/2)),x]
```

```
[Out] (2*B*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]) + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.46, size = 613, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d/a*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+a*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)-(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b+b*cos(d*
x+c)^2+cos(d*x+c)*a-b*cos(d*x+c)-a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d
*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)
^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}B\sqrt{\cos(dx+c)}}{b\cos(dx+c)^3 + a\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 +
a*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)
^(3/2)), x)
```

3.441 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$-\frac{\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{d}$$

```
[Out] -((Cot[c + d*x]*EllipticE[ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/d)
```

Rubi [A] time = 0.0876338, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {2994}

$$-\frac{\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] -((Cot[c + d*x]*EllipticE[ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = -\frac{\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{d}$$

Mathematica [F] time = 34.8373, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),
 x]
```

Maple [B] time = 0.444, size = 658, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/10/d/(2+3*cos(d*x+c))^(1/2)*(2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*
5^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*10^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)+4*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),1/5*5^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)*10^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)+2*cos(d*x+c
)^2*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(
1/2))+2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*10^(1/2)*((2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5
*5^(1/2))-5*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),1/5*5^(1/2))+2*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
```

```
F((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))-5*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))+30*cos(d*x+c)^3-10
*cos(d*x+c)^2-20*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\cos(dx+c)+2}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3+2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) + 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3*
cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)+1}{\sqrt{3\cos(c+dx)+2}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) + 2)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)),
 x)
```

3.442 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{5}\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

```
[Out] -((Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos
[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d)
```

Rubi [A] time = 0.103997, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {2994}

$$-\frac{\sqrt{5}\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos
[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = -\frac{\sqrt{5}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

Mathematica [F] time = 38.3501, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]])
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]])
, x]
```

Maple [B] time = 0.461, size = 601, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d/(-2+3*cos(d*x+c))^(1/2)*(-2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),5^(1/2))-4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(
(-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),5^(1/2))-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d
*x+c)^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)
^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)*sin(d*x+c)-(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)*sin(d*x+c)+3*cos(d*x+c)^3
-5*cos(d*x+c)^2+2*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)+1}{\sqrt{3\,\cos\left(dx+c\right)-2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\,\cos\left(dx+c\right)-2}(\cos\left(dx+c\right)+1)\sqrt{\cos\left(dx+c\right)}}{3\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) - 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3*
cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(c+dx\right)+1}{\sqrt{3\cos\left(c+dx\right)-2}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)+1}{\sqrt{3\,\cos\left(dx+c\right)-2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)),
 x)
```

3.443 $\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

```
[Out] (Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rubi [A] time = 0.206488, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$\frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```


```
[Out] (Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]])/d
```


Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+s}}{d}$$

Mathematica [F] time = 32.6107, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
 x]
```

Maple [B] time = 0.469, size = 611, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(2-3*cos(d*x+c))^(1/2)*(2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),5^(1/2))+4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((-
2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/
```

```
sin(d*x+c),5^(1/2))+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x
+c)^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*((-2+3*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))
+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)^2*sin(d*x+c)+2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)*sin(d*x+c)+(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)*sin(d*x+c)-3*cos(d*x+c)^3+5
*cos(d*x+c)^2-2*cos(d*x+c))/(-2+3*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3-2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(
3*cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)+1}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2 - 3*cos(c + d*x))*cos(c + d*x)**(3/2)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

3.444 $\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{d}$$

```
[Out] (Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[
-2 - 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]])/d
```

Rubi [A] time = 0.190973, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[
-2 - 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]])/d
```

Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-s}}{d}$$

Mathematica [F] time = 28.6932, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

Maple [B] time = 0.364, size = 705, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/10/d*(-2-3*cos(d*x+c))^(1/2)*(2*sin(d*x+c)*10^(1/2)*((2+3*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(1/5*
5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)^2*5^(1/2)+4*sin(d*x+
```

```
c)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2)
)*cos(d*x+c)*5^(1/2)+2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*5^(1/2)*10
^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*5^(1/2)*(-1+co
s(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)-2*sin(d*x+c)*cos(d*x+c)^2*10^(1/2)
*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)-s
in(d*x+c)*cos(d*x+c)^2*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(1/5*5^(1/2)*(-1+cos(d*x+c)
)/sin(d*x+c),5^(1/2))*5^(1/2)-2*sin(d*x+c)*cos(d*x+c)*10^(1/2)*((2+3*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellip
ticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)-sin(d*x+c)*cos
(d*x+c)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*EllipticE(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^
(1/2))*5^(1/2)+30*cos(d*x+c)^3-10*cos(d*x+c)^2-20*cos(d*x+c))/(2+3*cos(d*x+
c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3+2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(
3*cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(c + dx)} + 1}{\sqrt{-3\cos{(c + dx)} - 2}\cos^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(-3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(dx + c)} + 1}{\sqrt{-3\cos{(dx + c)} - 2}\cos{(dx + c)}^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

3.445 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rubi [A] time = 0.0825297, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {2994}

$$\frac{2\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3d}$$

Mathematica [F] time = 37.7417, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),
 x]
```

Maple [B] time = 0.448, size = 665, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d/(3+2*cos(d*x+c))^(1/2)*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*10^(1/2
)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*2^(1/2)+6*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*2^(1/2)+3*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)+3
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*cos(d*
x+c)^2*sin(d*x+c)-5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+
2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),1/
5*I*5^(1/2))*cos(d*x+c)^2*sin(d*x+c)+3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
```

```
x+c))/sin(d*x+c),1/5*I*5^(1/2))*cos(d*x+c)*sin(d*x+c)-5*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*cos(d*x+c)*sin(d*x+c)+20*c
os(d*x+c)^3+10*cos(d*x+c)^2-30*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\cos(dx+c)+3}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{2\cos(dx+c)^3+3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) + 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2*
cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)+1}{\sqrt{2\cos(c+dx)+3}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) + 3)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)),
 x)
```

3.446 $\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{5}\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

```
[Out] (2*Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rubi [A] time = 0.0993586, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {2994}

$$\frac{2\sqrt{5}\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (2*Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{5}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx}}{3d}$$

Mathematica [F] time = 38.0136, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
 x]
```

Maple [B] time = 0.458, size = 663, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*(3-2*cos(d*x+c))^(1/2)*(3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1
/2))*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(-2*(-3+2*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+6*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),I*5^(1/2))*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(-2*(-3
+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)+3*2^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c)^2-2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c)^2+3*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c)-2^(1
```

```
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c
)-4*cos(d*x+c)^3+10*cos(d*x+c)^2-6*cos(d*x+c))/(-3+2*cos(d*x+c))/cos(d*x+c)
^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)^3-3\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(
2*cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)+1}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3 - 2*cos(c + d*x))*cos(c + d*x)**(3/2)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)+1}{\sqrt{-2\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

3.447 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

```
[Out] (-2*Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rubi [A] time = 0.201563, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$-\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{(-\cos(c+dx))^{3/2}\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

Mathematica [F] time = 40.906, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
, x]
```

Maple [B] time = 0.352, size = 714, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d/(-3+2*cos(d*x+c))^(1/2)*(3*I*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/
sin(d*x+c),1/5*I*5^(1/2))*5^(1/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+6*I*(-2*(
```

```
-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*El
lipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*5^(1/2)*2^(1/2)
*sin(d*x+c)*cos(d*x+c)+5*I*5^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*(-3+2*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellip
ticE(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))+3*I*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*5^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2
))-3*I*5^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c
))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))+5*I*5^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*
(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*EllipticE(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))-3*I*5^
(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/
sin(d*x+c),1/5*I*5^(1/2))+20*cos(d*x+c)^3-50*cos(d*x+c)^2+30*cos(d*x+c))/co
s(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{2\cos(dx+c)-3}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{2\cos(dx+c)^3-3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) - 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2*
cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)+1}{\sqrt{2\cos(c+dx)-3}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)),
 x)
```

3.448 $\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rubi [A] time = 0.188941, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```


```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```


Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1}}{3d}$$

Mathematica [F] time = 30.1155, size = 0, normalized size = 0.

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

Maple [B] time = 0.366, size = 740, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*(-3-2*cos(d*x+c))^(1/2)*(3*I*5^(1/2)*cos(d*x+c)^2*sin(d*x+c)*10^(1/
2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))+6*I
```

```
*5^(1/2)*cos(d*x+c)*sin(d*x+c)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(1/5*I*(-1+cos(d*x+
c))*5^(1/2)/sin(d*x+c),I*5^(1/2))+I*5^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Ellipti
cE(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*10^(1/2)*((3+2*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*I*
5^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin
(d*x+c),I*5^(1/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*I*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*sin(d*x+c)*5^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))+I*5^(1/2)*cos(
d*x+c)*sin(d*x+c)*EllipticE(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1
/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)-3*I*5^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF(1/5*I*(-1+
cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-20*cos(d*x+c)^3-1
0*cos(d*x+c)^2+30*cos(d*x+c))/(3+2*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)^3+3\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(
2*cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c + dx) + 1}{\sqrt{-2\cos(c + dx) - 3}\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(-2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c) + 1}{\sqrt{-2\cos(dx + c) - 3}\cos(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

3.449 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left((A+B\cos(e+fx))(c\cos(e+fx))^m(a+b\cos(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])
, x]
```

Rubi [A] time = 0.0822421, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x]
```

```
[Out] Defer[Int][(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),
x]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+f$$

Mathematica [A] time = 7.55386, size = 0, normalized size = 0.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]), x
]
```

Maple [A] time = 2.795, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{n}\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\cos\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**n*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] Exception raised: AttributeError
```

3.450 $\int (c\cos(e+fx))^m (a+b\cos(e+fx))^4 (A+B\cos(e+fx))\,dx$

Optimal. Leaf size=595

$$-\frac{\sin(e+fx)\left(4a^3Ab\left(m^2+8m+15\right)+6a^2b^2B\left(m^2+7m+10\right)+a^4B\left(m^2+8m+15\right)+4aAb^3\left(m^2+7m+10\right)+b^4B\left(m\right.\right.}{c^2f(m+2)(m+3)(m+5)\sqrt{\sin^2(e+fx)}}$$

```
[Out] (b*(A*b^3*(15 + 8*m + m^2) + 4*a*b^2*B*(15 + 8*m + m^2) + 2*a^3*B*(28 + 10*
m + m^2) + a^2*A*b*(110 + 47*m + 5*m^2))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f
*x])/(c*f*(2 + m)*(4 + m)*(5 + m)) + (b^2*(b^2*B*(4 + m)^2 + 2*a*A*b*(5 + m
)^2 + a^2*B*(36 + 11*m + m^2))*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e
+ f*x])/(c*f*(3 + m)*(4 + m)*(5 + m)) + (b*(A*b*(5 + m) + a*B*(8 + m))*(c*C
os[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(4 + m)*(5 +
 m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^3*Sin[e + f*x])/(
c*f*(5 + m)) - ((A*b^4*(3 + 4*m + m^2) + 4*a*b^3*B*(3 + 4*m + m^2) + 6*a^2*
A*b^2*(4 + 5*m + m^2) + 4*a^3*b*B*(4 + 5*m + m^2) + a^4*A*(8 + 6*m + m^2))*
(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2])
 - ((b^4*B*(8 + 6*m + m^2) + 4*a*A*b^3*(10 + 7*m + m^2) + 6*a^2*b^2*B*(10 +
 7*m + m^2) + 4*a^3*A*b*(15 + 8*m + m^2) + a^4*B*(15 + 8*m + m^2))*(c*Cos[e
 + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^
2]*Sin[e + f*x])/(c^2*f*(2 + m)*(3 + m)*(5 + m)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 1.98357, antiderivative size = 595, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3049, 3033, 3023, 2748, 2643}

$$-\frac{\sin(e+fx)\left(4a^3Ab\left(m^2+8m+15\right)+6a^2b^2B\left(m^2+7m+10\right)+a^4B\left(m^2+8m+15\right)+4aAb^3\left(m^2+7m+10\right)+b^4B\left(m\right.\right.}{c^2f(m+2)(m+3)(m+5)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x]
```



```
[Out] (b*(A*b^3*(15 + 8*m + m^2) + 4*a*b^2*B*(15 + 8*m + m^2) + 2*a^3*B*(28 + 10*
m + m^2) + a^2*A*b*(110 + 47*m + 5*m^2))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f
*x])/(c*f*(2 + m)*(4 + m)*(5 + m)) + (b^2*(b^2*B*(4 + m)^2 + 2*a*A*b*(5 + m
```

```
)^2 + a^2*B*(36 + 11*m + m^2))*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e
+ f*x])/(c*f*(3 + m)*(4 + m)*(5 + m)) + (b*(A*b*(5 + m) + a*B*(8 + m))*(c*C
os[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(4 + m)*(5 +
 m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^3*Sin[e + f*x])/(
c*f*(5 + m)) - ((A*b^4*(3 + 4*m + m^2) + 4*a*b^3*B*(3 + 4*m + m^2) + 6*a^2*
A*b^2*(4 + 5*m + m^2) + 4*a^3*b*B*(4 + 5*m + m^2) + a^4*A*(8 + 6*m + m^2))*
(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2])
 - ((b^4*B*(8 + 6*m + m^2) + 4*a*A*b^3*(10 + 7*m + m^2) + 6*a^2*b^2*B*(10 +
 7*m + m^2) + 4*a^3*A*b*(15 + 8*m + m^2) + a^4*B*(15 + 8*m + m^2))*(c*Cos[e
 + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^
2]*Sin[e + f*x])/(c^2*f*(2 + m)*(3 + m)*(5 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
```

```
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^3\sin(e+fx)}{cf(5+m)} +$$

$$= \frac{b(Ab(5+m)+aB(8+m))(c\cos(e+fx))^{1+m}(a+b\cos}{cf(4+m)(5+m)}$$

$$= \frac{b^2\left(b^2B(4+m)^2+2aAb(5+m)^2+a^2B\left(36+11m+m\right.\right.}{cf(3+m)(4+m)}$$

$$= \frac{b\left(Ab^3\left(15+8m+m^2\right)+4ab^2B\left(15+8m+m^2\right)+2a^3\right.}{c}$$

$$= \frac{b\left(Ab^3\left(15+8m+m^2\right)+4ab^2B\left(15+8m+m^2\right)+2a^3\right.}{c}$$

$$= \frac{b\left(Ab^3\left(15+8m+m^2\right)+4ab^2B\left(15+8m+m^2\right)+2a^3\right.}{c}$$

Mathematica [A] time = 6.19836, size = 487, normalized size = 0.82

$$-\frac{a^3(aB+4Ab)\sin(e+fx)\cos^2(e+fx)(c\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{f(m+2)\sqrt{\sin^2(e+fx)}} - \frac{2a^2b(2aB+3Ab)\sin(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x]
```

```
[Out] -((a^4*A*Cos[e + f*x]*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2]))
- (a^3*(4*A*b + a*B)*Cos[e + f*x]^2*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/
2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 + m)*Sqrt[Sin[
e + f*x]^2]) - (2*a^2*b*(3*A*b + 2*a*B)*Cos[e + f*x]^3*(c*Cos[e + f*x])^m*H
ypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(
f*(3 + m)*Sqrt[Sin[e + f*x]^2]) - (2*a*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^4*(
c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*(4 + m)*Sqrt[Sin[e + f*x]^2]) - (b^3*(A*b + 4*a*B)*Cos
[e + f*x]^5*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2,
 Cos[e + f*x]^2]*Sin[e + f*x])/(f*(5 + m)*Sqrt[Sin[e + f*x]^2]) - (b^4*B*Co
s[e + f*x]^6*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2
, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(6 + m)*Sqrt[Sin[e + f*x]^2])
```

Maple [F] time = 2.33, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{4}\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{4}\left(c\cos\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^{4}\cos\left(fx+e\right)^{5}+Aa^{4}+\left(4\,Bab^{3}+Ab^{4}\right)\cos\left(fx+e\right)^{4}+2\left(3\,Ba^{2}b^{2}+2\,Aab^{3}\right)\cos\left(fx+e\right)^{3}+2\left(2\,Ba^{3}b+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**4*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^4\left(c\cos\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m, x
)
```

3.451 $\int (c\cos(e+fx))^m (a+b\cos(e+fx))^3 (A+B\cos(e+fx))\,dx$

Optimal. Leaf size=406

$$-\frac{\sin(e+fx)\left(3a^2Ab(m+3)+a^3B(m+3)+3ab^2B(m+2)+Ab^3(m+2)\right)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+\right.}{c^2f(m+2)(m+3)\sqrt{\sin^2(e+fx)}}$$

```
[Out] (b*(b^2*B*(3 + m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m))*(c*Cos[e + f*x])^(1
+ m)*Sin[e + f*x])/(c*f*(2 + m)*(4 + m)) + (b^2*(A*b*(4 + m) + a*B*(6 + m))
*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(3 + m)*(4 + m))
+ (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(
4 + m)) - ((a^2*(2 + m)*(b*B*(1 + m) + a*A*(4 + m)) + b*(1 + m)*(b^2*B*(3 +
 m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m)))*(c*Cos[e + f*x])^(1 + m)*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1
+ m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2]) - ((A*b^3*(2 + m) + 3*a*b^2*B*(2
 + m) + 3*a^2*A*b*(3 + m) + a^3*B*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hyperge
ometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*
(2 + m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 1.0534, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2990, 3033, 3023, 2748, 2643}

$$-\frac{\sin(e+fx)\left(3a^2Ab(m+3)+a^3B(m+3)+3ab^2B(m+2)+Ab^3(m+2)\right)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+\right.}{c^2f(m+2)(m+3)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(b^2*B*(3 + m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m))*(c*Cos[e + f*x])^(1
+ m)*Sin[e + f*x])/(c*f*(2 + m)*(4 + m)) + (b^2*(A*b*(4 + m) + a*B*(6 + m))
*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(3 + m)*(4 + m))
+ (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(
4 + m)) - ((a^2*(2 + m)*(b*B*(1 + m) + a*A*(4 + m)) + b*(1 + m)*(b^2*B*(3 +
 m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m)))*(c*Cos[e + f*x])^(1 + m)*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1
```

```
+ m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2]) - ((A*b^3*(2 + m) + 3*a*b^2*B*(2
 + m) + 3*a^2*A*b*(3 + m) + a^3*B*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hyperge
ometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*
(2 + m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^2\sin(e+fx)}{cf(4+m)} + \cdots$$

$$= \frac{b^2(Ab(4+m)+aB(6+m))\cos(e+fx)(c\cos(e+fx))^{1+\cdots}}{cf(3+m)(4+m)}$$

$$= \frac{b\left(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m)\right)(c\cos(e+\cdots}{cf(2+m)(4+m)}$$

$$= \frac{b\left(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m)\right)(c\cos(e+\cdots}{cf(2+m)(4+m)}$$

$$= \frac{b\left(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m)\right)(c\cos(e+\cdots}{cf(2+m)(4+m)}$$

Mathematica [A] time = 2.90557, size = 269, normalized size = 0.66

$$\frac{\sin(e+fx)\cos(e+fx)(c\cos(e+fx))^m\left(\cos(e+fx)\left(b\cos(e+fx)\left(b\cos(e+fx)\left(-\frac{(3aB+Ab)\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(e+fx)\right)}{m+4}-\frac{bB\cdots}{\cdots}\right.\right.\right.\right.}{f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*(c*Cos[e + f*x])^m*(-((a^3*A*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a^2*(3*A*b + a*B)
*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m)) + b
*Cos[e + f*x]*((-3*a*(A*b + a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/
2, Cos[e + f*x]^2])/(3 + m) + b*Cos[e + f*x]*(-(((A*b + 3*a*B)*Hypergeometr
ic2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m)) - (b*B*Cos[e + f
*x]*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Cos[e + f*x]^2])/(5 + m)))
```

```
))*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

Maple [F] time = 1.981, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{3}\left(A+B\cos\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{3}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^{3}\cos\left(fx+e\right)^{4}+Aa^{3}+\left(3\,Bab^{2}+Ab^{3}\right)\cos\left(fx+e\right)^{3}+3\left(Ba^{2}b+Aab^{2}\right)\cos\left(fx+e\right)^{2}+\left(Ba^{3}+3\,Aa^{2}b\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e))
```

```
*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**3*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{3}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m, x
)
```

3.452 $\int (c\cos(e+fx))^m (a+b\cos(e+fx))^2 (A+B\cos(e+fx))\,dx$

Optimal. Leaf size=287

$$-\frac{\sin(e+fx)\left(a^2A(m+2)+2abB(m+1)+Ab^2(m+1)\right)(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+f}{}$$

```
[Out] (b*(A*b*(3 + m) + a*B*(4 + m))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*
(2 + m)*(3 + m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])*Sin[e
 + f*x])/(c*f*(3 + m)) - ((A*b^2*(1 + m) + 2*a*b*B*(1 + m) + a^2*A*(2 + m))
*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2]) - ((b^
2*B*(2 + m) + a*(2*A*b + a*B)*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hypergeomet
ric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 +
 m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.541114, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2990, 3023, 2748, 2643}

$$-\frac{\sin(e+fx)\left(a^2A(m+2)+2abB(m+1)+Ab^2(m+1)\right)(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(A*b*(3 + m) + a*B*(4 + m))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*
(2 + m)*(3 + m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])*Sin[e
 + f*x])/(c*f*(3 + m)) - ((A*b^2*(1 + m) + 2*a*b*B*(1 + m) + a^2*A*(2 + m))
*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2]) - ((b^
2*B*(2 + m) + a*(2*A*b + a*B)*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hypergeomet
ric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 +
 m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))\sin(e+fx)}{cf(3+m)} +$$

$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin(e+f}{cf(2+m)(3+m)}$$

$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin(e+f}{cf(2+m)(3+m)}$$

$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin(e+f}{cf(2+m)(3+m)}$$

Mathematica [A] time = 1.74602, size = 217, normalized size = 0.76

$$\frac{\sin(e+fx)\cos(e+fx)(c\cos(e+fx))^m\left(\cos(e+fx)\left(b\cos(e+fx)\left(-\frac{(2aB+Ab)\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(e+fx)\right)}{m+3} - \frac{bB\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\right.}{m}\right.\right.\right.}{f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*(c*Cos[e + f*x])^m*(-((a^2*A*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a*(2*A*b + a*B)*H
ypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m)) + b*C
os[e + f*x]*(-(((A*b + 2*a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2,
Cos[e + f*x]^2])/(3 + m)) - (b*B*Cos[e + f*x]*Hypergeometric2F1[1/2, (4 + m
)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m))))*Sin[e + f*x])/(f*Sqrt[Sin[e + f
*x]^2])
```

Maple [F] time = 1.823, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^m \left(a+b\cos\left(fx+e\right)\right)^2 \left(A+B\cos\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\cos(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(fx+e)^3+Aa^2+(2\,Bab+Ab^2)\cos(fx+e)^2+(Ba^2+2\,Aab)\cos(fx+e)\right)(c\cos(fx+e))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2 +
 (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**2*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{2}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m, x
)
```

3.453 $\int (c\cos(e+fx))^m (a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=196

$$-\frac{(aB+Ab)\sin(e+fx)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)(aA(m+2)+bB(m+1))(c\cos(e}{cf(m+1)(m+2}$$

```
[Out] (b*B*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(2 + m)) - ((b*B*(1 + m) +
 a*A*(2 + m))*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*
x]^2]) - ((A*b + a*B)*(c*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 + m)*Sqrt[Sin[e +
f*x]^2])
```

Rubi [A] time = 0.246398, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2968, 3023, 2748, 2643}

$$-\frac{(aB+Ab)\sin(e+fx)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)(aA(m+2)+bB(m+1))(c\cos(e}{cf(m+1)(m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*B*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(2 + m)) - ((b*B*(1 + m) +
 a*A*(2 + m))*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*
x]^2]) - ((A*b + a*B)*(c*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 + m)*Sqrt[Sin[e +
f*x]^2])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m\left(aA+(Ab+aB)\cos(e+fx)+bB\,\mathrm{co}\right.$$

$$= \frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)} + \frac{\int (c\cos(e+fx))^m(c}{}$$

$$= \frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)} + \frac{(Ab+aB)\int (c\cos(e}{c}$$

$$= \frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)} - \frac{\left(aA+\frac{bB(1+m)}{2+m}\right)(c\,\mathrm{co}}{}$$

Mathematica [A] time = 0.32474, size = 151, normalized size = 0.77

$$-\frac{\sin(e+fx)\cos(e+fx)(c\cos(e+fx))^m\left((aA(m+2)+bB(m+1))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)+(m+1)\left((aB+A\right.\right.}{f(m+1)(m+2)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(c*Cos[e + f*x])^m*Sin[e + f*x]*((b*B*(1 + m) + a*A*(2 + m)
)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2] + (1 + m)*((
A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e
+ f*x]^2] - b*B*Sqrt[Sin[e + f*x]^2])))/(f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x
]^2]))
```

Maple [F] time = 1.793, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\cos\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(fx+e\right)^{2}+Aa+(Ba+Ab)\cos\left(fx+e\right)\right)\left(c\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*cos(f*x +
 e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

3.454 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=286

$$\frac{ac(Ab-aB)\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{bf\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(e}{}$$

```
[Out] (a*(A*b - a*B)*c*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Si
n[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1
- m)/2)*Sin[e + f*x])/(b*(a^2 - b^2)*f) - ((A*b - a*B)*AppellF1[1/2, -m/2,
1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x
])^m*Sin[e + f*x])/((a^2 - b^2)*f*(Cos[e + f*x]^2)^(m/2)) - (B*(c*Cos[e + f
*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*S
in[e + f*x])/(b*c*f*(1 + m)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.412338, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3002, 2643, 2823, 3189, 429}

$$\frac{ac(Ab-aB)\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{bf\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x]
```


```
[Out] (a*(A*b - a*B)*c*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Si
n[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1
- m)/2)*Sin[e + f*x])/(b*(a^2 - b^2)*f) - ((A*b - a*B)*AppellF1[1/2, -m/2,
1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x
])^m*Sin[e + f*x])/((a^2 - b^2)*f*(Cos[e + f*x]^2)^(m/2)) - (B*(c*Cos[e + f
*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*S
in[e + f*x])/(b*c*f*(1 + m)*Sqrt[Sin[e + f*x]^2])
```


Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \frac{B\int(c\cos(e+fx))^m\,dx}{b} - \frac{(-Ab+aB)\int\frac{(c\cos(e+fx))^m}{a+b\cos(e+fx)}\,dx}{b}$$

$$= -\frac{B(c\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{bcf(1+m)\sqrt{\sin^2(e+fx)}} + \frac{(a(Ab-}{}$$

$$= -\frac{B(c\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{bcf(1+m)\sqrt{\sin^2(e+fx)}} + \frac{\left(a(Ab-\right.}{}$$

$$= \frac{a(Ab-aB)cF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)(c\cos(e+fx))^{-1+m}}{b\left(a^2-b^2\right)f}$$

Mathematica [B] time = 27.0578, size = 10482, normalized size = 36.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.145, size = 0, normalized size = 0.

$$\int \frac{\left(c\cos\left(fx+e\right)\right)^m\left(A+B\cos\left(fx+e\right)\right)}{a+b\cos\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{b\cos\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

3.455 $\int (c\cos(e+fx))^m (a+b\cos(e+fx))^{3/2} (A+B\cos(e+fx))\,dx$

Optimal. Leaf size=180

$$\frac{2\text{Unintegrable}\left(\frac{(c\cos(e+fx))^m\left(\frac{1}{2}c\cos(e+fx)\left(a(2m+5)(aB+2Ab)+b^2B(2m+3)\right)+\frac{1}{2}bc\cos^2(e+fx)(2aB(m+3)+Ab(2m+5))+\frac{1}{2}ac\left(2aA\left(m+\frac{5}{2}\right)+2bB(m+1)\right)\right)}{\sqrt{a+b\cos(e+fx)}}\right)}{c(2m+5)}$$

```
[Out] (2*b*B*(c*Cos[e + f*x])^(1 + m)*Sqrt[a + b*Cos[e + f*x]]*Sin[e + f*x])/(c*f
*(5 + 2*m)) + (2*Unintegrable[((c*Cos[e + f*x])^m*((a*c*(2*b*B*(1 + m) + 2*
a*A*(5/2 + m)))/2 + (c*(b^2*B*(3 + 2*m) + a*(2*A*b + a*B)*(5 + 2*m))*Cos[e
+ f*x])/2 + (b*c*(2*a*B*(3 + m) + A*b*(5 + 2*m))*Cos[e + f*x]^2)/2))/Sqrt[a
 + b*Cos[e + f*x]], x])/(c*(5 + 2*m))
```

Rubi [A] time = 0.526552, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c\cos(e+fx))^m (a+b\cos(e+fx))^{3/2} (A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]),x]
```

```
[Out] (2*b*B*(c*Cos[e + f*x])^(1 + m)*Sqrt[a + b*Cos[e + f*x]]*Sin[e + f*x])/(c*f
*(5 + 2*m)) + (2*Defer[Int][((c*Cos[e + f*x])^m*((a*c*(2*b*B*(1 + m) + 2*a*
A*(5/2 + m)))/2 + (c*(b^2*B*(3 + 2*m) + a*(2*A*b + a*B)*(5 + 2*m))*Cos[e +
f*x])/2 + (b*c*(2*a*B*(3 + m) + A*b*(5 + 2*m))*Cos[e + f*x]^2)/2))/Sqrt[a +
 b*Cos[e + f*x]], x])/(c*(5 + 2*m))
```

Rubi steps

$$\int (c\cos(e+fx))^m (a+b\cos(e+fx))^{3/2} (A+B\cos(e+fx))\,dx = \frac{2bB(c\cos(e+fx))^{1+m}\sqrt{a+b\cos(e+fx)}\sin(e+fx)}{cf(5+2m)}$$

Mathematica [A] time = 66.2563, size = 0, normalized size = 0.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]
),x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]
), x]
```

Maple [A] time = 0.497, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\cos\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*cos(f*x + e))^
m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(fx+e\right)^{2}+Aa+(Ba+Ab)\cos\left(fx+e\right)\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(f
*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.456 $\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\cos(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x
]), x]
```

Rubi [A] time = 0.116255, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),x]
```

```
[Out] Defer[Int][(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])
, x]
```

Rubi steps

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx=\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))$$

Mathematica [A] time = 9.681, size = 0, normalized size = 0.

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),
x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),
 x]
```

Maple [A] time = 0.457, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(A+B\cos\left(fx+e\right)\right)\sqrt{a+b\cos\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c\cos\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)\sqrt{a+b\cos\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.457 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{(A+B\cos(e+fx))(c\cos(e+fx))^m}{\sqrt{a+b\cos(e+fx)}},x\right)$$

```
[Out] Unintegrable[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f
*x]], x]
```

Rubi [A] time = 0.123047, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]],x]
```

```
[Out] Defer[Int][((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x
]], x]
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Mathematica [A] time = 7.96176, size = 0, normalized size = 0.

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]
],x]
```

```
[Out] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]
], x]
```

Maple [A] time = 0.464, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(A+B\cos\left(fx+e\right)\right)\frac{1}{\sqrt{a+b\cos\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\cos\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)}{\sqrt{a+b\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

3.458 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{2\text{Unintegrable}\left(\frac{(c\cos(e+fx))^m\left(-\frac{1}{2}bc(2m+3)(Ab-aB)\cos^2(e+fx)-\frac{1}{2}ac(Ab-aB)\cos(e+fx)+\frac{1}{2}c\left(2b\left(m+\frac{1}{2}\right)(Ab-aB)+a(aA-bB)\right)\right)}{\sqrt{a+b\cos(e+fx)}},x\right)}{ac\left(a^2-b^2\right)}+\frac{2b(Ab-aB)}{acf\left(a^2\right.}$$

```
[Out] (2*b*(A*b - a*B)*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*(a^2 - b^2)*c*f*
Sqrt[a + b*Cos[e + f*x]]) + (2*Unintegrable[((c*Cos[e + f*x])^m*((c*(a*(a*A
 - b*B) + 2*b*(A*b - a*B)*(1/2 + m)))/2 - (a*(A*b - a*B)*c*Cos[e + f*x])/2
- (b*(A*b - a*B)*c*(3 + 2*m)*Cos[e + f*x]^2)/2))/Sqrt[a + b*Cos[e + f*x]],
x])/(a*(a^2 - b^2)*c)
```

Rubi [A] time = 0.499744, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(A*b - a*B)*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*(a^2 - b^2)*c*f*
Sqrt[a + b*Cos[e + f*x]]) + (2*Defer[Int][((c*Cos[e + f*x])^m*((c*(a*(a*A -
 b*B) + 2*b*(A*b - a*B)*(1/2 + m)))/2 - (a*(A*b - a*B)*c*Cos[e + f*x])/2 -
(b*(A*b - a*B)*c*(3 + 2*m)*Cos[e + f*x]^2)/2))/Sqrt[a + b*Cos[e + f*x]], x]
)/(a*(a^2 - b^2)*c)
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\frac{2b(Ab-aB)(c\cos(e+fx))^{1+m}\sin(e+fx)}{a\left(a^2-b^2\right)cf\sqrt{a+b\cos(e+fx)}}+\frac{2\int\frac{(c\cos(e+fx))^m\left(\frac{1}{2}c\left(a(aA-bB)+2b\right.\right.}{}}{}$$

Mathematica [A] time = 10.5573, size = 0, normalized size = 0.

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3
/2),x]
```

```
[Out] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3
/2), x]
```

Maple [A] time = 0.423, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(A+B\cos\left(fx+e\right)\right)\left(a+b\cos\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\cos\left(fx+e\right)\right)^{m}}{b^{2}\cos\left(fx+e\right)^{2}+2\,ab\cos\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m/(
b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\cos\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)}{\left(a+b\cos\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(3/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x))**(3/
2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

3.459 $\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=172

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{2a(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\Big(}{3d}$$

```
[Out] (-2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.222072, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{2a(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\Big(}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \int \sec^{\frac{3}{2}}(c + dx)(a + a\sec(c + dx))(B + A\sec(c + dx))\,dx$$
$$= \frac{2aA\sec^{\frac{5}{2}}(c + dx)\sin(c + dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c + dx)\left(\frac{1}{2}a(3A\right.$$
$$= \frac{2aA\sec^{\frac{5}{2}}(c + dx)\sin(c + dx)}{5d} + (a(A + B))\int \sec^{\frac{5}{2}}(c + dx)$$
$$= \frac{2a(3A + 5B)\sqrt{\sec(c + dx)}\sin(c + dx)}{5d} + \frac{2a(A + B)\sec^{\frac{3}{2}}(c}{3a}$$
$$= \frac{2a(3A + 5B)\sqrt{\sec(c + dx)}\sin(c + dx)}{5d} + \frac{2a(A + B)\sec^{\frac{3}{2}}(c}{3a}$$
$$= -\frac{2a(3A + 5B)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c + dx)}}{5d} +$$

Mathematica [C] time = 1.89684, size = 292, normalized size = 1.7

$$\frac{ae^{-ic}\left(-1 + e^{2ic}\right)\csc(c)(\cos(c + dx) + 1)\sec^2\left(\frac{1}{2}(c + dx)\right)\sqrt{\sec(c + dx)}\left((3A + 5B)e^{i(c+dx)}\left(1 + e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])*Csc[c]*(5*A + 5*B - 3*A*E^(I*(c +
d*x)) - 15*B*E^(I*(c + d*x)) - 24*A*E^((3*I)*(c + d*x)) - 30*B*E^((3*I)*(c
+ d*x)) - 5*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 9*A*E^((5*I)*
(c + d*x)) - 15*B*E^((5*I)*(c + d*x)) - (5*I)*(A + B)*(1 + E^((2*I)*(c + d*
x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (3*A + 5*B)*E^(I*(c +
 d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(30*d*E^(I*c)*(1
 + E^((2*I)*(c + d*x)))^2)
```

Maple [B] time = 10.024, size = 661, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+
2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/2*A
+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*A/(8*sin(1/2*d*x+1/2*
c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2
*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sec(d*x + c)^(
7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.460 $\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx$

Optimal. Leaf size=135

$$\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{}}{}$$

```
[Out] (-2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a
*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.192117, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a
*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))(B+A\sec(c+dx))\,dx$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2}{3}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a(A+3\right.$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + (a(A+B))\int \sec^{\frac{3}{2}}(c+dx)\,d$$

$$= \frac{2a(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$= \frac{2a(A+3B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a(}{}$$

$$= -\frac{2a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a}{}$$

Mathematica [C] time = 1.19416, size = 225, normalized size = 1.67

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(i\left((A+B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3Ae^{i(c+dx)}-\right.\right.}{3d\left(1+e^{2i(c+d\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*((A + 3*B)*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + I*(A - 3*A*E^(I*(c + d*x)) - 3*B*E^(I*(c + d
*x)) - A*E^((2*I)*(c + d*x)) - 3*A*E^((3*I)*(c + d*x)) - 3*B*E^((3*I)*(c +
d*x)) + (A + B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*Sec[(c + d*x)/2]^2*Sqrt[Sec[c
 + d*x]])/(3*d*(1 + E^((2*I)*(c + d*x))))
```

Maple [B] time = 7.894, size = 426, normalized size = 3.2

$$-4\,\frac{\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)(\sin(1/2\,dx+c/2))^2}a}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(1/2\,\frac{B\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\mathrm{Elli}}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(
1/2*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sec(d*x + c)^(
5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.461 $\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\,dx$

Optimal. Leaf size=106

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} - \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aA\sin}{}$$

```
[Out] (-2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.178832, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} - \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aA\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
```

```
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + 2\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B}{\sqrt{\sec(c+dx}}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - (a(A-B))\int \frac{1}{\sqrt{\sec(c+dx)}}\,d$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \Big(a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec}$$

$$= -\frac{2a(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a}{}$$

Mathematica [C] time = 1.05615, size = 157, normalized size = 1.48

$$\frac{2ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(i(A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3(A+B)\sqrt{\cos(c+dx)}F\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-3*I)*A*Cos[c + d*x] + (3
*I)*B*Cos[c + d*x] + 3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + I*(A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x]))/(3*d*E^(I*d*x))
```

Maple [A] time = 3.536, size = 240, normalized size = 2.3

$$-2\frac{a\left(A\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}+A\sqrt{(\sin(1/2\,dx+c/2))^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*a*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sec(d*x + c)^(
3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.462 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{2a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\,\mathrm{s}}{3d\sqrt{\mathrm{se}}}$$

```
[Out] (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.183918, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\,\mathrm{s}}{3d\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}a(A+B)-\frac{1}{2}a(3A+B)\sec(c+dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + (a(A+B))\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}(a(3A$$

$$= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \left(a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a(3}{}$$

Mathematica [C] time = 1.23907, size = 148, normalized size = 1.35

$$\frac{2ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-i(A+B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(B\sin(c+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((3*A + B)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - I*(A + B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*
(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c
+ d*x]*((3*I)*(A + B) + B*Sin[c + d*x])))/(3*d*E^(I*d*x))
```

Maple [B] time = 2.699, size = 321, normalized size = 2.9

$$-\frac{2a}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+3\,A\mathrm{EllipticF}\left(\cos(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-2*B
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos\left(dx+c\right)^2+(A+B)a\cos\left(dx+c\right)+Aa\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sqrt(sec(d*x +
 c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sqrt{\sec\left(c+dx\right)}\,dx+\int A\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int B\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int B\cos^2\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)*sqrt(sec(c +
 d*x)), x) + Integral(B*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(B*co
s(c + d*x)**2*sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.463 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{5d}$$

```
[Out] (2*a*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.202655, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(A+B)-\frac{1}{2}a(5A+3B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(a(5A+3B))\int \frac{}{\sqrt{s}}$$

$$= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(a(A+B))\int \sqrt{\sec(c+dx)}$$

$$= \frac{2a(5A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2aB\sin(c+d}{5d\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{2a(5A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(A+B)\sqrt{co}}{}$$

Mathematica [C] time = 1.59331, size = 148, normalized size = 1.05

$$\frac{a\sqrt{\sec(c+dx)}\left(-2i(5A+3B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(10(A+B)\sin(c+dx)+6i(5A+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(10*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2] - (2*I)*(5*A + 3*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((6*I)*(5*
A + 3*B) + 10*(A + B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(15*d)
```

Maple [B] time = 2.924, size = 355, normalized size = 2.5

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(20\,A+44\,B)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/sqrt(sec(d*x +
 c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{A}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{A\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{B\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{B\cos^{2}\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)/sqrt(sec(c +
 d*x)), x) + Integral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(B*co
s(c + d*x)**2/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.464 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+$$

```
[Out] (6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*d) + (2*a*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
(2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*B)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.224161, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*d) + (2*a*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
(2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*B)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(A+B)-\frac{1}{2}a(7A+5B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{7}(a(7A+5B))\int -$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{6a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(7A+5B)\sqrt{}}{}$$

Mathematica [C] time = 2.19619, size = 182, normalized size = 1.06

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-84i(A+B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(42(A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(7*A + 5*B)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] - (84*I)*(A + B)*E^(I*(c + d*x))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
+ Cos[c + d*x]*((252*I)*(A + B) + 5*(28*A + 23*B)*Sin[c + d*x] + 42*(A + B)
*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

Maple [A] time = 3.135, size = 383, normalized size = 2.2

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8+(-168\,A-528\,B)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-11
2*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/sec(d*x + c)^(
3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{A\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A*cos(c + d*x)/sec(c + d*x
)**(3/2), x) + Integral(B*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(B
*cos(c + d*x)**2/sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.465 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=199

$$\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3d}$$

```
[Out] (-4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*a^2*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
) + (2*A*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.345233, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*a^2*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
) + (2*A*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2(B+A\sec(c+dx))\,dx$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\text{s}}$$

$$= \frac{2a^2(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\left(\right.}{}$$

$$= \frac{2a^2(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\left(\right.}{}$$

$$= \frac{4a^2(4A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a^2(7A+5B)\sec}{1}$$

$$= \frac{4a^2(A+2B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{4}{}$$

$$= -\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 2.97918, size = 299, normalized size = 1.5

$$\frac{a^2e^{-ic}\left(-1+e^{2ic}\right)\csc(c)(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2(4A+5B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])^2*Csc[c]*(10*A + 5*B - 18*A*E^(I
*(c + d*x)) - 30*B*E^(I*(c + d*x)) - 54*A*E^((3*I)*(c + d*x)) - 60*B*E^((3*
I)*(c + d*x)) - 10*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 24*A*E
^((5*I)*(c + d*x)) - 30*B*E^((5*I)*(c + d*x)) - (10*I)*(A + 2*B)*(1 + E^((2
*I)*(c + d*x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(4*A + 5
*B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]])/(60
```

```
*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2)
```

Maple [B] time = 10.25, size = 741, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+(1/4*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*
x+1/2*c)^2-1)-1/20*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(
1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1
/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/2*A+1/4*B)*(-1/6*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2\right)\sec(dx+c)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c) + A)(a\cos(dx+c) + a)^2 \sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

3.466 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{s}}{}$$

```
[Out] (-4*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (4*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a^2*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*A*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.318199, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (4*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a^2*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*A*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 (A + B\cos(c+dx)) \sec^{\frac{5}{2}}(c+dx)\, dx = \int \frac{(a + a\sec(c+dx))^2 (B + A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\, dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\left(a^2 + a^2\sec(c+dx)\right)\sin(c+dx)}{3d} + \frac{2}{3}\int$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A\sqrt{\sec(c+dx)}}{}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A\sqrt{\sec(c+dx)}}{}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A\sqrt{\sec(c+dx)}}{}$$

$$= -\frac{4a^2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4a^2(2}{}$$

Mathematica [C] time = 2.24324, size = 279, normalized size = 1.74

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}(3\csc(c)\cos(dx)(4A - B\cos(2c) + B) + 2A\tan(c+dx) + 6B\cos(c)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(3*A*E^(I*
c)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*d*x)*((2*
A + 3*B)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
 d*x))] + A*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(3*(4*A + B
- B*Cos[2*c])*Cos[d*x]*Csc[c] + 6*B*Cos[c]*Sin[d*x] + 2*A*Tan[c + d*x])))/(
12*d)
```

Maple [B] time = 3.736, size = 513, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

3.467 $\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)\, dx$

Optimal. Leaf size=160

$$\frac{2a^2(3A - B)\sin(c + dx)\sqrt{\sec(c + dx)}}{3d} + \frac{4a^2(3A + 2B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d} + \frac{2B\sin(c + dx)\left(a^2 \mathrm{se}\right.}{3d\sqrt{\sec(c\ }}$$

```
[Out] (4*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (4*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a^2*(3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
 + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.322774, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(3A - B)\sin(c + dx)\sqrt{\sec(c + dx)}}{3d} + \frac{4a^2(3A + 2B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d} + \frac{2B\sin(c + dx)\left(a^2 \mathrm{se}\right.}{3d\sqrt{\sec(c\ }}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (4*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a^2*(3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
 + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\sec(c+d}{}$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2B\left(a^2+a^2\sec(c+\right.}{3d\sqrt{\sec(c}}$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2B\left(a^2+a^2\sec(c+\right.}{3d\sqrt{\sec(c}}$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2B\left(a^2+a^2\sec(c+\right.}{3d\sqrt{\sec(c}}$$

$$= \frac{4a^2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4a^2(3A}{}$$

Mathematica [C] time = 1.83939, size = 302, normalized size = 1.89

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}(6(A+2B)\cos(c)\sin(dx)-3\csc(c)\cos(dx)((A+2B)\cos(2c)-A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*Sqrt[E^(I*(c +
 d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(3*B*E^(I*c
)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*d*x)*(-((3
*A + 2*B)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c
+ d*x))]) + B*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(-3*(-A +
2*B + (A + 2*B)*Cos[2*c])*Cos[d*x]*Csc[c] + B*Cos[2*d*x]*Sin[2*c] + 6*(A +
2*B)*Cos[c]*Sin[d*x] + B*Cos[2*c]*Sin[2*d*x])))/(12*d)
```

Maple [A] time = 3.353, size = 388, normalized size = 2.4

$$-\frac{4a^2}{3d}\left(2B\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4-\sqrt{-2\left(\sin\left(1/2\,dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^2*(2*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(3*A+B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

3.468 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=166

$$\frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{5d}$$

```
[Out] (4*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt
[Sec[c + d*x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c +
d*x]^(3/2))
```

Rubi [A] time = 0.340205, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (4*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt
[Sec[c + d*x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c +
d*x]^(3/2))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\sec(c+$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 1.61995, size = 153, normalized size = 0.92

$$\frac{a^2\sqrt{\sec(c+dx)}\left(-4i(5A+4B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(10(A+2B)\sin(c+dx)+60iA}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(20*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] - (4*I)*(5*A + 4*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((60*I
)*A + (48*I)*B + 10*(A + 2*B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(15*d)
```

Maple [A] time = 3.26, size = 357, normalized size = 2.2

$$-\frac{4a^2}{15d}\sqrt{\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12B\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^6+(10A+32B)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

3.469 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=201

$$\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(}{}$$

```
[Out] (4*a^2*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(7*A + 9*B)*Sin[c + d*x])/(35*d*S
ec[c + d*x]^(3/2)) + (4*a^2*(7*A + 6*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.369115, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (4*a^2*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(7*A + 9*B)*Sin[c + d*x])/(35*d*S
ec[c + d*x]^(3/2)) + (4*a^2*(7*A + 6*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\sec(c+dx))\left(\frac{1}{2}a(7A\right.}{\sec}$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{5}(2$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2B\left(a^2+a^2\sec\right.}{7d\sec}$$

$$= \frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^2(7A+9}{35d\sec}$$

$$= \frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2(7A+6}{}$$

Mathematica [C] time = 2.29297, size = 193, normalized size = 0.96

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(4A+3B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(5(56A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(7*A + 6*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(4*A + 3*B)*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((672*I)*A + (504*I)*B + 5*(56*A + 51*B)*Sin[c + d*x]
+ 42*(A + 2*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x)
)
```

Maple [A] time = 2.94, size = 385, normalized size = 1.9

$$-\frac{4a^2}{105d}\sqrt{\left(2\left(\cos\left(1/2dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120B\cos\left(1/2dx+c/2\right)\left(\sin\left(1/2dx+c/2\right)\right)^8+\left(-84A-348B\right)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-9
1*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2\cos\left(dx+c\right)^3+\left(A+2B\right)a^2\cos\left(dx+c\right)^2+\left(2A+B\right)a^2\cos\left(dx+c\right)+Aa^2}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{A}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{2A\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{A\cos^2\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{B\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{2B\cos^2\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(2*A*cos(c + d*x)/sqrt(se
c(c + d*x)), x) + Integral(A*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integ
ral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(2*B*cos(c + d*x)**2/sq
rt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)**3/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

3.470 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(41A+42B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(11A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d}+\frac{4a^3(7A+9B$$

```
[Out] (-4*a^3*(7*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(7*A + 9*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(5*d) + (4*a^3*(41*A + 42*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (2*a*A*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/
(7*d) + (2*(11*A + 7*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c +
 d*x])/(35*d)
```

Rubi [A] time = 0.505158, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{4a^3(41A+42B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(11A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d}+\frac{4a^3(7A+9B$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```



```
[Out] (-4*a^3*(7*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(7*A + 9*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(5*d) + (4*a^3*(41*A + 42*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (2*a*A*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/
(7*d) + (2*(11*A + 7*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c +
 d*x])/(35*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3(B+A\sec(c+dx))\,dx$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{2(11A}{}$$

$$= \frac{4a^3(41A+42B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aA\sec^{\frac{3}{2}}(c+}{}$$

$$= \frac{4a^3(41A+42B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aA\sec^{\frac{3}{2}}(c+}{}$$

$$= \frac{4a^3(7A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^3(41A+42B)}{}$$

$$= \frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{21d}$$

$$= -\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 4.09606, size = 435, normalized size = 1.78

$$\frac{a^3\csc(c)e^{-idx}(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}\left(-1+e^{2ic}\right)(7A+9B)e^{2idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Csc[c]*Sec[(c + d*x)/2]^6*(7*Sqrt[2]*(7*A + 9*B)*
E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))] - ((-1 + E^((2*I)*c))*(21*B*(-5 + 16*E^(I*(c + d*x)) - 5*E^
((2*I)*(c + d*x)) + 54*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 56*E^(
(5*I)*(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 18*E^((7*I)*(c + d*x))) + 2*A*(-
65 + 84*E^(I*(c + d*x)) - 95*E^((2*I)*(c + d*x)) + 441*E^((3*I)*(c + d*x))
+ 95*E^((4*I)*(c + d*x)) + 504*E^((5*I)*(c + d*x)) + 65*E^((6*I)*(c + d*x))
 + 147*E^((7*I)*(c + d*x))) + (10*I)*(13*A + 21*B)*(1 + E^((2*I)*(c + d*x))
)^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(2*E^
(I*(c - d*x))*(1 + E^((2*I)*(c + d*x)))^3)))/(420*d*E^(I*d*x))
```

Maple [B] time = 11.729, size = 929, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+(1/8*A+3/8*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1)-1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2
*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(3/8*A+3/8*B)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/
2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
```

```
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3\right)\text{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(9/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

3.471 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(21A+20B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(3A+5B)}{}$$

```
[Out] (-4*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(21*A + 20*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + (2*(9*A + 5*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x
])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.494507, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(21A+20B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(3A+5B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-4*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(21*A + 20*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + (2*(9*A + 5*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x
])*Sin[c + d*x])/(15*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2}{5}\int$$

$$= \frac{2aA\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2(9A}{}$$

$$= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aA\sqrt{\sec(c+}}{}$$

$$= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aA\sqrt{\sec(c+}}{}$$

$$= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aA\sqrt{\sec(c+}}{}$$

$$= -\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 3.16145, size = 268, normalized size = 1.27

$$\frac{a^3\csc(c)\sec(c)e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(2\left(-1+e^{4ic}\right)(9A+5B)e^{-i(c-dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*Csc[c]*Sec[c]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((2*(9*A + 5*
B)*(-1 + E^((4*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c - d*x)) + (Sec[c + d*x]^2*Sin[2*c]
*((-18*I)*(9*A + 5*B)*Cos[c + d*x] - (54*I)*A*Cos[3*(c + d*x)] - (30*I)*B*C
os[3*(c + d*x)] + 40*(3*A + 5*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2,
2] + 66*A*Sin[c + d*x] + 45*B*Sin[c + d*x] + 30*A*Sin[2*(c + d*x)] + 10*B*S
in[2*(c + d*x)] + 54*A*Sin[3*(c + d*x)] + 45*B*Sin[3*(c + d*x)]))/2))/(30*d
*E^(I*d*x))
```

Maple [B] time = 10.273, size = 916, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(108*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+60*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6+60*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+100*B*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-108*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-60*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-100*B*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-72*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+15*B*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+25*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-50*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3\right)s\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(7/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

3.472 $\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)\, dx$

Optimal. Leaf size=199

$$\frac{4a^3(4A + B)\sin(c + dx)\sqrt{\sec(c + dx)}}{3d} + \frac{2(A - B)\sin(c + dx)\sqrt{\sec(c + dx)}\left(a^3\sec(c + dx) + a^3\right)}{3d} + \frac{20a^3(A + B)\sqrt{\cos(c +}}{}$$

```
[Out] (-4*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (20*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (4*a^3*(4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
3*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
 + (2*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*
d)
```

Rubi [A] time = 0.491089, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4017, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(4A + B)\sin(c + dx)\sqrt{\sec(c + dx)}}{3d} + \frac{2(A - B)\sin(c + dx)\sqrt{\sec(c + dx)}\left(a^3\sec(c + dx) + a^3\right)}{3d} + \frac{20a^3(A + B)\sqrt{\cos(c +}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-4*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (20*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (4*a^3*(4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
3*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
 + (2*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*
d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\sec(c+dx}{}$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2(A-B)\sqrt{\sec(c+dx}}{}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aB(a+a\sec(c+a}{3d\sqrt{\sec(c}}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aB(a+a\sec(c+a}{3d\sqrt{\sec(c}}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aB(a+a\sec(c+a}{3d\sqrt{\sec(c}}$$

$$= -\frac{4a^3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2}{}$$

Mathematica [C] time = 1.89163, size = 202, normalized size = 1.02

$$\frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(4i(A-B)\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+40(A+B)\cos^{\frac{3}{2}}(c+dx)F\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((-12*I)*A + (12*I)*B - (12
*I)*A*Cos[2*(c + d*x)] + (12*I)*B*Cos[2*(c + d*x)] + 40*(A + B)*Cos[c + d*x
```

```
]^(3/2)*EllipticF[(c + d*x)/2, 2] + (4*I)*(A - B)*(1 + E^((2*I)*(c + d*x)))
^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 4*A*Sin[c +
 d*x] + B*Sin[c + d*x] + 18*A*Sin[2*(c + d*x)] + 6*B*Sin[2*(c + d*x)] + B*S
in[3*(c + d*x)]))/(6*d*E^(I*d*x))
```

Maple [B] time = 3.417, size = 654, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(-4*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3\right)\text{se}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(5/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

3.473 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{3d}$$

```
[Out] (4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A - 6*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (2*(5*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.494971, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```



```
[Out] (4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A - 6*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (2*(5*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d
*Sqrt[Sec[c + d*x]])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\sec(c+d}{}$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5A+9B)\left(a^3+a^3 s\right.}{15d\sqrt{se}}$$

$$= \frac{4a^3(5A-6B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aB(a+a\sec(c+}{5d\sec^{\frac{3}{2}}(}$$

$$= \frac{4a^3(5A-6B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aB(a+a\sec(c+}{5d\sec^{\frac{3}{2}}(}$$

$$= \frac{4a^3(5A-6B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aB(a+a\sec(c+}{5d\sec^{\frac{3}{2}}(}$$

$$= \frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} +$$

Mathematica [C] time = 1.59662, size = 207, normalized size = 0.98

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-8i(5A+9B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+40(5A+3B)\sqrt{\cos(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((120*I)*A*Cos[c + d*x] + (
216*I)*B*Cos[c + d*x] + 40*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] - (8*I)*(5*A + 9*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 60*A*Sin[c + d*x]
+ 3*B*Sin[c + d*x] + 10*A*Sin[2*(c + d*x)] + 30*B*Sin[2*(c + d*x)] + 3*B*Si
n[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

Maple [B] time = 3.493, size = 519, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^3*(-12*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(5*A+21*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A+9*B)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-27*B*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3\right)\text{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(3/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

3.474 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{2(7A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{21d}$$

```
[Out] (4*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(42*A + 41*B)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2)) + (2*(7*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.507522, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(7A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (4*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(42*A + 41*B)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2)) + (2*(7*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\sec(c+$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(7A+11B)\left(a^3+a\right.}{35d\,\mathrm{s}}$$

$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 2.38709, size = 194, normalized size = 0.92

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(9A+7B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(5(84\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(21*A + 13*B)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(9*A + 7*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((168*I)*(9*A + 7*B) + 5*(84*A + 107*B)*Sin[c + d*x]
 + 42*(A + 3*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x
))
```

Maple [A] time = 3.111, size = 385, normalized size = 1.8

$$-\frac{4\,a^{3}}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(120\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-84\,A-432\,B\right)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{3}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ba^{3}\cos(dx+c)^{4}+(A+3\,B)a^{3}\cos(dx+c)^{3}+3\,(A+B)a^{3}\cos(dx+c)^{2}+(3\,A+B)a^{3}\cos(dx+c)+Aa^{3}\right)\sqrt{\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sqrt(sec(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

3.475 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^3(13A+}{}$$

```
[Out] (4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(24*A + 23*B)*Sin[c + d*x])/
(105*d*Sec[c + d*x]^(3/2)) + (4*a^3*(13*A + 11*B)*Sin[c + d*x])/(21*d*Sqrt[
Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (2*(9*A + 13*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d*S
ec[c + d*x]^(5/2))
```

Rubi [A] time = 0.535339, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^3(13A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```



```
[Out] (4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(24*A + 23*B)*Sin[c + d*x])/
(105*d*Sec[c + d*x]^(3/2)) + (4*a^3*(13*A + 11*B)*Sin[c + d*x])/(21*d*Sqrt[
Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (2*(9*A + 13*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d*S
ec[c + d*x]^(5/2))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c+dx))^2\left(\frac{1}{2}a(9A\right.}{\sec}$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(9A+13B)\left(a^3+a^3\sec(c+dx)\right)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(\cdots}{}$$

$$= \frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(\cdots}{}$$

$$= \frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\cdots}{}$$

$$= \frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^3(24A\cdots}{105d\cdots}$$

$$= \frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^3(13A\cdots}{}$$

Mathematica [C] time = 2.76805, size = 196, normalized size = 0.8

$$\frac{a^3\sqrt{\sec(c+dx)}\left(-112i(21A+17B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(30(107A+97B)\sin(c+dx)\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(240*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] - (112*I)*(21*A + 17*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*
x]*((7056*I)*A + (5712*I)*B + 30*(107*A + 97*B)*Sin[c + d*x] + 14*(54*A + 7
3*B)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 270*B*Sin[3*(c + d*x)] + 35
*B*Sin[4*(c + d*x)])))/(1260*d)
```

Maple [A] time = 3.016, size = 413, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-441*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+Aa^3}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/sqrt(sec(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

3.476 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(5A-3}{}$$

```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - (3*(A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(5*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A - B)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.304157, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(5A-3}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```



```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - (3*(A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(5*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A - B)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(5A-\right.}{a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(5A-3B)\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a} - \frac{(3(A-}{}$$

$$= -\frac{3(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(5A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} -$$

$$= -\frac{3(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(5A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} -$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-3B)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 7.18193, size = 650, normalized size = 3.37

$$\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{d} - \frac{3(A-B)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos(dx)}{d} + \frac{2\tan\left(\frac{c}{2}\right)\sec(c)(5A\cos(c)+2A-3B\,c}{3d}\right.}{a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x]))) + (B*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x])) + (5*A*Cos[c/2 +
(d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*S
qrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])) - (B*Cos[c/2 + (d*x)/2
]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec
[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Se
c[c + d*x]]*((-3*(A - B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*Sec[c/2]*Sec[c/
```

```
2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x
]*Sin[d*x])/(3*d) + (2*(2*A + 5*A*Cos[c] - 3*B*Cos[c])*Sec[c]*Tan[c/2])/(3*
d)))/(a + a*Cos[c + d*x])
```

Maple [B] time = 9.714, size = 493, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((A-B)*(cos(1/
2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*A+2*B)*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.477 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{ad}$$

```
[Out] -(((3*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) - ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
- B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.275871, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```


```
[Out] -(((3*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) - ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
- B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
```

```
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-B)+\frac{1}{2}a(3A-\right.}{a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{(3A-B)\int}{}$$

$$= \frac{(3A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A-}{}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(3A-B)\sqrt{\sec(c+dx)}}{ad}$$

$$= -\frac{(3A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-B)\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 4.36917, size = 400, normalized size = 2.52

$$\cos^2\left(\frac{1}{2}(c+dx)\right)\left(6\sqrt{\sec(c+dx)}\left(2(B-A)\tan\left(\frac{1}{2}(c+dx)\right)+2(3A-B)\csc(c)\cos(dx)\right)+6\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 6*Sqrt[Sec[c + d*x]]*(2*(3*A - B)*Cos[d*
x]*Csc[c] + 2*(-A + B)*Tan[(c + d*x)/2])))/(6*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 5.949, size = 319, normalized size = 2.

$$-\frac{1}{da}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1/2*c)^2
)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.478 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{ad}$$

```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rubi [A] time = 0.244608, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4019, 3787, 3771, 2639, 2641}

$$-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
```

```
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$
$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{(A+B)\int \sqrt{\mathrm{s}}}{2}$$
$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$
$$= \frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A+B)\sqrt{\cos(c+dx)}F}{}$$

Mathematica [C] time = 1.0928, size = 200, normalized size = 1.63

$$-\frac{\left(-1+e^{2ic}\right)e^{-\frac{1}{2}i(4c+dx)}\left(\csc\left(\frac{c}{2}\right)+i\sec\left(\frac{c}{2}\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((A-B)\left(e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F\right.\right.}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((-1 + E^((2*I)*c))*((3*I)*(A + B)*(1 + E^(I*(c + d*x)))*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + (A - B)*(-3*(1 + E^((2*I)*(c + d*x))) + E^(I*
(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*(Csc[c/2] + I*Sec[c/2])*Sec[(c
 + d*x)/2]*Sqrt[Sec[c + d*x]])/(24*a*d*E^((I/2)*(4*c + d*x)))
```

Maple [A] time = 3.193, size = 244, normalized size = 2.

$$\frac{1}{da}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+
(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1
/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x) + Integral(B*cos(c +
d*x)*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.479 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{ad}$$

```
[Out] -(((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[
c + d*x]))
```

Rubi [A] time = 0.252913, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] -(((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[
c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(A-3B)+\frac{1}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-3B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{(A-B)\int \sqrt{\sec(}}{2a}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left((A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\mathrm{co}}}{2a}$$

$$= -\frac{(A-3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A-B)\sqrt{\cos(c+dx)}F}{}$$

<u>Mathematica</u> [C] time = 2.57646, size = 422, normalized size = 3.38

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{6\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((A-2B)\cos\left(\frac{1}{2}(c-dx)\right)-B\cos\left(\frac{1}{2}(3c+dx)\right)\right)}{\sqrt{\sec(c+dx)}} + 2\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*((A - 2*B)*Cos[(c - d*x)/2] -
 B*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec[c + d*x
]] + 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] -
 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]))/(6*
a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 3.398, size = 244, normalized size = 2.

$$-\frac{1}{da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\left(AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)/sec(d*x+c)^(1/2),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos(c+dx)}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x) + In
tegral(B*cos(c + d*x)/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))
), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

3.480 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+$$

```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((3*A - 5*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
 + ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.28072, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4020, 3787, 3769, 3771, 2641, 2639}

$$-\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((3*A - 5*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
 + ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(3A-5B)+\frac{3}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{(3A-5B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(3(A-B))\int}{2a}$$

$$= -\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{(3A-5B)\int \sqrt{s}}{6a}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 4.93313, size = 444, normalized size = 2.72

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((12A-13B)\cos\left(\frac{1}{2}(c-dx)\right)+(6A-5B)\cos\left(\frac{1}{2}(3c+dx)\right)-2B\sin(c)\sin\left(\frac{3}{2}(c+dx)\right)\right)}{\sqrt{\sec(c+dx)}} - 6\sqrt{2}A\csc(c)e^{-id}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 20*B*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - (Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*((1
2*A - 13*B)*Cos[(c - d*x)/2] + (6*A - 5*B)*Cos[(3*c + d*x)/2] - 2*B*Sin[c]*
Sin[(3*(c + d*x))/2]))/Sqrt[Sec[c + d*x]]))/(6*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 3.086, size = 262, normalized size = 1.6

$$\frac{1}{3\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(3\,AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(-3*A+
7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

3.481 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3ad}$$

```
[Out] (-3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a*d) - ((5*A - 7*B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(
3/2)) + (5*(A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) + ((A - B)*Sin[
c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.300342, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4020, 3787, 3769, 3771, 2639, 2641}

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (-3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a*d) - ((5*A - 7*B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(
3/2)) + (5*(A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) + ((A - B)*Sin[
c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(5A-7B)+\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{(5A-7B)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a} + \frac{(5(A-B))\int -}{2a}$$

$$= -\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{3(5A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(A-B)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 3.21636, size = 518, normalized size = 2.64

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\left(40(A-B)\sin(2c)\cos(2dx)-12(20A-33B)\cos(c)\sin(dx)+40(A-B)\cos(2c)\sin(2dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((60*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) - (84*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 200*A*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 200*B*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c + d*x]]*(3*(40*A - 51*B +
 (20*A - 33*B)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 40*(A - B)*Cos[2*d*x]
*Sin[2*c] + 12*B*Cos[3*d*x]*Sin[3*c] - 120*(A - B)*Sec[c/2]*Sec[(c + d*x)/2
```

```
]*Sin[(d*x)/2] - 12*(20*A - 33*B)*Cos[c]*Sin[d*x] + 40*(A - B)*Cos[2*c]*Sin
[2*d*x] + 12*B*Cos[3*c]*Sin[3*d*x] - 120*(A - B)*Tan[c/2])))/(60*a*d*(1 + C
os[c + d*x]))
```

Maple [A] time = 3.461, size = 281, normalized size = 1.4

$$-\frac{1}{15\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1/2*c)
^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

3.482 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$-\frac{(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(4A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3a^2d}$$

```
[Out] -(((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((4*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*d) - ((5*A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c +
 d*x])) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x
])^2)
```

Rubi [A] time = 0.430758, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(4A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] -(((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((4*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*d) - ((5*A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c +
 d*x])) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x
])^2)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(7A-B)\sec(c+dx)\right)}{a+a\sec(c+dx)} d}{3a^2}$$

$$= -\frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \int$$

$$= -\frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - (5$$

$$= \frac{(4A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - (A$$

$$= -\frac{(5A-2B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{(4A-B)\sqrt{\sec(c+}}{a^2d}$$

$$= -\frac{(4A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(5A-2B)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 3.14467, size = 303, normalized size = 1.46

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-i(4A-B)e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*((29*I)*A - (5*I)*B + (2*I)*(25*A - 7
*B)*Cos[c + d*x] + (17*I)*A*Cos[2*(c + d*x)] - (5*I)*B*Cos[2*(c + d*x)] - (
I*(4*A - B)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 8*(5*A - 2
*B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c
 + d*x)/2] - I*Sin[(c + d*x)/2]) - 12*A*Sin[c + d*x] - 7*A*Sin[2*(c + d*x)]
 + B*Sin[2*(c + d*x)])*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*
d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 4.286, size = 494, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*x+1/2*c)^2)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a^2*cos(d*x + c)^2 + 2*a^
2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

3.483 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{a^2d}$$

```
[Out] (A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*a^2*d) - (A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x
])) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2
)
```

Rubi [A] time = 0.388056, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4019, 3787, 3771, 2639, 2641}

$$\frac{(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*a^2*d) - (A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x
])) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2
)
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-B)+\frac{1}{2}a(5A+B)\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{3a^2A}{2}+\frac{1}{2}a^2(\ldots}{\sqrt{se\ldots}}}{\ldots}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{A\int \frac{1}{\sqrt{\sec(c+\ldots}}}{2a^2}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(A\sqrt{\cos(c-\ldots}\right.}{\ldots}$$

$$= \frac{A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A+B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c-\ldots\right.}{3a^2d}$$

Mathematica [C] time = 2.00357, size = 256, normalized size = 1.59

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(2i\cos(c+dx)(i(A-B)\sin(c+dx)+(5A+B\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*A*(1 + E^(I*(c + d*x)))^3*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 8*(2*A + B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*Co
s[c + d*x]*(7*A - B + (5*A + B)*Cos[c + d*x] + I*(A - B)*Sin[c + d*x]))*(Co
s[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d
*x])^2)
```

Maple [A] time = 3.796, size = 350, normalized size = 2.2

$$\frac{1}{6\,a^2 d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6-4\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^3\sqrt{\left(\sin\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1
/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^
2*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sqrt{\sec(c+dx)}}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) +
 Integral(B*cos(c + d*x)*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*
x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

3.484 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=168

$$\frac{(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{a^2d}$$

```
[Out] -((B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d)) + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + ((A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rubi [A] time = 0.391055, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```



```
[Out] -((B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d)) + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + ((A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{3}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int -}{}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{B\int}{}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(B\sqrt{}}{}$$

$$= -\frac{B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A+2B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{3a^2}$$

Mathematica [C] time = 2.44334, size = 256, normalized size = 1.52

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(2\cos(c+dx)(-i(A-B)\sin(c+dx)+(A\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(8*(A + 2*B)*Cos[(c + d*x)/2]^3*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/
2]) + I*((B*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 2*Cos[c +
d*x]*(-A - 5*B + (A - 7*B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x])))*(Cos[(c
 + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])
^2)
```

Maple [A] time = 3.863, size = 350, normalized size = 2.1

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^3\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\left(\cos\left(1/2\,dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^6+4*B*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a
^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

3.485 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{a^2d}$$

```
[Out] -(((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.406233, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4020, 3787, 3771, 2639, 2641}

$$\frac{(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -(((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$$

$$=\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{-\frac{1}{2}a(A-7B)+\frac{3}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$=\frac{(2A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}+\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int -\frac{3}{2}$$

$$=\frac{(2A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}+\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(2A$$

$$=\frac{(2A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}+\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{((2A$$

$$=-\frac{(A-4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(2A-5B)\sqrt{\cos(c+d}$$

Mathematica [C] time = 6.37018, size = 475, normalized size = 2.7

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(5(A-4B)\cos\left(\frac{1}{2}(c-dx)\right)+4(A-4B)\cos\left(\frac{1}{2}(3c+dx)\right)+3A\cos\left(\frac{1}{2}(c+3dx)\right)-9B\cos\left(\frac{1}{2}(c+3dx)\right)-3B\cos\left(\frac{1}{2}(5c+3\right.\right.}{2\sqrt{\sec(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] (Cos[(c + d*x)/2]^4*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (8*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + ((5*(A - 4*B)*Cos[(c - d*x)/2] +
 4*(A - 4*B)*Cos[(3*c + d*x)/2] + 3*A*Cos[(c + 3*d*x)/2] - 9*B*Cos[(c + 3*d
*x)/2] - 3*B*Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(2
*Sqrt[Sec[c + d*x]]) + 8*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]] - 20*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
```

```
c[c + d*x]]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 3.718, size = 421, normalized size = 2.4

$$-\frac{1}{6\,a^2d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^3\sqrt{\left(\sin\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B*cos(
1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2*c)^4+9*A
*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)
^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a
^2)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

3.486 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=206

$$-\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+$$

```
[Out] ((4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a^2*d) - (5*(A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c
+ d*x]]) + ((4*A - 7*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[
c + d*x])) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
d*x])^2)
```

Rubi [A] time = 0.433914, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4020, 3787, 3769, 3771, 2641, 2639}

$$-\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a^2*d) - (5*(A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c
+ d*x]]) + ((4*A - 7*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[
c + d*x])) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
d*x])^2)
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{3}{2}a(A-3B)+\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} + \cdots$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} + \cdots$$

$$= -\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} + \frac{(A-\cdots}{3d\sqrt{\sec(c+\cdots}}$$

$$= \frac{(4A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5(A-2B)\sin(c+dx\cdots}{3a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{(4A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5(A-2B)\sqrt{\cos(c+\cdots}}{\cdots}$$

Mathematica [C] time = 6.84035, size = 777, normalized size = 3.77

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{8(A-2B)\cos(c)\sin(dx)}{d} - \frac{2\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{3d} - \frac{2(A-B)\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{4\sec\left(\frac{c}{2}\right)\cdots}{\cdots}\right.}{(a\cos\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] (-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) + (7
*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
```

```
(2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) - (10*A
*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (20*B*C
os[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (Cos[c/2
+ (d*x)/2]^4*Sqrt[Sec[c + d*x]]*((-2*(3*A - 5*B + A*Cos[2*c] - 2*B*Cos[2*c]
)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (4*B*Cos[2*d*x]*Sin[2*c])/(3*d) + (4*Sec[
c/2]*Sec[c/2 + (d*x)/2]*(7*A*Sin[(d*x)/2] - 10*B*Sin[(d*x)/2]))/(3*d) - (2*
Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (8
*(A - 2*B)*Cos[c]*Sin[d*x])/d + (4*B*Cos[2*c]*Sin[2*d*x])/(3*d) + (4*(7*A -
 10*B)*Tan[c/2])/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/
(a + a*Cos[c + d*x])^2
```

Maple [A] time = 3.851, size = 435, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2*
d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*co
s(1/2*d*x+1/2*c)^6-20*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-42*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+4
8*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a
^2)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

3.487 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$-\frac{(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{6a^3d}$$

```
[Out] -((49*A - 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(10*a^3*d) - ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Se
c[c + d*x])^3) - ((8*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a +
 a*Sec[c + d*x])^2) - ((13*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.612115, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((49*A - 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(10*a^3*d) - ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Se
c[c + d*x])^3) - ((8*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a +
 a*Sec[c + d*x])^2) - ((13*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int\frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{1}{2}a(11A-B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\int$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-(1$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-(1$$

$$=\frac{(49A-9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d}-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-(8$$

$$=-\frac{(13A-3B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}+\frac{(49A-9B)\sqrt{\sec(}}{10}$$

$$=-\frac{(49A-9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(}}{}$$

Mathematica [C] time = 5.39708, size = 358, normalized size = 1.37

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-i(49A-9B)e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(49*A - 9*B)*(1 + E^(I*(c + d*
x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 160*(13*A - 3*B)*Cos[(c + d*x)/2]^5
```

```
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c
+ d*x)/2]) + (2*I)*(642*A - 102*B + (1082*A - 207*B)*Cos[c + d*x] + 6*(87*A
 - 17*B)*Cos[2*(c + d*x)] + 106*A*Cos[3*(c + d*x)] - 21*B*Cos[3*(c + d*x)]
+ (161*I)*A*Sin[c + d*x] - (6*I)*B*Sin[c + d*x] + (148*I)*A*Sin[2*(c + d*x)
] - (18*I)*B*Sin[2*(c + d*x)] + (41*I)*A*Sin[3*(c + d*x)] - (6*I)*B*Sin[3*(
c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(120*a^3*d*E^(I*d*
x)*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 4.529, size = 685, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x
+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-9*B)*
sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*
d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a^3*cos(d*x + c)^3 + 3*a^
3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

3.488 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=222

$$-\frac{(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{10}$$

```
[Out] ((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - ((6*A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((9*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.581253, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4019, 3787, 3771, 2639, 2641}

$$-\frac{(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,x]
```



```
[Out] ((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - ((6*A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((9*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(9A+B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,d}{5a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(9A}{}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(9A}{}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(9A}{}$$

$$= \frac{(9A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A+B)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 6.88651, size = 793, normalized size = 3.57

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{5d} + \frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3A\sin\left(\frac{dx}{2}\right)+2B\sin\left(\frac{dx}{2}\right)\right)}{15d} + \frac{2(A-B)\tan\left(\frac{c}{2}\right)\mathrm{se}}{5d}\right.}{(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) - (S
qrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
```

```
*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (2*B*Cos[c/
2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/
2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*
x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(9*A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d)
 + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d
) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*
d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(3*A*Sin[(d*x)/2] + 2*B*Sin[(d*x)/2])
)/(15*d) + (4*(3*A + B)*Tan[c/2])/(3*d) + (4*(3*A + 2*B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
+ a*Cos[c + d*x])^3
```

Maple [A] time = 3.971, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+
24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^
3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

3.489 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=216

$$\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{10a^3d}$$

```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
10*a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a +
a*Sec[c + d*x])^3) - ((4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
 + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3
 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.566247, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
10*a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a +
a*Sec[c + d*x])^3) - ((4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
 + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3
 + a^3*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-B)+\frac{1}{2}a(7A+3B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \int$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+B)\sqrt{\cos(c+dx)}F}{}$$

Mathematica [C] time = 6.86473, size = 792, normalized size = 3.67

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{5d} + \frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2A\sin\left(\frac{dx}{2}\right)-7B\sin\left(\frac{dx}{2}\right)\right)}{15d} - \frac{2(A-B)\tan\left(\frac{c}{2}\right)}{5d}\right.}{(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] -(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (Sq
rt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
+ E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
```

```
I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*Co
s[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (2*B*Cos[c
/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c
/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d
*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(A - B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d)
+ (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] - 7*B*Sin[(d*x)/2]))/(
15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))
/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/
(3*d) + (4*(A + B)*Tan[c/2])/(3*d) + (4*(2*A - 7*B)*Sec[c/2 + (d*x)/2]^2*Ta
n[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*C
os[c + d*x])^3
```

Maple [A] time = 4.217, size = 451, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+6*A*
cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*
B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

3.490 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=222

$$\frac{(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{10a^3}$$

```
[Out] -((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Sec[c + d*x])^2) + ((A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.579967, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{10a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Sec[c + d*x])^2) + ((A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec(c+dx)}(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int\frac{\frac{1}{2}a(A-B)+\frac{5}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(2A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+$$

$$=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(2A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+$$

$$=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(2A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+$$

$$=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(2A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+$$

$$=-\frac{(A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(A+3B)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.95649, size = 793, normalized size = 3.57

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{5d}-\frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(7A\sin\left(\frac{dx}{2}\right)-12B\sin\left(\frac{dx}{2}\right)\right)}{15d}+\frac{2(A-B)\tan\left(\frac{c}{2}\right)\mathrm{s}}{5d}\right.}{(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (3*S
qrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
```

```
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*Co
s[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (2*B*Cos[c
/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c
/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*x
)/2]^6*Sqrt[Sec[c + d*x]]*((2*(A + 9*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(d*x)/2] - 12*B*Sin[(d*x)/2]))/(
15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2]))
/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2])
)/(5*d) + (4*(A - 9*B)*Tan[c/2])/(3*d) - (4*(7*A - 12*B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
+ a*Cos[c + d*x])^3
```

Maple [A] time = 3.7, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c
)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

3.491 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)\sqrt{\cos(c+d}}{}$$

```
[Out] -((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.576693, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4020, 3787, 3771, 2639, 2641}

$$\frac{(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```


```
[Out] -((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$$

$$=\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int \frac{-\frac{1}{2}a(A-11B)+\frac{5}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$=\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(3A-8B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\cdots$$

$$=\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(3A-8B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{(3A\cdots}{\cdots}$$

$$=\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(3A-8B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{(3A\cdots}{\cdots}$$

$$=\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(3A-8B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{(3A\cdots}{\cdots}$$

$$=-\frac{(9A-49B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(3A-13B)\sqrt{\cos(\cdots}}{\cdots}$$

Mathematica [C] time = 7.10992, size = 817, normalized size = 3.58

$$\frac{3\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\cdots\right.}{5d(\cos(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] (3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) - (49
*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
```

```
(2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A
*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) - (26*B*Cos
[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec
[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 +
(d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(-9*A + 39*B + 10*B*Cos[2*c])*Cos[d*x]*C
sc[c/2]*Sec[c/2])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2]
- 23*B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(12*A*Sin[(d
*x)/2] - 17*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*S
in[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*B*Cos[c]*Sin[d*x])/d - (4*(9*A -
 23*B)*Tan[c/2])/(3*d) + (4*(12*A - 17*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(1
5*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x
])^3
```

Maple [A] time = 3.719, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8-130*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2
*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*
x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

3.492 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=259

$$-\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}+\frac{7(7A-17B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{6a^3d}$$

```
[Out] (7*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((13*A - 33*B)*Sin[c + d*x])/(6*a^3*d*S
qrt[Sec[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a*
Sec[c + d*x])^3) + ((A - 2*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^2) + (7*(7*A - 17*B)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]]
*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.623547, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4020, 3787, 3769, 3771, 2641, 2639}

$$-\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}+\frac{7(7A-17B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```


```
[Out] (7*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((13*A - 33*B)*Sin[c + d*x])/(6*a^3*d*S
qrt[Sec[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a*
Sec[c + d*x])^3) + ((A - 2*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^2) + (7*(7*A - 17*B)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]]
*(a^3 + a^3*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(3A-13B)+\frac{7}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} +$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} +$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} +$$

$$= -\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} + \frac{(A}{3ad\sqrt{\sec(}}$$

$$= \frac{7(7A-17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-33B)\sin(c}{6a^3d\sqrt{\sec(c+d}}$$

$$= \frac{7(7A-17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-33B)\sqrt{\cos}}{}$$

Mathematica [C] time = 4.55882, size = 589, normalized size = 2.27

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left((806A-1961B)\cos\left(\frac{1}{2}(c-dx)\right)+(664A-1609B)\cos\left(\frac{1}{2}(3c+dx)\right)+470A\cos\left(\frac{1}{2}(c+3dx)\right)+265A\cos\left(\frac{1}{2}(5c-\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),
x]
```

```
[Out] (Cos[(c + d*x)/2]^6*((-98*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
```

```
^((2*I)*(c + d*x))]]))/E^(I*d*x) + (238*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (((806*A - 1961*B)*Cos[(c -
d*x)/2] + (664*A - 1609*B)*Cos[(3*c + d*x)/2] + 470*A*Cos[(c + 3*d*x)/2] -
1165*B*Cos[(c + 3*d*x)/2] + 265*A*Cos[(5*c + 3*d*x)/2] - 620*B*Cos[(5*c + 3
*d*x)/2] + 117*A*Cos[(3*c + 5*d*x)/2] - 292*B*Cos[(3*c + 5*d*x)/2] + 30*A*C
os[(7*c + 5*d*x)/2] - 65*B*Cos[(7*c + 5*d*x)/2] - 5*B*Cos[(5*c + 7*d*x)/2]
+ 5*B*Cos[(9*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*Sqrt[S
ec[c + d*x]]) - 260*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]] + 660*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 3.979, size = 465, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1/
2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-4
68*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+
1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1
/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B
)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(7/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

3.493 $\int \sqrt{a + a\cos(c+dx)}(A + B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=220

$$\frac{2a(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (32*a*(8*A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c
+ d*x]]) + (16*a*(8*A + 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Cos[c + d*x]]) + (4*a*(8*A + 9*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(10
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Sin[c +
 d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(9/2)*Sin[c +
d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.486495, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2772, 2771}

$$\frac{2a(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```


```
[Out] (32*a*(8*A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c
+ d*x]]) + (16*a*(8*A + 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Cos[c + d*x]]) + (4*a*(8*A + 9*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(10
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Sin[c +
 d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(9/2)*Sin[c +
d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}(A+B}{\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{2aA\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}\left((8A+9B)\sqrt{\cos(c+dx}\right.$$

$$= \frac{2a(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{9}{2}}(c+dx}{9d\sqrt{a+a\cos}}$$

$$= \frac{4a(8A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(8A+9B)\sec}{63d\sqrt{a+}}$$

$$= \frac{16a(8A+9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{4a(8A+9B)\sec}{105d\sqrt{a}}$$

$$= \frac{32a(8A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{16a(8A+9B)\sec}{315d\sqrt{a}}$$

Mathematica [A] time = 0.537771, size = 124, normalized size = 0.56

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(11(8A+9B)\cos(c+dx)+11(8A+9B)\cos(2(c+dx))+16A\cos(3}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(107*A + 81*B + 11*(8*A + 9*B)*Cos[c + d*x] +
 11*(8*A + 9*B)*Cos[2*(c + d*x)] + 16*A*Cos[3*(c + d*x)] + 18*B*Cos[3*(c +
d*x)] + 16*A*Cos[4*(c + d*x)] + 18*B*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*T
an[(c + d*x)/2])/(315*d)
```

Maple [A] time = 0.802, size = 138, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(128A(\cos(dx+c))^4+144B(\cos(dx+c))^4+64A(\cos(dx+c))^3+72B(\cos(dx+c))^3+48A\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(128*A*cos(d*x+c)^4+144*B*cos(d*x+c)^4+64*A*cos(d*
x+c)^3+72*B*cos(d*x+c)^3+48*A*cos(d*x+c)^2+54*B*cos(d*x+c)^2+40*A*cos(d*x+c
)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.98382, size = 890, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

```
[Out] 2/315*(A*(315*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 735*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1302*sqrt(2)*sqrt(a)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1206*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10
7*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(co
s(d*x + c) + 1)^10 + 1)) + 9*B*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x +
c) + 1) - 105*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 154*sqr
t(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 142*sqrt(2)*sqrt(a)*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 67*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 - 9*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.72264, size = 316, normalized size = 1.44

$$\frac{2\left(16\,(8\,A+9\,B)\cos\left(dx+c\right)^{4}+8\,(8\,A+9\,B)\cos\left(dx+c\right)^{3}+6\,(8\,A+9\,B)\cos\left(dx+c\right)^{2}+5\,(8\,A+9\,B)\cos\left(dx+c\right)+35}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(16*(8*A + 9*B)*cos(d*x + c)^4 + 8*(8*A + 9*B)*cos(d*x + c)^3 + 6*(8*
A + 9*B)*cos(d*x + c)^2 + 5*(8*A + 9*B)*cos(d*x + c) + 35*A)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x
 + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(11/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(11/2)
, x)
```

3.494 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{2a(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (16*a*(6*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c
+ d*x]]) + (8*a*(6*A + 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a*(6*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.405595, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2772, 2771}

$$\frac{2a(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```


```
[Out] (16*a*(6*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c
+ d*x]]) + (8*a*(6*A + 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a*(6*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*
Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\,\mathrm{co}}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}\left((6A+7B)\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a(6A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{7}{2}}(c+dx)\,\mathrm{si}}{7d\sqrt{a+a\cos(c}}$$

$$= \frac{8a(6A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sec^{\frac{5}{2}}(c}{35d\sqrt{a+a\,\mathrm{c}}}$$

$$= \frac{16a(6A+7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{8a(6A+7B)\sec^{\frac{3}{2}}}{105d\sqrt{a+a}}$$

Mathematica [A] time = 0.428378, size = 102, normalized size = 0.58

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos(3(c+}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] + 2
*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d*x
)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Maple [A] time = 0.801, size = 116, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(48A(\cos(dx+c))^3+56B(\cos(dx+c))^3+24A(\cos(dx+c))^2+28B(\cos(dx+c))^2+18A\,\mathrm{co}\right.}{105\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+28*B*cos(d*x+c)^2+18*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*(1/
cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.17877, size = 767, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/105*(3*A*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 7*B*(15*sq
rt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sqrt(2)*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 42*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 24*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sqrt
(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.64265, size = 273, normalized size = 1.56

$$\frac{2\left(8\,(6\,A+7\,B)\cos\left(dx+c\right)^{3}+4\,(6\,A+7\,B)\cos\left(dx+c\right)^{2}+3\,(6\,A+7\,B)\cos\left(dx+c\right)+15\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(d\right.}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*A
 + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(
d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
 x)
```

3.495 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=130

$$\frac{2a(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (4*a*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*(4*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*C
os[c + d*x]])
```

Rubi [A] time = 0.334, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2772, 2771}

$$\frac{2a(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (4*a*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*(4*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*C
os[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left((4A+5B)\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin}{5d\sqrt{a+a\cos(c}}$$

$$= \frac{4a(4A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a(4A+5B)\sec^{\frac{3}{2}}(c}{15d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.272196, size = 78, normalized size = 0.6

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx))+7A+5B)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
+ 5*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.759, size = 94, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(8A(\cos(dx+c))^2+10B(\cos(dx+c))^2+4A\cos(dx+c)+5B\cos(dx+c)+3A\right)\cos(dx+}{15d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+4*A*cos(d*x+c)+
5*B*cos(d*x+c)+3*A)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2
)/sin(d*x+c)
```

Maxima [B] time = 1.84246, size = 641, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
```

```
1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)
/(cos(d*x + c) + 1) - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - sqrt(2)*sqrt(a)*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.8969, size = 225, normalized size = 1.73

$$\frac{2\left(2\left(4A+5B\right)\cos\left(dx+c\right)^{2}+\left(4A+5B\right)\cos\left(dx+c\right)+3A\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt
(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
 x)
```

3.496 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*(2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x
]])
```

Rubi [A] time = 0.266136, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2961, 2980, 2771}

$$\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*(2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x
]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
```

```
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}(A+B}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\left((2A+3B)\sqrt{\cos(c+dx}\right.$$

$$= \frac{2a(2A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.182304, size = 57, normalized size = 0.67

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((2A+3B)\cos(c+dx)+A)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Sec[c + d*x]^(
3/2)*Tan[(c + d*x)/2])/(3*d)
```

Maple [A] time = 0.806, size = 70, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))(2A\cos(dx+c)+3B\cos(dx+c)+A)\cos(dx+c)}{3d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right)^{\frac{5}{2}}\sqrt{a(1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)+3*B*cos(d*x+c)+A)*cos(d*x+c)*(1/cos(
d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.75363, size = 513, normalized size = 6.04

$$\frac{2\left(\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{3B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(A*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 1)) + 3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(
2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.98353, size = 177, normalized size = 2.08

$$\frac{2\left((2A+3B)\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((
d*cos(d*x + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

3.497 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.272747, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2774, 216}

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
```

```
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}(A+B}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+a}\right.$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.}{}$$

$$= \frac{2\sqrt{a}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2}{}$$

Mathematica [A] time = 0.213843, size = 86, normalized size = 0.9

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(2A\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*B*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2])
)/d
```

Maple [B] time = 0.777, size = 171, normalized size = 1.8

$$2\,\frac{\cos(dx+c)\left(\left(\cos(dx+c)\right)^{-1}\right)^{3/2}\sqrt{a\,(1+\cos(dx+c))}}{d\,(1+\cos(dx+c))}\left(B\cos(dx+c)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cos}{1+\mathrm{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2/d*(B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+B*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*sin(d*x
+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c)
)
```

Maxima [B] time = 2.3093, size = 1223, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(B*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(s
```

```
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)) + 4*A*(sqrt(2
)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Fricas [A] time = 1.94673, size = 259, normalized size = 2.7

$$-\frac{2\left((B\cos(dx+c)+B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}A\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -2*((B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

3.498 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{a}(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.271111, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2981, 2774, 216}

$$\frac{\sqrt{a}(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```



```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
```

```
b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\,\mathrm{co}}{\sqrt{\cos(c+dx)}}$$

$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\left((2A+B)\sqrt{\cos(c+d\right.$$

$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left((2A+B)\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{\sqrt{a}(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.201082, size = 103, normalized size = 1.05

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(2A+B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(2*A + B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[
c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Maple [A] time = 0.766, size = 168, normalized size = 1.7

$$-\frac{(\cos(dx+c))^2-1}{d(\sin(dx+c))^2}\left(B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+B\arctan\left(\frac{\sin}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.81321, size = 1268, normalized size = 12.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
```

```
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A] time = 1.92259, size = 274, normalized size = 2.8

$$\frac{\sqrt{a\cos(dx+c)+a}B\sqrt{\cos(dx+c)}\sin(dx+c)-((2A+B)\cos(dx+c)+2A+B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A + B)*co
s(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sqrt(sec(c + d*x))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

3.499 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{a}(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*B*Sin[c + d*x])/(2*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(4*A + 3*B)*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.337594, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2981, 2770, 2774, 216}

$$\frac{\sqrt{a}(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*B*Sin[c + d*x])/(2*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(4*A + 3*B)*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\,\mathrm{c}$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left((4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a}(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{}{2d\sqrt{a+}}$$

Mathematica [A] time = 0.377992, size = 120, normalized size = 0.79

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(4A+3B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(4*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[
c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [A] time = 0.858, size = 238, normalized size = 1.6

$$\frac{(-1+\cos(dx+c))^2\cos(dx+c)}{4\,d\,(\sin(dx+c))^4}\left(2\,B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\cos(dx+c)+4\,A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*c
os(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))+3*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [B] time = 3.38504, size = 2499, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arc
```

```
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2
*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d
*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2
*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*
x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

<u>Fricas</u> [A] time = 1.9491, size = 348, normalized size = 2.3

$$-\frac{((4A+3B)\cos(dx+c)+4A+3B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2B\cos(dx+c)^2+(4A+3B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin}{\sqrt{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*cos(d*x + c)^2
+ (4*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(
d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(sec(c + d*x))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

3.500 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{a(6A+5B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)}{8d\sqrt{\sec(c+dx)}\sqrt{}}$$

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*B*Sin[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*(6*A + 5*B)*Sin[c + d*x])/(
12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(6*A + 5*B)*Sin[c +
d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.413737, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2981, 2770, 2774, 216}

$$\frac{a(6A+5B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)}{8d\sqrt{\sec(c+dx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*B*Sin[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*(6*A + 5*B)*Sin[c + d*x])/(
12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(6*A + 5*B)*Sin[c +
d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\,\mathrm{co}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{6}\left((6A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{\sqrt{a}(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{}{3d\sqrt{a+}}$$

Mathematica [A] time = 0.689976, size = 138, normalized size = 0.7

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(6A+5B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Co
s[c + d*x]]*(18*A + 19*B + 2*(6*A + 5*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)
])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [A] time = 0.833, size = 308, normalized size = 1.6

$$-\frac{(-1+\cos(dx+c))^3\cos(dx+c)}{24\,d\,(\sin(dx+c))^6}\left(8\,B\,(\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+12\,A\cos(dx+c)\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(8*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)+12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+
10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+18*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+18*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c))+15*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*
(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/co
s(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [B] time = 3.3366, size = 4024, normalized size = 20.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
```

```
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1)))*A + (4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin
(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
- 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sq
rt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
```

```
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*B)/d
```

Fricas [A] time = 1.97049, size = 397, normalized size = 2.03

$$-\frac{3\,((6\,A+5\,B)\cos(dx+c)+6\,A+5\,B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,B\cos(dx+c)^{3}+2\,(6\,A+5\,B)\cos(dx+c)^{2}+3\,(6\,A+5\,B)\right.}{\sqrt{\cos(dx+c)}}}{24\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((6*A + 5*B)*cos(d*x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*cos(d*x + c)
```

```
^3 + 2*(6*A + 5*B)*cos(d*x + c)^2 + 3*(6*A + 5*B)*cos(d*x + c))*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

3.501 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=275

$$\frac{2a^2(12A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(c+dx}{1155d\sqrt{a\cos(c+dx}}$$

```
[Out] (32*a^2*(168*A + 187*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (16*a^2*(168*A + 187*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(168*A + 187*B)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(168*A + 187*B)
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(12*A + 11*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(1
1*d)
```

Rubi [A] time = 0.723155, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(12A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(c+dx}{1155d\sqrt{a\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

```
[Out] (32*a^2*(168*A + 187*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (16*a^2*(168*A + 187*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(168*A + 187*B)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(168*A + 187*B)
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(12*A + 11*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(1
1*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}(A}{\cos^{\frac{13}{2}}(c+}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{1}{11}($$

$$= \frac{2a^2(12A+11B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a}}{}$$

$$= \frac{2a^2(168A+187B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(12A}{9}$$

$$= \frac{4a^2(168A+187B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(168A}{}$$

$$= \frac{16a^2(168A+187B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \frac{4a^2(168}{}$$

$$= \frac{32a^2(168A+187B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(16}{}$$

Mathematica [A] time = 0.774511, size = 146, normalized size = 0.53

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((6342A+6193B)\cos(c+dx)+13(168A+187B)\cos(2(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(2478*A + 2057*B + (6342*A + 6193*B)*Cos[c +
d*x] + 13*(168*A + 187*B)*Cos[2*(c + d*x)] + 2184*A*Cos[3*(c + d*x)] + 2431
*B*Cos[3*(c + d*x)] + 336*A*Cos[4*(c + d*x)] + 374*B*Cos[4*(c + d*x)] + 336
*A*Cos[5*(c + d*x)] + 374*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*Tan[(c +
```

```
d*x)/2])/(3465*d)
```

Maple [A] time = 0.777, size = 161, normalized size = 0.6

$$-\frac{2\,a\left(-1+\cos\left(dx+c\right)\right)\left(2688\,A\left(\cos\left(dx+c\right)\right)^{5}+2992\,B\left(\cos\left(dx+c\right)\right)^{5}+1344\,A\left(\cos\left(dx+c\right)\right)^{4}+1496\,B\left(\cos\left(dx+c\right)\right)^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(2688*A*cos(d*x+c)^5+2992*B*cos(d*x+c)^5+1344*A
*cos(d*x+c)^4+1496*B*cos(d*x+c)^4+1008*A*cos(d*x+c)^3+1122*B*cos(d*x+c)^3+8
40*A*cos(d*x+c)^2+935*B*cos(d*x+c)^2+735*A*cos(d*x+c)+385*B*cos(d*x+c)+315*
A)*cos(d*x+c)*(1/cos(d*x+c))^(13/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.3038, size = 961, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 4/3465*(21*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 495*sqrt(
2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1056*sqrt(2)*a^(3/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
299*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 46*sqrt(2)*a^(3
/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + 1)) + 11*(315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 11
55*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2184*sqrt(2)*a^(3/
2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2586*sqrt(2)*a^(3/2)*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 1759*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
```

```
+ 1)^9 - 611*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 94*sqr
t(2)*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*B*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.48124, size = 402, normalized size = 1.46

$$\frac{2\left(16\,(168\,A+187\,B)a\cos\left(dx+c\right)^{5}+8\,(168\,A+187\,B)a\cos\left(dx+c\right)^{4}+6\,(168\,A+187\,B)a\cos\left(dx+c\right)^{3}+5\,(168\,A+\right.}{3465\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(16*(168*A + 187*B)*a*cos(d*x + c)^5 + 8*(168*A + 187*B)*a*cos(d*x +
 c)^4 + 6*(168*A + 187*B)*a*cos(d*x + c)^3 + 5*(168*A + 187*B)*a*cos(d*x +
c)^2 + 35*(21*A + 11*B)*a*cos(d*x + c) + 315*A*a)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(13/
2), x)
```

3.502 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(10A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+dx)\sec}{315d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (16*a^2*(34*A + 39*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Co
s[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(34*A + 39*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 9*B)*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.649973, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(10A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+dx)\sec}{315d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```


```
[Out] (16*a^2*(34*A + 39*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Co
s[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(34*A + 39*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 9*B)*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}(A}{\cos^{\frac{11}{2}}(c+}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\right.$$

$$= \frac{2a^2(10A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\,c}}{}$$

$$= \frac{2a^2(34A+39B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(10A+}{63d}$$

$$= \frac{8a^2(34A+39B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+}{105}$$

$$= \frac{16a^2(34A+39B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{8a^2(34A+}{31}$$

Mathematica [A] time = 0.69188, size = 124, normalized size = 0.54

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((374A+324B)\cos(c+dx)+11(34A+39B)\cos(2(c+dx))+68A}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x]
 + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3*(
c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/
2)*Tan[(c + d*x)/2])/(315*d)
```

Maple [A] time = 0.684, size = 139, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(272A(\cos(dx+c))^4+312B(\cos(dx+c))^4+136A(\cos(dx+c))^3+156B(\cos(dx+c))^3+\right.}{315d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+136*A*cos
(d*x+c)^3+156*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+85*A*cos
(d*x+c)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.15245, size = 836, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 4/315*((315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 840*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)*a^(3/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 94*s
qrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 3*(105*sqrt(2)*a^(
3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 350*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 518*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 444*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 209*sqrt(2)*
a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 38*sqrt(2)*a^(3/2)*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/
((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.4335, size = 338, normalized size = 1.48

$$\frac{2\left(8\,(34\,A + 39\,B)a\cos\left(dx + c\right)^{4} + 4\,(34\,A + 39\,B)a\cos\left(dx + c\right)^{3} + 3\,(34\,A + 39\,B)a\cos\left(dx + c\right)^{2} + 5\,(17\,A + 9\,B)a\cos\left(\right.\right.}{315\left(d\cos\left(dx + c\right)^{5} + d\cos\left(dx + c\right)^{4}\right)\sqrt{\cos\left(dx + c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
+ 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A*a
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)
^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11/
2), x)
```

3.503 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)\sqrt{\sec(c}}{105d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (4*a^2*(52*A + 63*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.560834, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(8A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)\sqrt{\sec(c}}{105d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```


```
[Out] (4*a^2*(52*A + 63*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}(A+}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{c}\right.$$

$$= \frac{2a^2(8A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(}{}$$

$$= \frac{2a^2(52A+63B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+7B)}{35d\sqrt{a}}$$

$$= \frac{4a^2(52A+63B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(52A+63}{105d}$$

Mathematica [A] time = 0.544828, size = 102, normalized size = 0.56

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(3(78A+77B)\cos(c+dx)+(52A+63B)\cos(2(c+dx))+52A\cos(3}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x] +
 (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c + d
*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Maple [A] time = 0.652, size = 117, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(104A(\cos(dx+c))^3+126B(\cos(dx+c))^3+52A(\cos(dx+c))^2+63B(\cos(dx+c))^2+39A\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+52*A*cos(
d*x+c)^2+63*B*cos(d*x+c)^2+39*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)
*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.50775, size = 711, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 4/105*((105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)) + 21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c
) + 1) - 15*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2
)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sqrt(2)*a^(3/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.46307, size = 288, normalized size = 1.59

$$\frac{2\left(2\,(52\,A+63\,B)a\cos\left(dx+c\right)^{3}+(52\,A+63\,B)a\cos\left(dx+c\right)^{2}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)+15\,Aa\right)\sqrt{a\cos\left(dx+c\right)}}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2
), x)
```

3.504 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}$$

```
[Out] (2*a^2*(18*A + 25*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^2*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*
Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.468983, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2975, 2980, 2771}

$$\frac{2a^2(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*a^2*(18*A + 25*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^2*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*
Sin[c + d*x])/(5*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}(A+}{\cos^{\frac{7}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{c}\right.$$

$$= \frac{2a^2(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(}}{}$$

$$= \frac{2a^2(18A+25B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(6A+5B)}{15d\sqrt{a}}$$

Mathematica [A] time = 0.33013, size = 80, normalized size = 0.6

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(2(9A+5B)\cos(c+dx)+(18A+25B)\cos(2(c+dx))+24A+25B}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] + (
18*A + 25*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.569, size = 95, normalized size = 0.7

$$-\frac{2\,a\left(-1+\cos\left(dx+c\right)\right)\left(18\,A\left(\cos\left(dx+c\right)\right)^{2}+25\,B\left(\cos\left(dx+c\right)\right)^{2}+9\,A\cos\left(dx+c\right)+5\,B\cos\left(dx+c\right)+3\,A\right)\cos\left(dx\right.}{15\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(18*A*cos(d*x+c)^2+25*B*cos(d*x+c)^2+9*A*cos(d*x+
c)+5*B*cos(d*x+c)+3*A)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(
1/2)/sin(d*x+c)
```

Maxima [B] time = 1.99292, size = 589, normalized size = 4.4

$$\frac{4\left(\frac{3\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}+\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(d}{(\cos(dx+c)+}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\right.}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 4/15*(3*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(
3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) +
5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sqrt(2)*a^(3/2)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*B*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.4596, size = 234, normalized size = 1.75

$$\frac{2\left((18\,A+25\,B)a\cos\left(dx+c\right)^{2}+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2
)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2
), x)
```

3.505 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(}{}$$

```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.450702, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2774, 216}

$$\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}(A+}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{c}\right.$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(}}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(}}{}$$

$$= \frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2}{}$$

Mathematica [A] time = 0.394888, size = 106, normalized size = 0.73

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((5A+3B)\cos(c+dx)+A)+3\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt[2
]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3*B
)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] time = 0.666, size = 287, normalized size = 2.

$$-\frac{2\cos(dx+c)\,a\,(\sin(dx+c))^2}{3\,d\,(-1+\cos(dx+c))\,(1+\cos(dx+c))^2}\left(3\,B\,(\cos(dx+c))^2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*a*(3*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6*B*cos(d*x+c)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)+3*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+5*A*cos(d*x+c)*sin(d*x+c)+3*B*sin(d
*x+c)*cos(d*x+c)+A*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2)
*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [B] time = 2.87032, size = 1974, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(3*(6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*a^(3/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
((2*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
 - a*sin(2*d*x + 2*c) - 2*(a*cos(2*d*x + 2*c) + a)*sin(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + (2*a*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + a*cos(2*d*x + 2*c) + 2*(a*cos(2*d*x + 2*c) + a)*cos(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a)*sin(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + ((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*
x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a*cos(2*d*x + 2*c)^2 + a*
sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
```

```
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a*cos(2*d*x + 2*c
)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) + (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2
 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
1))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1) + 8*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [A] time = 1.61521, size = 355, normalized size = 2.45

$$-\frac{2\left(3\left(Ba\cos\left(dx+c\right)^{2}+Ba\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{((5A+3B)a\cos(dx+c)+Aa)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -2/3*(3*(B*a*cos(d*x + c)^2 + B*a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((5*A + 3*B)*a*co
s(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

3.506 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{\mathrm{se}}}{}$$

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(2*A - B)*Sin[c + d*x])/(
d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.465838, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2981, 2774, 216}

$$\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{\mathrm{se}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```


```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(2*A - B)*Sin[c + d*x])/(
d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}(A+}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{\cos}\right.$$

$$= -\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+}}{}$$

$$= -\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+}}{}$$

$$= \frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

Mathematica [A] time = 0.32242, size = 107, normalized size = 0.73

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(2A+3B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
(2*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A +
B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 0.655, size = 308, normalized size = 2.1

$$\frac{\cos(dx+c)\,a}{d\,(1+\cos(dx+c))}\left(2\,A\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+3\,B\cos(dx+c)\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d*a*(2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*cos(d*x+c)*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*sin(d*x+c)*cos(d*x+c)+3*B*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)
))^(1/2)/(1+cos(d*x+c))
```

Maxima [B] time = 2.97994, size = 2431, normalized size = 16.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
```

```
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A] time = 1.85919, size = 323, normalized size = 2.21

$$-\frac{((2A+3B)a\cos(dx+c)+(2A+3B)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(Ba\cos(dx+c)+2Aa)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -((((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (B*a*cos(d*x + c)
```

```
+ 2*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

3.507 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\cdots}}{2d\sqrt{\sec\cdots}}$$

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(4*A + 5*B)*Sin[c +
d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*B*Sqrt[a + a*C
os[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.455454, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\cdots}}{2d\sqrt{\sec\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(4*A + 5*B)*Sin[c +
d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*B*Sqrt[a + a*C
os[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}(A+}{\sqrt{\cos(c+dx}}$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{aB\sqrt{a+a\cos(c+dx}}{2d\sqrt{\sec(c+}}$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{aB\sqrt{a+a\cos(c+dx}}{2d\sqrt{\sec(c+}}$$

$$= \frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{4d}$$

Mathematica [A] time = 0.441364, size = 121, normalized size = 0.79

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(12A+7B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(Sqrt[2]*(12*A + 7*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[C
os[c + d*x]]*(4*A + 7*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [A] time = 0.696, size = 233, normalized size = 1.5

$$-\frac{a\left((\cos(dx+c))^2-1\right)}{4d(\sin(dx+c))^2}\left(2B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\cos(dx+c)+4A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+7B\sin(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*a*(2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+4*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+12*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))+7*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.7714, size = 2543, normalized size = 16.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x
```

```
 + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*B)/d
```

Fricas [A] time = 1.96677, size = 365, normalized size = 2.39

$$-\frac{((12A+7B)a\cos(dx+c)+(12A+7B)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(2Ba\cos(dx+c)^2+(4A+7B)a\cos(dx+c))\sqrt{a\cos(\cdots}}{\sqrt{\cos(dx+c)}}}{4(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*sqrt(a)*arctan(sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*a*cos(d
```

```
*x + c)^2 + (4*A + 7*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

3.508 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{a^2(6A+7B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(14A+1}{8d\sqrt{\sec(c+dx}}$$

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(6*A + 7*B)*Sin[c +
 d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*B*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a^2*(14*A + 11*B)*
Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.54217, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(6A+7B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(14A+1}{8d\sqrt{\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(6*A + 7*B)*Sin[c +
 d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*B*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a^2*(14*A + 11*B)*
Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+d}{3d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+d}{3d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+d}{3d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{}{12a}$$

Mathematica [A] time = 0.504107, size = 141, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(14A+11B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(\right.\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2
]*(14*A + 11*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (42*A
 + 37*B + 2*(6*A + 11*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*(-Sin[(c + d*
x)/2] + Sin[(3*(c + d*x))/2])))/(48*d)
```

Maple [A] time = 0.658, size = 309, normalized size = 1.5

$$\frac{a(-1+\cos(dx+c))^2\cos(dx+c)}{24d(\sin(dx+c))^4}\left(8B(\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+12A\cos(dx+c)\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d*a*(-1+cos(d*x+c))^2*(8*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
+22*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+42*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)+42*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))+33*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))
*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/c
os(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [B] time = 4.38399, size = 4081, normalized size = 20.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(
```

```
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*(a*cos(3/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(
3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) -
 (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
```

```
4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))) - 1) - a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))*B)/d
```

Fricas [A] time = 1.8698, size = 423, normalized size = 2.12

$$-\frac{3\left(\left(14\,A+11\,B\right)a\cos\left(dx+c\right)+\left(14\,A+11\,B\right)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,Ba\cos(dx+c)^{3}+2\left(6\,A+11\,B\right)a\cos(dx+c)\right.}{}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((14*A + 11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*a*
cos(d*x + c)^3 + 2*(6*A + 11*B)*a*cos(d*x + c)^2 + 3*(14*A + 11*B)*a*cos(d*
x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

3.509 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^2(88A+75B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+9B)\sin(c+dx)}{24d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(88A+75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{64d}$$

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(8*A + 9*B)*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*B*Sqrt[a +
a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a^2*(88*A + 75*B)
*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8
8*A + 75*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```

Rubi [A] time = 0.643638, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(88A+75B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+9B)\sin(c+dx)}{24d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(88A+75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(8*A + 9*B)*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*B*Sqrt[a +
a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a^2*(88*A + 75*B)
*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8
8*A + 75*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{}{2}$$

Mathematica [A] time = 0.773213, size = 158, normalized size = 0.64

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(88A+75B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2
]*(88*A + 75*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (296*
A + 285*B + 2*(88*A + 93*B)*Cos[c + d*x] + 4*(8*A + 15*B)*Cos[2*(c + d*x)]
```

```
+ 12*B*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(384*
d)
```

Maple [A] time = 0.549, size = 381, normalized size = 1.5

$$-\frac{a\left(-1+\cos\left(dx+c\right)\right)^{3}\cos\left(dx+c\right)}{192\,d\left(\sin\left(dx+c\right)\right)^{6}}\left(48\,B\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+64\,A\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d*a*(-1+cos(d*x+c))^3*(48*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+64*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)+120*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+176*A
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+150*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+264*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+225*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+264*
A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+225*B*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a
*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/
2)/sin(d*x+c)^6
```

Maxima [B] time = 6.19524, size = 12016, normalized size = 48.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arcta
```

n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
c))) + 1)^(1/4)((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(si
n(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1)
 - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arc
tan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s

in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(
a))*A + 3*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((5*a*cos(4*d*x + 4*c)^2*sin(4*
d*x + 4*c) + 5*a*sin(4*d*x + 4*c)^3 + 20*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d
*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 20*(a*sin(4*d*x + 4*c)^3 + (a*cos(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 5*(2*a*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c) - 2*(a*cos
(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*co
s(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 20*(a*sin(4*d*x + 4*c)
^3 + (a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*a*cos(4*d*x + 4*c)^2 + 32
*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c
)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c
) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 5*a*cos(4*d
*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 21*a*cos
(4*d*x + 4*c) + 5*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4
*c) + 21*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 20*(4*a*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a*sin
(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(
3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (5*a*cos(4*d*x + 4*c)^3
 - 8*a*cos(4*d*x + 4*c)^2 + 4*(5*a*cos(4*d*x + 4*c)^3 - 18*a*cos(4*d*x + 4*
c)^2 + (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 21*a*cos(4*d*x + 4
*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (5*a*co
s(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 4*(5*a*cos(4*d*x + 4*c)^3 + 2*a*
cos(4*d*x + 4*c)^2 + (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 - 11*a
*cos(4*d*x + 4*c) - 8*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + (8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4
*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d

```
*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 - 5*a*cos(4*d*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 +
 16*a*sin(4*d*x + 4*c)^2 - 21*a*cos(4*d*x + 4*c) + 5*a)*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 21*a*sin(4*d*x + 4*c))*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 4*(5*a*cos(4*d*x + 4*c)^3 - 13*a*cos(4*d*x + 4*c)^2
+ (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c))*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*(2*a*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c
) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(5*a*
cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)*sin(4*d*x + 4*c) + (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((3*a
*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a*sin(4*d*x + 4*c)^3 - 64*(a*cos(4
*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 12*(a*sin(4*d*x + 4*c)^3 + (
a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c) - 24*(a*c
os(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(1/
4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 3*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))*sin(4*d*x + 4*c) + 4*(3*a*sin(4*d*x + 4*c)^3 + 64*a*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a*cos(4*d*x
 + 4*c)^2 + 6*a*cos(4*d*x + 4*c) + 19*a)*sin(4*d*x + 4*c) - 72*(a*cos(4*d*x
 + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 6*(2*a*sin(4*d*x + 4*c)^3 + a*cos(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 2*(a*cos(4*d*x + 4*c)^2 -
a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (48*a*cos(4*d*x + 4*c)^2 + 48*a*sin(
4*d*x + 4*c)^2 - 47*a*cos(4*d*x + 4*c) - a)*sin(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - 2*(8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c
)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 14*a*sin(4*d*x + 4*c)^2 - 141*a*sin(4*d*x + 4*c)*sin(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*(4*a*cos(4*d*x + 4*c)^2 + 7*a*
sin(4*d*x + 4*c)^2 - 72*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - 4*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 3*(a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
```

+ 4*c))) - 3*(24*a*cos(4*d*x + 4*c)^2 + 24*a*sin(4*d*x + 4*c)^2 + a*cos(4*d
*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a*cos(4*d*x + 4*c)^3 - 64*
(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 56*a*cos(4*d*x + 4*c
)^2 + 4*(3*a*cos(4*d*x + 4*c)^3 + 34*a*cos(4*d*x + 4*c)^2 + (3*a*cos(4*d*x
+ 4*c) + 40*a)*sin(4*d*x + 4*c)^2 - 93*a*cos(4*d*x + 4*c) - 40*(a*cos(4*d*x
 + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 56*a)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + (3*a*cos(4*d*x + 4*c) + 56*a)*sin(4*d*x + 4*c)
^2 + 4*(3*a*cos(4*d*x + 4*c)^3 + 62*a*cos(4*d*x + 4*c)^2 + (3*a*cos(4*d*x +
 4*c) + 56*a)*sin(4*d*x + 4*c)^2 + 115*a*cos(4*d*x + 4*c) - 16*(a*cos(4*d*x
 + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 40*(a*cos(4*d*x + 4*c)^2 + a*sin(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 56*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 - 3*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 2*(6*a*cos(4*d*x + 4*c)^3 + 98*a*cos(4*d*x + 4*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 52*a)*sin(4*d*x + 4*c)^2 - 3*a*sin(4*d*x + 4*c)*sin(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 112*a*cos(4*d*x + 4*c) - (80*a*
cos(4*d*x + 4*c)^2 + 80*a*sin(4*d*x + 4*c)^2 - 77*a*cos(4*d*x + 4*c) - 3*a)
*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) - (40*a*cos(4*d*x + 4*c)^2 + 40*a*sin(4*d*
x + 4*c)^2 + 3*a*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 2*(128*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2*sin(4*d*x + 4*c) + 77*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(4*d*x + 4*c) + 8*(40*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))*sin(4*d*x + 4*c) - (3*a*cos(4*d*x + 4*c) + 52*a)*sin(4*d*x + 4*c
))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(3*a*cos(4*d*x
+ 4*c) + 56*a)*sin(4*d*x + 4*c) + 3*(a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*s
qrt(a) + 75*((a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x
+ 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*
cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*
(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
+ a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(s

in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a*cos(4*
d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d
*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*s
in(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*co
s(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*s
in(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a*cos(4*d*x + 4*c)^2 + 4*(a*co
s(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*
(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c
)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(

```
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
, cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (a*cos(
4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4
*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a
*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*
cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)),
(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*
cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 +
 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 2.33828, size = 473, normalized size = 1.91

$$-\frac{3\left((88\,A+75\,B)a\cos\left(dx+c\right)+(88\,A+75\,B)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,Ba\cos(dx+c)^{4}+8\,(8\,A+15\,B)a\cos(dx+c)\right.}{}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/192*(3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*B)*a)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*B*
a*cos(d*x + c)^4 + 8*(8*A + 15*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(
d*x + c)^2 + 3*(88*A + 75*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

3.510 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx$

Optimal. Leaf size=322

$$\frac{2a^2(16A+13B)\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{143d}+\frac{2a^3(280A+299B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{1287d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(4184A$$

```
[Out] (32*a^3*(4184*A + 4615*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a
+ a*Cos[c + d*x]]) + (16*a^3*(4184*A + 4615*B)*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(4184*A + 4615*B)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(4184*
A + 4615*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Cos[c + d*x
]]) + (2*a^3*(280*A + 299*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(1287*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*a^2*(16*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c
+ d*x]^(11/2)*Sin[c + d*x])/(143*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec
[c + d*x]^(13/2)*Sin[c + d*x])/(13*d)
```

Rubi [A] time = 0.938889, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(16A+13B)\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{143d}+\frac{2a^3(280A+299B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{1287d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(4184A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15/2),x]
```

```
[Out] (32*a^3*(4184*A + 4615*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a
+ a*Cos[c + d*x]]) + (16*a^3*(4184*A + 4615*B)*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(4184*A + 4615*B)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(4184*
A + 4615*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Cos[c + d*x
]]) + (2*a^3*(280*A + 299*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(1287*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*a^2*(16*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c
+ d*x]^(11/2)*Sin[c + d*x])/(143*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec
[c + d*x]^(13/2)*Sin[c + d*x])/(13*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
```

```
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}(A+}{\cos^{\frac{15}{2}}(c+d}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{13}{2}}(c+dx)\sin(c+dx)}{13d} + \frac{1}{13}$$

$$= \frac{2a^2(16A+13B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{11}{2}}(c+dx)\sin(c+}{143d}$$

$$= \frac{2a^3(280A+299B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{1287d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(16A+}{}$$

$$= \frac{2a^3(4184A+4615B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(280}{}$$

$$= \frac{4a^3(4184A+4615B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15015d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(418}{}$$

$$= \frac{16a^3(4184A+4615B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}} + \frac{4a^3(418}{}$$

$$= \frac{32a^3(4184A+4615B)\sqrt{\sec(c+dx)}\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}} + \frac{16a^3(41}{}$$

Mathematica [A] time = 0.890915, size = 171, normalized size = 0.53

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(35(5552A+5083B)\cos(c+dx)+14(15167A+15925B)\cos(2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(171806*A + 162955*B + 35*(5552*A + 5083*B)
*Cos[c + d*x] + 14*(15167*A + 15925*B)*Cos[2*(c + d*x)] + 62760*A*Cos[3*(c
+ d*x)] + 69225*B*Cos[3*(c + d*x)] + 62760*A*Cos[4*(c + d*x)] + 69225*B*Cos
[4*(c + d*x)] + 8368*A*Cos[5*(c + d*x)] + 9230*B*Cos[5*(c + d*x)] + 8368*A*
Cos[6*(c + d*x)] + 9230*B*Cos[6*(c + d*x)])*Sec[c + d*x]^(13/2)*Tan[(c + d*
x)/2])/(90090*d)
```

Maple [A] time = 0.651, size = 185, normalized size = 0.6

$$-\frac{2\,a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(66944\,A\left(\cos\left(dx+c\right)\right)^{6}+73840\,B\left(\cos\left(dx+c\right)\right)^{6}+33472\,A\left(\cos\left(dx+c\right)\right)^{5}+36920\,B\left(\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(66944*A*cos(d*x+c)^6+73840*B*cos(d*x+c)^6+3
3472*A*cos(d*x+c)^5+36920*B*cos(d*x+c)^5+25104*A*cos(d*x+c)^4+27690*B*cos(d
*x+c)^4+20920*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+18305*A*cos(d*x+c)^2+1456
0*B*cos(d*x+c)^2+11970*A*cos(d*x+c)+4095*B*cos(d*x+c)+3465*A)*cos(d*x+c)*(1
/cos(d*x+c))^(15/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.19606, size = 1030, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="maxima")
```

```
[Out] 8/45045*((45045*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 165165*sq
rt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*sqrt(2)*a^(5/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 289185*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 88
980*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11864*sqrt(2)*a
^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x +
```

```
c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c
) + 1)^10 + 1)) + 65*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
 3003*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6930*sqrt(2)*a^
(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10098*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 9053*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x +
 c) + 1)^9 - 4875*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 1
500*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 200*sqrt(2)*a^(
5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*B*(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 + 1)))/d
```

Fricas [A] time = 1.51198, size = 487, normalized size = 1.51

$$\frac{2\left(16\,(4184\,A+4615\,B)a^2\cos\left(dx+c\right)^6+8\,(4184\,A+4615\,B)a^2\cos\left(dx+c\right)^5+6\,(4184\,A+4615\,B)a^2\cos\left(dx+c\right)^4+5\,(\right.}{45045\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="fricas")
```

```
[Out] 2/45045*(16*(4184*A + 4615*B)*a^2*cos(d*x + c)^6 + 8*(4184*A + 4615*B)*a^2*
cos(d*x + c)^5 + 6*(4184*A + 4615*B)*a^2*cos(d*x + c)^4 + 5*(4184*A + 4615*
B)*a^2*cos(d*x + c)^3 + 35*(523*A + 416*B)*a^2*cos(d*x + c)^2 + 315*(38*A +
 13*B)*a^2*cos(d*x + c) + 3465*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/((d*cos(d*x + c)^7 + d*cos(d*x + c)^6)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

3.511 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=275

$$\frac{2a^2(14A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}+\frac{2a^3(194A+209B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+\dots}{11\dots}$$

```
[Out] (16*a^3*(710*A + 803*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(194*A + 209*B)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99
*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(
11*d)
```

Rubi [A] time = 0.848269, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(14A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}+\frac{2a^3(194A+209B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+\dots}{11\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```



```
[Out] (16*a^3*(710*A + 803*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(194*A + 209*B)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99
*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(
11*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}(A+}{\cos^{\frac{13}{2}}(c+dx}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{1}{11}($$

$$= \frac{2a^2(14A+11B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+d}{99d}$$

$$= \frac{2a^3(194A+209B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(14A+}{}$$

$$= \frac{2a^3(710A+803B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(194A}{6}$$

$$= \frac{8a^3(710A+803B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A}{11}$$

$$= \frac{16a^3(710A+803B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \frac{8a^3(710A}{3}$$

Mathematica [A] time = 1.27922, size = 147, normalized size = 0.53

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((25070A+24827B)\cos(c+dx)+(9230A+9284B)\cos(2(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos[
c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] + 1
0439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*x)]
 + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*T
```

```
an[(c + d*x)/2])/(6930*d)
```

Maple [A] time = 0.674, size = 163, normalized size = 0.6

$$-\frac{2\,a^2\,(-1+\cos(dx+c))\left(5680\,A\,(\cos(dx+c))^5+6424\,B\,(\cos(dx+c))^5+2840\,A\,(\cos(dx+c))^4+3212\,B\,(\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+2840
*A*cos(d*x+c)^4+3212*B*cos(d*x+c)^4+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3
+1775*A*cos(d*x+c)^2+1430*B*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*x+c)
+315*A)*cos(d*x+c)*(1/cos(d*x+c))^(13/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c
)
```

Maxima [B] time = 2.4238, size = 907, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 8/3465*(5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt(
2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt(2)*a
^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 11*(315*sqrt(2)*a^(5/2)*si
n(d*x + c)/(cos(d*x + c) + 1) - 1260*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 2394*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2736*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1859*sqrt(2)*a^(
```

```
5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 676*sqrt(2)*a^(5/2)*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 + 104*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x +
c) + 1)^13)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*
(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 1)))/d
```

Fricas [A] time = 1.48055, size = 417, normalized size = 1.52

$$\frac{2\left(8\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^5+4\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^4+3\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^3+5\,(355\,A+\right.}{3465\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d*
x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*cos
(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x
 + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

3.512 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(4A+3B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a^3(124A+135B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(292A+3}{315}$$

```
[Out] (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(124*A + 135*B)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.76767, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(4A+3B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a^3(124A+135B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(292A+3}{315}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```



```
[Out] (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(124*A + 135*B)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}(A+}{\cos^{\frac{11}{2}}(c+d}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\Big(2$$

$$= \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{21d}$$

$$= \frac{2a^3(124A+135B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(4A+3}{}$$

$$= \frac{2a^3(292A+345B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(124A+}{3}$$

$$= \frac{4a^3(292A+345B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(292A}{3}$$

Mathematica [A] time = 0.938351, size = 126, normalized size = 0.55

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((1396A+1215B)\cos(c+dx)+2(803A+870B)\cos(2(c+dx))+29}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
+ d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 345*
B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Sec[c
 + d*x]^(9/2)*Tan[(c + d*x)/2])/(630*d)
```

Maple [A] time = 0.654, size = 141, normalized size = 0.6

$$-\frac{2a^2(-1+\cos(dx+c))\left(584A(\cos(dx+c))^4+690B(\cos(dx+c))^4+292A(\cos(dx+c))^3+345B(\cos(dx+c))^3+2\right.}{315d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+292*A*c
os(d*x+c)^3+345*B*cos(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+130*A*
cos(d*x+c)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos
(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.3983, size = 782, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 8/315*((315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*
sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 1)) + 15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
77*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 119*sqrt(2)*a^(5/2
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 44*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.39322, size = 354, normalized size = 1.55

$$\frac{2\left(2\,(292\,A+345\,B)a^2\cos\left(dx+c\right)^4+(292\,A+345\,B)a^2\cos\left(dx+c\right)^3+3\,(73\,A+60\,B)a^2\cos\left(dx+c\right)^2+5\,(26\,A+9\,B \cdots\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x + c
) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*
cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

3.513 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(10A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{2a^3(10A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+3}{105}$$

```
[Out] (2*a^3*(230*A + 301*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(10*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.676108, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2975, 2980, 2771}

$$\frac{2a^2(10A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{2a^3(10A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+3}{105}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*a^3*(230*A + 301*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(10*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{5/2}(A + B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}(A}{\cos^{\frac{9}{2}}(c+d}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}($$

$$= \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+d}{35d}$$

$$= \frac{2a^3(10A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(10A+7}{}$$

$$= \frac{2a^3(230A+301B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(10A+}{1}$$

Mathematica [A] time = 0.700292, size = 104, normalized size = 0.57

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((930A+987B)\cos(c+dx)+2(115A+98B)\cos(2(c+dx))+230}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d*
x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*Cos
[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.655, size = 119, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(230A(\cos(dx+c))^3+301B(\cos(dx+c))^3+115A(\cos(dx+c))^2+98B(\cos(dx+c))^2+}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(230*A*cos(d*x+c)^3+301*B*cos(d*x+c)^3+115*A*c
os(d*x+c)^2+98*B*cos(d*x+c)^2+60*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x
+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.33033, size = 659, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 8/105*(5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 7*(15*sqrt(2)*a
^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 50*sqrt(2)*a^(5/2)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(
5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.44344, size = 300, normalized size = 1.66

$$\frac{2\left((230\,A+301\,B)a^2\cos\left(dx+c\right)^3+(115\,A+98\,B)a^2\cos\left(dx+c\right)^2+3\,(20\,A+7\,B)a^2\cos\left(dx+c\right)+15\,Aa^2\right)\sqrt{a\cos\left(dx\right.}}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c)
^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
```

```
in(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")

[Out] Timed out
```

3.514 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=192

$$\frac{2a^2(8A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}B\sqrt{\cos(c+}}{}$$

```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 35*B)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + a*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.622924, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2774, 216}

$$\frac{2a^2(8A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}B\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 35*B)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + a*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{5/2}(A + B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a + a\cos(c+dx))^{5/2}(A +}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA(a + a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\right.$$

$$= \frac{2a^2(8A+5B)\sqrt{a + a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a + a\cos(c+dx)}} + \frac{2a^2(8A+5B)}{}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a + a\cos(c+dx)}} + \frac{2a^2(8A+5B)}{}$$

$$= \frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + $$

Mathematica [A] time = 0.814521, size = 130, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(14A+5B)\cos(c+dx) + (43A+40B)\cos(2(c+\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(30*Sqr
t[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40*B
 + 2*(14*A + 5*B)*Cos[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c + d
*x)/2]))/(30*d)
```

Maple [B] time = 0.724, size = 389, normalized size = 2.

$$\frac{2a^2\cos(dx+c)(\sin(dx+c))^4}{15d(-1+\cos(dx+c))^2(1+\cos(dx+c))^3}\left(15B(\cos(dx+c))^3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\left(\frac{\cos(dx+c}{1+\cos(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] 2/15/d*a^2*(15*B*cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+45*B*cos(d*x+c)^2*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)+45*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*B*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+
c)))^(5/2)+43*A*sin(d*x+c)*cos(d*x+c)^2+40*B*sin(d*x+c)*cos(d*x+c)^2+14*A*c
os(d*x+c)*sin(d*x+c)+5*B*sin(d*x+c)*cos(d*x+c)+3*A*sin(d*x+c))*cos(d*x+c)*(
1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4/(-1+cos(d*x+c))^2
/(1+cos(d*x+c))^3
```

Maxima [B] time = 3.28966, size = 2313, normalized size = 12.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/30*(5*(10*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)*a^(5/2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c
) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2
*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
+ 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*c
os(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt
(a) + 2*((3*a^2*sin(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*sin(4*
d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c) + 4*a^2
)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos
(2*d*x + 2*c) + 4*a^2 + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c)
+ 4*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(3*a^2*si
n(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 15*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x +
 2*c) + a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt
(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
5/4) + 16*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(7/2)))/d
```

Fricas [A] time = 1.6166, size = 425, normalized size = 2.21

$$-\frac{2\left(15\left(Ba^2\cos(dx+c)^3+Ba^2\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left((43A+40B)a^2\cos(dx+c)^2+(14A+5B)a^2\cos(dx\right.}{\sqrt{\cos(dx+c}}\right.}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] -2/15*(15*(B*a^2*cos(d*x + c)^3 + B*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((43*A +
40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + 3*A*a^2)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*co
s(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.515 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=193

$$\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A+B)\sin(c$$

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(14*A + 3*B)*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt
[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*(a + a*Cos
[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.654143, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2981, 2774, 216}

$$\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A+B)\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```



```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(14*A + 3*B)*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt
[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*(a + a*Cos
[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}(A+}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\right.$$

$$= \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} +$$

$$= -\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+}}{}$$

$$= -\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+}}{}$$

$$= \frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

Mathematica [A] time = 0.733592, size = 130, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(2A+5B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+\sin\left(\frac{1}{2}(c+\right.\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt
[2]*(2*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A
+ 3*B + 4*(8*A + 3*B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]
))/(6*d)
```

Maple [B] time = 0.688, size = 492, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*a^2*(6*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*B*cos(d*x+c)^2*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)+12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+30*B*cos(d*x+c)*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*B*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
+3*B*sin(d*x+c)*cos(d*x+c)^2+16*A*cos(d*x+c)*sin(d*x+c)+6*B*sin(d*x+c)*cos(
d*x+c)+2*A*sin(d*x+c))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(
5/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [B] time = 3.74259, size = 3753, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2
*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3
*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
```

```
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqr
t(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
 + 3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)
^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2
*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) + 4*a
^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) +
(a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a
^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
```

```
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.93908, size = 432, normalized size = 2.24

$$-\frac{3\left((2A+5B)a^2\cos(dx+c)^2+(2A+5B)a^2\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(3Ba^2\cos(dx+c)^2+2(8A+3\cdots\right.}{\cdots}}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))) - (3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^2)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.516 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=198

$$\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin}{2}$$

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(4*A - 9*B)*Sin[c +
 d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(4*A - B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.664439, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```


```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(4*A - 9*B)*Sin[c +
 d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(4*A - B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}(A \cdots}{\cos^{\frac{3}{2}}(c+d \cdots}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\cdots\right.$$

$$= -\frac{a^2(4A-B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\cdots}{}$$

$$= -\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{a^2(4A-B)\sqrt{a+\cdots}}{2d\sqrt{\cdots}}$$

$$= -\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{a^2(4A-B)\sqrt{a+\cdots}}{2d\sqrt{\cdots}}$$

$$= \frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec\cdots}}{4d}$$

Mathematica [A] time = 0.780806, size = 126, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(20A+19B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\cdots\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2
]*(20*A + 19*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*
A + B + (4*A + 11*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/
(8*d)
```

Maple [B] time = 0.71, size = 344, normalized size = 1.7

$$\frac{a^2\cos(dx+c)}{4d(1+\cos(dx+c))}\left(20A\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+2B\sin(dx+c)(\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*a^2*(20*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*B*sin(d*x+c)*cos(d*x+c)
^2+19*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)*sin(d*x+c)+20*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))+11*B*sin(d*x+c)*cos(d*x+c)+19*B*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+8*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(
1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.91563, size = 393, normalized size = 1.98

$$-\frac{\left((20A+19B)a^2\cos(dx+c)+(20A+19B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2Ba^2\cos(dx+c)^2+(4A+11B)a^2\cos(dx+c)\right.}{\sqrt{\cos(dx+}}}{4(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((20*A + 19*B)*a^2*cos(d*x + c) + (20*A + 19*B)*a^2)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*a
^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.517 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\,\mathrm{s}}{4}$$

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*(2*A + 3
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (a*B*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.650332, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\,\mathrm{s}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*(2*A + 3
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (a*B*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}(A+}{\sqrt{\cos(c+dx}}$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.$$

$$= \frac{a^2(2A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{aB(a+a\cos}{3d}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2(2A+3B)\sqrt{a+a}}{4d\sqrt{s}}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2(2A+3B)\sqrt{a+a}}{4d\sqrt{s}}$$

$$= \frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{8d}$$

Mathematica [A] time = 0.968606, size = 141, normalized size = 0.7

$$\frac{a^2\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(38A+25B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Se
c[c + d*x]]*(3*Sqrt[2]*(38*A + 25*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*S
qrt[Cos[c + d*x]]*(66*A + 79*B + 2*(6*A + 17*B)*Cos[c + d*x] + 4*B*Cos[2*(c
 + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [A] time = 0.799, size = 305, normalized size = 1.5

$$-\frac{a^2\left((\cos(dx+c))^2-1\right)}{24\,d\,(\sin(dx+c))^2}\left(8\,B\,(\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+12\,A\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*a^2*(8*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+
12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+34*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+66*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)+75*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+114
*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*B*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c)
)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+
c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 3.18792, size = 4146, normalized size = 20.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
```

cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*
(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c)
 - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*
a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2

```
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a
^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1
))*sqrt(a))*B)/d
```

Fricas [A] time = 1.91022, size = 436, normalized size = 2.18

$$-\frac{3\left((38A+25B)a^2\cos(dx+c)+(38A+25B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a}\cos(dx+c)+a\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8Ba^2\cos(dx+c)^3+2(6A+17B)a^2\cos\right.}{}}{24\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arcta
n(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*
B*a^2*cos(d*x + c)^3 + 2*(6*A + 17*B)*a^2*cos(d*x + c)^2 + 3*(22*A + 25*B)*
a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.518 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=247

$$\frac{a^3(104A+95B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{24d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(200A+163B)\sqrt{\cos(c+dx)}\cdots}{6\cdots}$$

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*S
in[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8*A
 + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sec[c + d*x]^(3/2)) +
 (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + (
a^3*(200*A + 163*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]])
```

Rubi [A] time = 0.751629, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(104A+95B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{24d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(200A+163B)\sqrt{\cos(c+dx)}\cdots}{6\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```



```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*S
in[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8*A
 + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sec[c + d*x]^(3/2)) +
 (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + (
a^3*(200*A + 163*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
```

```
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
```

```
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{a^2(8A+11B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{aB(a+a\cos(c+dx))}{4d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\sqrt{a+a\cos(c+}}{24d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\sqrt{a+a\cos(c+}}{24d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\sqrt{a+a\cos(c+}}{24d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{a^{5/2}(200A+163B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} +$$

Mathematica [A] time = 0.968585, size = 159, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(200A+163B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\mathrm{si}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt
[2]*(200*A + 163*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (
632*A + 581*B + (272*A + 362*B)*Cos[c + d*x] + 4*(8*A + 23*B)*Cos[2*(c + d*
x)] + 12*B*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(
```

```
384*d)
```

Maple [A] time = 0.658, size = 383, normalized size = 1.6

$$\frac{a^2(-1+\cos(dx+c))^2\cos(dx+c)}{192\,d\,(\sin(dx+c))^4}\left(48\,B\,(\cos(dx+c))^3\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+64\,A\,(\cos(dx+c))^2\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/192/d*a^2*(-1+cos(d*x+c))^2*(48*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+64*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+184*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+272*
A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+326*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+600*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+489*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+600
*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+489*B*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(
a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1
/2)/sin(d*x+c)^4
```

Maxima [B] time = 5.21407, size = 12677, normalized size = 51.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/
```

3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
c))) + 1)^(1/4)(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(
1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2

*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) - 1))*sqrt(a))*A + (2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4
*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d
*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4
*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a
^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) +
a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 3*(2*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*
x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^
2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + (40*a^2*cos(4*d*x + 4*c)^2 + 40*a^2*sin(4*d*x + 4*c
)^2 - 3*a^2*cos(4*d*x + 4*c) + 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*(80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 - 8
3*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) - 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) + 83*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) - 12*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*
x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*
a^2*cos(4*d*x + 4*c)^3 - 40*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4
*c)^3 - 46*a^2*cos(4*d*x + 4*c)^2 + 83*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*
d*x + 4*c) - 40*a^2)*sin(4*d*x + 4*c)^2 - 40*a^2)*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + (3*a^2*cos(4*d*x + 4*c) - 40*a^2)*sin(4*d*x
 + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 34*a^2*cos(4*d*x + 4*c)^2 - 77*a^
2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 40*a^2)*sin(4*d*x + 4*c)^2 -
 40*a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (40*a^2*c

os(4*d*x + 4*c)^2 + 40*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 16
0*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c)
 + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 160*(a^2*c
os(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(80*a^2*cos(4*d*
x + 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 - 83*a^2*cos(4*d*x + 4*c) + 3*a^2)*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(320*a^2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 83*a^2*sin(4*
d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 -
 43*a^2*cos(4*d*x + 4*c)^2 + 40*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4
*c) - 40*a^2)*sin(4*d*x + 4*c)^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) - 3*(2*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(3*a^2*cos(4*d*x + 4*c) - 40*a^2)*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2
*cos(4*d*x + 4*c) - 40*a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)))*sqrt(a) + 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((a^2*cos(4*d*x + 4*c)^2*si
n(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)^3 + a^2*cos(1/4*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 176*(a^2*cos(4*d*x + 4*c)^2 + a^
2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d
*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c) + 164*(a^2*cos
(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*si
n(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*sin(4*d*x + 4*c)^3 - 176*a^2*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (a^2*cos
(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) - 43*a^2)*sin(4*d*x + 4*c) + 164*(
a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) +
a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(2*a^2*sin(4*d*x + 4*c)^3 + a^2*
cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 2*(
a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + (328*a^2*
cos(4*d*x + 4*c)^2 + 328*a^2*sin(4*d*x + 4*c)^2 - 329*a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(22*a^2*cos(4*d*x + 4*c)^2 + 20*
a^2*sin(4*d*x + 4*c)^2 - 329*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 88*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 8*(11*a^2*cos(4*d*x + 4*c)^2 + 10*a^2*sin(4*d*x + 4*c)

```
^2 - 164*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) - 11*a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) - (a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + (164*a^2*cos(4*d*x + 4*c)^2 + 164*a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x
 + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arct
an2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (a^2*cos(4*d*x + 4*c)^3 - 120*a
^2*cos(4*d*x + 4*c)^2 + 176*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^
2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^3 - a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 4*(a^2*cos(4*d*x + 4*c)^3 - 78*a^2*cos(4*d*x + 4*c)^2 + 19
7*a^2*cos(4*d*x + 4*c) + (a^2*cos(4*d*x + 4*c) - 76*a^2)*sin(4*d*x + 4*c)^2
 - 120*a^2 + 76*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (a^2*cos(4*d*x +
4*c) - 120*a^2)*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^3 - 118*a^2*co
s(4*d*x + 4*c)^2 - 239*a^2*cos(4*d*x + 4*c) + (a^2*cos(4*d*x + 4*c) - 120*a
^2)*sin(4*d*x + 4*c)^2 - 120*a^2 + 44*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d
*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 76*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(2*a
^2*cos(4*d*x + 4*c)^3 - 220*a^2*cos(4*d*x + 4*c)^2 - a^2*sin(4*d*x + 4*c)*s
in(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 240*a^2*cos(4*d*x + 4
*c) + 2*(a^2*cos(4*d*x + 4*c) - 109*a^2)*sin(4*d*x + 4*c)^2 + (152*a^2*cos(
4*d*x + 4*c)^2 + 152*a^2*sin(4*d*x + 4*c)^2 - 153*a^2*cos(4*d*x + 4*c) + a^
2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + (76*a^2*cos(4*d*x + 4*c)^2 + 76*a^2*si
n(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) - 2*(352*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2*sin(4*d*x + 4*c) + 153*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(76*a^2*cos(1/4*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (a^2*cos(4*d*x + 4*c) - 109*a^2)
*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2
*(a^2*cos(4*d*x + 4*c) - 120*a^2)*sin(4*d*x + 4*c) - (a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4
*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c
) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*co
s(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*a
```

```
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*si
n(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
 + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2*si
n(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a
^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d
*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4
*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x +
 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)^2
+ a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)
^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2
*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
```

```
))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x +
 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin
(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan
2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c
)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x
+ 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a
^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a
^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d
*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*
x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(c
os(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(
4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 2.37083, size = 500, normalized size = 2.02

$$-\frac{3\left(\left(200\,A+163\,B\right)a^2\cos\left(dx+c\right)+\left(200\,A+163\,B\right)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,Ba^2\cos(dx+c)^4+8\left(8\,A+23\,B\right)a\right.}{}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/192*(3*((200*A + 163*B)*a^2*cos(d*x + c) + (200*A + 163*B)*a^2)*sqrt(a)*
arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
- (48*B*a^2*cos(d*x + c)^4 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^3 + 2*(136*A +
 163*B)*a^2*cos(d*x + c)^2 + 3*(200*A + 163*B)*a^2*cos(d*x + c))*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

3.519 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{a^3(326A+283B)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(170A+157B)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\sqrt{a\cos(c+dx}}{40d\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(170*A + 157*B)
*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*(
10*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c + d*x]^(5/2
)) + (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(5/2))
 + (a^3*(326*A + 283*B)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c
 + d*x]^(3/2)) + (a^3*(326*A + 283*B)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.865567, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(326A+283B)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(170A+157B)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\sqrt{a\cos(c+dx}}{40d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(170*A + 157*B)
*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*(
10*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c + d*x]^(5/2
)) + (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(5/2))
 + (a^3*(326*A + 283*B)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c
 + d*x]^(3/2)) + (a^3*(326*A + 283*B)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{a^2(10A+13B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{40d\sec^{\frac{5}{2}}(c+dx)} + \frac{aB(a+a\cos(c+dx))}{5d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+a\cos(c+}}{40d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+a\cos(c+}}{40d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+a\cos(c+}}{40d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+a\cos(c+}}{40d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{a^{5/2}(326A+283B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{128d} +$$

Mathematica [A] time = 1.42732, size = 181, normalized size = 0.62

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(326A+283B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqr
t[2]*(326*A + 283*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] +
(5810*A + 5521*B + (3620*A + 3874*B)*Cos[c + d*x] + 4*(230*A + 331*B)*Cos[2
*(c + d*x)] + 120*A*Cos[3*(c + d*x)] + 348*B*Cos[3*(c + d*x)] + 48*B*Cos[4*
(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(3840*d)
```

Maple [A] time = 0.589, size = 455, normalized size = 1.6

$$-\frac{a^2(-1+\cos(dx+c))^3\cos(dx+c)}{1920\,d\,(\sin(dx+c))^6}\left(384\,B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos(dx+c))^4\sin(dx+c)+480\,A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/1920/d*a^2*(-1+cos(d*x+c))^3*(384*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^4*sin(d*x+c)+480*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*
sin(d*x+c)+1392*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+1840*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2264*B*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3260*A*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2830*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*cos(d*x+c)+4890*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+4245*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4890*A*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4245*B*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(
d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d
*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.43882, size = 562, normalized size = 1.91

$$-\frac{15\left((326\,A+283\,B)a^2\cos\left(dx+c\right)+(326\,A+283\,B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(384\,Ba^2\cos(dx+c)^5+48\,(10\,A+2}{}}{1920\,(d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/1920*(15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
) - (384*B*a^2*cos(d*x + c)^5 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^4 + 8*(23
0*A + 283*B)*a^2*cos(d*x + c)^3 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^2 + 1
5*(326*A + 283*B)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

3.520 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=295

$$-\frac{2(A-9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2(19A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A-93B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(257*A - 129*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) - (2*(29*A - 93*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 9*B)*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(9/2)*Sin[c +
 d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.05655, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2(19A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A-93B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(257*A - 129*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) - (2*(29*A - 93*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 9*B)*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(9/2)*Sin[c +
 d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-9B)+4aA}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a}}}{9a}$$

$$= -\frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos}\right.}{}$$

$$= \frac{2(19A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{2(29A-93B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(19A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2(}{}$$

Mathematica [C] time = 9.50608, size = 272, normalized size = 0.92

$$\frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(-315i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}\sin\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x
]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((-315*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-1279*A + 423*B + (214*A - 918*B)*
Cos[c + d*x] - 8*(157*A - 69*B)*Cos[2*(c + d*x)] + 58*A*Cos[3*(c + d*x)] -
186*B*Cos[3*(c + d*x)] - 257*A*Cos[4*(c + d*x)] + 129*B*Cos[4*(c + d*x)])*S
ec[c + d*x]^(9/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*x)/2])
/4))/(315*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.641, size = 793, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/315/d*2^(1/2)/a*(315*A*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))-315*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c
)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+1575*A*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1575*B*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+31
50*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))-3150*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))+3150*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3150*B*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+1575*A*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1575*B*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))+315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))-315*B*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))+257*A*cos(d*x+c)^4*2^(1/2)*sin(d*x+c)-129*B*cos(d*x+c)^4*2^(1/2)*sin(d*x
+c)-29*A*cos(d*x+c)^3*2^(1/2)*sin(d*x+c)+93*B*cos(d*x+c)^3*2^(1/2)*sin(d*x+
c)+57*A*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-9*B*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)
-5*A*cos(d*x+c)*2^(1/2)*sin(d*x+c)+45*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)+35*A*
2^(1/2)*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)^8/(-1+cos(d*x+c))^4/(1+cos(d*x+c))^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69998, size = 539, normalized size = 1.83

$$\frac{\frac{315\sqrt{2}\left((A-B)a\cos(dx+c)^5+(A-B)a\cos(dx+c)^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((257\,A-129\,B)\cos(dx+c)^4-(29\,A-93\,B)\cos(dx+c)^3+3\,(19\,A-3\,B)\,c\cdots\right.}{\sqrt{\cos(d\cdots}}}{315\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

```
[Out] 1/315*(315*sqrt(2)*((A - B)*a*cos(d*x + c)^5 + (A - B)*a*cos(d*x + c)^4)*ar
ctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c)))/sqrt(a) + 2*((257*A - 129*B)*cos(d*x + c)^4 - (29*A - 93*B)*cos(d*x +
 c)^3 + 3*(19*A - 3*B)*cos(d*x + c)^2 - 5*(A - 9*B)*cos(d*x + c) + 35*A)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^5
 + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(11/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{11}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(11/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

3.521 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=250

$$-\frac{2(A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(43*A - 91*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*(31*A - 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.837951, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(43*A - 91*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*(31*A - 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqr
t[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
```

```
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-7B)+3aA\,\mathrm{c}}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\,\mathrm{c}}}}{7a}$$

$$= -\frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos}\right.}{}$$

$$= \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2(43}{}$$

<u>Mathematica</u> [C] time = 6.79707, size = 250, normalized size = 1.

$$\frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(105i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((105*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-122*A + 14*B + 3*(47*A - 119*B)*Co
```

```
s[c + d*x] + (-62*A + 14*B)*Cos[2*(c + d*x)] + 43*A*Cos[3*(c + d*x)] - 91*B
*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2
])*Sin[(c + d*x)/2])/2))/(105*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x
])])
```

Maple [B] time = 0.769, size = 657, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/105/d*2^(1/2)/a*(105*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4*(c
os(d*x+c)/(1+cos(d*x+c)))^(7/2)-105*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+420*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-420*B*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+630*
A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(7/2)-630*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(7/2)+420*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-420*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+105*A*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-105*B*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+43*A*cos(d*x+c)^3*2^(1/2)*sin(d
*x+c)-91*B*cos(d*x+c)^3*2^(1/2)*sin(d*x+c)-31*A*cos(d*x+c)^2*2^(1/2)*sin(d*
x+c)+7*B*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+3*A*cos(d*x+c)*2^(1/2)*sin(d*x+c)-
21*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)-15*A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*sin(
d*x+c)^6*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^3/(1
+cos(d*x+c))^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72449, size = 493, normalized size = 1.97

$$-\frac{\frac{105\,\sqrt{2}\left((A-B)a\cos(dx+c)^{4}+(A-B)a\cos(dx+c)^{3}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((43\,A-91\,B)\cos(dx+c)^{3}-(31\,A-7\,B)\cos(dx+c)^{2}+3\,(A-7\,B)\cos(d\right.}{\sqrt{\cos(dx+c)}}}{105\left(ad\cos\left(dx+c\right)^{4}+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*a
rctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
+ c)))/sqrt(a) + 2*((43*A - 91*B)*cos(d*x + c)^3 - (31*A - 7*B)*cos(d*x + c
)^2 + 3*(A - 7*B)*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{9}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

3.522 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=207

$$-\frac{2(A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*C
os[c + d*x]]) - (2*(A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*
Cos[c + d*x]])
```

Rubi [A] time = 0.646825, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*C
os[c + d*x]]) - (2*(A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*
Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-5B)+2aA\,c}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}}{5a}$$

$$= -\frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos}\right.}{}$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2(1}{}$$

<u>Mathematica</u> [C] time = 7.96952, size = 1718, normalized size = 8.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(5*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(5/2)) + (8*B*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (
2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/15 - ((A - B)*Csc[
c/2 + (d*x)/2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 +
210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2
+ (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*
HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 +
```

```
2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^
12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1
500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hype
rgeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricP
FQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*(1 - 2*Sin[c
/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))))/(d*Sqrt[a*(1 + Cos[
c + d*x])])
```

Maple [B] time = 0.774, size = 521, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/15/d*2^(1/2)/a*(15*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))-15*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+45*A*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-45*B*cos(d*x+c)^2*(co
```

```
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+45*A*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
-45*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))-15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))+13*A*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-5*B*cos(d*x+c)^2*2^(1/2)*sin(
d*x+c)-A*cos(d*x+c)*2^(1/2)*sin(d*x+c)+5*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)+3*
A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)^4/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68812, size = 443, normalized size = 2.14

$$\frac{\frac{15\sqrt{2}\left((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((13A-5B)\cos(dx+c)^2-(A-5B)\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*arct
an(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c
)))/sqrt(a) + 2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

3.523 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=162

$$-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(\cdots}{3d\sqrt{\cdots}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rubi [A] time = 0.452988, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(\cdots}{3d\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-3B)+aA\,\mathrm{co}}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{c}}}}{3a}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos\right.}}{}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \left((A-B\right.$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2a(A-\right.}{}$$

$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2(A}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.774, size = 384, normalized size = 2.4

$$\frac{\sqrt{2}\cos(dx+c)(\sin(dx+c))^2}{3da(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(3A(\cos(dx+c))^2\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2} - 3B(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/3/d*2^(1/2)/a*(3*A*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)-3*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6*A*cos(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-6*B*cos(d*x+c)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-3*B*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+A*cos(d*x+c)*2^(1/2)*s
in(d*x+c)-3*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)-A*2^(1/2)*sin(d*x+c))*cos(d*x+c
)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c)
)/(1+cos(d*x+c))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62739, size = 393, normalized size = 2.43

$$-\frac{\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((A-3B)\cos(dx+c)-A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
/sqrt(a) + 2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

3.524 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=119

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.306133, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -\frac{a(A-B}{2\sqrt{\cos(c+dx)}\sqrt{a+}}}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c}}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\Big(}{d}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2A}{}$$

Mathematica [C] time = 1.50969, size = 203, normalized size = 1.71

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(10B-(A-B)\sec(c+dx)\left(\frac{1}{2}\sin(c+dx)\tan(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\sec(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sin[(c + d*x)/2]*(10*B - (A - B)*Sec
[c + d*x]*((-5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*(
1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c
+ d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -(Sec
[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Sqrt[a*
(1 + Cos[c + d*x])])
```

Maple [B] time = 0.717, size = 231, normalized size = 1.9

$$\frac{\sqrt{2}\cos(dx+c)}{da\,(1+\cos(dx+c))}\left(A\cos(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-B\cos(dx+c)\arcsin\left(\frac{-1+\cos(\right.}{\sin(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/a*(A*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+A*2^(1/2)*sin(d*x+c)+A*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*
(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61225, size = 306, normalized size = 2.57

$$\frac{\frac{\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}A\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*co
s(d*x + c) + a)*A*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

3.525 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rubi [A] time = 0.349669, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx + \cdots$$

$$= -\frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+\cdots}{\sqrt{\cos(c+dx)}\sqrt{a+\cdots}}\right.}{d}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{\cos\cdots}}\right.}{\cdots}$$

Mathematica [A] time = 0.204814, size = 102, normalized size = 0.73

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((A-B)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.753, size = 153, normalized size = 1.1

$$\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{da(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(-B\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+A\,\text{arcs}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/a*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-B*2^(1/2)*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+A*arcsin((-1+c
os(d*x+c))/sin(d*x+c))-B*arcsin((-1+cos(d*x+c))/sin(d*x+c)))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.4379, size = 278, normalized size = 1.99

$$-\frac{\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2B\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*B*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(a*cos(d*x + c) + a),
 x)
```

3.526 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{\sqrt{a}d}$$

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[
a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Sqrt[
a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.512543, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[
a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Sqrt[
a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{aB}{2}+\frac{1}{2}a(2A}{\sqrt{\cos(c+dx)}}}{a}$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)S}{}$$

$$= \frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tan^{-1}}{}$$

Mathematica [C] time = 1.36029, size = 467, normalized size = 2.58

$$\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-(2A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}Ae^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(B - B*E^(I*(c + d*x)) + B*E^((2*I)*(c + d*x))
 - B*E^((3*I)*(c + d*x)) - (2*A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^((2
*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))])] + 2*Sqrt[2]*A*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - Sqrt
[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(-1 + E^(I*(c +
 d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 2*A*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - B*E^(I*(
c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.727, size = 232, normalized size = 1.3

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^2}{2\,da(\sin(dx+c))^4}\sqrt{a(1+\cos(dx+c))}\left(B\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+2A\arctan\left(\frac{\sin(dx+c}{\cos(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)/a*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(B*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*A*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)-B*2^(1/2)*arctan(sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*arcsin((-1+cos(d*x+c
))/sin(d*x+c))-2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)))/(1/cos(d*x+c))^(1/2)
/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.4661, size = 466, normalized size = 2.57

$$\frac{\sqrt{a\cos(dx+c)+a}B\sqrt{\cos(dx+c)}\sin(dx+c)-((2A-B)\cos(dx+c)+2A-B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*co
s(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x)
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx + c) + A}{\sqrt{a\cos(dx + c) + a}\sqrt{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

3.527 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$-\frac{(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+\ldots}{\ldots}$$

```
[Out] -((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(
Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(2*
d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A - B)*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.699175, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(
Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(2*
d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A - B)*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(}{}}{2a}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{(\sqrt{c}}{}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{((4}{}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{((4}{}$$

$$= -\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tan^{-}}{}$$

Mathematica [C] time = 1.41316, size = 412, normalized size = 1.79

$$-\frac{ie^{-3i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-(4A-7B)e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-8\sqrt{2}(A-B)e^{2i(c+dx)}\sqrt{1+e^{2i(c+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] ((-I/16)*(1 + E^(I*(c + d*x)))*(-B - 4*A*E^(I*(c + d*x)) + 2*B*E^(I*(c + d*
x)) + 4*A*E^((2*I)*(c + d*x)) - 3*B*E^((2*I)*(c + d*x)) - 4*A*E^((3*I)*(c +
 d*x)) + 3*B*E^((3*I)*(c + d*x)) + 4*A*E^((4*I)*(c + d*x)) - 2*B*E^((4*I)*(
```

```
c + d*x)) + B*E^((5*I)*(c + d*x)) - (4*A - 7*B)*E^((2*I)*(c + d*x))*Sqrt[1
+ E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] - 8*Sqrt[2]*(A - B)*E^((2*I
)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 4*A*E^((2*I)*(c + d*x))*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 7*B*E^((2*I)*(
c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.786, size = 300, normalized size = 1.3

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^3}{8\,da(\sin(dx+c))^6}\sqrt{a(1+\cos(dx+c))}\left(-2\,B\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-4\,A\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/8/d*2^(1/2)/a*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-2*B
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*A*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+B*2^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))*2^(1/2)-7*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/cos(d*x+c))+8*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))-8*B*arc
sin((-1+cos(d*x+c))/sin(d*x+c)))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Fricas [A] time = 25.8801, size = 539, normalized size = 2.34

$$\frac{((4A-7B)\cos(dx+c)+4A-7B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{4\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cdots}}{\sqrt{a}\sin(dx+\cdots}\right.}{\sqrt{a}}}{4(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2)*((A - B)*a*
cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + (2*B*cos(d*x + c)^2 + (4*A - B)
*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

3.528 $\int \frac{\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=192

$$\frac{(2aB+2Ab-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(a-b)(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\Big(}{\sqrt{a}d}$$

```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(
A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (b*B*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.665248, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4221, 3045, 2982, 2782, 205, 2774, 216}

$$\frac{(2aB+2Ab-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(a-b)(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\Big(}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x
]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(
A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (b*B*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx)\right)\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \frac{aA + (Ab + aB)\,c}{\sqrt{\cos(c + d}}$$

$$= \frac{bB\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}} + \frac{\left(\sqrt{\cos(c + d\right.}}{}$$

$$= \frac{bB\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}} + \left((a - b)(A - \right.$$

$$= \frac{bB\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}} - \frac{\left(2a(a - b)(A\right.}{}$$

$$= \frac{(2Ab + 2aB - bB)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c +}}{\sqrt{a}d}$$

Mathematica [A] time = 0.449816, size = 143, normalized size = 0.74

$$\frac{\cos\left(\frac{1}{2}(c + dx)\right)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\left(\sqrt{2}(2aB + 2Ab - bB)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c + dx)\right)\right) + 2(a - b)(A - B)\tan^{-1}\left(\frac{\sin}{\sqrt{}}\right.\right.}{d\sqrt{a(\cos(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*(2*A*b + 2
*a*B - b*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(a - b)*(A - B)*ArcTan[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]] + 2*b*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/
2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.799, size = 317, normalized size = 1.7

$$-\frac{\sqrt{2}\left((\cos(dx + c))^2 - 1\right)}{2\,da\,(\sin(dx + c))^2}\left(B\sqrt{2}\sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}}b\sin(dx + c) + 2\,A\arctan\left(\frac{\sin(dx + c)}{\cos(dx + c)}\sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}}\right)\sqrt{2}b + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+cos(d*x
+c)*a)^(1/2),x)
```

```
[Out] -1/2/d*2^(1/2)/a*(B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b*sin(d*x+c)+
2*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)
*b+2*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1
/2)*a-B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(
1/2)*b-2*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a+2*A*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*b+2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a-2*B*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*b)*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))\,\sqrt{\sec(c+dx)}}{\sqrt{a\,(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a
+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(
a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{\sec(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d*
x + c))/sqrt(a*cos(d*x + c) + a), x)
```

3.529 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=317

$$\frac{(19A-15B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A-7B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-$$

```
[Out] ((19*A - 15*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((1201*A - 1029*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(210*a*d*S
qrt[a + a*Cos[c + d*x]]) + ((397*A - 273*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A - 63*B)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(7/2)
*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((11*A - 7*B)*Sec[c + d*x
]^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.10609, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(19A-15B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A-7B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((19*A - 15*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((1201*A - 1029*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(210*a*d*S
qrt[a + a*Cos[c + d*x]]) + ((397*A - 273*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A - 63*B)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(7/2)
*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((11*A - 7*B)*Sec[c + d*x
]^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(11A-7\ldots}{\cos^{\frac{9}{2}}(c+\ldots}}{2a^2}$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(11A-7B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{14ad\sqrt{a+a\cos(c+dx)}} + \ldots$$

$$= -\frac{(67A-63B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \ldots$$

$$= \frac{(397A-273B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} - \frac{(67A-63B)\sec^{\frac{5}{2}}(c+dx)\sin(c+\ldots}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{210ad\sqrt{a+a\cos(c+\ldots}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{210ad\sqrt{a+a\cos(c+\ldots}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{210ad\sqrt{a+a\cos(c+\ldots}}$$

$$= \frac{(19A-15B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \ldots$$

Mathematica [C] time = 10.3343, size = 2966, normalized size = 9.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*(-((A - B)*(1 - 2*Sin[c/2 + (d*x)/2]))/(28*(1 + Sin[c
/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) + ((A - B)*(1 + 2*Sin[c/
2 + (d*x)/2]))/(28*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7
/2)) - ((A - B)*(315*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 +
 (d*x)/2]^2]] + (5 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 - 2
*Sin[c/2 + (d*x)/2]^2)^(5/2)) - (11 + 17*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2
+ (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 + 71*Sin[c/2 + (d*x)/
2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sqrt
[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/70 + ((A - B)*(315
*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (5 -
 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(5/2)) - (11 - 17*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*S
in[c/2 + (d*x)/2]^2)^(3/2)) + (61 - 71*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 +
(d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sqrt[1 - 2*Sin[c/2 + (d*
x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/70 - ((-A - 3*B)*Csc[c/2 + (d*x)/2]^9*
(363825*Sin[c/2 + (d*x)/2]^2 - 4729725*Sin[c/2 + (d*x)/2]^4 + 26785605*Sin[
c/2 + (d*x)/2]^6 - 86790165*Sin[c/2 + (d*x)/2]^8 + 177677808*Sin[c/2 + (d*x
)/2]^10 - 239283044*Sin[c/2 + (d*x)/2]^12 + 52080*Hypergeometric2F1[2, 11/2
, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)
/2]^12 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2
 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 213120
160*Sin[c/2 + (d*x)/2]^14 - 168280*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2
 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 2240*H
ypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 121497024*Sin[c/2 +
(d*x)/2]^16 + 212520*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 3360*HypergeometricP
FQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 40125184*Sin[c/2 + (d*x)/2]^18 - 1
24320*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 2240*HypergeometricPFQ[{2, 2, 2, 2,
 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]*Sin[c/2 + (d*x)/2]^18 - 5840384*Sin[c/2 + (d*x)/2]^20 + 28000*Hypergeomet
ric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*S
in[c/2 + (d*x)/2]^20 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1,
13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/
2]^20 + 363825*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 5336100*Ar
cTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (
d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 346361
40*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/
2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1
31060160*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*
Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2

```
)] + 320535600*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] - 530671680*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
 + (d*x)/2]^2)] + 604296000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)] - 468948480*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 237726720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 70963200*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^18*Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 9461760*ArcTanh[Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^20*Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1120*Cos[(c + d*x)/2]^6*H
ypergeometricPFQ[{2, 2, 2, 11/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2
) + 280*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 11/2}, {1, 13/2}, Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(103 -
 164*Sin[c/2 + (d*x)/2]^2 + 70*Sin[c/2 + (d*x)/2]^4)))/(80850*(1 - 2*Sin[c/
2 + (d*x)/2]^2)^(9/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))))/(d*(a*(1 + Cos[c + d
*x]))^(3/2))
```

Maple [B] time = 0.72, size = 731, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/420/d*2^(1/2)/a^2*(1995*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^4*
sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1575*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+7980
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos
(d*x+c))/sin(d*x+c))-6300*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3*
sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+11970*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(7/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-945
0*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))+7980*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)*s
in(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-6300*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(7/2)*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+1995*A*
```

```
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x
+c))-1575*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))-1201*A*cos(d*x+c)^5*2^(1/2)+1029*B*cos(d*x+c)^5*2^(1/2)+39
7*A*cos(d*x+c)^4*2^(1/2)-273*B*cos(d*x+c)^4*2^(1/2)+1000*A*cos(d*x+c)^3*2^(
1/2)-840*B*cos(d*x+c)^3*2^(1/2)-232*A*cos(d*x+c)^2*2^(1/2)+168*B*cos(d*x+c)
^2*2^(1/2)+96*A*cos(d*x+c)*2^(1/2)-84*B*cos(d*x+c)*2^(1/2)-60*A*2^(1/2))*co
s(d*x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^5/(-1+cos
(d*x+c))^3/(1+cos(d*x+c))^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.90661, size = 645, normalized size = 2.03

$$-\frac{105 \sqrt{2}\left((19\,A-15\,B)\cos\left(dx+c\right)^{5}+2\,(19\,A-15\,B)\cos\left(dx+c\right)^{4}+(19\,A-15\,B)\cos\left(dx+c\right)^{3}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+}}{\sqrt{a}\,\mathrm{si}}\right.}{420\left(a^{2}d\cos\left(dx+c\right)^{5}+2\,a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/420*(105*sqrt(2)*((19*A - 15*B)*cos(d*x + c)^5 + 2*(19*A - 15*B)*cos(d*x
 + c)^4 + (19*A - 15*B)*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1201*A - 1029
*B)*cos(d*x + c)^4 + 12*(67*A - 63*B)*cos(d*x + c)^3 - 28*(7*A - 3*B)*cos(d
*x + c)^2 + 12*(3*A - 7*B)*cos(d*x + c) - 60*A)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)
^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

3.530 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{10ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-}{2a}$$

```
[Out] -((15*A - 11*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]
*a^(3/2)*d) + ((147*A - 95*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt
[a + a*Cos[c + d*x]]) - ((39*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(30
*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B)*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.887079, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{10ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((15*A - 11*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]
*a^(3/2)*d) + ((147*A - 95*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt
[a + a*Cos[c + d*x]]) - ((39*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(30
*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B)*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
```

```
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$=-\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(9A-5B)-}{\cos^{\frac{7}{2}}(c+dx}}{2a^2}$$

$$=-\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(9A-5B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}}+\frac{(\sqrt{}}{}$$

$$=-\frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(}{}$$

$$=\frac{(147A-95B)\sqrt{\sec(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{30ad\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{(147A-95B)\sqrt{\sec(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{30ad\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{(147A-95B)\sqrt{\sec(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{30ad\sqrt{a+a\cos(c+dx)}}$$

$$=-\frac{(15A-11B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}+\frac{(1}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.703, size = 595, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/60/d*2^(1/2)/a^2*(225*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3-165*B*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3+675*A*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*cos(d*x+c)^2-495*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2+675*A*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-495*B*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*c
os(d*x+c)+225*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(5/2)-165*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)-147*A*cos(d*x+c)^4*2^(1/2)+95*B*cos(d*x+c)^
4*2^(1/2)+39*A*cos(d*x+c)^3*2^(1/2)-35*B*cos(d*x+c)^3*2^(1/2)+120*A*cos(d*x
+c)^2*2^(1/2)-80*B*cos(d*x+c)^2*2^(1/2)-24*A*cos(d*x+c)*2^(1/2)+20*B*cos(d*
x+c)*2^(1/2)+12*A*2^(1/2))*cos(d*x+c)*sin(d*x+c)^3*(1/cos(d*x+c))^(7/2)*(a*
(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.77946, size = 590, normalized size = 2.19

$$\frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^4+2(15A-11B)\cos(dx+c)^3+(15A-11B)\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)}}{\sqrt{a}\sin(}\right.}{60\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^4 + 2*(15*A - 11*B)*cos(d*x +
c)^3 + (15*A - 11*B)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((147*A - 95*B)*co
s(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 - 4*(3*A - 5*B)*cos(d*x + c) +
 12*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos
(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

3.531 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B}{2d}$$

```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((19*A - 15*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*C
os[c + d*x])^(3/2)) + ((7*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.701828, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((19*A - 15*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*C
os[c + d*x])^(3/2)) + ((7*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*
Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(7A-3B}{\cos^{\frac{5}{2}}(c+}}{2a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} + -$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= \frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - (19$$

Mathematica [C] time = 6.84534, size = 981, normalized size = 4.4

$$\frac{2\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(\frac{(A+3B)\left(-12\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)-12\,{}_2F_1\left(2,\frac{7}{2}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*(-((A - B)*(1 - 2*Sin[c/2 + (d*x)/2]))/(12*(1 + Sin[c
/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + ((A - B)*(1 + 2*Sin[c/
```

```
2 + (d*x)/2]))/(12*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3
/2)) - ((A - B)*(5*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (
d*x)/2]^2]] + (1 + Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2
*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2
 + (d*x)/2])))/2 + ((A - B)*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2
*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2]
)*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/
(1 + Sin[c/2 + (d*x)/2])))/2 + ((A + 3*B)*Csc[c/2 + (d*x)/2]^5*(-12*Cos[(c
+ d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2
/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1
[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2
 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqr
t[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d
*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7
*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2
]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2)
)]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(126*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)
)))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.727, size = 457, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/12/d*2^(1/2)/a^2*(33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-21*B*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+66*A*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)-42*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-21*B*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-19*A*cos(d*x+c)^3*2^(1
/2)+15*B*cos(d*x+c)^3*2^(1/2)+7*A*cos(d*x+c)^2*2^(1/2)-3*B*cos(d*x+c)^2*2^(
1/2)+16*A*cos(d*x+c)*2^(1/2)-12*B*cos(d*x+c)*2^(1/2)-4*A*2^(1/2))*cos(d*x+c
)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))/
(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.17199, size = 531, normalized size = 2.38

$$-\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^3+2(11A-7B)\cos(dx+c)^2+(11A-7B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cdots}}{\sqrt{a}\sin(dx+c\cdots}\right.}{12\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^3 + 2*(11*A - 7*B)*cos(d*x + c)
^2 + (11*A - 7*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((19*A - 15*B)*cos(d*x
+ c)^2 + 12*(A - B)*cos(d*x + c) - 4*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

3.532 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)}{2d(}$$

```
[Out] -((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)) + ((5*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a
*Cos[c + d*x]])
```

Rubi [A] time = 0.519239, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)}{2d(}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)) + ((5*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a
*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(5A-B)\cdots}{\cos^{\frac{3}{2}}(c+d\cdots}}{2a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \cdots$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= -\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \cdots$$

Mathematica [C] time = 4.45078, size = 443, normalized size = 2.52

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(A+3B)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(30*(A - B)*ArcTa
n[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] - 30*(A - B)*ArcTan[(1 + 2*S
in[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] - (20*(A - B)*Sqrt[Cos[c + d*x]])/(-1
+ Sin[(c + d*x)/2]) - (20*(A - B)*Sqrt[Cos[c + d*x]])/(1 + Sin[(c + d*x)/2]
) + (5*(A - B)*(-1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(Cos[(c + d*x
)/4] + Sin[(c + d*x)/4])^2) - (5*(A - B)*(1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Co
s[c + d*x]]*(-1 + Sin[(c + d*x)/2])) + ((A + 3*B)*Csc[(c + d*x)/2]^3*(5*(1
+ 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec
[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2*
Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c +
```

```
d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))/(2*Cos[c + d*x]^(3/2))))/(10*d*(a*(1
+ Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.675, size = 312, normalized size = 1.8

$$-\frac{\sqrt{2}\cos(dx+c)}{4\,a^2d\sin(dx+c)\,(1+\cos(dx+c))}\left(-7\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(-7*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*A*cos(d*x+c)
^2*2^(1/2)-7*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)-B*cos(d*x+c)^2*2^(1/2)+3*B*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)*2^(1/2)+B*
cos(d*x+c)*2^(1/2)-4*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d
*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.0023, size = 435, normalized size = 2.47

$$\frac{\sqrt{2}\left((7\,A-3\,B)\cos(dx+c)^2+2\,(7\,A-3\,B)\cos(dx+c)+7\,A-3\,B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\,((5\,A-B)\cos}{}}{4\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A
 - 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) + 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

3.533 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3
/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c
+ d*x]])
```

Rubi [A] time = 0.337286, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2978, 12, 2782, 205}

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3
/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c
+ d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{2\sqrt{\cos(}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{4a}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{}{2d(a+}$$

Mathematica [C] time = 1.62216, size = 196, normalized size = 1.54

$$\frac{i\cos^3\left(\frac{1}{2}(c+dx)\right)\left((3A+B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{2}i(A-B)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.\right.}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (I*Cos[(c + d*x)/2]^3*(((3*A + B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) - (I/2)*(A - B)*Sec[(
c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))
)/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.656, size = 236, normalized size = 1.9

$$-\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{4a^2d(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-3A\arcsin\left(\frac{-1}{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(A*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*A*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A] time = 1.96492, size = 397, normalized size = 3.13

$$-\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{a\cos(dx}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

3.534 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=185

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx}{\sqrt{a\cos(c+dx)}}\right.}{a^{3/2}d}$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.543816, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx}{\sqrt{a\cos(c+dx)}}\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(}{\sqrt{\cos}}}{2a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{4a}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(A-5B)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{}$$

Mathematica [C] time = 1.64481, size = 243, normalized size = 1.31

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left((A-B)\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}-i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\right.}{2d(a(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(4*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[
2]*(A - 5*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
d*x))])] - 4*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*x))
 + (A - B)*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(
3*(c + d*x))/2])))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 0.657, size = 288, normalized size = 1.6

$$-\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^2}{4a^2d(\sin(dx+c))^5}\sqrt{a(1+\cos(dx+c))}\left(A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-B\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(A
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-B*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+B*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(
d*x+c))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [A] time = 43.2471, size = 571, normalized size = 3.09

$$-\frac{\sqrt{2}\left((A-5B)\cos(dx+c)^2+2(A-5B)\cos(dx+c)+A-5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos(dx+}}{4\left(a^2d\cos(dx+c)^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

3.535 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.}{2\sqrt{2}a^{3/2}d}$$

```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - ((A - 3*B)*Sin[
c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.736422, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.}{2\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - ((A - 3*B)*Sin[
c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c}}{}}{2a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= \frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{(5A-9B)\tan^{-}}{}$$

Mathematica [C] time = 6.69571, size = 836, normalized size = 3.53

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{\sec\left(\frac{c}{2}\right)\left(A\sin\left(\frac{c}{2}\right)-B\sin\left(\frac{c}{2}\right)\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2A\cos\left(\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)}{d}+\frac{2B\cos\left(\frac{3dx}{2}\right)\sin\left(\frac{3c}{2}\right)}{d}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] ((-I)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*
x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^
(3/2)) + ((3*I)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*
I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c
 + d*x]))^(3/2)) + ((2*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]
*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] +
ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(
c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2)) - ((3*I)*Sqrt[2]*B*Sqrt[E^(I*(c + d
*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(
I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^
((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d
*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2)) + (Cos[c/2 +
 (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((-2*A*Cos[(d*x)/2]*Sin[c/2])/d + (Sec[c/2]*
Sec[c/2 + (d*x)/2]*(A*Sin[c/2] - B*Sin[c/2]))/d + (2*B*Cos[(3*d*x)/2]*Sin[(
3*c)/2])/d - (2*A*Cos[c/2]*Sin[(d*x)/2])/d + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2
*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (2*B*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d
))/(a*(1 + Cos[c + d*x]))^(3/2)
```

Maple [A] time = 1.02, size = 370, normalized size = 1.6

$$-\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^3}{4a^2d(\sin(dx+c))^7}\sqrt{a(1+\cos(dx+c))}\left(-2B\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+4A\arctan\left(\frac{\sin}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-
2*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+4*A*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+A*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-6*B*2^(1/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-B*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+3*B*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-9*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^
7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [A] time = 47.8034, size = 667, normalized size = 2.81

$$\frac{\sqrt{2}\left((5A-9B)\cos(dx+c)^2+2(5A-9B)\cos(dx+c)+5A-9B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-4\left((2A-3\right.}{4\left(a^2d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A
 - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos
(d*x + c) + 2*A - 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
```

```
 + c))/(sqrt(a)*sin(d*x + c))) + 2*(2*B*cos(d*x + c)^2 - (A - 3*B)*cos(d*x
+ c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

3.536 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx$

Optimal. Leaf size=317

$$\frac{(157A-85B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}} - \frac{(787A-475B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}} + \frac{(2671A-1495B)\sin(c+dx)\sqrt{\sec(c}}{240a^2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -((283*A - 163*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt
[2]*a^(5/2)*d) + ((2671*A - 1495*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a
^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A - 475*B)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2)*
Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((21*A - 13*B)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((157*A - 85*B)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.12478, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(157A-85B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}} - \frac{(787A-475B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}} + \frac{(2671A-1495B)\sin(c+dx)\sqrt{\sec(c}}{240a^2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] -((283*A - 163*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt
[2]*a^(5/2)*d) + ((2671*A - 1495*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a
^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A - 475*B)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2)*
Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((21*A - 13*B)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((157*A - 85*B)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(13A-5B)\cdots}{\cos^{\frac{7}{2}}(c+dx)\cdots}}{4a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(787A-475B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \cdots$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\sec^{\frac{3}{2}}(c+dx)\sin(\cdots}{240a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\sec^{\frac{3}{2}}(c+dx)\sin(\cdots}{240a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\sec^{\frac{3}{2}}(c+dx)\sin(\cdots}{240a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(283A-163B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \cdots$$

Mathematica [C] time = 8.17732, size = 261, normalized size = 0.82

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(10(2605A-1381B)\cos(c+dx)+108(157A-85B)\cos(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-240*I)*(283*A - 163*B)*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*
x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (15053
*A - 7685*B + 10*(2605*A - 1381*B)*Cos[c + d*x] + 108*(157*A - 85*B)*Cos[2*
(c + d*x)] + 9110*A*Cos[3*(c + d*x)] - 5030*B*Cos[3*(c + d*x)] + 2671*A*Cos
[4*(c + d*x)] - 1495*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^3*Sec[c + d*x]^(5
/2)*Tan[(c + d*x)/2]))/(960*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.747, size = 729, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/480/d*2^(1/2)/a^3*(4245*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-2445*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+169
80*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(5/2)*cos(d*x+c)^3-9780*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3+25470*A*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2-1
4670*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)*cos(d*x+c)^2+16980*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-9780*B*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)+424
5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)-2445*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(5/2)-2671*A*cos(d*x+c)^5*2^(1/2)+1495*B*cos(d*x+c)^5*2^(1/2
)-1884*A*cos(d*x+c)^4*2^(1/2)+1020*B*cos(d*x+c)^4*2^(1/2)+2987*A*cos(d*x+c)
^3*2^(1/2)-1715*B*cos(d*x+c)^3*2^(1/2)+1728*A*cos(d*x+c)^2*2^(1/2)-960*B*co
s(d*x+c)^2*2^(1/2)-256*A*cos(d*x+c)*2^(1/2)+160*B*cos(d*x+c)*2^(1/2)+96*A*2
^(1/2))*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)
/(-1+cos(d*x+c))/(1+cos(d*x+c))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.84531, size = 732, normalized size = 2.31

$$\frac{15\sqrt{2}\left((283A-163B)\cos(dx+c)^5+3(283A-163B)\cos(dx+c)^4+3(283A-163B)\cos(dx+c)^3+(283A-163B\right.}{480\left(a^3d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^5 + 3*(283*A - 163*B)*cos(d
*x + c)^4 + 3*(283*A - 163*B)*cos(d*x + c)^3 + (283*A - 163*B)*cos(d*x + c)
^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqr
t(a)*sin(d*x + c))) + 2*((2671*A - 1495*B)*cos(d*x + c)^4 + 5*(911*A - 503*
B)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 - 160*(A - B)*cos(d*x +
 c) + 96*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*
d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*
cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

3.537 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{(95A-39B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{16\sqrt{2}}$$

```
[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((299*A - 147*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*
Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39*B)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.928702, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(95A-39B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((299*A - 147*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*
Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39*B)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
```

```
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(11A-3B)\cdots}{\cos^{\frac{5}{2}}(c+dx)\cdots}}{4a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(9\cdots}{\cdots}$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \cdots$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \cdots$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \cdots$$

$$= \frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{(2\cdots}{\cdots}$$

Mathematica [C] time = 3.58877, size = 243, normalized size = 0.9

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3(163A-75B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\frac{1}{8}i\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] ((I/12)*Cos[(c + d*x)/2]^5*((3*(163*A - 75*B)*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (I/8)*(
878*A - 510*B + (1537*A - 825*B)*Cos[c + d*x] + 2*(503*A - 255*B)*Cos[2*(c
+ d*x)] + 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]
^3*Sec[c + d*x]^(3/2)*Tan[(c + d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.638, size = 585, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/96/d*2^(1/2)/a^3*(489*A*sin(d*x+c)*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-225*B*sin(d*x+c)*cos(d*x+c)^3*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1467*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)-675*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1467*A*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-675*B*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)+489*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)-299*A*cos(d*x+c)^4*2^(1/2)-225*B*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+147*B*cos(d*x
+c)^4*2^(1/2)-204*A*cos(d*x+c)^3*2^(1/2)+108*B*cos(d*x+c)^3*2^(1/2)+343*A*c
os(d*x+c)^2*2^(1/2)-159*B*cos(d*x+c)^2*2^(1/2)+192*A*cos(d*x+c)*2^(1/2)-96*
B*cos(d*x+c)*2^(1/2)-32*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+co
s(d*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.83528, size = 672, normalized size = 2.49

$$-\frac{3\sqrt{2}\Big((163A-75B)\cos(dx+c)^4+3(163A-75B)\cos(dx+c)^3+3(163A-75B)\cos(dx+c)^2+(163A-75B)\cos}{96\left(a^3d\cos(dx+c)^4+3a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^4 + 3*(163*A - 75*B)*cos(d*x
+ c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + (163*A - 75*B)*cos(d*x + c))*sqr
t(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c))) + 2*((299*A - 147*B)*cos(d*x + c)^3 + (503*A - 255*B)*cos(d*x
+ c)^2 + 32*(5*A - 3*B)*cos(d*x + c) - 32*A)*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

3.538 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-}{1}$$

```
[Out] -((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a
+ a*Cos[c + d*x])^(3/2)) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.72912, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] -((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a
+ a*Cos[c + d*x])^(3/2)) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
```

tegerQ[n])

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(9A-B)-2}{\cos^{\frac{3}{2}}(c+dx)(}}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(\sqrt{}}{}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(4}{}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(4}{}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(4}{}$$

$$= -\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{(A}{}$$

Mathematica [C] time = 2.19252, size = 219, normalized size = 0.98

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(2(85A-13B)\cos(c+dx)+(49A-9B)\cos(2(c+dx)\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*(75*A - 19*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/E^((I/2)*(c + d*x)) + ((113*A - 9*
B + 2*(85*A - 13*B)*Cos[c + d*x] + (49*A - 9*B)*Cos[2*(c + d*x)])*Sec[(c +
d*x)/2]^3*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/4))/(4*d*(a*(1 + Cos[c + d*x
```

```
]))^(5/2))
```

Maple [B] time = 0.578, size = 457, normalized size = 2.1

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))\cos(dx+c)}{32\,da^3\,(\sin(dx+c))^3\,(1+\cos(dx+c))}\left(-75\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(-1+cos(d*x+c))*(-75*A*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19*B*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+49*A*cos(d*x+c)^3*2^(1/2)-150*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-9*B*cos(d*x+c)^3*2^
(1/2)+38*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+36*A*cos(d*x+c)^2*2^(1/2)-75*A*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-4*B*cos(d*x+
c)^2*2^(1/2)+19*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)-53*A*cos(d*x+c)*2^(1/2)+13*B*cos(d*x+c)*2^(1/2)-32*A
*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+
c)^3/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.89661, size = 570, normalized size = 2.56

$$\frac{\sqrt{2}\left((75\,A-19\,B)\cos(dx+c)^3+3\,(75\,A-19\,B)\cos(dx+c)^2+3\,(75\,A-19\,B)\cos(dx+c)+75\,A-19\,B\right)\sqrt{a}\arctan\left(\cdots\right.}{32\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)^
2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*arctan(sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((49*A
- 9*B)*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

3.539 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{(19A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(9A-B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}} - \frac{}{4d\sqrt{}}$$

```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Se
c[c + d*x]]) - ((9*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*
Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.526173, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2978, 12, 2782, 205}

$$\frac{(19A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(9A-B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}} - \frac{}{4d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Se
c[c + d*x]]) - ((9*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*
Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(7A}{\sqrt{\cos(c}}}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= \frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{}{4d(}$$

Mathematica [C] time = 1.73673, size = 216, normalized size = 1.23

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left((19A+5B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}i\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+\right.\right.\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] ((I/4)*Cos[(c + d*x)/2]^5*(((19*A + 5*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/E^((I/2)*(c + d*x)) - (I/4)*(13*A
- 5*B + (9*A - B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(Sin[
(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.682, size = 376, normalized size = 2.1

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(9A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2-B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
*(-1+cos(d*x+c))^2*(9*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)^2-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+4*A*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-19*A*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*sin(d*x+c)*cos(d*x+c)-4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-13
*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-19*A*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)+5*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*B*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.84354, size = 551, normalized size = 3.13

$$-\frac{\sqrt{2}\Big((19\,A+5\,B)\cos(dx+c)^3+3\,(19\,A+5\,B)\cos(dx+c)^2+3\,(19\,A+5\,B)\cos(dx+c)+19\,A+5\,B\Big)\sqrt{a}\arctan\Big(\frac{\sqrt{2}}{}}{32\,\Big(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
```

```
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((9*A - B)
*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

3.540 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+7B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{}{4d\ldots}$$

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a
^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec
[c + d*x]]) + ((A + 7*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.510908, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2977, 2978, 12, 2782, 205}

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+7B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{}{4d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a
^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec
[c + d*x]]) + ((A + 7*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*S
qrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(A}{\sqrt{\cos}}}{4a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= \frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{}{4a}$$

Mathematica [C] time = 1.76977, size = 213, normalized size = 1.22

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sec^4\left(\frac{1}{2}(c+dx)\right)((A+7B)\cos(c+dx)+5A+3B}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(5*A + 3*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((5*A + 3*B + (A
+ 7*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/
2] + Sin[(3*(c + d*x))/2]))/4))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.582, size = 375, normalized size = 2.2

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^3}{32\,da^3(\sin(dx+c))^7}\sqrt{a(1+\cos(dx+c))}\left(A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+7B\sqrt{2}\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(A
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+7*B*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*sin(d*x+c)*cos(d*x+c)+4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos
(d*x+c)+3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*B*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+5*A*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)-5*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*B*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/s
in(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [A] time = 1.71398, size = 544, normalized size = 3.13

$$-\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(}}{\sqrt{}}\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((A + 7*B)*cos
(d*x + c)^2 + (5*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

3.541 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{a\cos(c+dx}}\right.}{a^{5/2}d}$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((3*A - 11*B)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.738146, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{a\cos(c+dx}}\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```


```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((3*A - 11*B)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c}}{}}{4a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(3A-43B)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{}$$

Mathematica [C] time = 2.5089, size = 264, normalized size = 1.13

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}((7A-15B)\cos(c+dx)+3A-11B}{8d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(32*B*ArcSinh[E^(I*(c + d*x))] - Sqrt
```

```
[2]*(3*A - 43*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))])] - 32*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c +
d*x)) + ((3*A - 11*B + (7*A - 15*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[S
ec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/2))/(8*d*(a*(1 + C
os[c + d*x]))^(5/2))
```

Maple [B] time = 0.605, size = 476, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*cos(d*x+c)*(
7*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-15*B*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-32*B*2^(1/2)*sin(d*x+c)*cos(d*x
+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*A*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*sin(d*x+c)*cos(d*x+c)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)-32*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*sin(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*A*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*sin(d*x+c)+11*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c
))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [A] time = 77.0789, size = 759, normalized size = 3.24

$$-\frac{\sqrt{2}\left((3\,A-43\,B)\cos\left(dx+c\right)^{3}+3\,(3\,A-43\,B)\cos\left(dx+c\right)^{2}+3\,(3\,A-43\,B)\cos\left(dx+c\right)+3\,A-43\,B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{32\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2
 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 64*(B*cos(d*
x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((7*A - 1
5*B)*cos(d*x + c)^2 + (3*A - 11*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

3.542 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(43A-115B)\sqrt{\cos}\ldots}{\ldots}$$

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt
[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sq
rt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((7*A - 15*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - (
(11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.981494, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(43A-115B)\sqrt{\cos}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt
[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sq
rt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((7*A - 15*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - (
(11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}}{}}{4a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= \frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{(43A-115B)}{}$$

Mathematica [C] time = 7.15705, size = 929, normalized size = 3.25

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\left(B\sin\left(\frac{dx}{2}\right)-A\sin\left(\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} - \frac{(A-B)\tan\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} + \frac{\sec\left(\frac{c}{2}\right)\left(19A\sin\left(\frac{dx}{2}\right)-27B\sin\left(\frac{dx}{2}\right)\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} + \frac{(19A-}{}\right.}{(a(\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (((-11*I)/4)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c +
```

```
d*x]))^(5/2)) + (((35*I)/4)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))]])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(
a*(1 + Cos[c + d*x]))^(5/2)) + ((4*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x)
)] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
 d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^5)/(
d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2)) - ((10*I)*Sqrt[2]*B*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
 + (Cos[c/2 + (d*x)/2]^5*Sqrt[Sec[c + d*x]]*((15*(-A + B)*Cos[(d*x)/2]*Sin[
c/2])/(2*d) + (4*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d - (15*(A - B)*Cos[c/2]*Si
n[(d*x)/2])/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(19*A*Sin[(d*x)/2] - 27*
B*Sin[(d*x)/2]))/(4*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-(A*Sin[(d*x)/2])
+ B*Sin[(d*x)/2]))/(2*d) + (4*B*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d + ((19*A - 2
7*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) - ((A - B)*Sec[c/2 + (d*x)/2]^3*Tan
[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

Maple [B] time = 0.615, size = 609, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(
-16*B*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*A*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+32*A*cos(d*x+c)*2^(1/2)*sin(d
*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-39*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-80*B*2^(1/2)*sin(d*x
+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))+43*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+32*A*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-115*B*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+20*B*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)-80*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+43*A*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)-11*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-115*B*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+35*B*2^(1/2)*(cos(d*x+c)/(1+co
```

```
s(d*x+c)))^(1/2))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/si
n(d*x+c)^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Fricas [A] time = 99.5675, size = 853, normalized size = 2.98

$$\frac{\sqrt{2}\left((43\,A-115\,B)\cos(dx+c)^{3}+3\,(43\,A-115\,B)\cos(dx+c)^{2}+3\,(43\,A-115\,B)\cos(dx+c)+43\,A-115\,B\right)\sqrt{a}\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c
)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 32*((
2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*co
s(d*x + c) + 2*A - 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))/(sqrt(a)*sin(d*x + c))) + 2*(16*B*cos(d*x + c)^3 - 5*(3*A - 11*B)*c
os(d*x + c)^2 - (11*A - 35*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

3.543 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=317

$$\frac{(579A-199B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{(1887A-691B)\sin(c+dx)\sqrt{\sec(c}}{192a^3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt
[2]*a^(7/2)*d) - ((1887*A - 691*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^
3*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((109*A - 41*B)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((579*A - 199*B)
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.14665, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(579A-199B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{(1887A-691B)\sin(c+dx)\sqrt{\sec(c}}{192a^3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(7/2),x]
```


```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt
[2]*a^(7/2)*d) - ((1887*A - 691*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^
3*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((109*A - 41*B)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((579*A - 199*B)
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{3}{2}a(5A-B)\cdots}{\cos^{\frac{5}{2}}(c+dx\cdots}}{6a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \cdots$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \cdots$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \cdots$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\cdots}{6d(a+a\cos(c+dx))^{7/2}}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\cdots}{6d(a+a\cos(c+dx))^{7/2}}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\cdots}{6d(a+a\cos(c+dx))^{7/2}}$$

$$= \frac{(1015A-363B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}$$

Mathematica [C] time = 5.5, size = 267, normalized size = 0.84

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(4(9415A-3579B)\cos(c+dx)+8(3069A-1145B)\cos(2(c+dx))+10164A\cos(3(c+dx))+1887\cdots}{96d}\right.}{8(a(\cos(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((I*(1015*A - 363*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*E^((I/2)*(c + d*x))) - ((21641*
A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos[2*(
c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*A*Cos
[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Sec[c + d*x]^(3/
2)*Tan[(c + d*x)/2])/(96*d)))/(8*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.649, size = 729, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(-1+cos(d*x+c))*(-3045*A*cos(d*x+c)^4*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+1089*B*cos(
d*x+c)^4*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*sin(d*x+c)-12180*A*sin(d*x+c)*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+4356*B*sin(d*x+c)*cos(d*x+c)^3*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-18270*A*ar
csin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)+6534*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1887*A*cos(d*x+c)^5*2^(1/2)-12180*
A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)-691*B*cos(d*x+c)^5*2^(1/2)+4356*B*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3195*A*
cos(d*x+c)^4*2^(1/2)-3045*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-1183*B*cos(d*x+c)^4*2^(1/2)+1089*B*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-83
1*A*cos(d*x+c)^3*2^(1/2)+275*B*cos(d*x+c)^3*2^(1/2)-3355*A*cos(d*x+c)^2*2^(
1/2)+1215*B*cos(d*x+c)^2*2^(1/2)-1024*A*cos(d*x+c)*2^(1/2)+384*B*cos(d*x+c)
*2^(1/2)+128*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)^3/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.84682, size = 822, normalized size = 2.59

$$-\frac{3\sqrt{2}\left((1015\,A-363\,B)\cos\left(dx+c\right)^{5}+4\,(1015\,A-363\,B)\cos\left(dx+c\right)^{4}+6\,(1015\,A-363\,B)\cos\left(dx+c\right)^{3}+4\,(1015\,A\right.}{384\,\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^5 + 4*(1015*A - 363*B)*cos
(d*x + c)^4 + 6*(1015*A - 363*B)*cos(d*x + c)^3 + 4*(1015*A - 363*B)*cos(d*
x + c)^2 + (1015*A - 363*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1887*A - 69
1*B)*cos(d*x + c)^4 + 2*(2541*A - 937*B)*cos(d*x + c)^3 + 39*(109*A - 41*B)
*cos(d*x + c)^2 + 128*(7*A - 3*B)*cos(d*x + c) - 128*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(
d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

3.544 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=270

$$\frac{(691A-103B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{3(121A-21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{64}$$

```
[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqr
t[2]*a^(7/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos
[c + d*x])^(7/2)) - ((19*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*
(a + a*Cos[c + d*x])^(5/2)) - ((199*A - 43*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((691*A - 103*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.949955, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(691A-103B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{3(121A-21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{64}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(7/2),x]
```


```
[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqr
t[2]*a^(7/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos
[c + d*x])^(7/2)) - ((19*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*
(a + a*Cos[c + d*x])^(5/2)) - ((199*A - 43*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((691*A - 103*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
```

```
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(13A-B}{\cos^{\frac{3}{2}}(c+d}}{6a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} +$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} -$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} -$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} -$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} -$$

$$= -\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}$$

Mathematica [C] time = 3.18135, size = 242, normalized size = 0.9

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{16}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(9(941A-121B)\cos(c+dx)+4(937A-133B)\cos(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-9*I)*(121*A - 21*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x))
)/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((5284*A
- 532*B + 9*(941*A - 121*B)*Cos[c + d*x] + 4*(937*A - 133*B)*Cos[2*(c + d*x
)] + 691*A*Cos[3*(c + d*x)] - 103*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^5*Sq
rt[Sec[c + d*x]]*Tan[(c + d*x)/2])/16))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.647, size = 595, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*(-1+cos(d*x+c))^2*(1089*A*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin(d*x+c)-189*B*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
3*sin(d*x+c)+3267*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-691*A*cos(d*x+c)^4*2^(1/2)-567*B*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+103*B*cos(d*x+c)^4*2^(1/2)+3267*A*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1183*A*cos(
d*x+c)^3*2^(1/2)-567*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+163*B*cos(d*x+c)^3*2^(1/2)+1089*A*ar
csin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+275*A*cos(d*x+c)^2*2^(1/2)-189*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-71*B*cos(d*x+c)^2*2^(1/2)+1215*A*c
os(d*x+c)*2^(1/2)-195*B*cos(d*x+c)*2^(1/2)+384*A*2^(1/2))*cos(d*x+c)*(1/cos
(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.76961, size = 714, normalized size = 2.64

$$\frac{9\sqrt{2}\Big((121\,A-21\,B)\cos(dx+c)^4+4\,(121\,A-21\,B)\cos(dx+c)^3+6\,(121\,A-21\,B)\cos(dx+c)^2+4\,(121\,A-21\,B)\cos}{384\left(a^4d\cos(dx+c)^4+4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^4 + 4*(121*A - 21*B)*cos(d*x
+ c)^3 + 6*(121*A - 21*B)*cos(d*x + c)^2 + 4*(121*A - 21*B)*cos(d*x + c) +
121*A - 21*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(937*
A - 133*B)*cos(d*x + c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

3.545 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{(103A+5B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(63A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]
*a^(7/2)*d) - ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[S
ec[c + d*x]]) - ((5*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)
*Sqrt[Sec[c + d*x]]) - ((103*A + 5*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
 + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.727955, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2978, 12, 2782, 205}

$$-\frac{(103A+5B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(63A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]
*a^(7/2)*d) - ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[S
ec[c + d*x]]) - ((5*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)
*Sqrt[Sec[c + d*x]]) - ((103*A + 5*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
 + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(1}{\sqrt{\mathrm{co}}}}{6a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= \frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{}{6}$$

<u>Mathematica</u> [C] time = 3.02116, size = 228, normalized size = 1.02

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sec^6\left(\frac{1}{2}(c+dx)\right)((532A-4B)\cos(c+dx)+(103A}{384d(a(\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((48*I)*(63*A + 13*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*
I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))
/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (493*A - 7
3*B + (532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(c + d*x)])*Sec[(c +
 d*x)/2]^6*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(
384*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.707, size = 512, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)*(-1+cos(d*x+c))^3*(103*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d
*x+c)^3+5*B*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+163*A*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-189*A*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-7*B*2^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*cos(d*x+c)^2-39*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)^2*sin(d*x+c)-71*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3
78*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-37*B*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-78*B*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)*cos(d*x+c)-195*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)-189*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+39*B*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)-39*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*
x+c))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.73606, size = 686, normalized size = 3.08

$$-\frac{3\sqrt{2}\left((63\,A+13\,B)\cos(dx+c)^4+4\,(63\,A+13\,B)\cos(dx+c)^3+6\,(63\,A+13\,B)\cos(dx+c)^2+4\,(63\,A+13\,B)\cos(d\right.}{384\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x +
 c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 63*
A + 13*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) + 2*((103*A + 5*B)*cos(d*x + c)^3 + 2*(133*A - B)
*cos(d*x + c)^2 + 39*(5*A - B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
+ 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

3.546 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=221

$$-\frac{(5A-17B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(13A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+$$

```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*
a^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Se
c[c + d*x]]) + ((A + 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) - ((5*A - 17*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c +
 d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.725067, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2977, 2978, 12, 2782, 205}

$$-\frac{(5A-17B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(13A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*
a^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Se
c[c + d*x]]) + ((A + 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) - ((5*A - 17*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c +
 d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(A-}{\sqrt{\cos(}}}{6a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} + \frac{}{6a}$$

Mathematica [C] time = 2.87215, size = 233, normalized size = 1.05

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(4(A+35B)\cos(c+dx)+(17B-5A)\cos(2(c+dx))+73A+59B)}{48d} + \frac{i(13A+7B)e^{-}}{}\right.}{8(a(\cos(c+dx)+1))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((I*(13*A + 7*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(d*E^((I/2)*(c + d*x))) - ((73*A + 59*
B + 4*(A + 35*B)*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c + d*x)])*Sec[(c + d*
x)/2]^6*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/(48*d
```

```
)))/(8*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.661, size = 512, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^4*(
5*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-17*B*cos(d*x+c)^
3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-7*A*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2-39*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x
+c)^2*sin(d*x+c)-53*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
2-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-37*A*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-78*A*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+49*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)-42*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d
*x+c)+39*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-39*A*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)+21*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(1/2)/(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [A] time = 1.74693, size = 679, normalized size = 3.07

$$-\frac{3\sqrt{2}\left((13A+7B)\cos(dx+c)^4+4(13A+7B)\cos(dx+c)^3+6(13A+7B)\cos(dx+c)^2+4(13A+7B)\cos(dx+c\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x + c
)^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A +
7*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sq
rt(a)*sin(d*x + c))) + 2*((5*A - 17*B)*cos(d*x + c)^3 - 2*(A + 35*B)*cos(d*
x + c)^2 - 3*(13*A + 7*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a
^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c
))), x)
```

3.547 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=221

$$\frac{(17A+67B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{}{6d}$$

```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a
^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c +
d*x]^(3/2)) + ((A - 13*B)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) + ((17*A + 67*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
+ d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.723135, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2977, 2978, 12, 2782, 205}

$$\frac{(17A+67B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```


```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a
^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c +
d*x]^(3/2)) + ((A - 13*B)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) + ((17*A + 67*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
+ d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c}}{}}{6a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d}}$$

$$= \frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} + \frac{}{6d(}$$

Mathematica [C] time = 7.16513, size = 488, normalized size = 2.21

$$\frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{(17A+67B)\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{12d} + \frac{(17A+67B)\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d} + \frac{\sec\left(\frac{c}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{3d} + \frac{\sec\left(\frac{c}{2}\right)\mathrm{se}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] ((I/8)*(7*A + 5*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[
c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*(((17*A + 67*B
```

```
)*Cos[(d*x)/2]*Sin[c/2])/(12*d) + ((17*A + 67*B)*Cos[c/2]*Sin[(d*x)/2])/(12
*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(19*A*Sin[(d*x)/2] - 151*B*Sin[(d*x)/2
]))/(24*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2
]))/(3*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-17*A*Sin[(d*x)/2] + 29*B*Sin[(
d*x)/2]))/(12*d) + ((19*A - 151*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - ((
17*A - 29*B)*Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + ((A - B)*Sec[c/2 + (d*
x)/2]^5*Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x]))^(7/2)
```

Maple [B] time = 0.598, size = 512, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(
17*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+67*B*cos(d*x+c)
^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+53*A*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^2+21*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d
*x+c)^2*sin(d*x+c)-17*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)^2+15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-49*A*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+42*A*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-35*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*cos(d*x+c)+30*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos
(d*x+c)-21*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*A*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*sin(d*x+c)-15*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(3/2)
/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [A] time = 1.84914, size = 675, normalized size = 3.05

$$-\frac{3\sqrt{2}\Big((7A+5B)\cos(dx+c)^4+4(7A+5B)\cos(dx+c)^3+6(7A+5B)\cos(dx+c)^2+4(7A+5B)\cos(dx+c)+7}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)^
3 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) - 2*((17*A + 67*B)*cos(d*x + c)^3 + 10*(7*A + 5*B)*cos(d*x
+ c)^2 + 3*(7*A + 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*
d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/
2)), x)
```

3.548 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=281

$$\frac{(5A-49B)\sin(c+dx)}{64a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(5A-177B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+$$

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) + ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin[c + d*x])
/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)) + ((5*A - 17*B)*Sin[c
+ d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A - 49
*B)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.919124, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A-49B)\sin(c+dx)}{64a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(5A-177B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) + ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin[c + d*x])
/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)) + ((5*A - 17*B)*Sin[c
+ d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A - 49
*B)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c}{}}{6a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} + \frac{(5A-177B)\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

Mathematica [C] time = 3.91197, size = 281, normalized size = 1.

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{8}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(4(25A-181B)\cos(c+dx)+(67A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(128*B*ArcSinh[E^(I*(c + d*x))] - S
qrt[2]*(5*A - 177*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*
I)*(c + d*x))])] - 128*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*
(c + d*x)) + ((97*A - 541*B + 4*(25*A - 181*B)*Cos[c + d*x] + (67*A - 247*B
)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2
] + Sin[(3*(c + d*x))/2]))/8))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.675, size = 667, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(-1+cos(d*x+c))^6*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*
(67*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-247*B*cos(d*x+
c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-384*B*cos(d*x+c)^2*sin(d*x+c
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)-1
7*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+15*A*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-115*B*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-768*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-531*B*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-35*A*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)+30*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c)*cos(d*x+c)+215*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)-384*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))*sin(d*x+c)-1062*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d
*x+c)-15*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*A*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)+147*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)-531*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(5/2)
/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^13
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 98.8766, size = 927, normalized size = 3.3

$$-\frac{3\sqrt{2}\left((5A-177B)\cos(dx+c)^4+4(5A-177B)\cos(dx+c)^3+6(5A-177B)\cos(dx+c)^2+4(5A-177B)\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x +
 c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5*A
 - 177*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) + 768*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*
B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((67*A - 247*B)*cos(d*
x + c)^3 + 2*(25*A - 181*B)*cos(d*x + c)^2 + 3*(5*A - 49*B)*cos(d*x + c))*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/
2)), x)
```

3.549 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=333

$$\frac{(79A-259B)\sin(c+dx)}{192a^2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}+\frac{(2A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{7(7A-}{64a^3d\sqrt{\sec(c+}}$$

```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - ((177*A - 637*B)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin
[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)) + ((3*A - 7*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + (
(79*A - 259*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c +
d*x]^(3/2)) - (7*(7*A - 27*B)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.20498, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(79A-259B)\sin(c+dx)}{192a^2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}+\frac{(2A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{7(7A-}{64a^3d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```


```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - ((177*A - 637*B)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin
[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)) + ((3*A - 7*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + (
(79*A - 259*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c +
d*x]^(3/2)) - (7*(7*A - 27*B)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}}{}}{6a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+}$$

$$= \frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} - \frac{(177A-637B}{}$$

Mathematica [C] time = 7.5768, size = 1017, normalized size = 3.05

$$\sqrt{\sec(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{(A-B)\tan\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{\sec\left(\frac{c}{2}\right)\left(53B\sin\left(\frac{dx}{2}\right)-41A\sin\left(\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{12d} - \frac{(41A-}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/
2)),x]
```

```
[Out] (((-49*I)/8)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c +
d*x]))^(7/2)) + (((189*I)/8)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*
(a*(1 + Cos[c + d*x]))^(7/2)) + ((8*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x
))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
+ d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/
(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(7/2)) - ((28*I)*Sqrt[2]*B*Sq
rt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]
*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]
)*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(7/2)
) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*(((-247*A + 427*B)*Cos[(d*x)/2
]*Sin[c/2])/(12*d) + (8*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d - ((247*A - 427*B)
*Cos[c/2]*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(379*A*Sin[
(d*x)/2] - 703*B*Sin[(d*x)/2]))/(24*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*(A*
Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-41
*A*Sin[(d*x)/2] + 53*B*Sin[(d*x)/2]))/(12*d) + (8*B*Cos[(3*c)/2]*Sin[(3*d*x
)/2])/d + ((379*A - 703*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - ((41*A - 5
3*B)*Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + ((A - B)*Sec[c/2 + (d*x)/2]^5*
Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x]))^(7/2)
```

Maple [B] time = 0.71, size = 855, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)*
(-192*B*cos(d*x+c)^4*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+384*A*2^(1/2
)*sin(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c))+247*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3
-1344*B*cos(d*x+c)^2*sin(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))*2^(1/2)-907*B*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+768*A*cos(d*x+c)*2^(1/2)*sin(d*x+c)*arctan(sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+115*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2+531*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
```

```
c)^2*sin(d*x+c)-2688*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-343*B*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2-1911*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d
*x+c)^2*sin(d*x+c)+384*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-215*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*cos(d*x+c)+1062*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d
*x+c)-1344*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))*sin(d*x+c)+875*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)-3822*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-147*A*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+531*A*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*sin(d*x+c)+567*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1911*B*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(7/2)/(cos(d*x
+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^15
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 172.239, size = 1041, normalized size = 3.13

$$\frac{3\sqrt{2}\big((177\,A-637\,B)\cos(dx+c)^4+4\,(177\,A-637\,B)\cos(dx+c)^3+6\,(177\,A-637\,B)\cos(dx+c)^2+4\,(177\,A-637}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(d*
x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x + c
) + 177*A - 637*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) - 384*((2*A - 7*B)*cos(d*x + c)^4 + 4*(2
```

```
*A - 7*B)*cos(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 + 4*(2*A - 7*B)*cos
(d*x + c) + 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))) + 2*(192*B*cos(d*x + c)^4 - (247*A - 1099*B)
*cos(d*x + c)^3 - 2*(181*A - 721*B)*cos(d*x + c)^2 - 21*(7*A - 27*B)*cos(d*
x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*co
s(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*co
s(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/
2)), x)
```

3.550 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=180

$$\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(3aA+5bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] (-2*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a*A + 5*b*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*
a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.223326, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(3aA+5bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a*A + 5*b*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*
a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))(B+A\sec(c+dx))\,dx$$
$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}(3aA\right.$$
$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + (Ab+aB)\int \sec^{\frac{5}{2}}(c+dx)$$
$$= \frac{2(3aA+5bB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(Ab+aB)\sec^{\frac{3}{2}}(}{3}$$
$$= \frac{2(3aA+5bB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(Ab+aB)\sec^{\frac{3}{2}}(}{3}$$
$$= -\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [A] time = 1.82451, size = 132, normalized size = 0.73

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20(aB+Ab)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 12(3aA+5bB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\sin(c+dx)(\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a*A + 5*b*B)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 20*(A*b + a*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
2*(15*(a*A + b*B) + 10*(A*b + a*B)*Cos[c + d*x] + 3*(3*a*A + 5*b*B)*Cos[2*(
c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] time = 10.268, size = 663, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a*A/(8*sin(1/2*d*x+1/2*c)^6-
12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sec(d*x + c)
^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.551 $\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx$

Optimal. Leaf size=143

$$\frac{2(aB + Ab)\sin(c + dx)\sqrt{\sec(c + dx)}}{d} + \frac{2(aA + 3bB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d} - \frac{2(aB + Ab)\sqrt{\cos(c + d}}{}$$

```
[Out] (-2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
+ (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.201043, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2(aB + Ab)\sin(c + dx)\sqrt{\sec(c + dx)}}{d} + \frac{2(aA + 3bB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d} - \frac{2(aB + Ab)\sqrt{\cos(c + d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
+ (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}(b+a\sec(c+dx))(B+A\sec(c+dx))\,dx$$
$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2}{3}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}(aA+3$$
$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + (Ab+aB)\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2(Ab+aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin}{3d}$$
$$= \frac{2(aA+3bB)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2(}{}$$
$$= -\frac{2(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + -\frac{2(}{}$$

Mathematica [A] time = 0.796936, size = 104, normalized size = 0.73

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 3(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right) + \frac{\sin(c+dx)(3(aB+Ab)\cos(c+dx)+aA)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B)*EllipticE[(c + d*x
)/2, 2] + (a*A + 3*b*B)*EllipticF[(c + d*x)/2, 2] + ((a*A + 3*(A*b + a*B)*C
os[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 8.347, size = 428, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*A
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sec(d*x + c)
^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.552 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA}{}$$

```
[Out] (-2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.180414, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3997, 3787, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
```

```
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + 2\int \frac{\frac{1}{2}(-aA+bB)+\frac{1}{2}(Ab+}{\sqrt{\sec(c+a}}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + (Ab+aB)\int \sqrt{\sec(c+dx)}\,d$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \Big((Ab+aB)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}$$

$$= -\frac{2(aA-bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(}{}$$

Mathematica [A] time = 0.258839, size = 85, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left((aB+Ab)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-(aA-bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+aA\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]) + (A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*A*Si
n[c + d*x]))/d
```

Maple [A] time = 3.681, size = 244, normalized size = 2.2

$$-2\,\frac{A\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b+A\mathrm{EllipticE}\left(\cos\left(1/2\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-2*A*a*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-B*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sec(d*x + c)
^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.553 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bE}{3d}$$

```
[Out] (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.18779, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bE}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB) - \frac{1}{2}(3aA+bB)\sec(c+$$

$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - (-Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{3}(-3aA$$

$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \left((-Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(3}{}$$

Mathematica [A] time = 0.227436, size = 90, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2(3aA+bB)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(aB+Ab)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+bB\sin(2(c+dx)$$
$$3d$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*B*S
in[2*(c + d*x)]))/(3*d)
```

Maple [B] time = 3.579, size = 326, normalized size = 2.8

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,Bb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+3\,aA\sqrt{\left(\sin\left(1/2\,dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*b+B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a-2*B*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(B\cos(dx+c)+A)(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d*x
 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+B\cos\left(c+dx\right)\right)\left(a+b\cos\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.554 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{5d}$$

```
[Out] (2*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.208288, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB)-\frac{1}{2}(5aA+3bB)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx - \frac{1}{5}(-5aA-3bB)\int \frac{}{\sqrt{}}$$

$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-Ab-aB)\int \sqrt{\sec(c+d}$$

$$= \frac{2(5aA+3bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2bB\sin(c+}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2(5aA+3bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(Ab+aB)\sqrt{}}{}$$

Mathematica [A] time = 0.527425, size = 108, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5aB+5Ab+3bB\cos(c+dx))+10(aB+Ab)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(5aA+3bB\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5
*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 3.483, size = 371, normalized size = 2.5

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,Bb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(20\,Ab+20\,aB+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b-15*A*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a+
5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-9*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/sqrt(sec(d*x
 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))(a+b\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.555 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB$$

```
[Out] (6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a*A + 5*b
*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.233885, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a*A + 5*b
*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{1}{2}(7aA+5bB)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx - \frac{1}{7}(-7aA-5bB)\int$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$
$$= \frac{6(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(7aA+5bB)}{}$$

Mathematica [A] time = 0.966796, size = 125, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(42(aB+Ab)\cos(c+dx)+70aA+15bB\cos(2(c+dx))+65bB)+20(7aA+5bB)\sqrt{\cos(c}\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 20*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
(70*a*A + 65*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*b*B*Cos[2*(c + d*x)])*S
in[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 3.358, size = 413, normalized size = 2.3

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,Bb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-168\,Ab-168\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+35*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+25*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/sec(d*x + c)
^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.556 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=221

$$\frac{2\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(3a^2A+5b(2aB+Ab)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}$$

```
[Out] (-2*(3*a^2*A + 5*b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a^2*A +
5*b*(A*b + 2*a*B))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(7*A*b + 5
*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sec[c + d*x]^(3/2)*(
b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.379852, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4026, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(3a^2A+5b(2aB+Ab)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*(3*a^2*A + 5*b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a^2*A +
5*b*(A*b + 2*a*B))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(7*A*b + 5
*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sec[c + d*x]^(3/2)*(
b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int \sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2(B+A\sec(c+dx))\,dx$$

$$=\frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\text{se}$$

$$=\frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\text{se}$$

$$=\frac{2\left(3a^2A+5b(Ab+2aB)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a(7A}{}$$

$$=\frac{2\left(3a^2A+5b(Ab+2aB)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a(7A}{}$$

$$=-\frac{2\left(3a^2A+5b(Ab+2aB)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{s}}}{5d}$$

Mathematica [A] time = 2.36768, size = 171, normalized size = 0.77

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20\left(a^2B+2aAb+3b^2B\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-12\left(3a^2A+10abB+5Ab^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(5/2)*
EllipticE[(c + d*x)/2, 2] + 20*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(5/
2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a^2*A + A*b^2 + 2*a*b*B) + 10*a*(2*A*
b + a*B)*Cos[c + d*x] + 3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[2*(c + d*x)])*
Sin[c + d*x]))/(30*d)
```

Maple [B] time = 10.918, size = 750, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b+2*B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)-2/5*a^2*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin
(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

3.557 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=177

$$\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,}{d}$$

```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A*b + 3*a*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Se
c[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.351616, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4026, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A*b + 3*a*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Se
c[c + d*x])*Sin[c + d*x])/(3*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int -$$

$$= \frac{2aA\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int -$$

$$= \frac{2a(5Ab+3aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA\sqrt{\sec(c+}}{}$$

$$= \frac{2\left(a^2A+3Ab^2+6abB\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3d}$$

$$= -\frac{2\left(2aAb+a^2B-b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{d}$$

Mathematica [A] time = 1.11888, size = 125, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+d}{}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(2*a*A*b + a^2*B - b^2*B)*Elli
pticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2,
2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/
2)))/(3*d)
```

Maple [B] time = 7.943, size = 677, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*B*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a*(2*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)+2*a^2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

3.558 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-b(2aB+Ab)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}$$

```
[Out] (-2*(a^2*A - b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.320127, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4024, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-b(2aB+Ab)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2*A - b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB) + \left(-3aAb + \left(-\frac{3a^2}{2}\right.\right.}{\sqrt{\mathrm{se}}}$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB) - \frac{3}{2}a^2A\sec^2(c+dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(a^2A -\right.$$

$$= \frac{2\left(6aAb+3a^2B+b^2B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3d}$$

$$= -\frac{2\left(a^2A-b(Ab+2aB)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{d}$$

Mathematica [A] time = 0.728235, size = 124, normalized size = 0.77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(-6a^2A+12abB+6Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2\sin(c}{}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-6*a^2*A + 6*A*b^2 + 12*a*b*B)*Ell
ipticE[(c + d*x)/2, 2] + 2*(6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/
2, 2] + (2*(3*a^2*A + b^2*B*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d*x]])
)/(3*d)
```

Maple [B] time = 3.925, size = 404, normalized size = 2.5

$$-\frac{2}{3\,d}\left(4\,Bb^2\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+6\,Aab\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2-6*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*B*a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

3.559 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{2\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{5d}$$

```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.336487, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4024, 4047, 3771, 2639, 4045, 2641}

$$\frac{2\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB) + \left(-5aAb + \left(-\frac{5a}{2}\right.\right.}{s}$$

$$= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB) - \frac{5}{2}a^2A\sec^2(c+}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(-3a^2A\right.$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5d}$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5d}$$

Mathematica [A] time = 0.882372, size = 128, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(10\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c-\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 10*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + b*(5*A*b + 10*a*B + 3*b*B*Cos[c + d*x])*Sin[2*
(c + d*x)]))/(15*d)
```

Maple [B] time = 3.695, size = 487, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*b^2*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos (dx+c)+A)(b\cos (dx+c)+a)^2\sqrt{\sec (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos (dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos (dx+c)^2+\left(Ba^2+2\,Aab\right)\cos (dx+c)\right)\sqrt{\sec (dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

3.560 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{2\left(7a^2B+14aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2B+14aAb+5b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(5a^2A\right.}{}$$

```
[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b^2*B*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.372137, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4024, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(7a^2B+14aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2B+14aAb+5b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(5a^2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```



```
[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b^2*B*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB) + \left(-7aAb + \left(-\frac{7a^2}{2} - \frac{5b^2}{2}\right)B\right)\text{se}}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB) - \frac{7}{2}a^2A\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx - \frac{1}{7}\left(-\right.$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(14aAb+7a^2B+5b^2\right.}{21d\sqrt{\sec(c+}}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(14aAb+7a^2B+5b^2\right.}{21d\sqrt{\sec(c+}}$$

$$= \frac{2\left(5a^2A+3Ab^2+6abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \cdots$$

Mathematica [A] time = 1.33155, size = 161, normalized size = 0.76

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(5\left(14a^2B+28aAb+3b^2B\cos(2(c+dx))+13b^2B\right)+42b(2aB+Ab)\cos(c+dx)\right)+20\left(7a\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 20*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (42*b*(A*b + 2*a*B)*Cos[c + d*x] + 5*(28*a*
A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(
210*d)
```

Maple [B] time = 3.781, size = 548, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^2-336*B*a*b-360*B*b^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*B*a
*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*b^2-
70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+35*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c+dx))\,(a + b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c) + A)(b\cos(dx+c) + a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

3.561 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=295

$$\frac{2a\left(5a^2A+21abB+18Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(9a^2Ab+3a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+$$

```
[Out] (-2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x]
)^2*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.597226, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4026, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a\left(5a^2A+21abB+18Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(9a^2Ab+3a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x]
)^2*Sin[c + d*x])/(7*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}(b+a\sec(c+dx))^3(B+A\sec(c+dx))\,dx$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{}$$

$$= \frac{2a^2(11Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2aA\sec^{\frac{3}{2}}(c+}{}$$

$$= \frac{2a^2(11Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2aA\sec^{\frac{3}{2}}(c+}{}$$

$$= \frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\sec(c+dx)}\sin(c+}{5d}$$

$$= \frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\sec(c+dx)}\sin(c+}{5d}$$

$$= -\frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 3.50839, size = 225, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(21\left(9a^2Ab+3a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)+5a\left(5a^2A+21abB+21Ab^2\right)\tan(c+dx)+5\left(5a^3A+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*a^3*A + 21*a*A*b^2 + 21*a^
2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(9*a^2*
A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 5*a*(5*a^2*A + 21*A*b^
2 + 21*a*b*B)*Tan[c + d*x] + 21*a^2*(3*A*b + a*B)*Sec[c + d*x]*Tan[c + d*x]
 + 15*a^3*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```

Maple [B] time = 13.595, size = 944, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
^2*(A*b+3*B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*
x+1/2*c)^2-1)-2/5*a^2*(3*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/
2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^3*(-1/5
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
```

```
cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b*
(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa^2b\right)\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

3.562 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=244

$$\frac{2a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{3d}$$

```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a
^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*d) + (2*a^2*(9*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*
d)
```

Rubi [A] time = 0.577069, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4026, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a
^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*d) + (2*a^2*(9*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*
d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(}{}$$

$$= \frac{2a^2(9Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2aA\sqrt{\sec(c+}}{}$$

$$= \frac{2a^2(9Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2aA\sqrt{\sec(c+}}{}$$

$$= \frac{2a\left(3a^2A+14Ab^2+15abB\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2}{}$$

$$= \frac{2\left(3a^2Ab+3Ab^3+a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{3d}$$

$$= -\frac{2\left(3a^3A+15aAb^2+15a^2bB-5b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\right.}{5d}$$

Mathematica [A] time = 1.55486, size = 192, normalized size = 0.79

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(5\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(3*a^3*A + 15*a*A*b^2 + 15*a^2
*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*A*b + 3*A*b^3 + a^3*B
+ 9*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (a*(15*(a^2*A + 3*A*b^2 + 3*a*b*B)
 + 10*a*(3*A*b + a*B)*Cos[c + d*x] + 9*(a^2*A + 5*A*b^2 + 5*a*b*B)*Cos[2*(c
 + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(5/2))))/(15*d)
```

Maple [B] time = 10.882, size = 997, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
6*a*b*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1)-2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*
sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(3*A*b+B*a)*(-1/
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\Big(\big(Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa^2b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

3.563 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=239

$$\frac{2a\left(3a^2B+9aAb-2b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3d}$$

```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b
^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*a*(9*a*A*b + 3*a^2*B - 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*a^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b*B*(b
+ a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.565625, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4025, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(3a^2B+9aAb-2b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```



```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b
^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*a*(9*a*A*b + 3*a^2*B - 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*a^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b*B*(b
+ a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{(b+a\sec(c+a}{}$$

$$= \frac{2a^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2bB(b+a\sec(c-}{3d\sqrt{\sec}}$$

$$= \frac{2a^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2bB(b+a\sec(c-}{3d\sqrt{\sec}}$$

$$= \frac{2a\left(9aAb+3a^2B-2b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2(}{}$$

$$= \frac{2\left(a^3A+9aAb^2+9a^2bB+b^3B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3d}$$

$$= -\frac{2\left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}$$

Mathematica [A] time = 1.86882, size = 166, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(3a^2Ab+a^3B-3ab^2B-Ab^3\right)E\left(\frac{1}{2}(c\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a
*b^2*B)*EllipticE[(c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*
B)*EllipticF[(c + d*x)/2, 2] + ((2*a^3*A + b^3*B + 6*a^2*(3*A*b + a*B)*Cos[
c + d*x] + b^3*B*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 10.385, size = 1212, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(8*B*b^3*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*
x+1/2*c)^2+18*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+18*A*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-6*A*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*b^3*sin(1/2*d*x+1/2*c)^2-36*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+18*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+2*B*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*s
in(1/2*d*x+1/2*c)^2-18*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^
2-12*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-8*B*b^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3-9*A*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*a*b^2-9*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b+3*A*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*b^3+2*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*A*a^2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b-B*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*b^3-3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
```

```
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3+9*B*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*a*b^2+6*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*B*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa^2b\right)\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

3.564 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=237

$$\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\sqrt{c}}{5}$$

```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a
^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*(5*A*b + 9*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(5*a*A - b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*B*
(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.532947, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4025, 4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\sqrt{c}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a
^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*(5*A*b + 9*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(5*a*A - b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*B*
(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{(b+a\sec(c+}{}$$

$$= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2bB(b+a\sec(c+dx))^2\sin}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2bB(b+a\sec(c+dx))^2\sin}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(5aA-bB)\sqrt{\sec(c+dx}}{5d}$$

$$= \frac{2\left(9a^2Ab+Ab^3+3a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{3d}$$

$$= -\frac{2\left(5a^3A-15aAb^2-15a^2bB-3b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 1.37629, size = 172, normalized size = 0.73

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(20\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+12\left(-5a^3A+15a^2bB+15aAb^2+3b^3\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(12*(-5*a^3*A + 15*a*A*b^2 + 15*a^2*
b*B + 3*b^3*B)*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*A*b + A*b^3 + 3*a^3*B
+ 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (2*(10*b^2*(A*b + 3*a*B)*Cos[c + d
*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/Sqrt[Cos
[c + d*x]]))/(30*d)
```

Maple [B] time = 4.28, size = 867, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*b^2*(5*A*b+15*B*a+6*B*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^3+5*A*b
^3+15*B*a*b^2+3*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+45*A*a^2*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3-45*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*B*a*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-45*B*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2*b-9*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

3.565 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{21d}$$

```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2))
```

Rubi [A] time = 0.543859, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4025, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```



```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{(b+a\sec(c+$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\left(21aAb+18a^2B+5b^2\right.}{21d\sqrt{\sec(c+d}}$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 1.2559, size = 180, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(b\sin(2(c+dx))\left(5\left(42a^2B+42aAb+3b^2B\cos(2(c+dx))+13b^2B\right)+42b(3aB+Ab)\cos(c+dx)\right)+20\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(21*a^3*A + 21*a*A*b^2 + 21*a^2
*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(42*b*(A*b
 + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b + 42*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*
(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] time = 3.599, size = 664, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^3-504*B*a*b^2-360*B*b^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*b+5
04*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a*b^2
-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-315*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b-63*A*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*b^3+105*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3+105*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^
2-105*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3-189*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2+1
05*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b+25*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\cos\left(dx+c\right)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+3\left(Ba^2b+Aab^2\right)\cos\left(dx+c\right)^2+\left(Ba^3+3\,Aa^2b\right)\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

3.566 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=295

$$\frac{2b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(21a^2Ab+7a^3B+15a\right.}{}$$

```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 +
 7*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(21*d) + (2*b^2*(9*A*b + 13*a*B)*Sin[c + d*x])/(63*d*Sec[c + d
*x]^(5/2)) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.578549, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4025, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(21a^2Ab+7a^3B+15a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 +
 7*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(21*d) + (2*b^2*(9*A*b + 13*a*B)*Sin[c + d*x])/(63*d*Sec[c + d
*x]^(5/2)) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{(b+a\sec(c+dx))\left(-\frac{1}{2}b(9A\right.}{}$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} +$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} +$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27aAb+22a^2B+7b^2B\right)\sin(c+d}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27aAb+22a^2B+7b^2B\right)\sin(c+d}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

Mathematica [A] time = 1.78156, size = 219, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(7b\left(108a^2B+108aAb+43b^2B\right)\cos(c+dx)+5\left(252a^2Ab+84a^3B+18b^2(3aB+Ab)\cos(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(21*a^2*A*b + 5*A*b^3 + 7*a
^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*b*(108
*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(252*a^2*A*b + 78*A*b^3 + 8
4*a^3*B + 234*a*b^2*B + 18*b^2*(A*b + 3*a*B)*Cos[2*(c + d*x)] + 7*b^3*B*Cos
[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] time = 3.452, size = 745, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^3+2160*B*a*b^2+2240*B*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-1512*B
*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12
60*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+952*B
*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b^2-240
*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
```

```
(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\text{co}}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

3.567 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=210

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{\cos(c-}{}$$

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a*d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*(A*b - a*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d)
```

Rubi [A] time = 0.807839, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4033, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{\cos(c-}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a*d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*(A*b - a*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$=\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}+\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{Ab}{2}+\frac{1}{2}aA\sec(c+dx)-\frac{3}{2}(Ab-aB)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}}{3a}$$

$$=-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}+\frac{4\int \frac{\frac{3}{4}b}{}}{}$$

$$=-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}+\frac{4\int \frac{\frac{3}{4}b}{}}{}$$

$$=-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}+\frac{A\int \sqrt{}}{}$$

$$=\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d}-\frac{2(Ab-aB)\sqrt{\mathrm{se}}}{}$$

$$=\frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{}$$

Mathematica [A] time = 3.40825, size = 229, normalized size = 1.09

$$-\frac{\cot(c+dx)\left(-2\left(a^2(A-3B)+3ab(A-B)+3Ab^2\right)\sqrt{-\tan^2(c+dx)}F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-a^2A\sec^{\frac{5}{2}}(c+dx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]*(-(a^2*A*Sec[c + d*x]^(5/2)) + a^2*A*Cos[2*(c + d*x)]*Sec[c
+ d*x]^(5/2) - 6*a*(-(A*b) + a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[-Tan[c + d*x]^2] - 2*(3*A*b^2 + a^2*(A - 3*B) + 3*a*b*(A - B))*Ellipt
icF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*A*b^2*Ellipti
cPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a*b*
B*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
))/(3*a^3*d)
```

Maple [A] time = 9.955, size = 468, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b-B*a)*b^
2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a^2*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.568 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{ad}$$

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(a*d)
```

Rubi [A] time = 0.458501, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4033, 4106, 3849, 2805, 12, 3771, 2639}

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(a*d)
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
```

```
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{2\int \frac{-\frac{Ab}{2}-\frac{1}{2}aA\sec(c+dx)-\frac{1}{2}(Ab-aB)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}\,dx}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{2\int -\frac{Ab^2}{2\sqrt{\sec(c+dx)}}\,dx}{ab^2} + \frac{(-Ab+aB)\int \frac{\sec^{\frac{3}{2}}(c+dx}{b+a\sec(c+d}}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} + \frac{\left((-Ab+aB)\sqrt{\cos(c+dx}\right.}{}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a+b)d} + \frac{2A\sqrt{\sec(c+}}{}$$

$$= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\Gamma}{a}$$

Mathematica [A] time = 1.26548, size = 127, normalized size = 1.01

$$-\frac{2\cos(2(c+dx))\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(-(aA-aB+Ab)F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+(aB-Ab)\right.}{a^2d\left(\sec^2(c+dx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[2*(c + d*x)]*Csc[c + d*x]*(a*A*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] - (a*A + A*b - a*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (-(A*
b) + a*B)*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]
*Sqrt[-Tan[c + d*x]^2])/(a^2*d*(-2 + Sec[c + d*x]^2))
```

Maple [A] time = 7.079, size = 327, normalized size = 2.6

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-4\,\frac{\left(-Ab+aB\right)b\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}}}{a\left(-2\,ab+2\,b^{2}\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b+B*a)/a
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.569 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d)
```

Rubi [A] time = 0.282784, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4038, 3771, 2641, 3849, 2805}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```



```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
```

```
NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$= \frac{B\int \sqrt{\sec(c+dx)}\,dx}{b} - \frac{(-Ab+aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{b}$$

$$= \frac{\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{\left((-Ab+aB)\sqrt{\cos(c+dx)}\sqrt{s\ldots}\right.}{\ldots}$$

$$= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\ldots\right.}{b(a\ldots}$$

Mathematica [A] time = 0.52218, size = 78, normalized size = 0.77

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left((Ab-aB)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+AbF\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(A*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b - a*B
)*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2
])/(a*b*d)
```

Maple [A] time = 3.706, size = 217, normalized size = 2.2

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}}{b\left(a-b\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\left(A\mathrm{EllipticP}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))*a+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*b)/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos{(c + dx)})\sqrt{\sec{(c + dx)}}}{a + b\cos{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos{(dx + c)} + A)\sqrt{\sec{(dx + c)}}}{b\cos{(dx + c)} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.570 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(b^2*d) - (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rubi [A] time = 0.346273, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4038, 3771, 2639, 3848, 2803, 2641, 2805}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(b^2*d) - (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
```

```
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}\,dx$$
$$=\frac{B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b}-\frac{(-Ab+aB)\int \frac{\sqrt{\sec(c+dx)}}{b+a\sec(c+dx)}\,dx}{b}$$
$$=\frac{\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b}-\frac{\left((-Ab+aB)\sqrt{\cos(c+dx)}\sqrt{\right.}{}$$
$$=\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd}-\frac{\left((-Ab+aB)\sqrt{\cos(c+dx)}\sqrt{\mathrm{sec}}\right.}{b^2}$$
$$=\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{b^2d}$$

Mathematica [A] time = 6.14245, size = 224, normalized size = 1.5

$$\frac{\cot(c+dx)\left(-2Ab\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2aB\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cot[c + d*x]*(-(b*B*Sec[c + d*x]^(3/2)) - b*B*Cos[2*(c + d*x)]*Sec[c + d*x
]^(3/2) + b*B*Sec[c + d*x]^(7/2) + b*B*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2)
- 2*b*B*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2
*b*B*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*A*
b*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
 + 2*a*B*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c +
d*x]^2]))/(b^2*d)
```

Maple [A] time = 3.76, size = 295, normalized size = 2.

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}}{(a-b)\,b^2\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\left(A\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-A*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+B*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)\sqrt{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

3.571 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{2\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b}\right.}{b^3d(a+b)}$$

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) + (2*a^2*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b
^3*(a + b)*d) + (2*B*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.559735, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4034, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b}\right.}{b^3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) + (2*a^2*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b
^3*(a + b)*d) + (2*B*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{2\int \frac{-\frac{3}{2}(Ab-aB)-\frac{1}{2}bB\sec(c+dx)-\frac{1}{2}aB\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}\,dx}{3b}$$

$$= \frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{2\int \frac{-\frac{3}{2}b(Ab-aB)-\left(\frac{b^2B}{2}-\frac{3}{2}a(Ab-aB)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3b^3} + \frac{\left(a^2(Ab-aB)\right)}{}$$

$$= \frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b^2} - \frac{\left(3aAb-3a^2B-b^2B\right)\int \sqrt{\sec(}}{3b^3}$$

$$= \frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d} + \frac{2B\sin(c+d}{3bd\sqrt{\sec(c+}}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} - \frac{2\left(3aAb-3a^2B-b^2\right.}{}$$

Mathematica [A] time = 6.51574, size = 282, normalized size = 1.43

$$\frac{2\csc(c+dx)\left(-3a^2B\sqrt{-\tan^2(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+b(-3aB+3Ab+bB)\sqrt{-\tan^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Csc[c + d*x]*(-3*A*b^2 + 3*a*b*B + 3*A*b^2*Sec[c + d*x]^2 - 3*a*b*B*Sec[
c + d*x]^2 + b^2*B*Sin[c + d*x]*Tan[c + d*x] - 3*b*(A*b - a*B)*EllipticE[Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + b*
(3*A*b - 3*a*B + b*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] + 3*a*A*b*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*a^2*B*Ellipti
cPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c
 + d*x]^2]))/(3*b^3*d*Sec[c + d*x]^(3/2))
```

Maple [B] time = 3.815, size = 786, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-4*B*a*b^2+4*
B*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(2*B*a*b^2-2*B*b^3)*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*a*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*b^3-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b-3*a^3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b-B*a*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^2*b+3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))*a^3)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

3.572 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=405

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{\left(4a^2Ab-2a^3B+3ab^2B-5A\right.}{a^3d\left(a^2\right.}$$

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^
2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^3*(a - b)*(a + b)^2*d) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*
B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b
^2 + 3*a*b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(
A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c +
 d*x]))
```

Rubi [A] time = 1.28872, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{\left(4a^2Ab-2a^3B+3ab^2B-5A\right.}{a^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^
2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^3*(a - b)*(a + b)^2*d) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*
B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b
^2 + 3*a*b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(
A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c +
```

d*x]))

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
```

```
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}b(Ab-aB)+a(Ab-aB)\sec(c+dx)-\right.}{b+a\sec(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin}{a\left(a^2-b^2\right)d(b+a\sec(c}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2\right.}{}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2\right.}{}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2\right.}{}$$

$$= \frac{b\left(7a^2Ab-5Ab^3-5a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{a^3(a-b)(a+b)^2d}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} +$$

<u>Mathematica</u> [A] time = 7.05589, size = 741, normalized size = 1.83

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(-4a^2Ab+2a^3B-3ab^2B+5Ab^3\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)} + \frac{ab^2B\sin(c+dx)-Ab^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{2A\tan(c+dx)}{3a^2}\right)}{d} + \frac{-\frac{2\left(-28a^3Ab-24a^2b^2B+12a^4B+40aAb\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((-2*(-28*a^3*A*b + 40*a*A*b^3 + 12*a^4*B - 24*a^2*b^2*B)*Cos[c + d*x]^2*El
lipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt
[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x
```

```
]^2)) + (2*(-4*a^4*A - 44*a^2*A*b^2 + 45*A*b^4 + 30*a^3*b*B - 27*a*b^3*B)*C
os[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b
), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c +
d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-12
*a^2*A*b^2 + 15*A*b^4 + 6*a^3*b*B - 9*a*b^3*B)*Cos[2*(c + d*x)]*(b + a*Sec[
c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b
*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[
c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x
]]*(2 - Sec[c + d*x]^2)))/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]
*(((-4*a^2*A*b + 5*A*b^3 + 2*a^3*B - 3*a*b^2*B)*Sin[c + d*x])/(a^3*(a^2 - b
^2)) + (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a +
 b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] time = 16.78, size = 1031, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(2*A*b-B
*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-2*A*b+B*a)/a^3*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(A*b-B*a)*b/a^2*
(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-
b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^2*(-1/6*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

3.573 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=316

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{ad\left(a^2-b^2\right.}$$

```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d)) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - ((5*a^2*
A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 3*A*b^2 + a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*
Sec[c + d*x]))
```

Rubi [A] time = 0.941633, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{ad\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d)) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - ((5*a^2*
A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 3*A*b^2 + a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*
Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$=\frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))}-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}b(Ab-aB)+a(Ab-aB)\sec(c+dx\right.}{b+a\sec(c+d}}{a\left(a^2-b^2\right.}$$

$$=\frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d}+\frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin}{a\left(a^2-b^2\right)d(b+a\sec(c}$$

$$=\frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d}+\frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin}{a\left(a^2-b^2\right)d(b+a\sec(c}$$

$$=\frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d}+\frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin}{a\left(a^2-b^2\right)d(b+a\sec(c}$$

$$=-\frac{\left(5a^2Ab-3Ab^3-3a^3B+ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{a^2(a-b)(a+b)^2d}$$

$$=-\frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d}+\frac{(Ab-}{}$$

Mathematica [B] time = 6.93585, size = 687, normalized size = 2.17

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)}+\frac{Ab^2\sin(c+dx)-abB\sin(c+dx)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d}-\frac{-\frac{2\left(4a^3A+4a^2bB-8aAb^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\,\mathrm{se}}{b\left(1-\cos^2(c+dx)\right)(a+b\cos(c}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] -((-2*(4*a^3*A - 8*a*A*b^2 + 4*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^
2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(10*a^2
*A*b - 9*A*b^3 - 4*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt
[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])
*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[
```

```
c + d*x])*(1 - Cos[c + d*x]^2)) + ((2*a^2*A*b - 3*A*b^3 + a*b^2*B)*Cos[2*(c
 + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ellipt
icE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x
]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*Ellipt
icPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*a^2*(a - b)*(a + b)*d) + (
Sqrt[Sec[c + d*x]]*(((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a^2*(a^2 -
b^2)) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Cos
[c + d*x]))))/d
```

Maple [B] time = 10.849, size = 883, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^2/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2/a^2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b+B*a)/a*(-1/a*b^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos
(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
```

```
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

3.574 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=260

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{}}{ad\left(\right.}$$

```
[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*
B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.612991, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4029, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{}}{ad\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*
B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
```

```
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}b(Ab-aB)+a(Ab-aB)\sec(c+dx)-\frac{1}{2}\left(2a^2A-Ab\right.}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}}{a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}b^2(Ab-aB)+\frac{1}{2}ab(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{ab^2\left(a^2-b^2\right)} + \cdots$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a\left(a^2-b^2\right)} - \frac{(Ab-aB)}{2\cdots}$$

$$= \frac{\left(3a^2Ab-Ab^3-a^3B-ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a-b)b(a+b)^2d}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} - \frac{(Ab-aB)\sqrt{\cos(c+\cdots}}{\cdots}$$

Mathematica [B] time = 6.81852, size = 645, normalized size = 2.48

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{aB\sin(c+dx)-Ab\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{(aB-Ab)\sin(c+dx)}{a\left(a^2-b^2\right)}\right)}{d} + \frac{2\left(-4a^2A+abB+3Ab^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\sec(c+dx)+b)\left(\Pi(-\cdots\right.}{a\left(1-\cos^2(c+dx)\right)(a+b\cos(c+\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((-2*(4*a*A*b - 4*a^2*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-4*a^2*A + 3*A*b^2 +
a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Elliptic
Pi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 + ((A*b^2 - a*b*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*S
ec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Ellipti
cPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Se
c[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos
[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(
4*a*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-(((-(A*b) + a*B)*Sin[c + d*
x])/(a*(a^2 - b^2))) + (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/((a^2 - b^2
)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 8.523, size = 721, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)
^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^
2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
```

```
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos{(c + dx)})\sqrt{\sec{(c + dx)}}}{(a + b\cos{(c + dx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2, x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

3.575 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=258

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{c}}{}$$

```
[Out] ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B - 2*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A
*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.605312, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4027, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B - 2*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A
*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}(Ab-aB)+(aA-bB)\sec(c+dx)-\frac{1}{2}(Ab-aB)\sec}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}}{a^2-b^2}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}b(Ab-aB)-\left(\frac{1}{2}a(Ab-aB)-b(aA-bB)\right)\sec(c+d}{\sqrt{\sec(c+dx)}}}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2b\left(a^2-b^2\right)} + \frac{(aAb+a}{}$$

$$= -\frac{\left(a^2Ab+Ab^3+a^3B-3ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+a}}{(a-b)b^2(a+b)^2d}$$

$$= \frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d} + \frac{\left(aAb+a^2B-2b^2B\right)}{}$$

Mathematica [B] time = 6.77587, size = 632, normalized size = 2.45

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\sin(c+dx)}{b\left(b^2-a^2\right)}+\frac{a^2B\sin(c+dx)-aAb\sin(c+dx)}{b\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d} + \frac{(Ab-aB)\sin(c+dx)\cos(2(c+dx))(a\sec(c+dx)+b)\left(4a^2\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] ((-2*(4*a*A - 4*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b
*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-(A*b) + a*B)*Cos[c + d*x
]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin
[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Si
n[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((A*b - a*B)*Co
s[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*
EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c
+ d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*d) +
 (Sqrt[Sec[c + d*x]]*(((A*b - a*B)*Sin[c + d*x])/(b*(-a^2 + b^2)) + (-(a*A*
b*Sin[c + d*x]) + a^2*B*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))
))/d
```

Maple [B] time = 9.402, size = 808, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-1/a*b^2/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
```

```
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2
-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

3.576 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2Ab-3a^3B+4ab^2B-2Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{(-$$

```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B +
 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.65849, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4030, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2Ab-3a^3B+4ab^2B-2Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{(-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```



```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B +
 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-aAb+3a^2B-2b^2B\right)-b(Ab-aB)\sec(c+dx)+\frac{1}{2}}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx}}{b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}b\left(-aAb+3a^2B-2b^2B\right)-\left(b^2(Ab-aB)+\frac{1}{2}a\left(-aA\right.\right.}{\sqrt{\sec(c+dx)}}}{b^3\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\left(aAb-3a^2B+2b^2B\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{a\left(a^2Ab-3Ab^3-3a^3B+5ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{(a-b)b^3(a+b)^2d}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} + \frac{\left(a^2Ab\right.}{}$$

Mathematica [B] time = 6.88642, size = 661, normalized size = 2.33

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a^2Ab\sin(c+dx)-a^3B\sin(c+dx)}{b^2\left(b^2-a^2\right)(a+b\cos(c+dx))}-\frac{a(aB-Ab)\sin(c+dx)}{b^2\left(a^2-b^2\right)}\right)}{d} + \frac{2\left(a^2(-B)-aAb+2b^2B\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\sec(c+dx)+b)(}{a\left(1-\cos^2(c+dx)\right)(a+b\,\mathrm{co}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] ((-2*(4*A*b^2 - 4*a*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-(a*A*b) - a^2*B + 2*
b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Elliptic
Pi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 + ((a*A*b - 3*a^2*B + 2*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a
*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4
*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^
2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c +
 d*x]^2)))/(4*b*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((a*(-(A*b) + a
*B)*Sin[c + d*x])/(b^2*(a^2 - b^2))) + (a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c
+ d*x])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 10.895, size = 849, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-2
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-
B*a)/b^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
```

```
2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

3.577 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\left(9a^3Ab+16a^2b^2B-15a^4B-12a\right.}{}$$

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a
*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^4*(a + b)^2*d) - ((3*a*A*b
 - 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]
) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a
*Sec[c + d*x]))
```

Rubi [A] time = 0.962477, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4030, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\left(9a^3Ab+16a^2b^2B-15a^4B-12a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a
*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^4*(a + b)^2*d) - ((3*a*A*b
 - 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]
) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a
*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2}\,dx$$

$$=\frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}\left(-3aAb+5a^2B-2b^2B\right)-b(Ab-aB)\sec}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec}}{b\left(a^2-\right.}$$

$$=-\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}}+\frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(b+a\sec(c+}$$

$$=-\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}}+\frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(b+a\sec(c+}$$

$$=-\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}}+\frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(b+a\sec(c+}$$

$$=\frac{a^2\left(3a^2Ab-5Ab^3-5a^3B+7ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{(a-b)b^4(a+b)^2d}$$

$$=\frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3\left(a^2-b^2\right)d}-$$

Mathematica [A] time = 6.99334, size = 707, normalized size = 1.95

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a^2(aB-Ab)\sin(c+dx)}{b^3\left(a^2-b^2\right)}-\frac{a^3Ab\sin(c+dx)-a^4B\sin(c+dx)}{b^3\left(b^2-a^2\right)(a+b\cos(c+dx))}+\frac{B\sin(2(c+dx))}{3b^2}\right)}{d}-\frac{-\frac{2\left(8a^2bB-12aAb^2+4b^3B\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(}}{b\left(1-\cos^2(c+dx)\right)(}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] -((-2*(-12*a*A*b^2 + 8*a^2*b*B + 4*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-3*a
^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sq
rt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1
])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Co
```

```
s[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b + 6*A*b^3 + 15*a^3*B - 12*
a*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]
^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*
(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(12*(a - b)*
b^2*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((a^2*(-(A*b) + a*B)*Sin[c + d*x])/(b^
3*(a^2 - b^2)) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d*x])/(b^3*(-a^2 + b
^2)*(a + b*Cos[c + d*x])) + (B*Sin[2*(c + d*x)])/(3*b^2)))/d
```

Maple [B] time = 13.194, size = 1066, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-9*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b-
B*a)/b^4*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
```

```
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

3.578 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=480

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(11a^2Ab-7a^3B+ab^2B-5Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-29a^2Ab^2+8a^4A\right.}{}$$

```
[Out] -((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d)
 + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A*b
 - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B
 - 3*a*b^3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d) + (b
*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec
[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 1.45029, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2960, 4029, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(11a^2Ab-7a^3B+ab^2B-5Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-29a^2Ab^2+8a^4A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d)
 + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A*b
 - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B
 - 3*a*b^3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d) + (b
```

```
*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec
[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
```

```
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}b(Ab-aB)+2a(Ab-aB)\sec(c+d\ldots\right.}{(b+a\sec(c+d\ldots}}{2a\left(a^2-b\ldots\right.}$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+ab^2B\right)\sec\ldots}{4a^2\left(a^2-b^2\right)^2d(b+a\sec\ldots}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d} + \frac{b(\ldots}{2\ldots}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d} + \frac{b(\ldots}{2\ldots}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d} + \frac{b(\ldots}{2\ldots}$$

$$= -\frac{\left(35a^4Ab-38a^2Ab^3+15Ab^5-15a^5B+6a^3b^2B-3ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\ldots\right.}{4a^3(a-b)^2(a+b)^3d}$$

$$= -\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\ldots}{4a^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.16484, size = 850, normalized size = 1.77

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2} + \frac{Ab^2\sin(c+dx)-abB\sin(c+dx)}{2a\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{-5A\sin(c+dx)b^4+aB\sin(c+dx)b^3+11a^2A\sin(c+\ldots}{4a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx\ldots}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] -((-2*(16*a^5*A - 80*a^3*A*b^2 + 40*a*A*b^4 + 32*a^4*b*B - 8*a^2*b^3*B)*Cos
[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[
c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)) + (2*(56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B +
 19*a^3*b^2*B - 9*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*
x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + ((8*a^4*A*b - 29*a^2*A*b^3 + 15*A*b^5 + 9*a^3*b^2*B
- 3*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c +
d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a
/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a^3*
(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((8*a^4*A - 29*a^2*A*b^2 + 15
*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2) + (A*b^
2*Sin[c + d*x] - a*b*B*Sin[c + d*x])/(2*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^
2) + (11*a^2*A*b^2*Sin[c + d*x] - 5*A*b^4*Sin[c + d*x] - 7*a^3*b*B*Sin[c +
d*x] + a*b^3*B*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 19.237, size = 2002, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b+B*a)/a*(-1/2/a*b^2/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*c
```

os(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*
x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+
b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*
a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*A*b/a^2*(-1/
a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

3.579 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=405

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(7a^2Ab-3a^3B-3a\right.}{}$$

```
[Out] -((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A
*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 + 3*
A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b*(a + b)^
3*d) + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(
b + a*Sec[c + d*x])^2) + (b*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 1.03566, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(7a^2Ab-3a^3B-3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A
*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 + 3*
A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b*(a + b)^
3*d) + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(
b + a*Sec[c + d*x])^2) + (b*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}b(Ab-aB)+2a(Ab-aB)\sec(c+d}{(b+a\sec(c+d}}{2a\left(a^2-b\right.}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\text{se}}}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\text{se}}}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\text{se}}}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec}$$

$$= \frac{\left(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\right.}{4a^2(a-b)^2b(a+b)^3d}$$

$$= -\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 6.92288, size = 803, normalized size = 1.98

$$\frac{-\frac{2\left(16Ba^4-32Aba^3+8b^2Ba^2+8Ab^3a\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(16Aa^4-9bBa^3-19Ab^2a^2+3b\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((-2*(-32*a^3*A*b + 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B)*Cos[c + d*x]^2*Elli
pticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1
 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)) + (2*(16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4 - 9*a^3*b*B + 3*a*b^3*B)*Cos[c
+ d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -A
```

```
rcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^
2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A
*b^2 + 3*A*b^4 + 5*a^3*b*B + a*b^3*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])
*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])
/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - S
ec[c + d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-((-
9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2
) + (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/(2*(a^2 - b^2)*(a + b*Cos[c +
d*x])^2) + (-7*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 3*a^3*B*Sin[c +
d*x] + 3*a*b^2*B*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 15.027, size = 1744, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b*(
-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(
a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*
b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
```

```
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
5/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),
2^(1/2)))+2*B/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^
2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

3.580 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=402

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(3a^2Ab+a^3B-7a\right.}{}$$

```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*
b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^2*(a + b)^3*d)
+ (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a
*Sec[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 1.08968, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(3a^2Ab+a^3B-7a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*
b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^2*(a + b)^3*d)
+ (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a
*Sec[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\int \frac{\frac{1}{2}b(Ab-aB)+2a(Ab-aB)\sec(c+dx)-\frac{1}{2}\left(4a^2A-Ab^2\right.}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right)\sqrt{\sec(c}{4a\left(a^2-b^2\right)^2d(b+a\sec(c+}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right)\sqrt{\sec(c}{4a\left(a^2-b^2\right)^2d(b+a\sec(c+}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right)\sqrt{\sec(c}{4a\left(a^2-b^2\right)^2d(b+a\sec(c+}$$

$$= -\frac{\left(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(\right.}{4a(a-b)^2b^2(a+b)^3d}$$

$$= \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4ab\left(a^2-b^2\right)^2d} + \frac{(3}{}$$

Mathematica [A] time = 6.90651, size = 790, normalized size = 1.97

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(-5a^2Ab+a^3B+5ab^2B-Ab^3\right)\sin(c+dx)}{4ab\left(a^2-b^2\right)^2} - \frac{aAb\sin(c+dx)-a^2B\sin(c+dx)}{2b\left(b^2-a^2\right)(a+b\cos(c+dx))^2} + \frac{3a^2Ab\sin(c+dx)+a^3B\sin(c+dx)-7ab^2B\sin(c+dx)+3Ab^3\sin(c+}{4b\left(b^2-a^2\right)^2(a+b\cos(c+dx))}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] ((-2*(16*a^3*A + 8*a*A*b^2 - 24*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b),
-ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]
^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-9*a^
2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[
Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*
```

```
(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c
 + d*x])*(1 - Cos[c + d*x]^2)) + ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*C
os[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b
*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2
*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c
 + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a*(a - b)^2*(a + b
)^2*d) + (Sqrt[Sec[c + d*x]]*(((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Sin
[c + d*x])/(4*a*b*(a^2 - b^2)^2) - (a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]
)/(2*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (3*a^2*A*b*Sin[c + d*x] + 3*A
*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*x])/(4*b*(-a^2
 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 15.087, size = 1850, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*
x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^
2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b
^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^
2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/
a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(A*b-2*B*a)/b^2*(-1/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+
1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

3.581 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=400

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(a^3Ab-5a^2b^2B+3a^4\right.}{}$$

```
[Out] -((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7*a
*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*a^
2*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]]*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*
b^3*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 -
b^2)*d*(b + a*Sec[c + d*x])^2) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x])
)
```

Rubi [A] time = 0.955012, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4027, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(a^3Ab-5a^2b^2B+3a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7*a
*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*a^
2*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]]*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*
b^3*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 -
b^2)*d*(b + a*Sec[c + d*x])^2) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x])
```

```
)
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
```

```
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec(c+dx)}(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{\int \frac{\frac{1}{2}(Ab-aB)+2(aA-bB)\sec(c+dx)-\frac{3}{2}(Ab-aB)\,\mathrm{se}}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}}{2\left(a^2-b^2\right)}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2B\right)\sqrt{\sec}}{4b\left(a^2-b^2\right)^2d(b+a\sec(}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2B\right)\sqrt{\sec}}{4b\left(a^2-b^2\right)^2d(b+a\sec(}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2B\right)\sqrt{\sec}}{4b\left(a^2-b^2\right)^2d(b+a\sec(}$$

$$=-\frac{\left(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\right.}{4(a-b)^2b^3(a+b)^3d}$$

$$=-\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 6.91622, size = 792, normalized size = 1.98

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2}-\frac{a^3B\sin(c+dx)-a^2Ab\sin(c+dx)}{2b^2\left(b^2-a^2\right)(a+b\cos(c+dx))^2}+\frac{a^3Ab\sin(c+dx)+11a^2b^2B\sin(c+dx)-5a^4B\sin(c+dx)-7aAb^3}{4b^2\left(b^2-a^2\right)^2(a+b\cos(c+dx))}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] -((-2*(24*a*A*b^2 - 8*a^2*b*B - 16*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-5*a
^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[S
ec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(
```

```
b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c
+ d*x])*(1 - Cos[c + d*x]^2)) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*
Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*
b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^
2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[
c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*(a - b)^2*b*(a +
b)^2*d) + (Sqrt[Sec[c + d*x]]*(((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*S
in[c + d*x])/(4*b^2*(a^2 - b^2)^2) - (-(a^2*A*b*Sin[c + d*x]) + a^3*B*Sin[c
 + d*x])/(2*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (a^3*A*b*Sin[c + d*x
] - 7*a*A*b^3*Sin[c + d*x] - 5*a^4*B*Sin[c + d*x] + 11*a^2*b^2*B*Sin[c + d*
x])/(4*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 15.242, size = 1937, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-B*a)/b^3*(-1/2/a*b
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)
*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
)-2*a/b^3*(2*A*b-3*B*a)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2
/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*
a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2
))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

3.582 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=427

$$\frac{a\left(a^2Ab-5a^3B+11ab^2B-7Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(-5a^2Ab^3+3a^4A\right.}{}$$

```
[Out] -((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d)
 + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4
*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4
*(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*
a^3*b^2*B - 35*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + (a*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^
2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(4*b^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 1.05701, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4030, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\left(a^2Ab-5a^3B+11ab^2B-7Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(-5a^2Ab^3+3a^4A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d)
 + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4
*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4
*(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*
a^3*b^2*B - 35*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + (a*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^
2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c
```

```
 + d*x])/(4*b^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-aAb+5a^2B-4b^2B\right)-2b(Ab-aB)\sec(c+dx)+\frac{3}{2}a}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2B\right)\sqrt{\sec}}{4b^2\left(a^2-b^2\right)^2d(b+a\sec(}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2B\right)\sqrt{\sec}}{4b^2\left(a^2-b^2\right)^2d(b+a\sec(}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2B\right)\sqrt{\sec}}{4b^2\left(a^2-b^2\right)^2d(b+a\sec(}$$

$$= -\frac{a\left(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\right.}{4(a-b)^2b^4(a+b)^3d}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{4b^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.0666, size = 826, normalized size = 1.93

$$\frac{-\frac{2\left(16Ab^4-32aBb^3+8a^2Ab^2+8a^3Bb\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(5Ba^4-Aba^3-7b^2Ba^2-5Ab^3a+\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] ((-2*(8*a^2*A*b^2 + 16*A*b^4 + 8*a^3*b*B - 32*a*b^3*B)*Cos[c + d*x]^2*Ellip
ticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) + (2*(-(a^3*A*b) - 5*a*A*b^3 + 5*a^4*B - 7*a^2*b^2*B + 8*b^4*B)*Cos[c +
d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -Arc
Sin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]
```

```
*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-3*a^3*A*b
 + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*Cos[2*(c + d*x)]*(b + a*S
ec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b
)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*
Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*S
in[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c +
d*x]]*(2 - Sec[c + d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c +
 d*x]]*(-(a*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/(4*
b^3*(a^2 - b^2)^2) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d*x])/(2*b^3*(-a
^2 + b^2)*(a + b*Cos[c + d*x])^2) + (-5*a^4*A*b*Sin[c + d*x] + 11*a^2*A*b^3
*Sin[c + d*x] + 9*a^5*B*Sin[c + d*x] - 15*a^3*b^2*B*Sin[c + d*x])/(4*b^3*(-
a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 17.077, size = 1977, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-3
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+12*a/b^3*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b-B
*a)/b^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-
b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2
```

```
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2
)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)
^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2)))+2*a^2/b^4*(3*A*b-4*B*a)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*
x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

3.583 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=521

$$\frac{a\left(3a^2Ab-7a^3B+13ab^2B-9Ab^3\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}+\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\left(15a^3Ab+61a^2b^2B\right.}{12b^3d}$$

```
[Out] ((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*
B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*
(a^2 - b^2)^2*d) - ((45*a^5*A*b - 99*a^3*A*b^3 + 72*a*A*b^5 - 105*a^6*B + 2
23*a^4*b^2*B - 128*a^2*b^4*B - 8*b^6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^5*(a^2 - b^2)^2*d) + (a^2*(15*a^4*A*b -
 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(
a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b
^2*B - 8*b^4*B)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) +
 (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a*
Sec[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[c +
d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 1.52877, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2960, 4030, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\left(3a^2Ab-7a^3B+13ab^2B-9Ab^3\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}+\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\left(15a^3Ab+61a^2b^2B\right.}{12b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*
B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*
(a^2 - b^2)^2*d) - ((45*a^5*A*b - 99*a^3*A*b^3 + 72*a*A*b^5 - 105*a^6*B + 2
23*a^4*b^2*B - 128*a^2*b^4*B - 8*b^6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^5*(a^2 - b^2)^2*d) + (a^2*(15*a^4*A*b -
 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(
```

```
a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b
^2*B - 8*b^4*B)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) +
 (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a*
Sec[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[c +
d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x]))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
```

```
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^3}\,dx$$

$$=\frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}+\frac{\int \frac{\frac{1}{2}\left(-3aAb+7a^2B-4b^2B\right)-2b(Ab-a}{\sec^{\frac{3}{2}}(c+dx)(}}{2b(}$$

$$=\frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}+\frac{a\left(3a^2Ab-9Ab^3-7a^3B+\right.}{4b^2\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx}}$$

$$=-\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}+\frac{a}{2b\left(a^2-b^2\right)d}$$

$$=-\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}+\frac{a}{2b\left(a^2-b^2\right)d}$$

$$=-\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}+\frac{a}{2b\left(a^2-b^2\right)d}$$

$$=\frac{a^2\left(15a^4Ab-38a^2Ab^3+35Ab^5-35a^5B+86a^3b^2B-63ab^4B\right)\sqrt{\cos(c+dx)}}{4(a-b)^2b^5(a+b)^3d}$$

$$=\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.}{4b^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.29926, size = 871, normalized size = 1.67

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(11Ba^3-7Aba^2-17b^2Ba+13Ab^3\right)\sin(c+dx)a^2}{4b^4\left(a^2-b^2\right)^2}-\frac{a^5B\sin(c+dx)-a^4Ab\sin(c+dx)}{2b^4\left(b^2-a^2\right)(a+b\cos(c+dx))^2}+\frac{-13B\sin(c+dx)a^6+9Ab\sin(c+dx)a^5+19b^2B\sin(c+dx}{4b^4\left(b^2-a^2\right)^2(a+b\cos(c+dx))}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),
x]
```

```
[Out] -((-2*(-24*a^3*A*b^2 + 96*a*A*b^4 + 56*a^4*b*B - 112*a^2*b^3*B - 16*b^5*B)*
Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*S
ec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) + (2*(-15*a^4*A*b + 21*a^2*A*b^3 - 24*A*b^5 + 35*a^5
*B - 73*a^3*b^2*B + 56*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
 + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b +
a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)) + ((-45*a^4*A*b + 87*a^2*A*b^3 - 24*A*b^5 + 105*a
^5*B - 195*a^3*b^2*B + 72*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-
4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
+ 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a
*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[
c + d*x]^2)))/(48*(a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((a^2*(-
7*a^2*A*b + 13*A*b^3 + 11*a^3*B - 17*a*b^2*B)*Sin[c + d*x])/(4*b^4*(a^2 - b
^2)^2) - (-(a^4*A*b*Sin[c + d*x]) + a^5*B*Sin[c + d*x])/(2*b^4*(-a^2 + b^2)
*(a + b*Cos[c + d*x])^2) + (9*a^5*A*b*Sin[c + d*x] - 15*a^3*A*b^3*Sin[c + d
*x] - 13*a^6*B*Sin[c + d*x] + 19*a^4*b^2*B*Sin[c + d*x])/(4*b^4*(-a^2 + b^2
)^2*(a + b*Cos[c + d*x])) + (B*Sin[2*(c + d*x)])/(3*b^3)))/d
```

Maple [B] time = 19.167, size = 2195, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^5/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-18*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
```

))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-8*a^2/b^4*(3*A*b-5*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^4*(A*b-B*a)/b^5*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2/b^5*a^3*(4*A*b-5*B*a)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b

```
^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

3.584 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0371022, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3768, 3771, 2641}

$$\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx$$
$$= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c-}$$
$$= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3d}$$

Mathematica [A] time = 0.0684642, size = 47, normalized size = 0.73

$$\frac{2B\sec^{\frac{3}{2}}(c+dx)\left(\sin(c+dx)+\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sec[c + d*x]^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin
[c + d*x]))/(3*d)
```

Maple [B] time = 3.359, size = 214, normalized size = 3.3

$$-\frac{2B}{3d}\left(-2\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\left(\sin(1/2\,dx+c/2)\right)^2+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*B*((2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(B\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

3.585 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0361397, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3768, 3771, 2639}

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a + b\cos(c+dx)}\,dx = B\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+a}$$
$$= -\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\sec(c+dx)}\sin(c+}{d}$$

Mathematica [A] time = 0.0441791, size = 46, normalized size = 0.77

$$\frac{2B\sqrt{\sec(c+dx)}\left(\sin(c+dx) - \sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sin[c + d*x]))/d
```

Maple [A] time = 2.691, size = 102, normalized size = 1.7

$$-2\,\frac{B\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right) - 2\,(\sin(1/2\,dx+c/2))^2\cos}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(B\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

3.586 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.0234589, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 3771, 2641}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx &= B\int\sqrt{\sec(c+dx)}\,dx\\
&= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0286205, size = 37, normalized size = 1.

$$
\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Maple [B] time = 2.184, size = 134, normalized size = 3.6

$$
-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\,B\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\,\mathrm{EllipticF}\left(c\cdots\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\,\sin(1/2\,dx+c/2)\,\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}\,d\cdots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(B\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] B*Integral(sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

3.587 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.0227405, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 3771, 2639}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.033936, size = 37, normalized size = 1.

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 1.898, size = 134, normalized size = 3.6

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}B\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\mathrm{EllipticE}\left(\cos\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(B/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] B*Integral(1/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

3.588 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0390212, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3769, 3771, 2641}

$$\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx &= B\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0435626, size = 50, normalized size = 0.78

$$\frac{B\sqrt{\sec(c+dx)}\left(\sin(2(c+dx)) + 2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (B*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin
[2*(c + d*x)]))/(3*d)
```

Maple [B] time = 2.876, size = 180, normalized size = 2.8

$$-\frac{2B}{3d}\sqrt{\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Bigg(4(\sin(1/2dx+c/2))^4\cos(1/2dx+c/2)+\sqrt{2(\sin(1/2dx+c/2))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(B/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

3.589 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.036836, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3769, 3771, 2639}

$$\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx &= B\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}(3B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\
&= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}\left(3B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx\\
&= \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.070115, size = 56, normalized size = 0.88

$$\frac{B\sqrt{\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)
),x]
```

```
[Out] (B*Sqrt[Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [B] time = 2.808, size = 203, normalized size = 3.2

$$-\frac{2B}{5d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-8\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}\cos\left(1/2\,dx+c/2\right)+8\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B}{\sec\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(B/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c) + a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

3.590 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=473

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)+2ab(3A-7B)\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*
x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7
*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[
Sec[c + d*x]]) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^2*d) + (2*(A*b + 7*a*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (2*A*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.43775, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2999, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)+2ab(3A-7B)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*
x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7
*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
```

```
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[
Sec[c + d*x]]) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^2*d) + (2*(A*b + 7*a*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (2*A*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}(A+B}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\mathrm{co}}\right.$$

$$= \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35ad} +$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}{105a^2d}$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}{105a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(19a^2Ab+8Ab^3+63a^3B-14ab^2B\right)\sqrt{\cos(}}{}$$

Mathematica [B] time = 24.4085, size = 3321, normalized size = 7.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(19*a^2*A*b + 8*A*b^3 + 63
*a^3*B - 14*a*b^2*B)*Sin[c + d*x])/(105*a^3) + (2*Sec[c + d*x]^2*(A*b*Sin[c
 + d*x] + 7*a*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d
*x] - 4*A*b^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]))/(105*a^2) + (2*A*Sec[c
+ d*x]^2*Tan[c + d*x])/7))/d + (2*((-19*A*b)/(105*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (3*a*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b
^2*B)/(15*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*A*Sqrt[Sec[
c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (17*A*b^2*Sqrt[Sec[c + d*x]])/(1
05*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt
[a + b*Cos[c + d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d
*x]]) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]) - (1
9*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]
```

```
]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt[a + b*Cos[
c + d*x]]) - (3*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[
c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a +
 b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(19*a^
2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A
+ 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*
Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(10
5*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 6
3*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A
+ 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b
*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^3*(a + b*Cos[c
+ d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*
b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (19*a^
2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((19*a^
2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*
Sec[(c + d*x)/2]^4)/2 - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^
2*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + C
os[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])] + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B)
)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[
c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])] - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[
c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A +
7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[A
```

```
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c
 + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(
19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]
^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*
b^2*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2
] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(8*A*b^2 - 2*a
*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/
(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b
)]) - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/S
qrt[1 - Tan[(c + d*x)/2]^2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c
 + d*x)/2]^2]) + ((-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
+ 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (19*a^2*A*b
 + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d
*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^3*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]))))
```

Maple [B] time = 0.706, size = 3436, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d/a^3*(-25*A*cos(d*x+c)^5*a^3*b-19*A*cos(d*x+c)^5*a^2*b^2+4*A*cos(d*x
+c)^5*a*b^3-63*B*cos(d*x+c)^5*a^3*b-7*B*cos(d*x+c)^5*a^2*b^2+14*B*cos(d*x+c
)^5*a*b^3-19*A*cos(d*x+c)^4*a^3*b+20*A*cos(d*x+c)^4*a^2*b^2-8*A*cos(d*x+c)^
4*a*b^3-14*B*a*b^3*cos(d*x+c)^4+4*A*cos(d*x+c)^3*a*b^3-A*cos(d*x+c)^2*a^2*b
^2+18*A*cos(d*x+c)*a^3*b-7*B*cos(d*x+c)^3*a^2*b^2+28*B*cos(d*x+c)^2*a^3*b+3
5*B*cos(d*x+c)^4*a^3*b+14*B*cos(d*x+c)^4*a^2*b^2+26*A*cos(d*x+c)^3*a^3*b+8*
A*b^4*cos(d*x+c)^4+15*A*a^4+19*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
```

*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^2-63*B*cos(d*x+c)^4*a
^4+42*B*cos(d*x+c)^3*a^4-25*A*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4+21*B*c
os(d*x+c)*a^4-8*A*cos(d*x+c)^5*b^4+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4-25*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+
c)*a^4+63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4-25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^4+63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4-63*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4+
8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^3*sin(d*x+c)*a*b^3-19*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-2*A*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b
^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^3*sin(d*x+c)*a*b^3+63*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14*B*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^2-14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-49*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+14*B*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^2+19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s

```
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^3-19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-2*A*cos(d*x+c)^4*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*co
s(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^3+63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-14*B*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-14
*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3-49*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+14*B*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b
^2+19*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^3*sin(d*x+c)*a^3*b)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)/(a+b*cos(d*x+
c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
 x)
```

3.591 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=390

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+5abB-2Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*
Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(A*b + 5*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*Sqrt[a + b*C
os[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.05711, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2999, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+5abB-2Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*
Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(A*b + 5*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*Sqrt[a + b*C
```

```
os[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}(A+B}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\text{co}}\right.$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} +$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} +$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A-2Ab^2+5abB\right)\sqrt{\cos(c+dx)}\csc(c}{15a^3}$$

Mathematica [A] time = 17.9216, size = 423, normalized size = 1.08

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(9a^2A+5abB-2Ab^2)\sin(c+dx)}{15a^2}+\frac{2\sec(c+dx)(5aB\sin(c+dx)+Ab\sin(c+dx))}{15a}+\frac{2}{5}A\tan(c+dx)\sec(c+$$
$$d$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A - 2*A*b^2 + 5
*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(9*a*A - 2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A - 2*A*b^2 + 5*a*b*B)*C
os[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*
a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sin[c + d*x]
)/(15*a^2) + (2*Sec[c + d*x]*(A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a
) + (2*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.576, size = 2489, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d/a^2*(3*A*a^3-2*A*b^3*cos(d*x+c)^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2+5*B*a*b^2
*cos(d*x+c)^3-A*a*b^2*cos(d*x+c)^2+10*B*a^2*b*cos(d*x+c)^2+4*A*a^2*b*cos(d*
x+c)-5*B*cos(d*x+c)^3*a^2*b-9*A*cos(d*x+c)^4*a^2*b-A*cos(d*x+c)^4*a*b^2-5*B
*cos(d*x+c)^4*a^2*b-5*B*cos(d*x+c)^4*a*b^2+5*A*cos(d*x+c)^3*a^2*b+2*A*cos(d
*x+c)^3*a*b^2-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
```

)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-2*A*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2-7*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+5*B*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+9*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin
(d*x+c)*a^2*b-2*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-7*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b+5*B*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3-9*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^3*sin(d*x+c)*a^3+9*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-2*A*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a^3-5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+

```
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3+5*
B*a^3*cos(d*x+c)+2*A*cos(d*x+c)^4*b^3-9*A*cos(d*x+c)^3*a^3+6*A*cos(d*x+c)^2
*a^3-5*B*cos(d*x+c)^3*a^3)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)/(a+b*cos(d*x+c))
^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
 x)
```

3.592 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=324

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2(a-$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(A -
3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[
c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*d)
```

Rubi [A] time = 0.675137, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2999, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2(a-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(A -
3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[
c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\,\mathrm{co}}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(\right.$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left((a-b)(\right.$$

$$= \frac{2(a-b)\sqrt{a+b}(Ab+3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\right.}{3a^2d\sqrt{\sec(c}}$$

Mathematica [A] time = 14.977, size = 346, normalized size = 1.07

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(3aB+Ab)\sin(c+dx)}{3a}+\frac{2}{3}A\tan(c+dx)\right)}{d} + \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-(3aB+Ab)\,\mathrm{c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(A*b + 3*a*B)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a +
 b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b + 3*a*B)*Si
n[c + d*x])/(3*a) + (2*A*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.467, size = 1735, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a*(A*cos(d*x+c)^2*a^2-A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*B*
cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2+3*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*B*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a^2-A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b+A*cos
(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+A*cos(d*x+c)^3*b^2-A*cos(d*x+c)^2*b^2+3*B*
cos(d*x+c)^2*a^2-3*B*cos(d*x+c)*a^2+A*cos(d*x+c)^3*a*b-A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*
cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-3*B*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-a^2*A-A*sin(d*x+c)*cos(
```

```
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1
/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

3.593 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=411

$$\frac{2\sqrt{a+b}(Ab-a(A-B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}+\frac{2A(a}{}$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(A*b - a*(A - B))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sq
rt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.684636, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2991, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(Ab-a(A-B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}+\frac{2A(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(A*b - a*(A - B))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sq
rt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
```

```
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(d*Sqrt[Sec[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2991

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(B*d
)/b^2, Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\,\mathrm{co}}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aA+(Ab+aB)\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}}$$

$$= -\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{d\sqrt{\sec(c+dx)}}$$

$$= \frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.2097, size = 639, normalized size = 1.55

$$\frac{2\left(-(a(A+B)+b(A-B))\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\left(\tan\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a*A*
Tan[(c + d*x)/2] + A*b*Tan[(c + d*x)/2] - 2*A*b*Tan[(c + d*x)/2]^3 - a*A*Ta
n[(c + d*x)/2]^5 + A*b*Tan[(c + d*x)/2]^5 + 2*b*B*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*B*EllipticPi
[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + A*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (b*(A - B) + a*(A +
 B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1
 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.57, size = 1361, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*a+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*
```

```
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-A*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b*sin(d*x+c)+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*b+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b
-a*A)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

3.594 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=445

$$\frac{\sqrt{a+b}(2A+B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}(aB+2\ldots}{}$$

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A + B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b*d*Sqrt[Sec[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d
```

Rubi [A] time = 0.902686, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 3003, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(2A+B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}(aB+2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A + B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
```

```
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b*d*Sqrt[Sec[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}(A+B}{\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\left(\sqrt{\cos(c\right.$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\left(\sqrt{\cos(c\right.$$

$$= -\frac{\sqrt{a+b}(2Ab+aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\right.}{bd\sqrt{\sec(c}}$$

$$= -\frac{(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{co}}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{ad\sqrt{\sec(c+d}}$$

Mathematica [A] time = 17.4229, size = 795, normalized size = 1.79

$$\frac{aB\tan^5\left(\frac{1}{2}(c+dx)\right) - bB\tan^5\left(\frac{1}{2}(c+dx)\right) + 2bB\tan^3\left(\frac{1}{2}(c+dx)\right) + 4Ab\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\sqrt{1-\mathrm{t}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (-(a*B*Tan[(c + d*x)/2]) - b*B*Tan[(c + d*x)/2] + 2*b*B*Tan[(c + d*x)/2]^3
+ a*B*Tan[(c + d*x)/2]^5 - b*B*Tan[(c + d*x)/2]^5 + 4*A*b*EllipticPi[-1, -A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*B*El
lipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 4*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*B*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
(a + b)*B*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
```

```
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(A*b + a*(-A + B))*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)])/(d*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(-1 + Tan[(c + d*x)/2]^4))
```

Maple [B] time = 0.701, size = 1369, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(2*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-2*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b+4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*B*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+2*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*s
```

```
in(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b*sin(d*x+c)+B*cos(d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*x+c)^2-B*
cos(d*x+c)*a)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

3.595 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=533

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{4b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(4*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b + (a + 2*
b)*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec
[c + d*x]]) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]])
+ (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + ((4*
A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*d
)
```

Rubi [A] time = 1.26508, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 3003, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{4b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(4*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b + (a + 2*
b)*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
```

```
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec
[c + d*x]]) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]])
+ (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + ((4*
A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*d
)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\,\mathrm{c}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aB}{}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c}}{4bd}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c}}{4bd}$$

$$= -\frac{\sqrt{a+b}\left(4aAb-a^2B+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{4b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab+aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{4abd\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 18.6295, size = 1133, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*Tan[
(c + d*x)/2] + a^2*B*Tan[(c + d*x)/2] + a*b*B*Tan[(c + d*x)/2] - 8*A*b^2*Ta
n[(c + d*x)/2]^3 - 2*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c + d*x)/2]^5
+ 4*A*b^2*Tan[(c + d*x)/2]^5 - a^2*B*Tan[(c + d*x)/2]^5 + a*b*B*Tan[(c + d*
x)/2]^5 - 8*a*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)] + 2*a^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d
*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*B*EllipticPi[-1, -ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a*A*b*Ellipt
```

```
icPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)] + 2*a^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*B*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x
)/2]^2)/(a + b)] + (a + b)*(4*A*b + a*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(4
*a*A - a*B + 2*b*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*b*d*(1 + Tan[(c + d*x)/2
]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 +
Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.565, size = 2054, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/b*(8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+3*B*cos(d*x+c)^3*a*b-B*cos(d*x+c)^2*a*b-
2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b+4*A*cos(d*x+c)^3
*b^2-4*A*cos(d*x+c)^2*b^2+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2+B*cos(d
*x+c)^2*a^2-B*cos(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b-8*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+4*A*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
```

```
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*B*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^2-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a*b+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x
+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

3.596 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=620

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24b^2d} - \frac{(a-b)\sqrt{a+b}\left(-3a^2B+6aAb+16b^2B\right)\sqrt{\cos(c}}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a +
 b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(2*a^2*A*b -
8*A*b^3 - a^3*B - 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (B*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]]) + ((6*a*A*b - 3*a^2*
B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24
*b^2*d)
```

Rubi [A] time = 1.74001, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24b^2d} - \frac{(a-b)\sqrt{a+b}\left(-3a^2B+6aAb+16b^2B\right)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a +
```

```
 b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(2*a^2*A*b -
8*A*b^3 - a^3*B - 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (B*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]]) + ((6*a*A*b - 3*a^2*
B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24
*b^2*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos$$

$$= \frac{B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+}}{}}{}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx))^{3/2}\sin(}{3bd\sqrt{\sec(c+dx)}}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx))^{3/2}\sin(}{3bd\sqrt{\sec(c+dx)}}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx))^{3/2}\sin(}{3bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(2a^2Ab-8Ab^3-a^3B-4ab^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\right.}{8b^3d\sqrt{\sec(c+d}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(6aAb-3a^2B+16b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-}\right.}{24ab^2d\sqrt{\sec(c+d}}$$

Mathematica [B] time = 14.6712, size = 1549, normalized size = 2.5

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sin[c + d*x])/12 + ((6*A*b
 + a*B)*Sin[2*(c + d*x)])/(24*b) + (B*Sin[3*(c + d*x)])/12))/d + (Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]
*(6*a^2*A*b*Tan[(c + d*x)/2] + 6*a*A*b^2*Tan[(c + d*x)/2] - 3*a^3*B*Tan[(c
+ d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + 16*a*b^2*B*Tan[(c + d*x)/2] + 16*b
^3*B*Tan[(c + d*x)/2] - 12*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a^2*b*B*Tan[(c +
d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3 - 6*a^2*A*b*Tan[(c + d*x)/2]^5 + 6*
a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B*Tan[(c
+ d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d*x)/2]^5 + 16*b^3*B*Tan[(c + d*x)/2]^5 +
 12*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d
*x)/2]^2)/(a + b)] - 48*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*B*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 24*a*b^2*B*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] +
 12*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*A*b^3*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
- 6*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(
c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 24*a*b^2*B*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
- (a + b)*(-6*a*A*b + 3*a^2*B - 16*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(
-12*A*b^2 + 2*a*b*(3*A - 7*B) + a^2*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*b^2*
d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x
```

```
)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.787, size = 2957, normalized size = 4.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d/b^2*(6*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-6*A*a*b^2*cos(d*x+c)^2+3*B*a^2*b*cos(d*
x+c)^2-6*A*a^2*b*cos(d*x+c)-B*cos(d*x+c)^3*a^2*b+10*B*cos(d*x+c)^4*a*b^2+18
*A*cos(d*x+c)^3*a*b^2-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^3-
3*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-12*A*cos(d*x+c)^2*b^3+6*A*cos(d*
x+c)^2*a^2*b+6*B*cos(d*x+c)^2*a*b^2-12*A*cos(d*x+c)*a*b^2-2*B*cos(d*x+c)*a^
2*b-16*B*cos(d*x+c)*a*b^2+6*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+12*A*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-12
*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^2*b-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+16*B*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+2*
B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b-28*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*B*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-24
*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*b^3+48*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
```

```
*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-3*B*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+16*B*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^3+6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+6*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+12*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)
-12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^2*b*sin(d*x+c)-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b*sin(d*x+c)+16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-28*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+24*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*s
in(d*x+c)+3*B*a^3*cos(d*x+c)+12*A*cos(d*x+c)^4*b^3+48*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-3*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)
+16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3
*sin(d*x+c)+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*a^3*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*
cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

3.597 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=562

$$\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad}+\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)\sin(c+d}{315a^2d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*
d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 - a^3*(147*A - 75*B
) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(88*a^2*A*b -
 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*a*d) + (2*(10*A*b + 9*
a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*
a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 2.07837, antiderivative size = 562, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad}+\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)\sin(c+d}{315a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*
d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 - a^3*(147*A - 75*B
```

```
) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(88*a^2*A*b -
 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*a*d) + (2*(10*A*b + 9*
a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*
a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}(A}{\cos^{\frac{11}{2}}(c+}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\right.$$

$$= \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+}{63d}$$

$$= \frac{2\left(49a^2A+3Ab^2+72abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+}{315ad}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315a^2d}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{a+b\cos(c+dx)}}{315a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB\right.}{}$$

<u>Mathematica</u> [B] time = 25.675, size = 3739, normalized size = 6.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 33*a^2*A*b^2
+ 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sin[c + d*x])/(315*a^3) + (2*Sec[c +
d*x]^3*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(
49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 72*a*b*B*Sin[c + d*x]))/(315
*a) + (2*Sec[c + d*x]*(88*a^2*A*b*Sin[c + d*x] - 4*A*b^3*Sin[c + d*x] + 75*
a^3*B*Sin[c + d*x] + 9*a*b^2*B*Sin[c + d*x]))/(315*a^2) + (2*a*A*Sec[c + d*
x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^2*A)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (11*A*b^2)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (8*A*b^4)/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (82
```

*a*b*B)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^3*B)/(35*a *Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (13*a*A*b*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Sqrt[Sec[c + d*x]])/(315*a*Sq rt[a + b*Cos[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b* Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (31*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B* Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) - (7*a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (11*A*b^3*Cos[2* (c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5* Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*Cos[c + d*x]]) - (82*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(147*a^4*A + 33 *a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos [c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a *b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d* x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]] *Sin[c + d*x]*(-2*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d* x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25* B) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d* x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b *B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(147 *a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d* x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8* A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sq rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (1 47*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]* (a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Se c[c + d*x]]*(-((147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3 *B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(147 *a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[(a + b*Cos [c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]

, (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin [c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b *B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan [(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^ 2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))* Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b* Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B) *Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13 *A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]) /((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/ 2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(147*a^4* A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*S ec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x) /2]^2]) + ((-2*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]) /((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b) /(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b* Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/ 2]], (-a + b)/(a + b)] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S ec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))

Maple [B] time = 0.969, size = 4399, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d/a^3*(186*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^
5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^4+246*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+153*B*sin(d*x+c)*cos(d*x+c)^5
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-
18*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^3-246*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-246*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+186*A*sin(d*
x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^4*b+33*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)
^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*A*s
in(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^4-147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*sin(d*x+c)*cos(d*x+c)^4
```

*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-33*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+153*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-18*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+18*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-35*A*a^5-81*B*cos(d*x+c)^3*a^3*b^2-117*B*cos(d*x+c)^2*a^4*b+75*B*cos(d*x+c)^6*a^4*b+246*B*cos(d*x+c)^6*a^3*b^2+9*B*cos(d*x+c)^6*a^2*b^3-18*B*cos(d*x+c)^6*a*b^4+246*B*cos(d*x+c)^5*a^4*b-165*B*cos(d*x+c)^5*a^3*b^2-18*B*cos(d*x+c)^5*a^2*b^3+18*B*cos(d*x+c)^5*a*b^4-204*B*cos(d*x+c)^4*a^4*b+9*B*cos(d*x+c)^4*a^2*b^3-85*A*cos(d*x+c)*a^4*b-4*A*cos(d*x+c)^6*a*b^4-10*A*cos(d*x+c)^5*a^4*b+33*A*cos(d*x+c)^5*a^3*b^2-34*A*cos(d*x+c)^5*a^2*b^3+8*A*cos(d*x+c)^5*a*b^4-68*A*cos(d*x+c)^4*a^3*b^2-4*A*cos(d*x+c)^4*a*b^4-52*A*cos(d*x+c)^3*a^4*b+A*cos(d*x+c)^3*a^2*b^3-53*A*cos(d*x+c)^2*a^3*b^2+147*A*cos(d*x+c)^6*a^4*b+88*A*cos(d*x+c)^6*a^3*b^2+33*A*cos(d*x+c)^6*a^2*b^3+8*A*cos(d*x+c)^6*b^5+147*A*cos(d*x+c)^5*a^5-8*A*cos(d*x+c)^5*b^5-98*A*cos(d*x+c)^4*a^5-14*A*cos(d*x+c)^2*a^5+75*B*cos(d*x+c)^5*a^5-30*B*cos(d*x+c)^3*a^5-45*B*cos(d*x+c)*a^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*co

```
s(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b^2+75*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*sin(d*x+c)*c
os(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^5-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^5+75*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5)*cos(d*x+c)/(a+b*cos(
d*x+c))^(1/2)*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{11}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11/
2), x)
```

3.598 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=473

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}-\frac{2(a-b)\sqrt{a+b}\left(a^2(-(25A-63B))+3ab(19A\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*
x]]) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2 - a^2*(25*A - 63*B) + 3*a*b*(19*A -
7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt
[Sec[c + d*x]]) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sqrt[a +
b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.52908, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}-\frac{2(a-b)\sqrt{a+b}\left(a^2(-(25A-63B))+3ab(19A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*
x]]) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2 - a^2*(25*A - 63*B) + 3*a*b*(19*A -
7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
```

```
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt
[Sec[c + d*x]]) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sqrt[a +
b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}(A\ \cdots}{\cos^{\frac{9}{2}}(c+d\ \cdots}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cdots}\right.$$

$$= \frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx\ \cdots}{35d}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+\ \cdots}{105ad}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+\ \cdots}{105ad}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(82a^2Ab-6Ab^3+63a^3B+21ab^2B\right)\sqrt{\mathrm{co}\cdots}}{\cdots}$$

Mathematica [B] time = 24.3363, size = 3318, normalized size = 7.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-82*a^2*A*b + 6*A*b^3 -
63*a^3*B - 21*a*b^2*B)*Sin[c + d*x])/(105*a^2) + (2*Sec[c + d*x]^2*(8*A*b*S
in[c + d*x] + 7*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d
*x] + 3*A*b^2*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x]))/(105*a) + (2*a*A*Sec[c
 + d*x]^2*Tan[c + d*x])/7))/d + (2*((-82*a*A*b)/(105*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/(35*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (3*a^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b
^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*A*Sqrt[Sec[c
 + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^2*Sqrt[Sec[c + d*x]])/(10
5*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a +
 b*Cos[c + d*x]]) + (a*b*B*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]])
 - (b^3*B*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (82*A*b^2*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^
```

4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) -
(3*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]])
- (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]
))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3
+ 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(
25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*d*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a
*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82
*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*(a + b*Cos[c + d*x])^(3/2
)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c
 + d*x)/2]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82*a^2*A*b - 6*
A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
 d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((82*a^2*A*b - 6*
A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d
x)/2]^4)/2 - ((a + b)(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x
])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])] + (a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x]
)^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)] - ((a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x]
)*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) +
 a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[

```
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x
])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(82*a^2
*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin
[c + d*x]*Tan[(c + d*x)/2] + (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)
*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (8
2*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-6*A*b^2 + 3*a*b*(1
9*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])
- ((a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[
1 - Tan[(c + d*x)/2]^2]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d
*x)/2]^2]) + ((-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*
a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82*a^2*A*b -
 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x
)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^2*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]))
```

Maple [B] time = 0.64, size = 3421, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d/a^2*(25*A*cos(d*x+c)^5*a^3*b+82*A*cos(d*x+c)^5*a^2*b^2+3*A*cos(d*x
+c)^5*a*b^3+63*B*cos(d*x+c)^5*a^3*b+42*B*cos(d*x+c)^5*a^2*b^2+21*B*cos(d*x+
c)^5*a*b^3+82*A*cos(d*x+c)^4*a^3*b-55*A*cos(d*x+c)^4*a^2*b^2-6*A*cos(d*x+c)
^4*a*b^3-21*B*a*b^3*cos(d*x+c)^4+3*A*cos(d*x+c)^3*a*b^3-27*A*cos(d*x+c)^2*a
^2*b^2-39*A*cos(d*x+c)*a^3*b-63*B*cos(d*x+c)^3*a^2*b^2-63*B*cos(d*x+c)^2*a^
3*b+21*B*cos(d*x+c)^4*a^2*b^2-68*A*cos(d*x+c)^3*a^3*b+6*A*b^4*cos(d*x+c)^4-
15*A*a^4-82*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
```

1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^2+63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*a^4+25*A*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4-21*B*cos(d*x+c)*a^4-6*A*cos(d*x+c)^5*b^4+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+25*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4-63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4+63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+82*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+51*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^2-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-63*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-21*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-21*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+84*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+21*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-82*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/

```
(a+b))^(1/2))*a^3*b-82*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82
*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b+51*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)^4*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3-63*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^4*sin(d*x+c)*a^3*b-21*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-21*B*cos(d*x+c)^4*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^3+84*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+21*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2-82*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)*a^3*b)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x
+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2
), x)
```

3.599 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=393

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{15a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)
*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B + 15*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*(6*A*b + 5*a*B)*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sq
rt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.09051, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2989, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{15a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)
*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B + 15*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*(6*A*b + 5*a*B)*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sq
```

```
rt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}(A\ \cdots}{\cos^{\frac{7}{2}}(c+d\ \cdots}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cdots}\right.$$

$$= \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\ \cdots}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\ \cdots}{15d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+3Ab^2+20abB\right)\sqrt{\cos(c+dx)}\csc\cdots}{15\cdots}$$

Mathematica [A] time = 18.5459, size = 427, normalized size = 1.09

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(9a^2A+20abB+3Ab^2)\sin(c+dx)}{15a}+\frac{2}{15}\sec(c+dx)(5aB\sin(c+dx)+6Ab\sin(c+dx))+\frac{2}{5}aA\,\mathrm{t}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A + 3*A*b^2 + 2
0*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + 2*a*(a + b)*(3*b*(A + 5*B) + a*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 3*A*b^2 + 2
0*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(15*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sin
[c + d*x])/(15*a) + (2*Sec[c + d*x]*(6*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x
]))/15 + (2*a*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.605, size = 2674, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d/a*(-3*A*a^3+15*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*a*b^2*cos(d*x+c)^2-25*B*a
^2*b*cos(d*x+c)^2-9*A*a^2*b*cos(d*x+c)+20*B*cos(d*x+c)^3*a^2*b-20*B*a*b^2*c
os(d*x+c)^3+20*B*cos(d*x+c)^4*a*b^2+3*A*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4
*a^2*b+6*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2*b-3*A*b^3*cos(d*x+c)^3+1
5*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^2+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
```

b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-20*B*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
-20*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-9*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2
*b-3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^2+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+3*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*
b^2-20*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+20*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)
*a^2*b-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^3*sin(d*x+c)*a^2*b-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3+9*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a
^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^3*sin(d*x+c)*a^3-9*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*cos(d*x+c)^2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*a^3+5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x

```
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-5*B*a^3*c
os(d*x+c)+3*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+5*
B*cos(d*x+c)^3*a^3-3*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+12*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b)*cos(
d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2
), x)
```

3.600 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=479

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(3*A*b^2 + a^
2*(A - 3*B) - a*(4*A*b - 6*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*B*Sqrt[Cos[c + d*
x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) +
 (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.0759, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2961, 2989, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(3*A*b^2 + a^
2*(A - 3*B) - a*(4*A*b - 6*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
```

```
)/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*B*Sqrt[Cos[c + d*
x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) +
 (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}(A}{\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\right.$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\right.$$

$$= -\frac{2b\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{d\sqrt{\sec(c+}}$$

$$= \frac{2(a-b)\sqrt{a+b}(4Ab+3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\mathrm{s}\right.}{3ad\sqrt{\mathrm{se}}}$$

Mathematica [B] time = 24.2648, size = 6011, normalized size = 12.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.694, size = 2326, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*b^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
```

b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2+6*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c
)*b^2+3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*b^2+A*cos(d*x+c)^2*a^2-4*A*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2-3*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+A*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*a^2+3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^2-4*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b-3*B*c
os(d*x+c)^2*a*b+4*A*cos(d*x+c)^2*a*b-5*A*cos(d*x+c)*a*b+4*A*cos(d*x+c)^3*b^
2-4*A*cos(d*x+c)^2*b^2+3*B*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)*a^2+A*cos(d*x+c)
^3*a*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b-3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*a*b-a^2*A-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip

```
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+
c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

3.601 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=509

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}(2a(A-B)-b(4A+B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{}}{d\sqrt{\sec(}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A
 + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c
+ d*x]]) - (Sqrt[a + b]*(2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a*A - b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.38004, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}(2a(A-B)-b(4A+B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{}}{d\sqrt{\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A
 + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
```

```
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c
+ d*x]]) - (Sqrt[a + b]*(2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a*A - b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}(A+}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{\cos}\right.$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \frac{(2aA-}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \frac{(2aA-}{}$$

$$= -\frac{\sqrt{a+b}(2Ab+3aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-}\right.}{d\sqrt{\sec(c}}$$

$$= \frac{(a-b)\sqrt{a+b}(2aA-bB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\right.}{ad\sqrt{\sec(c}}$$

Mathematica [A] time = 18.3951, size = 935, normalized size = 1.84

$$\frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2a^2A\tan^5\left(\frac{1}{2}(c+dx)\right) - 2aAb\tan^5\left(\frac{1}{2}(c+dx)\right) + b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a^2*A*Tan[(c + d*x)/2] - 2*a*A*b*Tan[(c
+ d*x)/2] + a*b*B*Tan[(c + d*x)/2] + b^2*B*Tan[(c + d*x)/2] + 4*a*A*b*Tan[(
c + d*x)/2]^3 - 2*b^2*B*Tan[(c + d*x)/2]^3 + 2*a^2*A*Tan[(c + d*x)/2]^5 - 2
*a*A*b*Tan[(c + d*x)/2]^5 - a*b*B*Tan[(c + d*x)/2]^5 + b^2*B*Tan[(c + d*x)/
2]^5 - 4*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
```

```
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] - 6*a*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*A*b^2*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
- 6*a*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(
c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(2*a*A - b*B)*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + 2*(-(A*b^2) + 2*a*b*(A - B) + a^2*(A + B))*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)]))/(d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.476, size = 2193, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+2*B*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*a^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^2-2*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2
+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
```

```
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+B*b^2*cos(d*x+c)^3+B*cos(d*x+c)^2*a*b-B*c
os(d*x+c)*a*b+2*A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b-B*cos(d*x+c)^2*b^2-2*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*a*b+2*A*cos(d*x+c)*a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*a^2*A-2*A*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*b^2*sin(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*sin(d*x+c)+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2)*cos(d*x+c)/(a+b*cos(d*x+c))
^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
```

```
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c) + A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

3.602 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=532

$$-\frac{\sqrt{a+b}\left(3a^2B+12aAb+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{4bd\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a*A + 4*A*b +
 5*a*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*
Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c
+ d*x]]) + (b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*
x]]) + ((4*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(4*d)
```

Rubi [A] time = 1.36707, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2B+12aAb+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{4bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a*A + 4*A*b +
 5*a*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*
```

```
Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c
+ d*x]]) + (b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*
x]]) + ((4*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(4*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}(A+}{\sqrt{\cos(c+dx}}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+5aB)\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+5aB)\sqrt{a+b\,\mathrm{co}}}{}$$

$$= -\frac{\sqrt{a+b}\left(12aAb+3a^2B+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)}{4ba}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab+5aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{4ad\sqrt{\sec}}$$

Mathematica [B] time = 18.5723, size = 1146, normalized size = 2.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (b*B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) +
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*Ta
n[(c + d*x)/2] + 5*a^2*B*Tan[(c + d*x)/2] + 5*a*b*B*Tan[(c + d*x)/2] - 8*A*
b^2*Tan[(c + d*x)/2]^3 - 10*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c + d*x
)/2]^5 + 4*A*b^2*Tan[(c + d*x)/2]^5 - 5*a^2*B*Tan[(c + d*x)/2]^5 + 5*a*b*B*
Tan[(c + d*x)/2]^5 - 24*a*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*B*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
```

```
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 24
*a*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^
2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(4*A*b + 5*a*B)*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 2*(4*a^2*(A - B) - 2*b^2*B + a*b*(-8*A + B))*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
 d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)]))/(4*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.547, size = 2432, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(8*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+24*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a*b+8*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-8*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+7*B*cos(d*x+c)^3*a*b
-5*B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c
)*a*b+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+2*B*cos(d*x+c)^4*b^2-2*B*co
s(d*x+c)^2*b^2+5*B*cos(d*x+c)^2*a^2-5*B*cos(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
```

```
-b)/(a+b))^(1/2))*a*b+5*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-16*A*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-16*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)
+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+4*A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+5
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

3.603 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=626

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\left(3a^2B+30aAb+14abB+12Ab^2+16b\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a +
b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14*a*b*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a +
 b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + ((6*A*b +
7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Sec[c + d*x]]) + (
(30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(24*b*d)
```

Rubi [A] time = 1.97347, antiderivative size = 626, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\left(3a^2B+30aAb+14abB+12Ab^2+16b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a +
```

```
b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14*a*b*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a +
 b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + ((6*A*b +
7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Sec[c + d*x]]) + (
(30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(24*b*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{12d\sqrt{\sec(c+dx)}}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{12d\sqrt{\sec(c+dx)}}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{12d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(6a^2Ab+8Ab^3-a^3B+12ab^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Gamma}{8b^2d\sqrt{\sec}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(30aAb+3a^2B+16b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(}{24abd\sqrt{\sec(c}}$$

Mathematica [B] time = 19.2171, size = 1505, normalized size = 2.4

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Sin[c + d*x])/12 + ((6*A
*b + 7*a*B)*Sin[2*(c + d*x)])/24 + (b*B*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1
 - Tan[(c + d*x)/2]^2)^(-1)]*(30*a^2*A*b*Tan[(c + d*x)/2] + 30*a*A*b^2*Tan[
(c + d*x)/2] + 3*a^3*B*Tan[(c + d*x)/2] + 3*a^2*b*B*Tan[(c + d*x)/2] + 16*a
*b^2*B*Tan[(c + d*x)/2] + 16*b^3*B*Tan[(c + d*x)/2] - 60*a*A*b^2*Tan[(c + d
*x)/2]^3 - 6*a^2*b*B*Tan[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3 - 30*
a^2*A*b*Tan[(c + d*x)/2]^5 + 30*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^3*B*Tan[(c
```

```
 + d*x)/2]^5 + 3*a^2*b*B*Tan[(c + d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d*x)/2]^5
 + 16*b^3*B*Tan[(c + d*x)/2]^5 - 36*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*A*b^3*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^
3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] - 72*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] - 36*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*
A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 72*a*
b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(30*a*A*b + 3*a^2*B + 16*b^2*B)*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)] - 2*b*(12*A*b^2 + a^2*(24*A - 7*B) + a*(-6*A*b + 2
6*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*
x)/2]^2)])
```

Maple [B] time = 0.65, size = 3141, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d/b*(-30*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
```

(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+30*A*a*b^2*cos(d*x+c)^2+3*B*a^2*b*cos(d*x+c)^2+30*A*a^2*b*cos(d*x+c)-17*B*cos(d*x+c)^3*a^2*b-22*B*cos(d*x+c)^4*a*b^2-42*A*cos(d*x+c)^3*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-8*B*cos(d*x+c)^5*b^3-8*B*cos(d*x+c)^3*b^3-3*B*cos(d*x+c)^2*a^3+16*B*cos(d*x+c)^2*b^3+12*A*cos(d*x+c)^2*b^3-30*A*cos(d*x+c)^2*a^2*b+6*B*cos(d*x+c)^2*a*b^2+12*A*cos(d*x+c)*a*b^2+14*B*cos(d*x+c)*a^2*b+16*B*cos(d*x+c)*a*b^2-30*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-36*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-16*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-14*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+52*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-72*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-48*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-16*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(

```
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-36*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-3*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c
)-16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2*sin(d*x+c)-14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b*sin(d*x+c)+52*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*B*a^3*cos(d*x+c
)-12*A*cos(d*x+c)^4*b^3-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-16*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+6*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c))*
(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

3.604 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=730

$$\frac{\left(24a^2Ab-9a^3B+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192b^2d}+\frac{\left(-3a^2B+8aAb+12b^2B\right)\sin(c+d}{32bd\sqrt{\sec(c+d}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d*Sqrt[Sec[c +
 d*x]]) - (Sqrt[a + b]*(9*a^3*B - 6*a^2*b*(4*A + B) - 8*b^3*(16*A + 9*B) -
4*a*b^2*(28*A + 39*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(192*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 - 3
*a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(64*b^3*d*Sqrt[Sec[c + d*x]]) + ((8*a*A*b - 3*a^2*
B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c + d
*x]]) + ((8*A*b - 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b*d*S
qrt[Sec[c + d*x]]) + (B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d*Sqr
t[Sec[c + d*x]]) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d)
```

Rubi [A] time = 2.44076, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(24a^2Ab-9a^3B+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192b^2d}+\frac{\left(-3a^2B+8aAb+12b^2B\right)\sin(c+d}{32bd\sqrt{\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
```

```
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d*Sqrt[Sec[c +
 d*x]]) - (Sqrt[a + b]*(9*a^3*B - 6*a^2*b*(4*A + B) - 8*b^3*(16*A + 9*B) -
4*a*b^2*(28*A + 39*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(192*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 - 3
*a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(64*b^3*d*Sqrt[Sec[c + d*x]]) + ((8*a*A*b - 3*a^2*
B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c + d
*x]]) + ((8*A*b - 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b*d*S
qrt[Sec[c + d*x]]) + (B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d*Sqr
t[Sec[c + d*x]]) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+$$

$$= \frac{B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{}$$

$$= \frac{(8Ab-3aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx))^{5/2}}{4bd\sqrt{\sec(c+}}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} + \frac{(8Ab-3aB)}{}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} + \frac{(8Ab-3aB)}{}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} + \frac{(8Ab-3aB)}{}$$

$$= \frac{\sqrt{a+b}\left(8a^3Ab-96aAb^3-3a^4B-24a^2b^2B-48b^4B\right)\sqrt{\cos(c+dx)}\csc}{64b^3d\sqrt{}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24a^2Ab+128Ab^3-9a^3B+156ab^2B\right)\sqrt{\cos(c+dx)}\csc}{192ab^2d\sqrt{}}$$

<u>Mathematica</u> [B] time = 21.6206, size = 1907, normalized size = 2.61

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((8*A*b + 9*a*B)*Sin[c + d*x]
)/96 + ((56*a*A*b + 3*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/(192*b) + ((8*A*b
 + 9*a*B)*Sin[3*(c + d*x)])/96 + (b*B*Sin[4*(c + d*x)])/32))/d + (Sqrt[(1 -
 Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-24*a^3*A*b*Tan[(c + d*x)/2] - 24*a^2
```

*A*b^2*Tan[(c + d*x)/2] - 128*a*A*b^3*Tan[(c + d*x)/2] - 128*A*b^4*Tan[(c + d*x)/2] + 9*a^4*B*Tan[(c + d*x)/2] + 9*a^3*b*B*Tan[(c + d*x)/2] - 156*a^2*b^2*B*Tan[(c + d*x)/2] - 156*a*b^3*B*Tan[(c + d*x)/2] + 48*a^2*A*b^2*Tan[(c + d*x)/2]^3 + 256*A*b^4*Tan[(c + d*x)/2]^3 - 18*a^3*b*B*Tan[(c + d*x)/2]^3 + 312*a*b^3*B*Tan[(c + d*x)/2]^3 + 24*a^3*A*b*Tan[(c + d*x)/2]^5 - 24*a^2*A*b^2*Tan[(c + d*x)/2]^5 + 128*a*A*b^3*Tan[(c + d*x)/2]^5 - 128*A*b^4*Tan[(c + d*x)/2]^5 - 9*a^4*B*Tan[(c + d*x)/2]^5 + 9*a^3*b*B*Tan[(c + d*x)/2]^5 + 156*a^2*b^2*B*Tan[(c + d*x)/2]^5 - 156*a*b^3*B*Tan[(c + d*x)/2]^5 - 48*a^3*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 576*a*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 18*a^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 144*a^2*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 288*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*a^3*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 576*a*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 18*a^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 144*a^2*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 288*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-24*a^2*A*b - 128*A*b^3 + 9*a^3*B - 156*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(2*a^2*b*(28*A - 57*B) - 4*a*b^2*(52*A - 9*B) + 3*a^3*B - 72*b^3*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(192*b^2*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))

Maple [B] time = 0.964, size = 4056, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/192/d/b^2*(9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4*sin(d*x+c)-120*B*cos(d*x+c)^5*a*b^3-176*A*cos(d*x+c)^4*a*b^3+24
*A*cos(d*x+c)^2*a^2*b^2+24*A*cos(d*x+c)*a^3*b-9*B*cos(d*x+c)^2*a^3*b-78*B*c
os(d*x+c)^4*a^2*b^2-48*B*cos(d*x+c)^6*b^4-24*B*cos(d*x+c)^4*b^4+72*B*cos(d*
x+c)^2*b^4+128*A*cos(d*x+c)^2*b^4-64*A*cos(d*x+c)^3*b^4+48*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)*a^3*b-288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b^4*sin(d*x+c)+48*A*cos(d*x+c)^2*a*b^3-78*B*cos(d*x+c)^2*a^2*b^2+
156*B*cos(d*x+c)^2*a*b^3+112*A*cos(d*x+c)*a^2*b^2+128*A*cos(d*x+c)*a*b^3+6*
B*cos(d*x+c)*a^3*b+156*B*cos(d*x+c)*a^2*b^2+72*B*cos(d*x+c)*a*b^3-136*A*cos
(d*x+c)^3*a^2*b^2+3*B*cos(d*x+c)^3*a^3*b-108*B*cos(d*x+c)^3*a*b^3-24*A*cos(
d*x+c)^2*a^3*b-9*B*cos(d*x+c)*a^4+9*B*cos(d*x+c)^2*a^4-128*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-18*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*sin(
d*x+c)-64*A*cos(d*x+c)^5*b^4-576*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-24*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
*a^3*b-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-112*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*a^2*b^2+416*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+9*B*(cos(
```

d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a^3*b-156*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-156*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-6*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a^3*b+228*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-128*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)*b^4-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4+9*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a^4+144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-576*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-24*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin
(d*x+c)-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^2*sin(d*x+c)-128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-112*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+416*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-144*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2
*sin(d*x+c)+9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^

```
(1/2))*a^3*b*sin(d*x+c)-156*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-156*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-6*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+228*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(
d*x+c)-72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^3*sin(d*x+c)+144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^4*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

3.605 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=662

$$\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{693d}+\frac{2\left(1145a^2Ab+539a^3B+825ab^2B+15Ab^3\right.}{34}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3465*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(40*A
*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b*(505*A - 209*B) + 15*a^2*b^2*(19*A -
 121*B) + 10*a*b^3*(3*A - 11*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3465*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(675*a^4*A + 1025*a^2*A*b^
2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a
^3*B + 825*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*a*A*(a + b
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 2.91098, antiderivative size = 662, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{693d}+\frac{2\left(1145a^2Ab+539a^3B+825ab^2B+15Ab^3\right.}{34}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE
```

```
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3465*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(40*A
*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b*(505*A - 209*B) + 15*a^2*b^2*(19*A -
 121*B) + 10*a*b^3*(3*A - 11*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3465*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(675*a^4*A + 1025*a^2*A*b^
2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a
^3*B + 825*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*a*A*(a + b
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}(A}{\cos^{\frac{13}{2}}(c+}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{1}{11}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c-}{99d}$$

$$= \frac{2\left(81a^2A+113Ab^2+209abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(}{693d}$$

$$= \frac{2\left(1145a^2Ab+15Ab^3+539a^3B+825ab^2B\right)\sqrt{a+b\cos}}{3465ad}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B\right.}{3465a^2d}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B\right.}{3465a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3705a^4Ab+255a^2Ab^3+40Ab^5+1617\right.}{}$$

Mathematica [B] time = 26.7664, size = 4198, normalized size = 6.34

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3705*a^4*A*b + 255*a^2*A*
b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sin[c + d*x])/(
3465*a^3) + (2*Sec[c + d*x]^4*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c + d*x
]))/99 + (2*Sec[c + d*x]^3*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x]
+ 209*a*b*B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(1145*a^2*A*b*Sin[c + d*
x] + 15*A*b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin[c + d
*x]))/(3465*a) + (2*Sec[c + d*x]*(675*a^4*A*Sin[c + d*x] + 1025*a^2*A*b^2*S
in[c + d*x] - 20*A*b^4*Sin[c + d*x] + 1793*a^3*b*B*Sin[c + d*x] + 55*a*b^3*
B*Sin[c + d*x]))/(3465*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/11))/d
+ (2*((-247*a^2*A*b)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (1
7*A*b^3)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^5)/(693
*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*a^3*B)/(15*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a*b^2*B)/(35*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) + (2*b^4*B)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) + (15*a^3*A*Sqrt[Sec[c + d*x]])/(77*Sqrt[a + b*Cos[c + d*x]])
- (26*a*A*b^2*Sqrt[Sec[c + d*x]])/(231*Sqrt[a + b*Cos[c + d*x]]) - (7*A*b^4
*Sqrt[Sec[c + d*x]])/(99*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Sqrt[Sec[c
+ d*x]])/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) + (38*a^2*b*B*Sqrt[Sec[c + d*x]
])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*b^3*B*Sqrt[Sec[c + d*x]])/(315*Sqr
t[a + b*Cos[c + d*x]]) + (2*b^5*B*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Co
s[c + d*x]]) - (247*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[
a + b*Cos[c + d*x]]) - (17*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*
a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])
/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[
c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (31*b^3*B*Cos[2*(c + d*x)]*Sqrt[
Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (2*b^5*B*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*(-2*(a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617
*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*
A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(22
5*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*
a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3465*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)
*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 1
10*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)

/(a + b)] + 2*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*
A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b
 + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Co
s[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465
a^3(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(3705*a^4*A*b + 255*a^
2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)
*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(5
05*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A
*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*a^3*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*(-((3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B
- 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((
a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2
*B - 110*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c +
 d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) +
 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B
))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos
[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])] - ((a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1
 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)] + (a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*
B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((
b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c
 + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))] + b*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*
a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c
 + d*x]*Tan[(c + d*x)/2] + (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*
a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5
 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d

```
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(40*A*b^4 - 10*a*b^
3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^
4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(3
705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*
a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c +
d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3465*a^3*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(3705*a^4*A*b + 255*a^
2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)
*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(5
05*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A
*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d
*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(346
5*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]])))
```

Maple [B] time = 1.276, size = 5381, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(13/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(13/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

3.606 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=562

$$\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5Ab^3\right)s}{315}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^
3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A
- 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b^2*(11*A - 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(163*
a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(315*a*d) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*a*(
4*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2
1*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(
9*d)
```

Rubi [A] time = 2.07156, antiderivative size = 562, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5Ab^3\right)s}{315}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^
```

```
3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A
- 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b^2*(11*A - 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(163*
a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(315*a*d) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*a*(
4*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2
1*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(
9*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}(A+}{\cos^{\frac{11}{2}}(c+dx}$$

$$=\frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{1}{9}\Big(2$$

$$=\frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx}{21d}$$

$$=\frac{2\left(49a^2A+75Ab^2+135abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+}{315d}$$

$$=\frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{a+b\cos(c+a}}{315ad}$$

$$=\frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{a+b\cos(c+a}}{315ad}$$

$$=\frac{2(a-b)\sqrt{a+b}\left(147a^4A+279a^2Ab^2-10Ab^4+435a^3bB\right.}{}$$

Mathematica [B] time = 25.8229, size = 3755, normalized size = 6.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sin[c + d*x])/(315*a^2) + (2*Sec[c
+ d*x]^3*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin[c + d*x]))/63 + (2*Sec[c + d*
```

x]^2*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*x] + 135*a*b*B*Sin[c + d*x
]))/315 + (2*Sec[c + d*x]*(163*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d*x]
+ 75*a^3*B*Sin[c + d*x] + 135*a*b^2*B*Sin[c + d*x]))/(315*a) + (2*a^2*A*Sec
[c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^3*A)/(15*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (31*a*A*b^2)/(35*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + (2*A*b^4)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
- (29*a^2*b*B)/(21*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^3*B)/(
7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (38*a^2*A*b*Sqrt[Sec[c + d
*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*A*b^3*Sqrt[Sec[c + d*x]])/(315*
Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b
*Cos[c + d*x]]) + (5*a^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]
) - (2*a*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (b^4*B*S
qrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*A*b*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]) - (29*a*b
^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (
b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]))*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(147*a^4*A + 279*a^2*A*b^
2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-10*A*b^3 + 15*a*b^2*
(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 27
9*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^2*d*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sin[c + d*x]*(-2*(a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b
*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)] + 2*a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49
*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 +
 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(315*a^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d
*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a
 + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a
(a + b)(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b
*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d
x])/((a + b)(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*

```
b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*(-((147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 +
435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2
]^4)/2 - ((a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*
a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1
 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])] + (a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(
49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((
Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c +
 d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(147*a^4*A + 279*
a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*
x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))] + (a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(
49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] + b*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B
)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (147*a^4*
A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*(a + b*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (147*a^4*A + 279*a^2*
A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-10*A*b^3 + 15*a*b^
2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*
Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*
b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[
1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/
(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*
(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a +
b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A
 + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(
Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c
```

```
+ d*x]*Tan[c + d*x]))/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/
2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.887, size = 4400, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] 2/315/d/a^2*(-261*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)
^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^4-435*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-405*B*sin(d*x+c)*cos(d*x+c)
^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^
2-45*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^3+435*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+435*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3*b^2+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-261*A*sin(
d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^4*b-279*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-155*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d
*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+
```

```
10*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^4+147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*A*sin(d*x+c)*cos(d
*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b^2+279*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-435*B*sin(d*x
+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4*b-405*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-45*B*sin(d*x+c)*cos(d*x+c)^4*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+435*
B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^4*b+435*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+45*B*sin(d*x+c)*cos(d*x
+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^3+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^4+35*A*a^5+270*B*cos(d*x+c)^3*a^3*b^2+180*B*co
s(d*x+c)^2*a^4*b-75*B*cos(d*x+c)^6*a^4*b-435*B*cos(d*x+c)^6*a^3*b^2-135*B*c
os(d*x+c)^6*a^2*b^3-45*B*cos(d*x+c)^6*a*b^4-435*B*cos(d*x+c)^5*a^4*b+165*B*
cos(d*x+c)^5*a^3*b^2-45*B*cos(d*x+c)^5*a^2*b^3+45*B*cos(d*x+c)^5*a*b^4+330*
B*cos(d*x+c)^4*a^4*b+180*B*cos(d*x+c)^4*a^2*b^3+130*A*cos(d*x+c)*a^4*b-5*A*
cos(d*x+c)^6*a*b^4-65*A*cos(d*x+c)^5*a^4*b-279*A*cos(d*x+c)^5*a^3*b^2+199*A
*cos(d*x+c)^5*a^2*b^3+10*A*cos(d*x+c)^5*a*b^4+272*A*cos(d*x+c)^4*a^3*b^2-5*
A*cos(d*x+c)^4*a*b^4+82*A*cos(d*x+c)^3*a^4*b+80*A*cos(d*x+c)^3*a^2*b^3+170*
A*cos(d*x+c)^2*a^3*b^2-147*A*cos(d*x+c)^6*a^4*b-163*A*cos(d*x+c)^6*a^3*b^2-
279*A*cos(d*x+c)^6*a^2*b^3+10*A*cos(d*x+c)^6*b^5-147*A*cos(d*x+c)^5*a^5-10*
A*cos(d*x+c)^5*b^5+98*A*cos(d*x+c)^4*a^5+14*A*cos(d*x+c)^2*a^5-75*B*cos(d*x
+c)^5*a^5+30*B*cos(d*x+c)^3*a^5+45*B*cos(d*x+c)*a^5-147*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^5-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
```

```
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-279*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*
cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b^2-155*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+147*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^4*b+279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-75*B*sin(d*x+c)*cos(d*x+c)^5*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*sin(
d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^5+147*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-10*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-75*B*sin(d*
x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^5)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(11/2)/sin(d
*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

3.607 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=474

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)-8ab(15A\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt[Sec[c +
 d*x]]) + (2*(a - b)*Sqrt[a + b]*(a^2*(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*
a*b*(15*A - 7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(10
5*a*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*a*(10*A*b + 7*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a
*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.50085, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)-8ab(15A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt[Sec[c +
 d*x]]) + (2*(a - b)*Sqrt[a + b]*(a^2*(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*
a*b*(15*A - 7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
```

```
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(10
5*a*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*a*(10*A*b + 7*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a
*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}(A+}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\Big(2$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{35d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{105d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{105d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(145a^2Ab+15Ab^3+63a^3B+161ab^2B\right)\sqrt{}}{}$$

Mathematica [B] time = 24.9105, size = 3348, normalized size = 7.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(145*a^2*A*b + 15*A*b^3 +
63*a^3*B + 161*a*b^2*B)*Sin[c + d*x])/(105*a) + (2*Sec[c + d*x]^2*(15*a*A*b
*Sin[c + d*x] + 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Sin[c
 + d*x] + 45*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x]))/105 + (2*a^2*A*Se
c[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-29*a^2*A*b)/(21*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^3)/(7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) - (3*a^3*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (23
*a*b^2*B)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^3*A*Sqrt[
Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (2*a*A*b^2*Sqrt[Sec[c + d*x]
])/(21*Sqrt[a + b*Cos[c + d*x]]) - (A*b^4*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a +
 b*Cos[c + d*x]]) + (8*a^2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d
*x]]) - (8*b^3*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (29*a*
A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) -
```

(A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]])
- (3*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x
]]) - (23*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c +
d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(145*a^2*A*b + 1
5*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15
*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
))/(105*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(145*a^2*A*b + 15*A*b
^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A +
7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
05*a*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(145*a^2*A*b + 15*A*b^
3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7
B) + a^2(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*
Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(10
5*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(-((145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(145*a^
2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Co
s[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(15*b^2*(A
+ 7*B) + 8*a*b*(15*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/
(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(145*
a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))] + (a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7*B) + a^2*(25

```
*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(145*a^2*A*b +
15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c +
d*x]*Tan[(c + d*x)/2] + (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*(
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (145
*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(15*b^2*(A + 7*B) +
 8*a*b*(15*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2
]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(
a + b)]) - ((a + b)*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a
+ b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 16
1*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7*B) + a^2*(25*A +
 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*(a +
 b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*S
ec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x
]))/(105*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.774, size = 3636, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d/a*(25*A*cos(d*x+c)^5*a^3*b+145*A*cos(d*x+c)^5*a^2*b^2+45*A*cos(d*x
+c)^5*a*b^3+63*B*cos(d*x+c)^5*a^3*b+77*B*cos(d*x+c)^5*a^2*b^2+161*B*cos(d*x
+c)^5*a*b^3+145*A*cos(d*x+c)^4*a^3*b-55*A*cos(d*x+c)^4*a^2*b^2+15*A*cos(d*x
+c)^4*a*b^3-161*B*a*b^3*cos(d*x+c)^4-60*A*cos(d*x+c)^3*a*b^3-90*A*cos(d*x+c
)^2*a^2*b^2-60*A*cos(d*x+c)*a^3*b-238*B*cos(d*x+c)^3*a^2*b^2-98*B*cos(d*x+c
)^2*a^3*b+35*B*cos(d*x+c)^4*a^3*b+161*B*cos(d*x+c)^4*a^2*b^2-110*A*cos(d*x+
```

c)^3*a^3*b-15*A*b^4*cos(d*x+c)^4-15*A*a^4-145*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^2+63*B
*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*a^4+25*A*cos(d*x+c)^4*a^4-10*A*cos(d*x+
c)^2*a^4-21*B*cos(d*x+c)*a^4+15*A*cos(d*x+c)^5*b^4-15*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+
25*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)^4*sin(d*x+c)*a^4-63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*cos(d*x+c)^4*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*
A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4+63*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^3*sin(d*x+c)*a^4-15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+145*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+135*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*a^2*b^2+15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-63*B*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
b-161*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b^2-161*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+119*B*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b+161*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-145*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d

```
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*cos
(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b^2-15*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+145*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+13
5*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^2+15*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-63*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^
3*b-161*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-161*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+119*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d
*x+c)*a^3*b+161*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2-145*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+105*B*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^3+105*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3)*cos(d*x+c)/(a+b*cos(d*x+c))
^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
```

```
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\mathrm{sec}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

[Out] Timed out

3.608 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=553

$$\frac{2\sqrt{a+b}\left(a^2b(17A-35B)+a^3(-(9A-5B))-ab^2(23A-45B)+15Ab^3\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{15ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a +
 b]*(15*A*b^3 - a*b^2*(23*A - 45*B) + a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B)
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c
+ d*x]]) - (2*b^2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b + 5*a*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.47274, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2b(17A-35B)+a^3(-(9A-5B))-ab^2(23A-45B)+15Ab^3\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{15ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a +
 b]*(15*A*b^3 - a*b^2*(23*A - 45*B) + a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B)
```

```
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c
+ d*x]]) - (2*b^2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b + 5*a*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}(A+}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\right.$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}$$

$$= -\frac{2b^2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{d\sqrt{\sec(c+d}}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+23Ab^2+35abB\right)\sqrt{\cos(c+dx)}\csc}{15a}$$

Mathematica [B] time = 25.0543, size = 7062, normalized size = 12.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.713, size = 3282, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+5*A*cos(d*x+c)^3*a^2*b+45*B*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-34*A*a*b
^2*cos(d*x+c)^2-40*B*a^2*b*cos(d*x+c)^2-14*A*a^2*b*cos(d*x+c)+35*B*cos(d*x+
c)^3*a^2*b-35*B*a*b^2*cos(d*x+c)^3+35*B*cos(d*x+c)^4*a*b^2+23*A*cos(d*x+c)^
3*a*b^2+9*A*cos(d*x+c)^4*a^2*b+11*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2
*b-23*A*b^3*cos(d*x+c)^3+45*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+23*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^
2-35*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b-35*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+35*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a
^2*b-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a^2*b-23*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+17*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)
*a^2*b+23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-35*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-35*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*a*b^2+35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-23*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)*b^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
```

```
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^3*sin(d*x+c)*a^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-9*A*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3-23*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*b^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+5*B*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^3*sin(d*x+c)*a^3-5*B*a^3*cos(d*x+c)+23*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+
c)^3*a^3-6*A*cos(d*x+c)^2*a^3+5*B*cos(d*x+c)^3*a^3-23*A*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
2+17*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^3*sin(d*x+c)*a^2*b+15*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-15*B*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3
+30*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*b^3+15*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-15*B*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
3+30*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*b^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x
+c))^(7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.609 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=596

$$-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{3d}-\frac{\sqrt{a+b}\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2
*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3*b^2*(6*A + B))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[a + b]*
(2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(2*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.89549, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{3d}-\frac{\sqrt{a+b}\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2
```

```
*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3*b^2*(6*A + B))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[a + b]*
(2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(2*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}(A}{\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\Big($$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx}{d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx}{d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx}{d}$$

$$= -\frac{b\sqrt{a+b}(2Ab+5aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\mathrm{s}\right.}{d\sqrt{\mathrm{sec}}}$$

$$= \frac{(a-b)\sqrt{a+b}\left(14aAb+6a^2B-3b^2B\right)\sqrt{\cos(c+dx)}\csc(}{3a}$$

Mathematica [B] time = 25.5968, size = 7752, normalized size = 13.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.612, size = 3212, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)

[Out] -1/3/d*(-2*A*a^3+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+3*B*b^3*cos(d*x+c)^4+2*A*cos(d*x+c)^3*a^2*b-14*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+14*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-14*A*a*b^2*cos(d*x+c)^2-6*B*a^2*b*cos(d*x+c)^2-16*A*a^2*b*cos(d*x+c)+6*B*cos(d*x+c)^3*a^2*b+3*B*a*b^2*cos(d*x+c)^3+14*A*cos(d*x+c)^3*a*b^2-3*B*cos(d*x+c)^3*b^3+6*B*cos(d*x+c)^2*a^3+14*A*cos(d*x+c)^2*a^2*b-3*B*cos(d*x+c)^2*a*b^2-18*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-14*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+18*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-6*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-14*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+18*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+18*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-18*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*

```
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+30*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-6*A*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^3+12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+3*B*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*b^3-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^2*a^3+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+2*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+6
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+6*B*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-6*B*a^3*c
os(d*x+c)+2*A*cos(d*x+c)^2*a^3+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3-6*A*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.610 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=607

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{4d}-\frac{\sqrt{a+b}\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*
a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(20*a
*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((8*a^2*A - 4*A*b^2 -
9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d) +
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.87928, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2989, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{4d}-\frac{\sqrt{a+b}\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*
```

```
a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(20*a
*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((8*a^2*A - 4*A*b^2 -
9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d) +
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}(A+}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{c}\right.$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b}{}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} - \frac{\left(8a^2A-4.\right.}{}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} - \frac{\left(8a^2A-4.\right.}{}$$

$$= -\frac{\sqrt{a+b}\left(20aAb+15a^2B+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx}{4a}$$

$$= \frac{(a-b)\sqrt{a+b}\left(8a^2A-4Ab^2-9abB\right)\sqrt{\cos(c+dx)}\csc(c+}{4ad}$$

Mathematica [B] time = 19.3882, size = 1290, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(2*a^2*A*Sin[c + d*x] + (b^2*B
*Sin[2*(c + d*x)])/4))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-8*a^3*A*T
an[(c + d*x)/2] - 8*a^2*A*b*Tan[(c + d*x)/2] + 4*a*A*b^2*Tan[(c + d*x)/2] +
 4*A*b^3*Tan[(c + d*x)/2] + 9*a^2*b*B*Tan[(c + d*x)/2] + 9*a*b^2*B*Tan[(c +
 d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2]^3 - 8*A*b^3*Tan[(c + d*x)/2]^3 - 18*
a*b^2*B*Tan[(c + d*x)/2]^3 + 8*a^3*A*Tan[(c + d*x)/2]^5 - 8*a^2*A*b*Tan[(c
+ d*x)/2]^5 - 4*a*A*b^2*Tan[(c + d*x)/2]^5 + 4*A*b^3*Tan[(c + d*x)/2]^5 - 9
*a^2*b*B*Tan[(c + d*x)/2]^5 + 9*a*b^2*B*Tan[(c + d*x)/2]^5 - 40*a*A*b^2*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)] - 30*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] - 8*b^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 40*a*A*b^2*EllipticPi[-1, -ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] - 30*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
 d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^3*B*EllipticPi[-1, -ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] - (a + b)*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*
(12*a^2*b*(A - B) - 2*b^3*B + a*b^2*(-12*A + B) + 4*a^3*(A + B))*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)]))/(4*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.735, size = 3278, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(-8*A*a^3+8*A*cos(d*x+c)*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+2*B*b^3*cos(d*x+c)^4+8*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c
)-8*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*a^2*b+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+4*A*a*b^2*cos(d*x+c)^2+9*B
*a^2*b*cos(d*x+c)^2-8*A*a^2*b*cos(d*x+c)+11*B*a*b^2*cos(d*x+c)^3+24*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+
c)-2*B*cos(d*x+c)^2*b^3-4*A*cos(d*x+c)^2*b^3+4*A*b^3*cos(d*x+c)^3+8*A*cos(d
*x+c)^2*a^2*b-9*B*cos(d*x+c)^2*a*b^2-4*A*cos(d*x+c)*a*b^2-9*B*cos(d*x+c)*a^
2*b-2*B*cos(d*x+c)*a*b^2+4*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-24*A*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+9*B
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a^2*b+9*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-24*B*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+2*B*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b*sin(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+9*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+9*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)
```

```
+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*a^3+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-24*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-8*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a^3+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^3+40*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a*b^2*sin(d*x+c)+40*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+30*B*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^
2*b+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a^2*b*sin(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*B*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c))*cos(d*x+c
)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.611 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=624

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(a^2(48A+33B)+a(54Ab+26bB)+4b\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b
]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*B) + a*(54*A*b + 26*b*B))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*Sqrt[Sec[c + d*x]]) - (Sqr
t[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) +
(b*(2*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c +
 d*x]]) + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]]) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 1.94606, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(a^2(48A+33B)+a(54Ab+26bB)+4b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b
]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*B) + a*(54*A*b + 26*b*B))*Sqrt[Cos[c
```

```
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*Sqrt[Sec[c + d*x]]) - (Sqr
t[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) +
(b*(2*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c +
 d*x]]) + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]]) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(24*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}(A}{\sqrt{\cos(c+d}}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\right.$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{bB(a+b}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{bB(a+b}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{bB(a+b}{}$$

$$= -\frac{\sqrt{a+b}\left(30a^2Ab+8Ab^3+5a^3B+20ab^2B\right)\sqrt{\cos(c+d}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(54aAb+33a^2B+16b^2B\right)\sqrt{\cos(c+dx)}}{}$$

Mathematica [B] time = 19.3676, size = 1521, normalized size = 2.44

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*B*Sin[c + d*x])/12 + (b*
(6*A*b + 13*a*B)*Sin[2*(c + d*x)])/24 + (b^2*B*Sin[3*(c + d*x)])/12))/d - (
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-54*a^2*A*b*Tan[(c + d*x)/2] - 54*a*A*
b^2*Tan[(c + d*x)/2] - 33*a^3*B*Tan[(c + d*x)/2] - 33*a^2*b*B*Tan[(c + d*x)
/2] - 16*a*b^2*B*Tan[(c + d*x)/2] - 16*b^3*B*Tan[(c + d*x)/2] + 108*a*A*b^2
*Tan[(c + d*x)/2]^3 + 66*a^2*b*B*Tan[(c + d*x)/2]^3 + 32*b^3*B*Tan[(c + d*x
)/2]^3 + 54*a^2*A*b*Tan[(c + d*x)/2]^5 - 54*a*A*b^2*Tan[(c + d*x)/2]^5 + 33
*a^3*B*Tan[(c + d*x)/2]^5 - 33*a^2*b*B*Tan[(c + d*x)/2]^5 + 16*a*b^2*B*Tan[
(c + d*x)/2]^5 - 16*b^3*B*Tan[(c + d*x)/2]^5 + 180*a^2*A*b*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^
3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/
(a + b)] + 30*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a^2*A*b*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] + 48*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^3*B*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] + 120*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(54*a*A*b + 3
3*a^2*B + 16*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(-12*A*b^3 + 2*a*b^2*(3*A
 - 19*B) + 24*a^3*(A - B) + a^2*(-72*A*b + 13*b*B))*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)]))/(24*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
```

```
2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.727, size = 3514, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(48*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+54*A*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^2*b-144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-54*A*a*b^2*cos(d*x+c)^2-33*B*a^2*b*co
s(d*x+c)^2-54*A*a^2*b*cos(d*x+c)+59*B*cos(d*x+c)^3*a^2*b+34*B*cos(d*x+c)^4*
a*b^2+66*A*cos(d*x+c)^3*a*b^2-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-144*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*B*cos(d*x+c)^5*b^3
+8*B*cos(d*x+c)^3*b^3+33*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-12*A*cos(
d*x+c)^2*b^3+54*A*cos(d*x+c)^2*a^2*b-18*B*cos(d*x+c)^2*a*b^2-12*A*cos(d*x+c
)*a*b^2-26*B*cos(d*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+54*A*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2
+12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*a*b^2+180*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+33*B*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2
*b+16*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*a*b^2+26*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-76*B*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
```

llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+
120*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b^2-24*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+48*A*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^
3+33*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^3+16*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+30*B*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+54*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*s
in(d*x+c)+54*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^2*sin(d*x+c)+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+180*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+33*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+16*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*
x+c)+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)*a^3-48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+26*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-76*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+
120*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a*b^2*sin(d*x+c)-33*B*a^3*cos(d*x+c)+12*A*cos(d*x+c)^4*b^3+48*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-
48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
sin(d*x+c)+33*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))

```
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3*sin(d*x+c)+16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.612 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=724

$$\frac{(264a^2Ab+15a^3B+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{(5a^2B+24aAb+12b^2B)\sin(c}{32d\sqrt{\sec(c}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d*Sqrt[Sec[c +
 d*x]]) + (Sqrt[a + b]*(15*a^3*B + 8*b^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59
*B) + 4*a*b^2*(52*A + 71*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b
^3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B*(a + b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + ((24*a*A*b +
5*a^2*B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c
 + d*x]]) + ((8*A*b + 11*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*
d*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rubi [A] time = 2.51753, antiderivative size = 724, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(264a^2Ab+15a^3B+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{(5a^2B+24aAb+12b^2B)\sin(c}{32d\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
```

```
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d*Sqrt[Sec[c +
 d*x]]) + (Sqrt[a + b]*(15*a^3*B + 8*b^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59
*B) + 4*a*b^2*(52*A + 71*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b
^3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B*(a + b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + ((24*a*A*b +
5*a^2*B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c
 + d*x]]) + ((8*A*b + 11*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*
d*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{(8Ab+11aB)(a+b\cos(c+dx}{24d\sqrt{\sec(c+dx}}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(24aAb+5a^2B+12b^2B\right)\sqrt{a+}}{32d\sqrt{\sec(c}}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(24aAb+5a^2B+12b^2B\right)\sqrt{a+}}{32d\sqrt{\sec(c}}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(24aAb+5a^2B+12b^2B\right)\sqrt{a+}}{32d\sqrt{\sec(c}}$$

$$= -\frac{\sqrt{a+b}\left(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B\right)\sqrt{\cos(c+}}{64}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{\cos(c+d}}{192ab}$$

Mathematica [B] time = 20.1522, size = 1877, normalized size = 2.59

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*(8*A*b + 17*a*B)*Sin[c + d
*x])/96 + ((104*a*A*b + 59*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/192 + (b*(8*
A*b + 17*a*B)*Sin[3*(c + d*x)])/96 + (b^2*B*Sin[4*(c + d*x)])/32))/d + (Sqr
t[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(264*a^3*A*b*Tan[(c + d*x)/2] + 264*a^2*A*
```

b^2*Tan[(c + d*x)/2] + 128*a*A*b^3*Tan[(c + d*x)/2] + 128*A*b^4*Tan[(c + d*x)/2] + 15*a^4*B*Tan[(c + d*x)/2] + 15*a^3*b*B*Tan[(c + d*x)/2] + 284*a^2*b^2*B*Tan[(c + d*x)/2] + 284*a*b^3*B*Tan[(c + d*x)/2] - 528*a^2*A*b^2*Tan[(c + d*x)/2]^3 - 256*A*b^4*Tan[(c + d*x)/2]^3 - 30*a^3*b*B*Tan[(c + d*x)/2]^3 - 568*a*b^3*B*Tan[(c + d*x)/2]^3 - 264*a^3*A*b*Tan[(c + d*x)/2]^5 + 264*a^2*A*b^2*Tan[(c + d*x)/2]^5 - 128*a*A*b^3*Tan[(c + d*x)/2]^5 + 128*A*b^4*Tan[(c + d*x)/2]^5 - 15*a^4*B*Tan[(c + d*x)/2]^5 + 15*a^3*b*B*Tan[(c + d*x)/2]^5 - 284*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 284*a*b^3*B*Tan[(c + d*x)/2]^5 - 240*a^3*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 960*a*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 288*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 240*a^3*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 960*a*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 288*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(a^3*(192*A - 59*B) + 4*a*b^2*(76*A - 9*B) + 72*b^3*B + a^2*(-104*A*b + 322*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(192*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])

Maple [B] time = 0.883, size = 4240, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/192/d/b*(15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4*sin(d*x+c)+184*B*cos(d*x+c)^5*a*b^3+272*A*cos(d*x+c)^4*a*b^3-26
4*A*cos(d*x+c)^2*a^2*b^2-264*A*cos(d*x+c)*a^3*b-15*B*cos(d*x+c)^2*a^3*b+254
*B*cos(d*x+c)^4*a^2*b^2+48*B*cos(d*x+c)^6*b^4+24*B*cos(d*x+c)^4*b^4-72*B*co
s(d*x+c)^2*b^4-128*A*cos(d*x+c)^2*b^4+64*A*cos(d*x+c)^3*b^4+240*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a^3*b+288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*b^4*sin(d*x+c)-144*A*cos(d*x+c)^2*a*b^3+30*B*cos(d*x+c)^2*a^
2*b^2-284*B*cos(d*x+c)^2*a*b^3-208*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*
b^3-118*B*cos(d*x+c)*a^3*b-284*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+4
72*A*cos(d*x+c)^3*a^2*b^2+133*B*cos(d*x+c)^3*a^3*b+172*B*cos(d*x+c)^3*a*b^3
+264*A*cos(d*x+c)^2*a^3*b-15*B*cos(d*x+c)*a^4+15*B*cos(d*x+c)^2*a^4+128*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4*sin(d*
x+c)-30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a^4*sin(d*x+c)+64*A*cos(d*x+c)^5*b^4-384*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-384*A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b+960*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+264*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+264*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)*a^2*b^2+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+208*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-608*A*(cos(
```

d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a*b^3+720*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+15*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+284*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*a^2*b^2+284*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+118*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-644*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)*a^2*b^2+72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+128*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4-30*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)*a^4+288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4+15*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-144*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*b^4+240*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+960*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+264*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+264*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*si
n(d*x+c)+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^3*sin(d*x+c)+208*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-608*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c

```
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+720*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+15
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*
sin(d*x+c)+284*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b^2*sin(d*x+c)+284*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+118*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-644*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+72*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*si
n(d*x+c)-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4*sin(d*x+c))*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

3.613 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=839

$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}+\frac{(10Ab-3aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}+\frac{\left(-15Ba^2+50Aba+64b^2B\right)\sin}{240bd\sqrt{\sec}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2
*B + 1024*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192
0*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*B - 30*a^3*b*(5*A + B)
 - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355*A + 193*B) - 4*a^2*b^2*(295*A + 423*
B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqrt[
Sec[c + d*x]]) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a^
5*B - 40*a^3*b^2*B - 240*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(128*b^3*d*Sqrt[Sec[c + d*x]]) + ((50*a^2*A*b + 12
0*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
20*b*d*Sqrt[Sec[c + d*x]]) + ((50*a*A*b - 15*a^2*B + 64*b^2*B)*(a + b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x]]) + ((10*A*b - 3*a*
B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c + d*x]]) + (
B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*x]]) + ((1
50*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1920*b^2*d)
```

Rubi [A] time = 3.6028, antiderivative size = 839, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}+\frac{(10Ab-3aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}+\frac{\left(-15Ba^2+50Aba+64b^2B\right)\sin}{240bd\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2
*B + 1024*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192
0*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*B - 30*a^3*b*(5*A + B)
 - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355*A + 193*B) - 4*a^2*b^2*(295*A + 423*
B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqrt[
Sec[c + d*x]]) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a^
5*B - 40*a^3*b^2*B - 240*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(128*b^3*d*Sqrt[Sec[c + d*x]]) + ((50*a^2*A*b + 12
0*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
20*b*d*Sqrt[Sec[c + d*x]]) + ((50*a*A*b - 15*a^2*B + 64*b^2*B)*(a + b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x]]) + ((10*A*b - 3*a*
B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c + d*x]]) + (
B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*x]]) + ((1
50*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1920*b^2*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A$$

$$= \frac{B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{(10Ab-3aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx)}{5bd\sqrt{\sec(c}}$$

$$= \frac{\left(50aAb-15a^2B+64b^2B\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{240bd\sqrt{\sec(c+dx)}} + \frac{(10Ab}{}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5B-40a^3b^2B-240ab^4B\right)}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b\right.}{}$$

Mathematica [A] time = 15.5365, size = 705, normalized size = 0.84

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{1}{960}\left(93a^2B+170aAb+88b^2B\right)\sin(c+dx)+\frac{1}{960}\left(93a^2B+170aAb+100b^2B\right)\sin(3(c\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((170*a*A*b + 93*a^2*B + 88*b
^2*B)*Sin[c + d*x])/960 + ((590*a^2*A*b + 480*A*b^3 + 15*a^3*B + 1024*a*b^2
*B)*Sin[2*(c + d*x)])/(1920*b) + ((170*a*A*b + 93*a^2*B + 100*b^2*B)*Sin[3*
(c + d*x)])/960 + (b*(10*A*b + 21*a*B)*Sin[4*(c + d*x)])/320 + (b^2*B*Sin[5
*(c + d*x)])/80))/d - (-(b*(a + b)*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B +
 1692*a^2*b^2*B + 1024*b^4*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)]) + a*(a + b)*(45*a^4*B - 30*a^3*b*(5*A + 3*B) + 60*a^2*b^2*(5*A +
11*B) + 16*b^4*(45*A + 64*B) + 8*a*b^3*(265*A + 129*B))*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 15*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*
b^5 - 3*a^5*B - 40*a^3*b^2*B - 240*a*b^4*B)*((a - b)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^2)/(a + b)] - b*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a
^2*b^2*B + 1024*b^4*B)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2])/(1920*b^3*d*Sqrt[a + b*Cos[c + d*x]
]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.967, size = 5172, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\cos\left(dx+c\right)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos\left(dx+c\right)^2 + \left(Ba^2 + 2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

3.614 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=403

$$-\frac{2\sqrt{a+b}\left(a^2(9A-5B)-2ab(A+5B)+8Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a
+ b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*b*(A + 5*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*b - 5
*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
+ (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 1.02566, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\left(a^2(9A-5B)-2ab(A+5B)+8Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a
+ b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*b*(A + 5*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*b - 5
*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
```

```
+ (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2A\sqrt{a+b\cos(c}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2A\sqrt{a+b\cos(c}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+8Ab^2-10abB\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right.\right.}{15a^4d\sqrt{\sec(c+dx)}}$$

<u>Mathematica</u> [B] time = 22.5069, size = 2987, normalized size = 7.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 8*A*b^2 - 10*a*
b*B)*Sin[c + d*x])/(15*a^3) + (2*Sec[c + d*x]*(-4*A*b*Sin[c + d*x] + 5*a*B*
Sin[c + d*x]))/(15*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(5*a)))/d + (2*((
-3*A)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^2)/(15*a^2*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*B)/(3*a*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]) - (7*A*b*Sqrt[Sec[c + d*x]])/(15*a*Sqrt[a + b*
Cos[c + d*x]]) - (8*A*b^3*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*
x]]) + (B*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[
Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^3*Cos[2*(c + d*x
)]*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*B*Cos[2*(
c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
```

```
8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
5*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a
*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2
- 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))/(15*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(9*a^2*A +
 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a
^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((9*a^2*A + 8*A*b^2 -
10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b
)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c
 + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]
)))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(8*A*b^2 + 2*a*b*(A - 5*B) +
 a^2*(9*A + 5*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c +
 d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])] - ((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[
c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2
*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))
)/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(9*a^2*A + 8*
A*b^2 - 10*a*b*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x
)/2] + (9*a^2*A + 8*A*b^2 - 10*a*b*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(8*A*b^
2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^
```

```
2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a
+ b)]) - ((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c
+ d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(
c + d*x)/2]^2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
 + ((-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*
B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c
+ d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(
15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.671, size = 2488, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/a^3*(-3*A*a^3-10*A*cos(d*x+c)^3*a^2*b-4*A*a*b^2*cos(d*x+c)^2+5*B*a^
2*b*cos(d*x+c)^2+A*a^2*b*cos(d*x+c)-10*B*cos(d*x+c)^3*a^2*b+10*B*a*b^2*cos(
d*x+c)^3-10*B*cos(d*x+c)^4*a*b^2+8*A*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4*a^
2*b-4*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2*b-8*A*b^3*cos(d*x+c)^3+8*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)^3*sin(d*x+c)*a*b^2+10*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*B*cos(d*x+c)^3*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-10
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^3*sin(d*x+c)*a^2*b-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-8*A*cos(d*x+c)^2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2
```

```
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)*a^2*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+10*B*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+
10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)*a*b^2-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-9*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*
b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*sin(d*x+c)*b^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3+5*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-9*A*
cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3-8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+5*B*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-5*B*a^3*cos(d*x+c)+8*A*cos(d*x+c)^4*b^3
+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+5*B*cos(d*x+c)^3*a^3-8*A*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)/(a+
b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

3.615 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=330

$$\frac{2\sqrt{a+b}(a(A-3B)+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{2(a}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a
*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*
Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*a*d)
```

Rubi [A] time = 0.663952, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 3000, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a(A-3B)+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{2(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a
*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*
Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*a*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left((2Ab+a(A-3B))\sqrt{\cos(c+\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{3a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 16.0303, size = 355, normalized size = 1.08

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(3aB-2Ab)\sin(c+dx)}{3a^2}+\frac{2A\tan(c+dx)}{3a}\right)}{d} + \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left((2Ab-3aB)\cos(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
-2*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
```

```
ec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-2*
A*b + 3*a*B)*Sin[c + d*x])/(3*a^2) + (2*A*Tan[c + d*x])/(3*a)))/d
```

Maple [B] time = 0.769, size = 1544, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a^2*(A*cos(d*x+c)^2*a^2+2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-
3*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*a^2+3*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+A*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*B*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^2+2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b
-2*A*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b-2*A*cos(d*x+c)^3*b^2+2*A*cos(d*x+c)^
2*b^2+3*B*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)*a^2+A*cos(d*x+c)^3*a*b+2*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-3*B*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b-a^2*A+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
```

```
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)*co
s(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

3.616 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=270

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}(A-$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A - B)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.456611, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2998, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}(A-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A - B)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
```

```
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + \left((-A+\right.$$

$$= \frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a}{}}}{a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.9796, size = 279, normalized size = 1.03

$$\frac{2\left(A\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\cos(c+dx)) - \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2a(A+B)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}\right.}{ad\sqrt{a+b\cos(c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*(A*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] - (Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*(2*A*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sq
rt[(1 + Sec[c + d*x])^(-1)] - 2*a*(A + B)*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[(1 + Sec[c + d*x])^(-1)] + A*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2]))/(a*d*Sqrt[a + b
*Cos[c + d*x]])
```

Maple [B] time = 0.708, size = 812, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d/a*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*sin(d*x+c)+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A
)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

3.617 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=268

$$\frac{2A\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\sqrt{\cos(c+}{}$$

```
[Out] (2*A*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.399286, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 3006, 2809, 2816}

$$\frac{2A\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\sqrt{\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*A*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```


Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
```

```
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3006

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[B/d, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist
[(B*c - A*d)/d, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx + \left(B\sqrt{\mathrm{co}\right.$$

$$= \frac{2A\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 2.64605, size = 159, normalized size = 0.59

$$\frac{2\sqrt{\sec(c+dx)+1}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left((A-B)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2B\Pi\left(-\right.\right.}{d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((A - B)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*B*EllipticPi[-1, -ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2
]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]
^2])
```

Maple [A] time = 0.674, size = 199, normalized size = 0.7

$$2\,\frac{\sqrt{(\cos(dx+c))^{-1}}(\sin(dx+c))^2}{d\sqrt{a+b\cos(dx+c)}(-1+\cos(dx+c))}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\left(A\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-B*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))+2*B*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/(a+b*cos(d*x+c))^(1/2)*
(1/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

3.618 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=487

$$-\frac{\sqrt{a+b}(2Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{aB\sin}{bd}$$

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b
- a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b
^2*d*Sqrt[Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Co
s[c + d*x]])
```

Rubi [A] time = 1.26536, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 3003, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}(2Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{aB\sin}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b
```

```
- a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b
^2*d*Sqrt[Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Co
s[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
```

```
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aB+2}{}$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{abB+(2aAb-}{\sqrt{\cos(c+d}}}{2b}$$

$$= -\frac{\sqrt{a+b}(2Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}(2Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{abd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 18.2758, size = 1091, normalized size = 2.24

$$\frac{\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(-a\sqrt{\frac{a-b}{a+b}}B\tan^5\left(\frac{1}{2}(c+dx)\right)+b\sqrt{\frac{a-b}{a+b}}B\tan^5\left(\frac{1}{2}(c+dx)\right)-2b\sqrt{\frac{a-b}{a+b}}B\tan^3\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]*(a*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] + b*Sqrt[(a - b)/(a
+ b)]*B*Tan[(c + d*x)/2] - 2*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 -
 a*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + b*Sqrt[(a - b)/(a + b)]*B*T
an[(c + d*x)/2]^5 - (4*I)*A*b*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a
 - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*a*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Ta
n[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
```

```
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*A*b*Elli
pticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
-((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a*B*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*
B*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
 - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(A*b - a*B)*
EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*Sqrt[(a - b)/(a +
 b)]*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c
+ d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.708, size = 1004, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/b*(4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+B*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+B*(cos(d*x+
```

```
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)
+B*cos(d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*x+c)^2-B*cos(d*x+c)*a)*(1/cos(
d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

3.619 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=539

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b - 3*a
*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*
Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Sqrt[Cos[c
 + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*Sqrt[Sec[c
 + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt[Sec[c + d
*x]]) + ((4*A*b - 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(4*b^2*d)
```

Rubi [A] time = 1.25292, antiderivative size = 539, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b - 3*a
*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
```

```
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*
Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Sqrt[Cos[c
 + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*Sqrt[Sec[c
 + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt[Sec[c + d
*x]]) + ((4*A*b - 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(4*b^2*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{aB}{2}+bB\cos(c+dx}{\sqrt{\cos(c+}}}{2b}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx}}{4b^2d}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx}}{4b^2d}$$

$$= \frac{\sqrt{a+b}\left(4aAb-3a^2B-4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{4ab^2d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 19.7937, size = 1169, normalized size = 2.17

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) +
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-4*a*A*b*Tan[(c + d*x)/2] -
 4*A*b^2*Tan[(c + d*x)/2] + 3*a^2*B*Tan[(c + d*x)/2] + 3*a*b*B*Tan[(c + d*x
)/2] + 8*A*b^2*Tan[(c + d*x)/2]^3 - 6*a*b*B*Tan[(c + d*x)/2]^3 + 4*a*A*b*Ta
n[(c + d*x)/2]^5 - 4*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^2*B*Tan[(c + d*x)/2]^5
+ 3*a*b*B*Tan[(c + d*x)/2]^5 - 8*a*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
```

```
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*El
lipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] - 8*a*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, -ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] + 8*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-4*A*b + 3*a*B)*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] - 2*(a - 2*b)*b*B*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*b^2*d*Sqrt
[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x
)/2]^2)))
```

Maple [B] time = 0.648, size = 1878, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/b^2*(8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b
+2*B*cos(d*x+c)*a*b-4*A*cos(d*x+c)^2*a*b+4*A*cos(d*x+c)*a*b-4*A*cos(d*x+c)^
3*b^2+4*A*cos(d*x+c)^2*b^2-2*B*cos(d*x+c)^4*b^2+2*B*cos(d*x+c)^2*b^2+3*B*co
s(d*x+c)^2*a^2-3*B*cos(d*x+c)*a^2-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*B*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d
```

```
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-6*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*a^2-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+8*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-4
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-6*B*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-8*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^
2+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
```

```
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

3.620 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=433

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5a^2Ab\right.}{}$$

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a
 + 2*b)*(4*A*b + a*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
(a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.15943, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5a^2Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a
 + 2*b)*(4*A*b + a*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c
```

```
+ d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
(a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(a^2}{}}{}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a+b\cos(c}}{3a^2\left(a^2-\right.}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a+b\cos(c}}{3a^2\left(a^2-\right.}$$

$$= -\frac{2\left(5a^2Ab-8Ab^3-3a^3B+6ab^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\sqrt{}}\right.\right.}{3a^4\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 24.5402, size = 3433, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-5*a^2*A*b + 8*A*b^3 + 3*
a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)) + (2*(-(A*b^3*Sin[c +
d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*
A*Tan[c + d*x])/(3*a^2)))/d + (2*((5*A*b)/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(3*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) - (a*B)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (2*b^2*B)/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) + (a*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]) + (7*A*b^2*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]
) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])
 + (5*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a +
b*Cos[c + d*x]]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^
```

2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x
]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt
[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^
2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 +
 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(
c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(
a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)
*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a
^2 - b^2)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-5*a^2*A*b + 8*A
*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
x]](-((-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*
B - 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*
x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])] + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d
*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d
x])] - ((a + b)(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x
])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))] + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*
B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((

```
a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(-5*a^2*A*b + 8*A*b^3 +
 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
 d*x)/2] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-5*a^2*A*b + 8*A*b^3
+ 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]^2 + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a
 + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^
3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*T
an[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*a^3*(a^2 - b
^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-5*a
^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a
*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Se
c[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]
))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqr
t[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.651, size = 3343, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/(a+b)/(a-b)/a^3*(-4*A*cos(d*x+c)^3*a*b^3+4*A*cos(d*x+c)^2*a^2*b^2+4*
A*cos(d*x+c)*a^3*b+3*B*cos(d*x+c)^3*a^2*b^2-3*B*cos(d*x+c)^2*a^3*b+A*cos(d*
x+c)^3*a^3*b-8*A*cos(d*x+c)^2*b^4+8*A*cos(d*x+c)^3*b^4+A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4
-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^2*b^4+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
```

a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4-3*B*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+A*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^2*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3+5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^2*a^3*b+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^2*a^3*b-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2-A*a^4+8
*A*cos(d*x+c)^2*a*b^3-6*B*cos(d*x+c)^2*a^2*b^2+6*B*cos(d*x+c)^2*a*b^3-4*A*c
os(d*x+c)*a*b^3+3*B*cos(d*x+c)*a^2*b^2-5*A*cos(d*x+c)^3*a^2*b^2+3*B*cos(d*x
+c)^3*a^3*b-6*B*cos(d*x+c)^3*a*b^3-5*A*cos(d*x+c)^2*a^3*b+A*cos(d*x+c)^2*a^
4-3*B*cos(d*x+c)*a^4+3*B*cos(d*x+c)^2*a^4+A*a^2*b^2+6*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^
3-5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+5*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-8*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)

```
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)*a*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-3*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)*a^3*b+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-3*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a^3*b-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4-3*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
*a^4)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

3.621 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A - B))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[
Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.790547, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 3000, 2998, 2816, 2994}

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A - B))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[
Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\left(a^2\right.}{}}{a\left(a^2-\right.}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left((a-b)(2Ab+a(A-B))\sqrt{\cos(c+d\right.}{a}$$

$$= \frac{2\left(a^2A-2Ab^2+abB\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\right.}{a^3\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 18.5707, size = 433, normalized size = 1.26

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(a^2A+abB-2Ab^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)}-\frac{2\left(abB\sin(c+dx)-Ab^2\sin(c+dx)\right)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d}+\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+d}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2*A - 2*A*b^2 + a*b*B)*
Sin[c + d*x])/(a^2*(a^2 - b^2)) - (2*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c +
 d*x]))/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*(-2*(a + b)*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
```

```
llipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b
+ a*(A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] - (a^2*A - 2*A*b^2 + a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] time = 0.668, size = 2291, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/a^2/(a-b)/(a+b)*(-A*a^3+A*cos(d*x+c)*a^3-A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-A*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^2*b-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*a^2*b+A*a*b^2*cos(d*x+c)^2-B*a^2*b*cos(d*x+c)^2-A*a^
2*b*cos(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b*sin(d*x+c)-2*A*cos(d*x+c)^2*b^3+A*cos(d*x+c)^2*a^2*b+B*cos(d*
x+c)^2*a*b^2-2*A*cos(d*x+c)*a*b^2+B*cos(d*x+c)*a^2*b-B*cos(d*x+c)*a*b^2+A*a
*b^2+2*A*cos(d*x+c)*b^3+2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*A*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-B*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^2*b-B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
```

```
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*
x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*s
in(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+2*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c))*cos(d*
x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

3.622 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=324

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{a^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A + B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rubi [A] time = 0.68498, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2993, 2998, 2816, 2994}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{a^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A + B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{Ab-a}{\cos^{\frac{3}{2}}(}}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left((a-b)(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{a^2-}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{}}}{a^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.6734, size = 305, normalized size = 0.94

$$\frac{2\left(\frac{b(Ab-aB)\sin(c+dx)}{\sqrt{\sec(c+dx)}} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-(Ab-aB)\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+2a(a+b)(A-B)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{(a}\right.}{\sqrt{\sec^2\left(\frac{1}{2}\right.}}\right.}{d\left(a^3-ab^2\right)\sqrt{a+b\cos(c+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*((b*(A*b - a*B)*Sin[c + d*x])/Sqrt[Sec[c + d*x]] + (Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*(2*(a + b)*(-(A*b) + a*B)*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(a + b)*(A - B)*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b +
a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (A*b - a*B)*Cos[c + d*x]*(a
 + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)
/2]^2]))/((a^3 - a*b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.684, size = 1636, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)

[Out] -2/d/(a+b)/(a-b)/a*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(A*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*a^2+A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(
d*x+c)-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)^2*a*b+A
*cos(d*x+c)^2*b^2+B*cos(d*x+c)^2*a^2-B*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b-A*
cos(d*x+c)*b^2-B*cos(d*x+c)*a^2+B*cos(d*x+c)*a*b)/sin(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

3.623 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=476

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+}}\right.\right.}{abd\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*
Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*S
qrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(b^2*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.775266, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2961, 2992, 2809, 2794, 2795, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+}}\right.\right.}{abd\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*
Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*S
qrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
```

```
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(b^2*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2992

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
```

```
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{\left((Ab-aB)\sqrt{\cos(c+dx}\right.}{}$$

$$= -\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{a(}}{ab\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 13.9953, size = 1403, normalized size = 2.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b - a*B)*Sin[c + d*x])/
(b*(-a^2 + b^2)) - (2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(b*(-a^2 +
 b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*b*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2] + A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - a^2*Sqrt
[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - a*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c +
 d*x)/2] - 2*A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 2*a*b*Sqrt[(a
 - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 - a*A*b*Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]^5 + A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + a^2*Sqrt[(a -
b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x
)/2]^5 - (2*I)*a^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*
b^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*B*EllipticP
i[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
+ b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*B*Ellip
ticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -
((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(-(
A*b) + a*B)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -(
(a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b
)*(-(A*b) + (2*a + b)*B)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)]))/(b*Sqrt[(a - b)/(a + b)]*(a^2 - b^2)*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[
(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2
]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.596, size = 2016, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/d/(a+b)/(a-b)/b*(-A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+B*cos(d*x+c)^2*a*b-B*cos(d*
x+c)*a*b+A*cos(d*x+c)^2*a*b-A*cos(d*x+c)*a*b-A*cos(d*x+c)^2*b^2-B*cos(d*x+c
)^2*a^2+B*cos(d*x+c)*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b-A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+A*cos(d*x+c)*b^2+A*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-B*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
```

```
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-2*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2
-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

3.624 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=560

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+ -$$

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - ((2*A*b - (3*a + b)*B)*S
qrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d*Sqr
t[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A
*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.50902, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+ -$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```



```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - ((2*A*b - (3*a + b)*B)*S
qrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
```

```
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d*Sqr
t[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A
*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+}}{b^2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+}}{b^2}$$

$$= -\frac{\sqrt{a+b}(2Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ab^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 19.578, size = 1567, normalized size = 2.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-(A*b) + a*B)*Sin[c +
d*x])/(b^2*(a^2 - b^2)) + (2*(a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x]))/(
b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (Sqrt[(1 - Tan[(c + d*x)/2]^2)
^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[
(c + d*x)/2]^2)]*(2*a^2*A*b*Tan[(c + d*x)/2] + 2*a*A*b^2*Tan[(c + d*x)/2] -
```

```
 3*a^3*B*Tan[(c + d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + a*b^2*B*Tan[(c + d
*x)/2] + b^3*B*Tan[(c + d*x)/2] - 4*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a^2*b*B*
Tan[(c + d*x)/2]^3 - 2*b^3*B*Tan[(c + d*x)/2]^3 - 2*a^2*A*b*Tan[(c + d*x)/2
]^5 + 2*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B
*Tan[(c + d*x)/2]^5 - a*b^2*B*Tan[(c + d*x)/2]^5 + b^3*B*Tan[(c + d*x)/2]^5
 + 4*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)] - 4*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*B*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a*b^2*B*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] +
4*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(
c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6
*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a
 + b)*(-2*a*A*b + 3*a^2*B - b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(a + b)*
(-(A*b) + a*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b^2*(-a^2 + b^2)*d*Sqrt[1 + Tan
[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2))
)
```

Maple [B] time = 0.549, size = 2890, normalized size = 5.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d/(a+b)/(a-b)/b^2*(2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
```

x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-2*A*a*b^2*cos(d*x+c)^2+3*B*a^2*
b*cos(d*x+c)^2-2*A*a^2*b*cos(d*x+c)-B*cos(d*x+c)^3*a^2*b-2*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+B*cos(d*
x+c)^3*b^3-3*B*cos(d*x+c)^2*a^3-B*cos(d*x+c)^2*b^3+2*A*cos(d*x+c)^2*a^2*b+B
*cos(d*x+c)^2*a*b^2+2*A*cos(d*x+c)*a*b^2-2*B*cos(d*x+c)*a^2*b-B*cos(d*x+c)*
a*b^2+2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b^2-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-4*A*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
2*b-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^2*b+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+2*B*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+2*B*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a*b^2-6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*A*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+4*A*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*b^3-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*B*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b*sin(d*x+c)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-3*B*(cos(d*x+c)/

```
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d
*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^2*sin(d*x+c)-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*B*a^3*cos(d*x+c)+4*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x
+c)-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^3*sin(d*x+c)+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*
x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

3.625 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=607

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{2b\left(10a^2Ab-7a^3B+3ab^2B\right.}{3a^2d\left(a^2-b^2\right)}$$

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a
 + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*
(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*b^2*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec
[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B
+ 3*a*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
 + b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*
b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^
2 - b^2)^2*d)
```

Rubi [A] time = 2.20318, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{2b\left(10a^2Ab-7a^3B+3ab^2B\right.}{3a^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a
```

```
 + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*
(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*b^2*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec
[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B
+ 3*a*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
 + b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*
b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^
2 - b^2)^2*d)
```

<u>Rule 2961</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

<u>Rule 3000</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

<u>Rule 3055</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{3}{2}(a^2}{}}{}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2B\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b}}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2B\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b}}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2B\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b}}$$

$$= -\frac{2\left(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B\right)\sqrt{\cos(c+dx)}\,\mathrm{csc}}{3a^5(a-b)(a+b)^{3/2}}$$

Mathematica [B] time = 26.5591, size = 4316, normalized size = 7.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-8*a^4*A*b + 28*a^2*A*b^3
 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sin[c + d*x])/(3*a^4*(a^2
 - b^2)^2) + (2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2
 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-11*a^2*A*b^3*Sin[c + d*x] + 7*A*b^5*
Sin[c + d*x] + 8*a^3*b^2*B*Sin[c + d*x] - 4*a*b^4*B*Sin[c + d*x]))/(3*a^3*(
a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d + (2*((
8*a*A*b)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2
8*A*b^3)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) +
(16*A*b^5)/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
```

) - (a^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*b^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*b^4*B)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*A*b^2*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (32*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^6*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (3*a*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (17*b^3*B*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*b^5*B*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (8*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (28*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*b^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]

```
 - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4
*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))
/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) +
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((-8*a^4*A*b + 28*a^2*A*b^3 - 16
*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5
+ 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((
Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c +
 d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(-16*A*b^4 + 2*
a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B)
)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[
c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])] - ((a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*
a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a
+ b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A +
2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d
*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(-8*a
^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[
c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-8*a^4*A*b + 2
8*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(a + b*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-8*a^4*A*b + 28*
a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a
+ b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-16*A
*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*
(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-8*a^4*A*
b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)
])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 -
16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a
^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
```

```
 (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B
)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-
(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c
 + d*x]*Tan[c + d*x]))/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.757, size = 8101, normalized size = 13.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

3.626 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=496

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3A+B)\right.}{}$$

```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqr
t[Sec[c + d*x]]) + (2*(8*A*b^3 - 3*a^3*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*
b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*S
qrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b
*(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.38254, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3A+B)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqr
t[Sec[c + d*x]]) + (2*(8*A*b^3 - 3*a^3*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*
b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
```

```
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*S
qrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b
*(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(3a}{}}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B+ab^2B\right)\sqrt{}}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,c}}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B+ab^2B\right)\sqrt{}}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,c}}$$

$$= \frac{2\left(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(}}$$

Mathematica [B] time = 26.0868, size = 3891, normalized size = 7.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4*A - 15*a^2*A*b^2 +
8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) - (2*(
-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c
+ d*x])^2) - (2*(-8*a^2*A*b^2*Sin[c + d*x] + 4*A*b^4*Sin[c + d*x] + 5*a^3*b
*B*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c
+ d*x]))))/d + (2*(-((a^2*A)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])) + (5*A*b^2)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (8*A*b^4)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]) - (2*a*b*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + (2*b^3*B)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (3*a*A*b*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c
 + d*x]]) + (17*A*b^3*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos
[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*
```

Cos[c + d*x]]) + (a^2*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*b^2*B*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[(a + b*Cos[c + d*

```
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*
x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(3
*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(
-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Si
n[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^
3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
+ b*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]
*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (3*a^4*A - 15*a^2*A*b^2
 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*
b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]^2 + (a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) -
2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(3
*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1
 - Tan[(c + d*x)/2]^2]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*
a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A +
 B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*
a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^
2*Sec[c + d*x]*Tan[c + d*x]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.824, size = 6506, normalized size = 13.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

3.627 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=469

$$-\frac{2\left(6a^2Ab-3a^3B-ab^2B-2Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2\right.}{}$$

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]])
- (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x
]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)*Sqrt[Sec[c + d*x]]) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rubi [A] time = 1.19007, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 2993, 2998, 2816, 2994}

$$-\frac{2\left(6a^2Ab-3a^3B-ab^2B-2Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]])
- (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
```

```
 Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x
]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)*Sqrt[Sec[c + d*x]]) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2\left(6a^2Ab-2Ab^3-3a^3B-\right.}{3a\left(a^2-b^2\right)^2d}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2\left(6a^2Ab-2Ab^3-3a^3B-\right.}{3a\left(a^2-b^2\right)^2d}$$

$$= \frac{2\left(6a^2Ab-2Ab^3-3a^3B-ab^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 24.3229, size = 3493, normalized size = 7.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-6*a^2*A*b + 2*A*b^3 + 3
*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) + (2*(-(A*b*Sin[c + d
*x]) + a*B*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-5*a
^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 2*a^3*B*Sin[c + d*x] + 2*a*b^2*B
*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*((-2*a*A*
b)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/
(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*B)/(
(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b^2*B)/(3*(a^
2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[c
 + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b^2*Sqrt[Sec[c +
d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c + d
*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*B*Sqrt[Sec[c +
d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*B*Sqrt[Sec[c + d*x
]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*A*b^2*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
 d*x]]) + (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a
 + b*Cos[c + d*x]]) + (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2
 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(2*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2
 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B
)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(
3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-6*a^2*A*b +
2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3
*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*(a + b*
```

Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a
*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (-6*a^2*A*b
+ 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-6*a^2
*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*
x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])] + (a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Si
n[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a
 + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*
x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))] + (a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b
*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b*(-6*a^2*A*b + 2*A*b^3 + 3*a^
3*B + a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2
] - (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B
 + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
x)/2]^2 + (a(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a +
b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B
 + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c
 + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*(a^3 - a*b^2)^2*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-6*a^2*A*b
 + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b
*(3*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)

```
/(a + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[
c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))
/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.641, size = 5205, normalized size = 11.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

3.628 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=431

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4abB+\right.}{}$$

```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a*(3*A
+ B) - b*(A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*
x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3
*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.08094, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2999, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4abB+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a*(3*A
+ B) - b*(A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*
x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3
```

```
*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2A+Ab^2-4abB\right)}{3\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2A+Ab^2-4abB\right)}{3\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= -\frac{2\left(3a^2A+Ab^2-4abB\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 18.9916, size = 528, normalized size = 1.23

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)}{3a\left(a^2-b^2\right)^2}+\frac{2\left(a^2B\sin(c+dx)-aAb\sin(c+dx)\right)}{3b\left(b^2-a^2\right)(a+b\cos(c+dx))^2}+\frac{2\left(2a^2Ab\sin(c+dx)+a^3B\sin(c+dx)-5ab^2\right.}{3b\left(b^2-a^2\right)^2(a+b\cos\right.}}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2*A + A*b^2 - 4*a*b*
B)*Sin[c + d*x])/(3*a*(a^2 - b^2)^2) + (2*(-(a*A*b*Sin[c + d*x]) + a^2*B*Si
n[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(2*a^2*A*b*Sin[
c + d*x] + 2*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 5*a*b^2*B*Sin[c + d*
x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - (2*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(-2*(a + b)*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(3*a
*A + A*b - a*B - 3*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (3*a^2*A + A*b^2 - 4*a*b*B)*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*(a^2 - b^2)^2*d*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] time = 0.681, size = 4243, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/3/d/a/(a+b)^2/(a-b)^2*(-2*A*cos(d*x+c)^3*a*b^3-4*A*cos(d*x+c)^2*a^2*b^2-4
*A*cos(d*x+c)*a^3*b+5*B*cos(d*x+c)^3*a^2*b^2+4*B*cos(d*x+c)^2*a^3*b-2*A*cos
(d*x+c)^3*a^3*b-A*cos(d*x+c)^2*b^4+A*cos(d*x+c)^3*b^4-A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^4+
3*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
```

b)/(a+b))^(1/2))*a^4-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^2*a^2*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^2*a^2*b^2-A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b-4*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^2*a^2*b^2+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3+2*A*cos(
d*x+c)^2*a*b^3-8*B*cos(d*x+c)^2*a^2*b^2+4*B*cos(d*x+c)^2*a*b^3+A*cos(d*x+c)
*a^2*b^2-4*B*cos(d*x+c)*a^3*b+3*B*cos(d*x+c)*a^2*b^2+3*A*cos(d*x+c)^3*a^2*b
^2-4*B*cos(d*x+c)^3*a*b^3+6*A*cos(d*x+c)^2*a^3*b+3*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*A*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^4-B*cos(d*x+c)^3*a^4-3*A*cos(d*x+c)^2*a^4+B*cos(d*x+c)*a^4+3*A*cos(d*x+c
)*a^4+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^2*a*b^3-3*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+7*A*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*si

n(d*x+c)-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)*a^3*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-2*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a
*b^3+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)*a^2*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+4*B*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+8
B(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a^2*b^2+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-5*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-7*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*a^2*b^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4-3*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*
sin(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3*sin(d*x+c)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-4*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-3*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*s

```
in(d*x+c))*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c)))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

3.629 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=602

$$-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2bB+3a\right.}{}$$

```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*
A*b^3 + 3*a^3*B + a^2*b*B - a*b^2*(A + 6*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]
) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (2*a*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.64539, antiderivative size = 602, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2bB+3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```



```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*
```

```
A*b^3 + 3*a^3*B + a^2*b*B - a*b^2*(A + 6*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]
) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (2*a*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}}{}$$

$$= -\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{}}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{}}{b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(4Ab^3+3a^3B-7ab^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{3a(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 16.1467, size = 1994, normalized size = 3.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(4*A*b^3 + 3*a^3*B - 7*a*b
^2*B)*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)^2) - (2*(-(a^2*A*b*Sin[c + d*x]) +
a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-(a^
3*A*b*Sin[c + d*x]) + 5*a*A*b^3*Sin[c + d*x] + 4*a^4*B*Sin[c + d*x] - 8*a^2
*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - (2*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*
```

```
x)/2]^2)]*(4*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] + 4*A*b^4*Sqrt[
(a - b)/(a + b)]*Tan[(c + d*x)/2] + 3*a^4*Sqrt[(a - b)/(a + b)]*B*Tan[(c +
d*x)/2] + 3*a^3*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - 7*a^2*b^2*Sqrt
[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - 7*a*b^3*Sqrt[(a - b)/(a + b)]*B*Tan[
(c + d*x)/2] - 8*A*b^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 - 6*a^3*b*S
qrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 14*a*b^3*Sqrt[(a - b)/(a + b)]*
B*Tan[(c + d*x)/2]^3 - 4*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 +
 4*A*b^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*a^4*Sqrt[(a - b)/(a +
 b)]*B*Tan[(c + d*x)/2]^5 + 3*a^3*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2
]^5 + 7*a^2*b^2*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 7*a*b^3*Sqrt[(
a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + (6*I)*a^4*B*EllipticPi[(a + b)/(a -
b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] - (12*I)*a^2*b^2*B*EllipticPi[(a + b)/(a - b), I*ArcS
inh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] + (6*I)*b^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b
)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (
6*I)*a^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 - (12*I)*a^2*b^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a +
 b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] + (6*I)*b^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + I*(a - b)*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*EllipticE[I*ArcSi
nh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(3*b^3*(A - B) + 6*a^3*B +
4*a^2*b*B - a*b^2*(A + 9*B))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)]))/(3*b^2*Sqrt[(a - b)/(a + b)]*(a^2 - b^2)^2*d*(-1 + Tan[(c + d*x)/2
]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(
c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.571, size = 5757, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

3.630 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=733

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} - \frac{\left(6a^3Ab+26a^2b^2B-15a^4B-14aAb^3-3b^4B\right)\sin(c+dx)\sqrt{\sec(c+dx}}{3b^3d\left(a^2-b^2\right)^2}$$

```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d*S
qrt[Sec[c + d*x]]) + ((3*b^3*(4*A - B) + 15*a^3*B - a*b^2*(2*A + 21*B) - a^
2*(6*A*b - 5*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^3*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 5*
a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4
*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3
- 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^
2*b^2*B - 3*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*b^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 2.49364, antiderivative size = 733, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} - \frac{\left(6a^3Ab+26a^2b^2B-15a^4B-14aAb^3-3b^4B\right)\sin(c+dx)\sqrt{\sec(c+dx}}{3b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
```

```
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d*S
qrt[Sec[c + d*x]]) + ((3*b^3*(4*A - B) + 15*a^3*B - a*b^2*(2*A + 21*B) - a^
2*(6*A*b - 5*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^3*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 5*
a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4
*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3
- 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^
2*b^2*B - 3*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*b^3*(a^2 - b^2)^2*d)
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos}}$$

$$= -\frac{\sqrt{a+b}(2Ab-5aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{b^4d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{3a(a-b)b^3(a+b)^{3/2}d\sqrt{\sec(}}$$

Mathematica [B] time = 22.4906, size = 2342, normalized size = 3.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-3*a^2*A*b + 7*A*b^3 +
 6*a^3*B - 10*a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2) + (2*(-(a^3*A*b*
Sin[c + d*x]) + a^4*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x
])^2) + (2*(-4*a^4*A*b*Sin[c + d*x] + 8*a^2*A*b^3*Sin[c + d*x] + 7*a^5*B*Si
n[c + d*x] - 11*a^3*b^2*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(a + b*Cos[c
```

+ d*x]))))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(6*a^4*A*b*Tan[(c + d*x)/2] + 6*a^3*A*b^2*Tan[(c + d*x)/2] - 14*a^2*A*b^3*Tan[(c + d*x)/2] - 14*a*A*b^4*Tan[(c + d*x)/2] - 15*a^5*B*Tan[(c + d*x)/2] - 15*a^4*b*B*Tan[(c + d*x)/2] + 26*a^3*b^2*B*Tan[(c + d*x)/2] + 26*a^2*b^3*B*Tan[(c + d*x)/2] - 3*a*b^4*B*Tan[(c + d*x)/2] - 3*b^5*B*Tan[(c + d*x)/2] - 12*a^3*A*b^2*Tan[(c + d*x)/2]^3 + 28*a*A*b^4*Tan[(c + d*x)/2]^3 + 30*a^4*b*B*Tan[(c + d*x)/2]^3 - 52*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 6*b^5*B*Tan[(c + d*x)/2]^3 - 6*a^4*A*b*Tan[(c + d*x)/2]^5 + 6*a^3*A*b^2*Tan[(c + d*x)/2]^5 + 14*a^2*A*b^3*Tan[(c + d*x)/2]^5 - 14*a*A*b^4*Tan[(c + d*x)/2]^5 + 15*a^5*B*Tan[(c + d*x)/2]^5 - 15*a^4*b*B*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*B*Tan[(c + d*x)/2]^5 + 26*a^2*b^3*B*Tan[(c + d*x)/2]^5 + 3*a*b^4*B*Tan[(c + d*x)/2]^5 - 3*b^5*B*Tan[(c + d*x)/2]^5 + 12*a^4*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 24*a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 12*A*b^5*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^5*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 60*a^3*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 12*a^4*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 24*a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 12*A*b^5*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^5*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 60*a^3*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-6*a^3*A*b + 14*a*A*b^3 + 15*a^4*B - 26*a^2*b^2*B + 3*b^4*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(a + b)*(3*A*b^3 + 3*a*b^2*(A - 2*B) + 5*a^3*B -

```
a^2*b*(2*A + 3*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[
1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)
/2]^2)))
```

Maple [B] time = 0.714, size = 8621, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

3.631 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{2B(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\sqrt{\cos}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.370149, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {21, 4222, 2801, 2816, 2994}

$$\frac{2B(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= -\left(\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx\right)+($$
$$= \frac{2(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 6.0999, size = 298, normalized size = 1.12

$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{ad}-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] B*((2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2]))
```

Maple [B] time = 0.583, size = 621, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d/a*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+a*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)-(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+b*cos(d*
x+c)^2+cos(d*x+c)*a-b*cos(d*x+c)-a)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*co
s(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

3.632 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.155044, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {21, 4222, 2816}

$$\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2
),x]
```


```
[Out] (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]])
```


Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a+}}}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.134802, size = 104, normalized size = 0.8

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.679, size = 126, normalized size = 1.

$$2\frac{B\sqrt{(\cos(dx+c))^{-1}}\,(\sin(dx+c))^{2}}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)\,(1+\cos(dx+c))}}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*B/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))/(a+
b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

3.633 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=137

$$-\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.155048, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {21, 4222, 2809}

$$-\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]
),x]
```


```
[Out] (-2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```


Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= -\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{bd\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.178368, size = 149, normalized size = 1.09

$$-\frac{2B\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right.\right.\right.\right.}{d\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c +
 d*x]]),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[
1 + Sec[c + d*x]])/(d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*x]
])
```

Maple [A] time = 0.642, size = 144, normalized size = 1.1

$$2\frac{B}{d\sqrt{a+b\cos(dx+c)}\sqrt{(\cos(dx+c))^{-1}}}\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*B/d/(a+b*cos(d*x+c))^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2)))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(1/cos(d*x+c))^(1/2)/(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x
 + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x
 + c))), x)
```

3.634 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=479

$$\frac{aB\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{aB\sin(c+dx)}{bd\sqrt{a+b\,\mathrm{c}}}$$

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*B*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[
Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rubi [A] time = 0.922362, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {21, 4222, 2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{aB\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{aB\sin(c+dx)}{bd\sqrt{a+b\,\mathrm{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
```

```
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*B*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[
Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
```

```
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = B\int \frac{1}{\sqrt{a + b\cos(c + dx)}\sec^{\frac{3}{2}}(c + dx)}\,dx$$

$$= \left(B\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx$$

$$= \frac{\left(B\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{\left(aB\sqrt{\cos(c + dx\right.}{}$$

$$= \frac{a\sqrt{a + b}B\sqrt{\cos(c + dx)}\csc(c + dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^2d\sqrt{\sec(c + dx)}}$$

$$= \frac{a\sqrt{a + b}B\sqrt{\cos(c + dx)}\csc(c + dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^2d\sqrt{\sec(c + dx)}}$$

$$= \frac{a\sqrt{a + b}B\sqrt{\cos(c + dx)}\csc(c + dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^2d\sqrt{\sec(c + dx)}}$$

$$= \frac{a\sqrt{a + b}B\sqrt{\cos(c + dx)}\csc(c + dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^2d\sqrt{\sec(c + dx)}}$$

$$= -\frac{(a - b)\sqrt{a + b}B\sqrt{\cos(c + dx)}\csc(c + dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{abd\sqrt{\sec(c + dx)}}$$

Mathematica [C] time = 6.11691, size = 5018, normalized size = 10.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]
^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.697, size = 634, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] B/d/b*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)
-b*cos(d*x+c)^3-a*cos(d*x+c)^2+b*cos(d*x+c)^2+cos(d*x+c)*a)*cos(d*x+c)*(1/c
os(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)
^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)
^(3/2)), x)
```

3.635 $\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=58

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Unintegrable}\left((A+B\cos(e+fx))(c\cos(e+fx))^{-m}(a+b\cos(e+fx))^n,x\right)$$

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Unintegrable[((a + b*Cos[e + f*x])^n*
(A + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi [A] time = 0.18063, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][((a + b*Cos[e + f*x])^n*(A
 + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi steps

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int (c\cos(e+fx))^{-m}($$

Mathematica [A] time = 8.45503, size = 0, normalized size = 0.

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m, x
]
```

Maple [A] time = 3.207, size = 0, normalized size = 0.

$$\int \left(a+b\cos\left(fx+e\right)\right)^{n}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**n*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^n (c\sec(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x
)
```

3.636 $\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=644

$$-\frac{c^6\sin(e+fx)\left(4a^3Ab\left(m^2-8m+15\right)+6a^2b^2B\left(m^2-7m+10\right)+a^4B\left(m^2-8m+15\right)+4aAb^3\left(m^2-7m+10\right)+b^4B\right.}{f(2-m)(4-m)(6-m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -((c^6*(4*a^3*A*b*(15 - 8*m + m^2) + a^4*B*(15 - 8*m + m^2) + 4*a*A*b^3*(10
 - 7*m + m^2) + 6*a^2*b^2*B*(10 - 7*m + m^2) + b^4*B*(8 - 6*m + m^2))*Hyper
geometric2F1[1/2, (6 - m)/2, (8 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-
6 + m)*Sin[e + f*x])/(f*(2 - m)*(4 - m)*(6 - m)*Sqrt[Sin[e + f*x]^2])) - (c
^5*(a^4*A*(8 - 6*m + m^2) + 6*a^2*A*b^2*(4 - 5*m + m^2) + 4*a^3*b*B*(4 - 5*
m + m^2) + A*b^4*(3 - 4*m + m^2) + 4*a*b^3*B*(3 - 4*m + m^2))*Hypergeometri
c2F1[1/2, (5 - m)/2, (7 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*S
in[e + f*x])/(f*(1 - m)*(3 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^5*(4*a
^2*A*b*(3 - 4*m + m^2) + a^3*B*(3 - 4*m + m^2) + 2*A*b^3*(4 - 2*m + m^2) +
a*b^2*B*(8 - 13*m + 5*m^2))*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1 -
 m)*(2 - m)*(4 - m)) - (a^2*c^5*(2*a*b*B*(1 - m)^2 + a^2*A*(2 - m)^2 + A*b^
2*(6 - m + m^2))*Sec[e + f*x]*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1
 - m)*(2 - m)*(3 - m)) - (a*c^5*(a*B*(1 - m) - A*b*(2 + m))*(c*Sec[e + f*x]
)^(-5 + m)*(b + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*
c^5*(c*Sec[e + f*x])^(-5 + m)*(b + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(1 -
m))
```

Rubi [A] time = 2.04236, antiderivative size = 644, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4026, 4096, 4076, 4047, 3772, 2643, 4046}

$$-\frac{c^6\sin(e+fx)\left(4a^3Ab\left(m^2-8m+15\right)+6a^2b^2B\left(m^2-7m+10\right)+a^4B\left(m^2-8m+15\right)+4aAb^3\left(m^2-7m+10\right)+b^4B\right.}{f(2-m)(4-m)(6-m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^6*(4*a^3*A*b*(15 - 8*m + m^2) + a^4*B*(15 - 8*m + m^2) + 4*a*A*b^3*(10
 - 7*m + m^2) + 6*a^2*b^2*B*(10 - 7*m + m^2) + b^4*B*(8 - 6*m + m^2))*Hyper
```

```
geometric2F1[1/2, (6 - m)/2, (8 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-
6 + m)*Sin[e + f*x])/(f*(2 - m)*(4 - m)*(6 - m)*Sqrt[Sin[e + f*x]^2])) - (c
^5*(a^4*A*(8 - 6*m + m^2) + 6*a^2*A*b^2*(4 - 5*m + m^2) + 4*a^3*b*B*(4 - 5*
m + m^2) + A*b^4*(3 - 4*m + m^2) + 4*a*b^3*B*(3 - 4*m + m^2))*Hypergeometri
c2F1[1/2, (5 - m)/2, (7 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*S
in[e + f*x])/(f*(1 - m)*(3 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^5*(4*a
^2*A*b*(3 - 4*m + m^2) + a^3*B*(3 - 4*m + m^2) + 2*A*b^3*(4 - 2*m + m^2) +
a*b^2*B*(8 - 13*m + 5*m^2))*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1 -
 m)*(2 - m)*(4 - m)) - (a^2*c^5*(2*a*b*B*(1 - m)^2 + a^2*A*(2 - m)^2 + A*b^
2*(6 - m + m^2))*Sec[e + f*x]*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1
 - m)*(2 - m)*(3 - m)) - (a*c^5*(a*B*(1 - m) - A*b*(2 + m))*(c*Sec[e + f*x]
)^(-5 + m)*(b + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*
c^5*(c*Sec[e + f*x])^(-5 + m)*(b + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(1 -
m))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^5\int (c\sec(e+fx))^{-5+m}(b+a\sec(e+fx))^4(B+A\sec($$

$$= -\frac{aAc^5(c\sec(e+fx))^{-5+m}(b+a\sec(e+fx))^3\tan(e+}{f(1-m)}$$

$$= -\frac{ac^5(aB(1-m)-Ab(2+m))(c\sec(e+fx))^{-5+m}(b+a}{f(1-m)(2-m)}$$

$$= -\frac{a^2c^5\left(2abB(1-m)^2+a^2A(2-m)^2+Ab^2\left(6-m+m\right.\right.}{f(3-m)\left(2-3\right.}$$

$$= -\frac{a^2c^5\left(2abB(1-m)^2+a^2A(2-m)^2+Ab^2\left(6-m+m\right.\right.}{f(3-m)\left(2-3\right.}$$

$$= -\frac{ac^5\left(4a^2Ab\left(3-4m+m^2\right)+a^3B\left(3-4m+m^2\right)+2A\right.}{f}$$

$$= -\frac{\left(a^4A\left(8-6m+m^2\right)+6a^2Ab^2\left(4-5m+m^2\right)+4a^3b\right.}{}$$

$$= -\frac{\left(a^4A\left(8-6m+m^2\right)+6a^2Ab^2\left(4-5m+m^2\right)+4a^3b\right.}{}$$

Mathematica [A] time = 3.95873, size = 317, normalized size = 0.49

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left(\frac{b^3(4aB+Ab)\cos^4(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-4}{2};\frac{m-2}{2};\sec^2(e+fx)\right)}{m-4}+a\left(\frac{2b^2(3aB+2Ab)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\right.}{m-3}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^4*B*Cos[e + f*x]^5*Hypergeometric2F1[1/2, (-5 + m)/2, (-3
 + m)/2, Sec[e + f*x]^2])/(-5 + m) + (b^3*(A*b + 4*a*B)*Cos[e + f*x]^4*Hype
rgeometric2F1[1/2, (-4 + m)/2, (-2 + m)/2, Sec[e + f*x]^2])/(-4 + m) + a*((
2*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2, (-1
 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a*((2*b*(3*A*b + 2*a*B)*Cos[e + f*x]^2
*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) + a*(((4
```

```
*A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[
e + f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e +
 f*x]^2])/m))))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F] time = 2.862, size = 0, normalized size = 0.

$$\int \left(a+b\cos\left(fx+e\right)\right)^{4}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{4}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(Bb^{4}\cos\left(fx+e\right)^{5}+Aa^{4}+\left(4\,Bab^{3}+Ab^{4}\right)\cos\left(fx+e\right)^{4}+2\left(3\,Ba^{2}b^{2}+2\,Aab^{3}\right)\cos\left(fx+e\right)^{3}+2\left(2\,Ba^{3}b+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**4*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^4 (c\sec(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m, x
)
```

3.637 $\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=455

$$-\frac{c^5\sin(e+fx)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m+4\right)+b^3B\left(m^2-4m+3\right)\right)(c\sec(e+fx}{f(1-m)(3-m)(5-m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -((c^5*(a^3*A*(8 - 6*m + m^2) + 3*a*A*b^2*(4 - 5*m + m^2) + 3*a^2*b*B*(4 -
5*m + m^2) + b^3*B*(3 - 4*m + m^2))*Hypergeometric2F1[1/2, (5 - m)/2, (7 -
m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*Sin[e + f*x])/(f*(1 - m)*(3
 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2])) - (c^4*(A*b^3*(2 - m) + 3*a*b^2*B*(2 -
 m) + 3*a^2*A*b*(3 - m) + a^3*B*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2,
(6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e + f*x])/(f*(2 -
m)*(4 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^4*(3*a*b*B*(1 - m) + a^2*A*(2 - m)
- 2*A*b^2*m)*(c*Sec[e + f*x])^(-4 + m)*Tan[e + f*x])/(f*(1 - m)*(3 - m)) -
(a^2*c^4*(a*B*(1 - m) - A*b*(1 + m))*Sec[e + f*x]*(c*Sec[e + f*x])^(-4 + m)
*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^4*(c*Sec[e + f*x])^(-4 + m)*(b
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m))
```

Rubi [A] time = 1.14548, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4026, 4076, 4047, 3772, 2643, 4046}

$$-\frac{c^5\sin(e+fx)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m+4\right)+b^3B\left(m^2-4m+3\right)\right)(c\sec(e+fx}{f(1-m)(3-m)(5-m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^5*(a^3*A*(8 - 6*m + m^2) + 3*a*A*b^2*(4 - 5*m + m^2) + 3*a^2*b*B*(4 -
5*m + m^2) + b^3*B*(3 - 4*m + m^2))*Hypergeometric2F1[1/2, (5 - m)/2, (7 -
m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*Sin[e + f*x])/(f*(1 - m)*(3
 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2])) - (c^4*(A*b^3*(2 - m) + 3*a*b^2*B*(2 -
 m) + 3*a^2*A*b*(3 - m) + a^3*B*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2,
(6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e + f*x])/(f*(2 -
m)*(4 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^4*(3*a*b*B*(1 - m) + a^2*A*(2 - m)
```

```
- 2*A*b^2*m)*(c*Sec[e + f*x])^(-4 + m)*Tan[e + f*x])/(f*(1 - m)*(3 - m)) -
(a^2*c^4*(a*B*(1 - m) - A*b*(1 + m))*Sec[e + f*x]*(c*Sec[e + f*x])^(-4 + m)
*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^4*(c*Sec[e + f*x])^(-4 + m)*(b
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m))
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^4\int (c\sec(e+fx))^{-4+m}(b+a\sec(e+fx))^3(B+A\sec(e$$

$$= -\frac{aAc^4(c\sec(e+fx))^{-4+m}(b+a\sec(e+fx))^2\tan(e+fx}{f(1-m)}$$

$$= -\frac{a^2c^4(aB(1-m)-Ab(1+m))\sec(e+fx)(c\sec(e+fx))}{f(1-m)(2-m)}$$

$$= -\frac{a^2c^4(aB(1-m)-Ab(1+m))\sec(e+fx)(c\sec(e+fx))}{f(1-m)(2-m)}$$

$$= -\frac{ac^4\left(3abB(1-m)+a^2A(2-m)-2Ab^2m\right)(c\sec(e+fx}{f(1-m)(3-m)}$$

$$= -\frac{\left(Ab^3(2-m)+3ab^2B(2-m)+3a^2Ab(3-m)+a^3B(3\right.}{f(2}$$

$$= -\frac{\left(Ab^3(2-m)+3ab^2B(2-m)+3a^2Ab(3-m)+a^3B(3\right.}{f(2}$$

Mathematica [A] time = 2.16282, size = 259, normalized size = 0.57

$$\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left(\frac{b^2(3aB+Ab)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-3}{2};\frac{m-1}{2};\sec^2(e+fx)\right)}{m-3}+a\left(\frac{3b(aB+Ab)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-2}{2}\right.}{m-2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^3*B*Cos[e + f*x]^4*Hypergeometric2F1[1/2, (-4 + m)/2, (-2
 + m)/2, Sec[e + f*x]^2])/(-4 + m) + (b^2*(A*b + 3*a*B)*Cos[e + f*x]^3*Hype
rgeometric2F1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a*((
3*b*(A*b + a*B)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[
e + f*x]^2])/(-2 + m) + a*(((3*A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/
2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F
1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2])/m)))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e
 + f*x]^2])/f
```

Maple [F] time = 2.351, size = 0, normalized size = 0.

$$\int \left(a+b\cos\left(fx+e\right)\right)^{3}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{3}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\cos\left(fx+e\right)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos\left(fx+e\right)^3 + 3\left(Ba^2b + Aab^2\right)\cos\left(fx+e\right)^2 + \left(Ba^3 + 3\,Aa^2b\right)\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e))
*(c*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**3*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^3\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m, x
)
```

3.638 $\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=327

$$-\frac{c^4\sin(e+fx)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)(c\sec(e+fx))^{m-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(e+fx)\right)}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}-\frac{c^3\sin(e}{}$$

```
[Out] -((c^4*(b^2*B*(2 - m) + 2*a*A*b*(3 - m) + a^2*B*(3 - m))*Hypergeometric2F1[
1/2, (4 - m)/2, (6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e
+ f*x])/(f*(2 - m)*(4 - m)*Sqrt[Sin[e + f*x]^2])) - (c^3*(A*b^2*(1 - m) + 2
*a*b*B*(1 - m) + a^2*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2
, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3 - m
)*Sqrt[Sin[e + f*x]^2]) - (a*c^3*(a*B*(1 - m) - A*b*m)*(c*Sec[e + f*x])^(-3
 + m)*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^3*(c*Sec[e + f*x])^(-3 + m
)*(b + a*Sec[e + f*x])*Tan[e + f*x])/(f*(1 - m))
```

Rubi [A] time = 0.642796, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4026, 4047, 3772, 2643, 4046}

$$-\frac{c^4\sin(e+fx)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)(c\sec(e+fx))^{m-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(e+fx)\right)}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}-\frac{c^3\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```


```
[Out] -((c^4*(b^2*B*(2 - m) + 2*a*A*b*(3 - m) + a^2*B*(3 - m))*Hypergeometric2F1[
1/2, (4 - m)/2, (6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e
+ f*x])/(f*(2 - m)*(4 - m)*Sqrt[Sin[e + f*x]^2])) - (c^3*(A*b^2*(1 - m) + 2
*a*b*B*(1 - m) + a^2*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2
, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3 - m
)*Sqrt[Sin[e + f*x]^2]) - (a*c^3*(a*B*(1 - m) - A*b*m)*(c*Sec[e + f*x])^(-3
 + m)*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^3*(c*Sec[e + f*x])^(-3 + m
)*(b + a*Sec[e + f*x])*Tan[e + f*x])/(f*(1 - m))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^3\int (c\sec(e+fx))^{-3+m}(b+a\sec(e+fx))^2(B+A\sec($$

$$= -\frac{aAc^3(c\sec(e+fx))^{-3+m}(b+a\sec(e+fx))\tan(e+f}{f(1-m)}$$

$$= -\frac{aAc^3(c\sec(e+fx))^{-3+m}(b+a\sec(e+fx))\tan(e+f}{f(1-m)}$$

$$= -\frac{ac^3(aB(1-m)-Abm)(c\sec(e+fx))^{-3+m}\tan(e+fx}{f(1-m)(2-m)}$$

$$= -\frac{\left(Ab^2(1-m)+2abB(1-m)+a^2A(2-m)\right)\cos^3(e+f}{f(1-m)(3}$$

$$= -\frac{\left(Ab^2(1-m)+2abB(1-m)+a^2A(2-m)\right)\cos^3(e+f}{f(1-m)(3}$$

Mathematica [A] time = 0.935299, size = 205, normalized size = 0.63

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left(\frac{b(2aB+Ab)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-2}{2};\frac{m}{2};\sec^2(e+fx)\right)}{m-2}+a\left(\frac{(aB+2Ab)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2}\right.}{m-1}\right.\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^2*B*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2, (-1
 + m)/2, Sec[e + f*x]^2])/(-3 + m) + (b*(A*b + 2*a*B)*Cos[e + f*x]^2*Hyperg
eometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) + a*(((2*A*b +
a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x
]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2
])/m))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F] time = 2.099, size = 0, normalized size = 0.

$$\int \left(a+b\cos\left(fx+e\right)\right)^2\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{2}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^{2}\cos\left(fx+e\right)^{3}+Aa^{2}+\left(2\,Bab+Ab^{2}\right)\cos\left(fx+e\right)^{2}+\left(Ba^{2}+2\,Aab\right)\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2 +
 (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**2*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m, x
)
```

3.639 $\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=217

$$-\frac{c^3\sin(e+fx)(aA(2-m)+bB(1-m))(c\sec(e+fx))^{m-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(e+fx)\right)}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}}-\frac{c^2(aB+Ab)\sin(e+fx)(c\,\mathrm{se}}{f(2}$$

```
[Out] -((c^3*(b*B*(1 - m) + a*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m
)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3
- m)*Sqrt[Sin[e + f*x]^2])) - ((A*b + a*B)*c^2*Hypergeometric2F1[1/2, (2 -
m)/2, (4 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-2 + m)*Sin[e + f*x])/(f
*(2 - m)*Sqrt[Sin[e + f*x]^2]) - (a*A*c^2*(c*Sec[e + f*x])^(-2 + m)*Tan[e +
 f*x])/(f*(1 - m))
```

Rubi [A] time = 0.359662, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2960, 3997, 3787, 3772, 2643}

$$-\frac{c^3\sin(e+fx)(aA(2-m)+bB(1-m))(c\sec(e+fx))^{m-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(e+fx)\right)}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}}-\frac{c^2(aB+Ab)\sin(e+fx)(c\,\mathrm{se}}{f(2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```


```
[Out] -((c^3*(b*B*(1 - m) + a*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m
)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3
- m)*Sqrt[Sin[e + f*x]^2])) - ((A*b + a*B)*c^2*Hypergeometric2F1[1/2, (2 -
m)/2, (4 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-2 + m)*Sin[e + f*x])/(f
*(2 - m)*Sqrt[Sin[e + f*x]^2]) - (a*A*c^2*(c*Sec[e + f*x])^(-2 + m)*Tan[e +
 f*x])/(f*(1 - m))
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^2\int (c\sec(e+fx))^{-2+m}(b+a\sec(e+fx))(B+A\sec(e+$$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} - \frac{c^2\int (c\sec(e+f$$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} + ((Ab+aB)c)\int ($$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} + \Bigg((Ab+aB)c\Bigg(\frac{\text{co}}{}$$

$$= -\frac{(Ab+aB)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(e+fx)\right)(}{f(2-m)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.361708, size = 163, normalized size = 0.75

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left((m-2)\left(m(aB+Ab)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sec^2(e+fx)\right)+aA(m-\right.\right.}{f(m-2)(m-1)m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*(b*B*(-1 + m)*m*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 + m
)/2, m/2, Sec[e + f*x]^2] + (-2 + m)*((A*b + a*B)*m*Cos[e + f*x]*Hypergeome
tric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2] + a*A*(-1 + m)*Hypergeo
metric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2]))*(c*Sec[e + f*x])^m*Sqrt[-T
an[e + f*x]^2])/(f*(-2 + m)*(-1 + m)*m)
```

Maple [F] time = 2.181, size = 0, normalized size = 0.

$$\int \left(a+b\cos\left(fx+e\right)\right)\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos\left(fx+e\right)^{2}+Aa+(Ba+Ab)\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*sec(f*x +
 e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sec\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)\left(a+b\cos\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

3.640 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=299

$$-\frac{(Ab-aB)\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{m/2}(c\sec(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{cf\left(a^2-b^2\right)}+\frac{a(Ab$$

```
[Out] -(((A*b - a*B)*AppellF1[1/2, m/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*
x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(Cos[e + f*x]^2)^(m/2)*(c*Sec[e + f*x])^(1
 + m)*Sin[e + f*x])/((a^2 - b^2)*c*f)) + (a*(A*b - a*B)*AppellF1[1/2, (1 +
m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(Cos[e +
 f*x]^2)^((1 + m)/2)*(c*Sec[e + f*x])^(1 + m)*Sin[e + f*x])/(b*(a^2 - b^2)*
c*f) - (B*c*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(c
*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(b*f*(1 - m)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.575894, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4038, 3772, 2643, 3869, 2823, 3189, 429}

$$-\frac{(Ab-aB)\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{m/2}(c\sec(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{cf\left(a^2-b^2\right)}+\frac{a(Ab$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x]
```


```
[Out] -(((A*b - a*B)*AppellF1[1/2, m/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*
x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(Cos[e + f*x]^2)^(m/2)*(c*Sec[e + f*x])^(1
 + m)*Sin[e + f*x])/((a^2 - b^2)*c*f)) + (a*(A*b - a*B)*AppellF1[1/2, (1 +
m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(Cos[e +
 f*x]^2)^((1 + m)/2)*(c*Sec[e + f*x])^(1 + m)*Sin[e + f*x])/(b*(a^2 - b^2)*
c*f) - (B*c*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(c
*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(b*f*(1 - m)*Sqrt[Sin[e + f*x]^2])
```


Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
```

```
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \int \frac{(c\sec(e+fx))^m(B+A\sec(e+fx))}{b+a\sec(e+fx)}\,dx$$

$$= \frac{B\int (c\sec(e+fx))^m\,dx}{b} + \frac{(Ab-aB)\int \frac{(c\sec(e+fx))^{1+m}}{b+a\sec(e+fx)}\,dx}{bc}$$

$$= \frac{\left(B\left(\frac{\cos(e+fx)}{c}\right)^m (c\sec(e+fx))^m\right)\int \left(\frac{\cos(e+fx)}{c}\right)^{-m}\,dx}{b} + \frac{\left((Ab-aB)\cos^{1+m}(\right.}{}$$

$$= -\frac{B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx))^m\sin(e+fx}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx))^m\sin(e+fx}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{(Ab-aB)F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+}{\left(a^2-b^2\right)cf}$$

Mathematica [B] time = 26.5004, size = 10630, normalized size = 35.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.394, size = 0, normalized size = 0.

$$\int \frac{\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}}{a+b\cos\left(fx+e\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

```
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{b\cos\left(fx+e\right)+a}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{b\cos\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sec\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)}{a+b\cos\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e)),x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

3.641 $\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=209

$$\frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Unintegrable}\left(\frac{(c\cos(e+fx))^{-m}\left(\frac{1}{2}c\cos(e+fx)\left(a(5-2m)(aB+2Ab)+b^2B(3-2m)\right)+\frac{1}{2}bc\cos^2(e+fx)(2aB(3-m)+\ldots}{\sqrt{a+b\cos(e+fx)}}\right.}{c(5-2m)}$$

```
[Out] (2*b*B*Cos[e + f*x]*Sqrt[a + b*Cos[e + f*x]]*(c*Sec[e + f*x])^m*Sin[e + f*x
])/(f*(5 - 2*m)) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Unintegrable[((
a*c*(2*b*B*(1 - m) + 2*a*A*(5/2 - m)))/2 + (c*(b^2*B*(3 - 2*m) + a*(2*A*b +
 a*B)*(5 - 2*m))*Cos[e + f*x])/2 + (b*c*(A*b*(5 - 2*m) + 2*a*B*(3 - m))*Cos
[e + f*x]^2)/2)/((c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(c*(5 -
2*m))
```

Rubi [A] time = 0.721117, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (2*b*B*Cos[e + f*x]*Sqrt[a + b*Cos[e + f*x]]*(c*Sec[e + f*x])^m*Sin[e + f*x
])/(f*(5 - 2*m)) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][((a*
c*(2*b*B*(1 - m) + 2*a*A*(5/2 - m)))/2 + (c*(b^2*B*(3 - 2*m) + a*(2*A*b + a
*B)*(5 - 2*m))*Cos[e + f*x])/2 + (b*c*(A*b*(5 - 2*m) + 2*a*B*(3 - m))*Cos[e
 + f*x]^2)/2)/((c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(c*(5 - 2*
m))
```

Rubi steps

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int (c\cos(e+fx))^{-m}$$

$$= \frac{2bB\cos(e+fx)\sqrt{a+b\cos(e+fx)}(c\sec(e+fx))^m \operatorname{si}}{f(5-2m)}$$

Mathematica [A] time = 57.0767, size = 0, normalized size = 0.

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^
m,x]
```

```
[Out] Integrate[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^
m, x]
```

Maple [A] time = 0.714, size = 0, normalized size = 0.

$$\int \left(a+b\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))^
m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(fx+e\right)^{2}+Aa+(Ba+Ab)\cos\left(fx+e\right)\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(f
*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))^
m, x)
```

3.642 $\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=60

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Unintegrable}\left(\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\cos(e+fx))^{-m},x\right)$$

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Unintegrable[(Sqrt[a + b*Cos[e + f*x]
]*(A + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi [A] time = 0.238709, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][(Sqrt[a + b*Cos[e + f*x]]*
(A + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int (c\cos(e+fx))^{-m}\sqrt{a}$$

Mathematica [A] time = 12.3413, size = 0, normalized size = 0.

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,
x]
```

```
[Out] Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,
 x]
```

Maple [A] time = 0.773, size = 0, normalized size = 0.

$$\int \sqrt{a+b\cos\left(fx+e\right)}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sec \left(e+fx\right)\right)^{m} \left(A+B\cos \left(e+fx\right)\right) \sqrt{a+b\cos \left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos \left(fx+e\right)+A\right) \sqrt{b\cos \left(fx+e\right)+a}\left(c\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
 x)
```

3.643 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal. Leaf size=60

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Unintegrable}\left(\frac{(A+B\cos(e+fx))(c\cos(e+fx))^{-m}}{\sqrt{a+b\cos(e+fx)}},x\right)$$

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Unintegrable[(A + B*Cos[e + f*x])/((c
*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x]
```

Rubi [A] time = 0.241677, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][(A + B*Cos[e + f*x])/((c*C
os[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x]
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int \frac{(c\cos(e+fx))^{-m}(A+B\cos(e+fx)}{\sqrt{a+b\cos(e+fx)}}$$

Mathematica [A] time = 8.2039, size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]
],x]
```

```
[Out] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]
], x]
```

Maple [A] time = 0.715, size = 0, normalized size = 0.

$$\int \left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\frac{1}{\sqrt{a+b\cos\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sec\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)}{\sqrt{a+b\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

3.644 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Unintegrable}\left(\frac{(c\cos(e+fx))^{-m}\left(\frac{1}{2}c\left(a^2A-2abB(1-m)+Ab^2(1-2m)\right)-\frac{1}{2}bc(3-2m)(Ab-aB)\cos^2(e+fx)-\frac{1}{2}ac(Ab\right.}{\sqrt{a+b\cos(e+fx)}}\right.}{ac\left(a^2-b^2\right)}$$

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x]*(c*Sec[e + f*x])^m*Sin[e + f*x])/(a*(a^2 - b^
2)*f*Sqrt[a + b*Cos[e + f*x]]) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*U
nintegrable[((c*(a^2*A + A*b^2*(1 - 2*m) - 2*a*b*B*(1 - m)))/2 - (a*(A*b -
a*B)*c*Cos[e + f*x])/2 - (b*(A*b - a*B)*c*(3 - 2*m)*Cos[e + f*x]^2)/2)/((c*
Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(a*(a^2 - b^2)*c)
```

Rubi [A] time = 0.686432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x]*(c*Sec[e + f*x])^m*Sin[e + f*x])/(a*(a^2 - b^
2)*f*Sqrt[a + b*Cos[e + f*x]]) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*D
efer[Int][((c*(a^2*A + A*b^2*(1 - 2*m) - 2*a*b*B*(1 - m)))/2 - (a*(A*b - a*
B)*c*Cos[e + f*x])/2 - (b*(A*b - a*B)*c*(3 - 2*m)*Cos[e + f*x]^2)/2)/((c*Co
s[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(a*(a^2 - b^2)*c)
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int \frac{(c\cos(e+fx))^{-m}(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)(c\sec(e+fx))^m\sin(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\cos(e+fx)}} + \frac{\left(2(c\cos(e+fx))^m(c\right.}{}$$

Mathematica [A] time = 10.5023, size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3
/2),x]
```

```
[Out] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3
/2), x]
```

Maple [A] time = 0.716, size = 0, normalized size = 0.

$$\int \left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

```
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)\right)^{m}}{b^{2}\cos\left(fx+e\right)^{2}+2\,ab\cos\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m/(
b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```